A Balloonborne Instrument for the Measurement of Vertical Profiles of Supercooled Liquid Water Concentration

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ABSTRACT

A vibrating wire placed in the humidity duct of a standard U.S. rawinsonde is used to measure vertical profiles of the concentration of supercooled liquid water in clouds. The natural frequency of vibration varies according to the mass of ice accumulated by contact freezing. By monitoring the natural frequency and the airspeed relative to the wire, the supercooled liquid water concentration can be found.

Suitable electronics are developed for both individual expendable rawinsonde units and a ground-based receiver so that the vibration frequency can be recorded. Calibration of the frequency change versus mass accumulation is done theoretically along with measurements made in a wind tunnel with supercooled water present. Further verification is found by the use of paint, uniformly sprayed on the “upwind” side of an exposed wire. For exposition of the instrument, a sample sounding is described.

1. Introduction

Measurements of liquid water concentrations (LWC) are often made by aircraft equipped with a Johnson-Williams hot wire device (Neel and Steinmetz, 1952; Neel, 1955) or a Particle Measuring Service probe (Knollenberg, 1970, 1972). Recently, a dual-frequency passive radiometer has been developed at NOAA to obtain continuous vertically integrated values of LWC (Guiraud et al., 1979).

In the present article the development of a balloonborne “cloudsonde” instrument is described. In this development vertical profiles of supercooled liquid water concentration (SLWC) and vertical air motion (w) along with the usual measurements of temperature, humidity, pressure-height and wind are obtained. Currently, instrumentation for liquid water measurements is fully developed; a newer type vertical air motion instrument is not yet operational. In the meantime the SLWC device is used concurrently with parachute dropsondes such as described by Hill (1978).

The capability of obtaining vertical profiles of SLWC and vertical air motion makes possible the acquisition of critical measurements in otherwise inaccessible regions of the lower atmosphere. Furthermore, vertical profiles of SLWC and vertical air motion should lead to improved understanding of mesoscale and orographic cloud systems.

2. Balloonborne measurement system

a. Supercooled liquid water measurement: concept

The basic concept utilized for measuring SLWC is a vibrating wire exposed to the airstream during ascent of a balloonborne package. The wire is fixed at one end and free at the other. The package consists of a standard NWS rawinsonde, modified to accommodate the vibrating wire and related electronics. The natural frequency of vibration varies according to the mass of ice accumulated by contact freezing. Thus, by monitoring the rate of change of the natural frequency and the airspeed relative to the wire, the supercooled liquid water concentration can be determined.

In selecting a suitable ice collector, an important consideration is the collection efficiency. It is desirable to choose a collector of sufficiently small diameter so that the collection efficiency does not vary much for the drop sizes encountered. Collection efficiencies for various drop sizes and collector diameters were calculated according to Langmuir and Blodgett (1946). Based on these calculations, a (piano) wire of 0.60 mm diameter was chosen. Slightly smaller diameters would have been acceptable but the stiffness and vibration properties were not as suitable as the value chosen. The length of the wire was set at 90 mm, so the natural vibration frequency is around 53 Hz. The collection ef-
problem in measuring supercooled water at large cloud droplet sizes.
In addition to the collection efficiency, consideration must also be given to the heat economy of the collecting wire for reasons discussed by Ludlam (1951) and others. ¹ In exposed cylinders mounted on aircraft, there is a temperature dependent limit of supercooled water concentration beyond which there is very little additional rate of accumulation. While such limits exist for a balloonborne vibrating wire, the critical concentrations are far above the concentrations expected in the atmosphere except at temperatures within a degree below freezing. The reason for the very limited below-freezing temperature range where incomplete freezing takes place is that both the airstream velocity and the probe diameter are much lower than in the case of aircraft.

b. Mechanical and electronic design

The design of SLWC measuring system consists of two parts: the balloonborne package and the ground based rawinsonde (R/S) receiver-recording system. In the balloonborne package the SLWC signal is generated by the vibrating wire which is driven by a coil mounted on the outside of the humidity duct. The vibrating wire extends from the driving coil into the humidity duct; 65 mm of the 90 mm wire are exposed to the airflow. A very small pickup coil is mounted near the free end of the wire; the pickup coil serves to feed back the natural frequency, so the drive coil acts at the natural fre-

frequency which is allowed to vary. As indicated in Fig. 2 the signal from the driving oscillator is fed into a Schmidt trigger and a monostable multivibrator to eliminate noise and enhance the signal. Then the signal is fed into a phase-locked loop and the frequency is multiplied by 10. The multiplication by 10 not only separates the received signal from other parameters, but amplifies the response to collected ice prior to transmission. After multiplication the signal is passed through a modulation shaping circuit to properly modulate the radio frequency (rf) carrier.

