Doppler Radar Measurements of Turbulent Kinetic Energy Dissipation Rates in a Northeastern Colorado Convective Storm

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ABSTRACT

In this paper we report the results of measurements of turbulent kinetic energy dissipation rates within a convective storm. The measurements were obtained with two Doppler radars, one scanning the storm from a distance at low elevation angles and the other pointing vertically with the storm passing overhead. With the scanning radar we measured the wind shear in the radial velocity field and the turbulent kinetic energy dissipation rates within the storm. These dissipation rates showed good agreement with those measured by the zenith-pointing radar data; dissipation rates ranged from 30 cm$^3$ s$^{-2}$ to greater than 3500 cm$^3$ s$^{-2}$ in the region between the updraft and downdraft.

1. Introduction

The spatial distribution of turbulence within severe storms is of interest in studies of hazards to aircraft, the diffusion of seed material within storms, the microphysics of drop-size growth and hail formation, and the basic physics of turbulence itself. In the past, turbulence measurements in convective storms have been made primarily with sensors on aircraft (Steiner and Rhyne, 1962; Sand et al. 1974). However, the volume of the storm that can be sampled by an aircraft is small compared with the total volume, and a long time is required to collect even such a limited sample. On the other hand, Doppler radars can sample almost the complete volume of a storm in a very short time, collecting enormous amounts of spectral information about the movements of scatterers within the storm. This information includes the radial component of the mean velocity of the scatterers in the pulse volume, obtained from the Doppler shift of the mean (first moment) frequency, and the variance of scatterer velocity within the pulse volume, obtained from the width (second central moment; hereinafter referred to as the second moment) of the Doppler spectrum.

The second moment of the spectrum contains information about turbulence that can be used to calculate the turbulent kinetic energy dissipation rate when other contributors to the second moment can be ignored or calculated (Frisch and Clifford, 1974; Gorelik and Mel’nikhuk, 1963, 1968). We used this method to calculate the spatial distribution of turbulent kinetic energy dissipation rate in a severe storm in northeastern Colorado on 9 July 1973. With one Doppler radar we scanned the storm from a distance at low elevation angles. The storm passed almost directly over a second radar, whose antenna was pointed vertically to obtain a time-height cross section of vertical velocity through the storm as it passed overhead. From the second moment of the spectra, obtained with the distant radar, we calculated turbulent dissipation rates; these were compared with dissipation rates calculated from the first moment data acquired at a height of 4.5 km by the vertically pointing radar. In one region common to the two radars the values from the second moment of the Doppler spectra varied between 280 and 800 cm$^3$ s$^{-2}$, showing good agreement with the first moment value which was 700 cm$^3$ s$^{-2}$.

2. Method

There are three potentially important contributors to the width or second moment of the Doppler spectrum for a narrow beam radar: turbulence, wind shear and the spread of particle fall speeds. If we can determine the contributions of the last two, we can isolate the contribution of turbulence for use in our calculations. If we assume the turbulence to be homogeneous and isotropic we can then calculate the turbulent kinetic energy dissipation rate $\varepsilon$.

The contribution of fallspeed to the second moment is $\sigma_p \sin \theta$, where $\theta$ is the elevation angle and $\sigma_p$ the variance of particle fall velocity in quiet air. The spread of particle fall speeds is nearly independent of rainfall rate and is about 1 m$^3$ s$^{-2}$ (Lhermitte, 1963). Thus, for the low elevation angles ($< 15^\circ$) at which the distant radar scanned the storm, the fall velocity contribution is less than \(\frac{1}{10} \) m$^3$ s$^{-2}$ and may be ignored.

The contribution of wind shear to the second moment is most easily discussed in terms of (i) shear in the radial velocity along the radial direction, i.e., parallel
to the beam axis, and (ii) shear in the radial velocity \textit{transverse} to the beam axis.

The contribution of shear parallel to the beam axis is (Sirmans and Dovin, 1973)

$$\sigma_s^2 = \left(k_s^2 \right) / 12,$$  \hspace{1cm} (1)

where $k_s$ is the component of the shear parallel to the beam (s$^{-1}$), and $h$ is the radar range resolution (m).