The remaining circuits act to intersperse the liquid water measurements in place of alternate temperature samples. This is done by sensing each baro-switch closure and actuating the modulation control relay just after every other closure. Each closure represents either humidity, low reference or high reference and, because a temperature sample normally follows each, it is necessary to sense and count all three types of closures so the liquid water samples can be adequately monitored.2

The signal output from the RD65CS receiver is a series of narrow pulses occurring at the frequency of the modulation on the radiosonde transmitter carrier. This signal frequency is 0–200 Hz for a standard R/S, but when driven by the SLWC sensor it is 470–550 Hz. A block diagram of the modified R/S receiver circuits is shown in Fig. 3. The pulses representing SLWC are quite narrow and are therefore unsuitable for the frequency-to-voltage (F/V) converter which requires a duty cycle of at least 20% for frequencies <1 kHz. Therefore, the pulses are stretched so that their duty cycle is from 45 to 55%. The output of the F/V converter is amplified and recorded on a strip-chart recorder. The deflection on the recorder is approximately proportional to the mass of ice accumulated by the sensor.

Calibration of the recorder is achieved by inserting first a 480 Hz signal and then a 540 Hz signal. These two frequencies define a range which represents 60 cycles of frequency change. Thus the chart is calibrated in terms of chart divisions per cycle.

c. Preliminary testing

A radiosonde unit with the vibrating wire and electric coils attached was tested at the University of Alberta, Edmonton, in a low-speed wind tunnel with supercooled water injected continuously. While a calibration of the vibrating wire system was not carried out at that time, certain important information concerning the characteristics of the system was acquired during 19 separate tests wherein vibration frequency versus time was measured for various liquid water concentrations.

During each of these tests the rate of supply of liquid water was held constant. The actual liquid water concentration in the test chamber was estimated by exposing a small wire over a short time interval and then finding its weight change; the flow velocity was also monitored. However, these measurements could only be used as a rough estimate of the liquid water concentration because an unknown amount of ice sublimated during the weighings.

On the other hand, the rate of frequency change of

\[2\text{ Modification of circuits is made for newer type radiosonde units; in this case FM modulation allows for continuous transmission of supercooled water information.} \]
the vibrating wire is a measure of the supercooled water concentration. As will be discussed in Section 3, the calibration of the vibrating wire is based in part on these data. Therefore, the frequency versus time curves are shown in Figs. 4a, 4b and 4c for three successive days of testing. It is clear from these figures that the rate of change of frequency with time is very nearly constant, a fact which will be utilized later. Variations in the initial value of frequency for the various samples are due to differences in the starting time after the rawinsonde unit was placed within the test area. (Ice had to be removed after each test was made.) Also, a different unit was used on the second and third test days; a very small difference in wire length causes a small frequency difference.

Another observation was that no difference in frequency change was observed when the wire was directly exposed to the airflow as compared to when the wire was placed in the humidity duct, below and upstream from the hygristor. It was also found that there was a negligible change in the collection cross section of the wire due to ice accumulation over the whole dynamic range of the instrument. Yet ice was observed to accumulate in the upwind direction by as much as 15 wire diameters.

The dynamic range of the instrument was found to be sufficient for most conditions normally expected in operational situations. Frequency changes up to 8 Hz from an initial value of ~52 Hz were measured. This change is equivalent to a liquid water concentration of ~1.4 g m⁻³ over 1 km cloud depth. In no test sample was there evidence of failure due to exceeding the dynamic range of the instrument.

In one instance, after long exposure (10 min) to a liquid water concentration of about 0.3 g m⁻³, a small amount of ice broke off from the vibrating wire.
However, following breakoff, the rate of frequency change remained as before. (Of 21 actual soundings during February and March 1980, ice breakoff occurred only once.)

Interference by the presence of precipitating ice crystals is minimized by the placement of the wire within the humidity duct. Such ice crystals pass beneath the wire as the air flows through the humidity duct. With smaller ice crystals, we expect they would not adhere to the vibrating wire. In several instances of actual soundings when precipitation was occurring, no frequency reductions were observed, which indicates not only was supercooled water virtually absent but that no measurable accumulation of ice crystals accumulated on the wire. However, the problem of ice crystal interference in mixed phase clouds will be further investigated in the future. We believe at present the effects of ice crystals on the measurements is small or negligible.

3. Calibration

Calibration of the vibration frequency as a function of accumulated rime ice is accomplished both theoretically and experimentally. An exact expression for the fundamental vibration frequency of a uniform cantilevered wire is

\[
f = \frac{0.5596 \left( \frac{EI^3}{L^2 \rho} \right)^{1/2}}{\sqrt{\pi}},
\]

where \( f \) is the frequency (Hz), \( L \) the wire length, \( E \) the Young's modulus (dyn cm\(^{-2}\)), \( I \) the moment of inertia (cm), and \( \rho \) the density (g cm\(^{-3}\)). For a piano wire with diameter \( D = 0.06 \) cm, density 7.7 g cm\(^{-3}\), \( E = 20 \times 10^{11} \) dyn cm\(^{-2}\) and \( L = 9.0 \) cm, the vibration frequency is 52.815 Hz.

However, as soon as a portion of the wire becomes loaded with a different material, such as rime ice, the problem becomes more complex. The configuration of the loaded wire is illustrated in Fig. 5. The length of wire \( L_1 \) is unloaded and the remaining outer portion of the wire is allowed to accumulate ice with a collecting thickness \( D \), an accumulation thickness \( \alpha D \) and a length \( L - L_1 \). To find the vibration frequency under these conditions, we make use of the Rayleigh method, in which the maximum potential energy (PE) is equated with the maximum kinetic energy (KE) during the course of vibration.

The potential and kinetic energies must be integrated over the two wire segments.

Thus, Rayleigh's quotient is written

\[
\omega^2 = \frac{PE_{\text{max}}}{\omega^2 KE_{\text{max}}} = \frac{\int_0^{L_1} [(E_1 I_1/2)(\partial^2 y/\partial x^2)^2]dx + \int_{L_1}^{L} [(E_2 I_2/2)(\partial^2 y/\partial x^2)^2]dx}{\omega^2 \int_0^{L_1} [M_1/2(\partial y/\partial t)^2]dx + \omega^2 \int_{L_1}^{L} [M_2/2(\partial y/\partial t)^2]dx},
\]

where \( \omega = 2\pi f \), \( y \) is the displacement from a stationary wire, \( x \) the distance from the fixed end of the wire, \( M \) denotes the mass per unit length, the subscripts 1 and 2 refer to the unloaded and loaded wires, respectively and \( t \) is time. An admissible function for the \( y \) displacement as a function of \( x \) and \( t \) is assumed as

\[
y = B[\cos(\pi x/2L) - 1],
\]

where \( B \) is some unspecified amplitude.

Because the loaded portion of the wire is a composite structure, the steel wire in the loaded portion will be treated as an equivalent ice section as illustrated in Fig. 6. With this assumption, we write \( \beta = E_{\text{steel}}/E_{\text{ice}} \) and \( E_2 I_2 \) becomes \( E_1 I_2/\beta \). Integration of (2) yields

\[
\omega^2 = (\pi^4 D^4/32 L^4) \frac{(I_1 - I_2/\beta)[L_1/L + (1/\pi) \sin\pi L_1/L] + I_2/\beta}{(M_1 - M_2)[3 L_1/2L + (1/2\pi) \sin\pi L_1/L - (4/\pi) \sin\pi L_1/L - (2L/2)] + M_2(3/2 - 4/\pi)},
\]

where

\[
I_1 = (\pi/64) D^4,
\]

\[
I_2 = (\alpha^3/12) D^4 + \alpha D^2[(\alpha/2 + 1)D - \bar{y}]^2
+ (\pi/64) \beta D^4 + \beta D^2[\bar{y} - D/2]^2,
\]

\[
\bar{y} = [\alpha(\alpha/2 + 1) + \beta/2] D/\alpha + \beta,
\]

For our particular case, we have \( L_1 = 2.5 \) cm, \( L = 9.0 \) cm, \( M_1 = 0.2025 \) g cm\(^{-1}\), \( M_2 = 0.0225 \) g cm\(^{-3}\), \( \alpha = 0.2025 \), \( \beta = 0.0225 \), \( \rho = 0.2 \) g cm\(^{-3}\).