The contribution of shear transverse to the beam axis is given by (Sloss and Atlas, 1968)

$$\sigma_c^2 = (0.3 R \phi)^2,$$  \hspace{1cm} (2)

where $k$ is the shear of the radial velocity across the beam (s$^{-1}$), $R$ range (m), and $\phi$ the one-way, half-power beamwidth. It is convenient to divide the transverse shear into two perpendicular components, $k^2 = k_r^2 + k_t^2$. Then the total contribution of shear to the second moment is given by

$$\sigma_s^2 = (k_r^2 + k_t^2) (0.3 R \phi)^2 + k_r^2 h^2 / 12.$$ \hspace{1cm} (3)

A scanning Doppler radar can measure the mean radial velocity throughout a storm, so $k_r$, $k_t$, and $k_s$ can be calculated. The beamwidth of our antenna was 0.9°, the range $R$ was between 40 and 55 km, and the resolution $h$ was 75 m. Therefore, it is clear from Eqs. (1) and (3) that the second moment contributions from $k_r$ and $k_t$ are about an order of magnitude greater than that from $k_s$. The second moment remaining after removal of the shear contribution is a result of small-scale velocity fluctuations within the radar pulse volume. For narrow beam radars the second moment produced by these fluctuations is related to the turbulent structure by

$$\sigma_k^2 = \int d k \phi_{11}(k) \left[ 1 - (2\pi)^2 |F_p(k)|^2 \right],$$ \hspace{1cm} (4)

where $\phi_{11}(k)$ is the radial component of the spectral density tensor of the velocity field, $F_p(k)$ the filter function of the radar illumination pattern, and $k$ the vector wavenumber. Thus the Doppler spectrum width depends not only on the velocity spectrum along the radar beam axis, but on the beam illumination function. The beam illumination function depends on the physical width of the beam (Frisch and Clifford, 1974) and thus varies with distance from the radar.

If we assume that the scales of motion within the pulse volume that cause the measured $\sigma_k^2$ lie within an inertial subrange, we can relate the turbulent kinetic energy dissipation rate $\epsilon$ to $\sigma_k^2$ (Frisch and Clifford, 1974):

$$\epsilon \approx \left( \frac{\sigma_k^2}{\alpha} \right)^1,$$ \hspace{1cm} (5)

where $\alpha = 0.53$ (Wyngaard and Pao, 1972) and $\alpha$, the beamwidth parameter, is $R \phi / 1.66$ for a Gaussian beam or $0.0094 R$ for our radar, where $R$ is the pulse volume range from the radar (m) and $\phi$ the one-way half-power beamwidth. Eq. (5) applies when the antenna beamwidth is much larger than the range resolution. To confirm the validity of their method of estimating dissipation rates, Frisch and Clifford (1974) compared their radar-measured dissipation rates with those measured from an instrumented tower and found good agreement.

In applying this method to a thunderstorm, we assume that the outer scale of the inertial subrange is larger than the maximum dimension of the beam illumination function. This assumption seems reasonable since aircraft observations in thunderstorm inflow regions indicate that the velocity spectra behave in the manner characteristic of an inertial subrange (Foote and Fankhauser, 1973), with the inertial subrange existing at wavelengths as large as 4 km. Aircraft penetrations of thunderstorms by Steiner and Rhine (1962) have also yielded one-dimensional spectra characterized by an inertial subrange having an outer scale of several kilometers.

3. Experiment and results

Data were collected for a storm on 9 July 1973 during the field program of the National Hail Research Experiment (NHRE). Two Doppler radars from the Wave Propagation Laboratory (WPL), situated 50 km apart along an approximately north-south line, observed the storm. Additional observations made with instrumented aircraft, a 10 cm radar and a surface meteorological network assisted in interpreting the Doppler radar data. The storm passed over the south radar which was zenith-pointing, and moved in a southerly direction at approximately 7 m s$^{-1}$ (Chalon et al., 1976). This storm produced pea-sized hail, mixed with rain. A few larger hailstones, up to 1.5 cm in diameter, were collected on the ground. [The storm is described by Browning et al., (1976).]

The moments of the Doppler spectra were estimated from the power spectral densities calculated from 128 complex radar samples that were acquired in 65 ms. A Hamming weighting function was used. Prior to calculating the moments a velocity window was applied to the signal plus noise power spectrum so that the noise spectrum was eliminated except for those spectral points that also contained signal power. The velocity window extended from both sides of the mean velocity to the points where the spectral power exceeded the mean noise level by 3 dB.

Fig. 1 shows radar reflectivity, radial velocity and the width of the spectra measured by the north radar in a vertical plane through the two radar locations. (The reflectivity contours are not corrected for attenuation.) This plane is approximately parallel to the direction of echo motion. The direction of echo motion, to the right in Fig. 1, is away from the radar so that
radial velocity field shows air motion toward the region of high reflectivity. Behind and at the same level of the 44 dBZ contour, the radial velocity field shows the air motion to be toward the high-reflectivity region. At the top and behind the 44 dBZ contour the radial velocity field shows motion away from the high-reflectivity region. Data obtained by the zenith-pointing radar showed a strong updraft just ahead of this high-reflectivity region (>44 dBZ). The data in Fig. 1

![Radar Reflectivity](image)

![Radial Velocity Field](image)

![Velocity Variance Field](image)

**Fig. 1.** Contours of radar reflectivity, radial velocity and second moments of the Doppler spectrum in a vertical plane through the two Doppler radars. The echo motion (measured by the north radar) was approximately parallel to this plane and the storm was moving away from the north radar. Positive radial velocity is in the direction of echo motion. Shaded region is the trailing edge of the updraft. Data are measured with a 3 cm radar located about 55 km NNW of the storm. Zenith-pointing radar was located at y=3.7 km.

positive velocities are in the direction of echo motion. When we use the radar echo as the frame of reference, i.e., subtract 7 m s⁻¹ toward the south from the total velocity, the reflectivity and radial velocity measurements show that within the highest reflectivity regions (>44 dBZ), the N-S component of relative air motion was approximately zero. Ahead of (i.e., in the direction of storm motion) and below the 44 dBZ contour the

![Reflectivity Field](image)

![Radial Velocity Field](image)

![Velocity Variance Field](image)

**Fig. 2.** Radar reflectivity, radial velocity, and second moments of the Doppler spectrum at 5 km altitude. Data are measured with a 3 cm radar located about 55 km NNW of the storm. Positive radial velocity is in the direction of echo motion. Zenith-pointing radar was located at x=8.3, y=3.7 km.
suggest convergence of the air entering the storm at low- and mid-level, and divergence of the air at the top of the updraft.

The radial velocity fields shown in Fig. 1 can be used to calculate the shear components $k_x$ and $k_y$ in the vertical plane passing through the two radars. Calculating $k_n$, we find it to be largest in the rear of the storm with a maximum value of about $6 \times 10^{-3} \text{s}^{-1}$.

The radial shear component $k_n$, also calculated from the radial velocity in Fig. 1, has a maximum value of $10^{-5} \text{ s}^{-1}$ at a height of 4 km, between $y = 3 \text{ km}$ and $y = 4 \text{ km}$, i.e., between the low- and mid-levels of the storm and slightly in front of the high-reflectivity region in the direction of echo motion. Part of an updraft (denoted by the shaded area in Fig. 1) was observed by the south radar with its antenna pointing vertically as the storm passed overhead (Strauch and Merrem, 1976). This region contained updraft velocities as high as 16 m s$^{-1}$. After the echo advected less than 3 km, the zenith-pointing radar observed downdrafts as large as 12 m s$^{-1}$. This horizontal shear of the vertical wind does not contribute appreciably to the width of the Doppler spectrum observed by the north radar because the elevation angle of the radar antenna in the region of high shear was less than $8^\circ$. Second-moment contours measured by the north radar show a distinct maximum at altitudes between 4 and 8 km, just ahead of the highest reflectivity region which is also the region between the updraft and downdraft.

Fig. 2 shows mid-level contours in a horizontal plane at a height of 5 km. Comparing contours of reflectivity and radial velocity we note strong convergence of air overtaking the storm from the north and air entering the storm from the south near the highest reflectivities. The velocity variance is large in a region about 5 km in diameter, just ahead of the high-reflectivity core (at $x = 7 \text{ km}$, $y = 3 \text{ km}$). The third component of shear, $k_z$, was calculated from the radial velocity fields in Fig. 2. These values are generally very small but show a maximum of $4 \times 10^{-3} \text{ s}^{-1}$ at the southwest edge of the coverage area. The total shear contribution to the spectrum width measured by the north (low-angle) radar was only a small portion of the total width in the region where the highest second moments were observed. Therefore, the high second moment values shown in Figs. 1 and 2 portray regions with high turbulent kinetic energy dissipation rates. The largest spectral widths were measured just ahead of the high-reflectivity region, where a transition between updraft and downdraft occurred (Browning et al., 1976).

From the second moments measured by the north radar scanning at low elevation angles (maximum of $15^\circ$), we calculated the turbulent kinetic energy dissipation rate $\epsilon$ using Eq. (5) after correcting for the shear contribution to the velocity second moment. If the total shear contribution to the width of the Doppler spectrum is less than 50%, it was removed from the total width and a corrected variance for turbulence only was calculated. If the shear contribution was greater than 50% of the total, the width measurement was not used for dissipation rate calculations. The measured first and second moments of the Doppler spectrum, obtained with a range spacing of 600 m and an elevation and azimuth spacing of about 900 m, were interpolated to a 900 m Cartesian grid using linear interpolation. The dissipation rates were calculated from the interpolated values of the first and second moments at various locations throughout the storm. The wind shear was calculated from finite differences of the interpolated mean radial velocity at adjacent grid points. The interpolation process filters the measured data field and thereby attenuates scale sizes on the order of the grid spacing. Hence velocity fluctuations at scale sizes larger compared to the pulse volume are treated as shear and their contributions to the width of the spectrum are removed. Velocity fluctuations on scale sizes of the order of the pulse volume are not treated as shear because mean velocity fluctuations on these scale sizes are not distinguishable from statistical uncertainties in the estimate of the mean velocity. This means that we may not have removed the effects of all velocity fluctuations at scale size greater than the pulse volume. For this reason we did not attempt to calculate the dissipation rate when the shear contribution was greater than 50% of the total.
vertical velocity \([D_{vw}(r)]\) at an altitude of 4.5 km was calculated from the vertical velocity measurements taken every \(\frac{1}{2}\) s as the storm advected overhead. Assuming Taylor's hypothesis to be valid, the temporal record can be treated as a spatial record with \(r = \bar{u}t\), where \(\bar{u}\) is the average horizontal motion of the storm (7 m s\(^{-1}\)) and \(t\) time. The radar pulse volume was 75 m in depth by about 65 m in diameter at the height of 4.5 km. For values of \(r\) where \(D_{vw}(r)\) varied as \(r^4\), \(D_{vw}(r)\) is related to the dissipation rate by

\[
D_{vw}(r) = B e^{\epsilon r^4} = B e^{\epsilon (\bar{u} t)^4},
\]

where

\[
D_{vw}(r) = [w(x) - w(x+r)]^4 = [w(x) - w(x+\bar{u} t)]^4
\]

(6)

and \(B\) is a constant equal to 1.4 (Record and Cramer, 1966; Tatarski, 1961). The vertical velocity time record was 240 s long, and in computing \(D_{vw}(r)\) we used delays up to 40 s. We assumed that the changes in the particle fall velocity in still air were unimportant. At a height of 4.5 km (above ground level) and with a mean horizontal wind speed of 7 m s\(^{-1}\), Eq. (6) yielded an average value of \(\epsilon = 700\) cm\(^2\) s\(^{-3}\). The temporal record of vertical velocity corresponds to the location (Fig. 3) where the scanning radar measured \(\epsilon\) values ranging from 280 to almost 800 cm\(^2\) s\(^{-3}\). Thus there is reasonable agreement between the two entirely different radar measurement techniques.

Although we had no direct measurements with which to compare our calculated results for this storm, data taken by aircraft penetrating other storms verify that our indirectly derived values were consistent with direct measurements. For example, using storm data from Steiner and Rhyne (1962), we found maximum values of \(\epsilon \approx 10^4\) cm\(^2\) s\(^{-3}\). Using Foote and Fankhauser's (1973) velocity spectra from another storm, we calculated values for \(\epsilon\) of about 620 cm\(^2\) s\(^{-3}\) in the outflow and 68 cm\(^2\) s\(^{-3}\) in the inflow regions. Dissipation rates of 3000 cm\(^2\) s\(^{-3}\) were measured by Sand et al. (1974) in another northeast Colorado thunderstorm. Thus, our values lie within the range of values found by other investigators.

Another check on the order of magnitude of the dissipation rate is given by \(\epsilon = g^2/l\) (Tennekes and Lumley, 1972) where \(g\) is the average velocity variation within the storm and \(l\) a characteristic scale for the storm. Examination of Figs. 1 and 2 indicates that the variation in the mean radial velocity field is \(\sim 8-10\) m s\(^{-1}\) and the scale of the storm is \(\sim 10\) km. This yields \(\epsilon \approx 500\) to \(10^5\) cm\(^2\) s\(^{-3}\), which is in the range of values we calculated from our storm measurements.

4. Discussion

Aircraft measurements of the turbulent kinetic energy dissipation rate with in situ sensors provide data along the flight path, but during the time required for the aircraft to make a single pass through the storm,
the radar can obtain estimates throughout the entire storm. For example, the radar data we used were acquired in 160 s. In addition, the probability is small that the aircraft will pass through the most turbulent region of a storm, and the safety of doing so is questionable. Kyle (1974) used an aircraft measurement to arrive at an average dissipation rate of 512 cm$^2$ s$^{-3}$ throughout the height of a storm to compute the diffusion of seed material. An aircraft flying through the storm we studied might well have measured this average value depending on the track and record length used. However, the radar measurements show that the distribution of $\varepsilon$ is not constant, and any model for diffusion within this storm must include a small but intense turbulent region at mid-level.

Dissipation rates measured by aircraft in clear air turbulence are summarized by Trout and Panofsky (1969). The highest rate they report is 1160 cm$^2$ s$^{-3}$ with a geometric mean of 675 cm$^2$ s$^{-3}$ for “severe turbulence.” They believe the average estimates to be conservative and may easily be in error by 50%. The rates we calculated using the Doppler radar measurements range from their classification for “light turbulence” to greater than their numbers for “severe turbulence.” The highest dissipation rates we measured were in the area between the updraft and the downdraft; an aircraft would certainly have experienced severe turbulence in that region.

5. Conclusion

The three-dimensional scalar field of the second moment of the Doppler spectrum, measured in a convective storm by a Doppler radar scanning at low elevation angles, shows regions of high shear and severe turbulence. Shear of the radial velocity can be estimated from the gradients of the radial velocity field, so the contribution of shear to the width of the spectra can be assessed. Turbulent kinetic energy dissipation rates can be derived from the portion of the second moment that is not related to the shear. Dissipation rates derived from the second moment of the Doppler spectra measured in a northeast Colorado thunderstorm ranged from rates associated with little or no turbulence to rates associated with severe turbulence. The highest dissipation rates were found in the mid-levels of the storm where there was large shear between the updraft and downdraft. Remote measurement of the dissipation rate throughout the storm provides essential data for calculating the diffusion of seed material injected along a line or at a point. It is also potentially important in identifying hazardous areas for aircraft.

REFERENCES