Thus we may compute the vibration frequency for various values of \( \alpha \) and \( \beta \). \( \alpha \) is the accumulation
thickness measured in wire diameters of ice and $\beta$ is the stiffness factor of steel compared to ice.) For hard ice $\beta$ is denoted as $\beta_0$; and $\beta_0 \approx 30$. When $a = 0$, we have the equivalent case of a uniform steel wire. For this case Eq. (4) yields a value of $f = 54.136$. This value is higher than the exact solution by 2.50%, which is to be expected with the Rayleigh approximation. To correct for this difference in result, frequency calculations according to Eq. (4) are multiplied by 0.975. Values of the adjusted vibration frequency ($f^*$) as a function of $\alpha$ are shown in Fig. 7 for several values of $\beta$, expressed as $(a^* \beta_0)$ where $a = 16$. It is readily apparent from Fig. 7 that as the accumulation thickness increases, the vibration frequency reaches a minimum and then increases. However, when the stiffness approaches zero (large $\beta$), the frequency decreases only in accordance with the mass increase.

With negligible stiffness from the added mass the frequency decrease is nearly linear. Thus, the laboratory findings of an approximately linear decrease in frequency lead to the conclusion that the accumulated rime ice contributes a negligible amount of added stiffness. The mass of rime ice is apparently either very soft or laced with tiny cracks.

As a further verification, a known amount of paint was sprayed on one side of the wire and the vibration frequency was recorded. While the thickness of the paint is very much less than that of ice, the paint mass is compared with the same ice mass. Values of the vibration frequency with paint added to the wire are included in Fig. 7 as a function of added mass expressed as $\alpha$ for ice. The result is a close agreement with the measured and computed values of frequency when the stiffness of the added mass is negligible.

Because the stiffness of the rime ice can be assumed negligible, i.e., $\beta \to \infty$, we write (4) accordingly, i.e.,

$$\omega^2 = \left(\frac{\pi^4E_i}{32L^4}\right) \frac{1}{(M_1 - M_2)(3L_i/2L + (1/2\pi) \sin\pi L_i/L - (4/\pi) \sin\pi L_i/(2L) + M_2(3/2 - 4/\pi))}.$$  \hspace{1cm} \text{(8)}$$

This result reduces to

$$f^2 = \frac{c}{1 + b_1 \alpha},$$ \hspace{1cm} \text{(9)}$$

where in our particular case $c = 2930.22$ and $b_1 = 0.03207$ with $\rho_i = 0.2$. When (9) is expressed in terms of total mass of ice $m_i$ accumulated by the wire, i.e., $(L - L_i)$ times $(M_1 - M_2)$, we find

$$f^2 = \frac{c}{1 + bm_i},$$ \hspace{1cm} \text{(10)}$$

![Fig. 7. Adjusted frequency versus ice width for various values of stiffness, and calibration data for painted wire with equivalent mass.](image-url)
where $c$ is unchanged and $b = 6.8528$. As before we adjust the approximate solution by a factor of 0.975 so that conformity to the exact solution is obtained. Thus we have

$$f^* = f_0(1 + bm_i)^{-1/2},$$

where $f_0 = 52.78$, and from (11) we obtain

$$m_i = (f_0^2 - f^*{}^2)/(bf^*{}^2).$$

As noted earlier the measured frequency is multiplied by 10 to give a transmitted frequency $f''$. That is, $f^* = f''/10$. From such measurements the accumulation of rime ice can readily be found from (12).

The calculation of SLWC is

$$SLWC = \epsilon^{-1}dm_i/dv,$$

where $\epsilon$ is the collection efficiency, $dm_i$ the change in ice mass accumulated, and $dv$ the incremental volume swept out by the wire. In turn $dv$ is equal to $(L - L_i)Dwdt$, where $w$ is the air velocity passing the wire and $dt$ the time over which the change in ice mass is found. The result is

$$SLWC = [e(L - L_i)Dw]^{-1}dm_i/dt$$

and in terms of frequency change, we find with the aid of (12), that

$$SLWC = -{(2f_0^2/eb(L - L_i)Dwf^*{}^2)}df^*/dt.$$  

Fig. 8. Skew-T diagram of temperature and dew point for 0010 GMT 8 February 1980.

The air velocity in the humidity duct of the rawinsonde unit has been found to be approximately 96% of the rise rate of the balloon relative to the air (Shinnors et al., 1971). No additional velocity due to the vibration of the wire need be added because the vibration-velocity vectors of the wire are parallel to the duct airflow vector.

A small frequency dependence on temperature of the wire has been determined from several soundings made during the absence of clouds. That is, $df^*/dT$ is $-0.0053$ Hz °C$^{-1}$. This small frequency dependence on temperature can easily be removed prior to the solution of (15).

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### Table 1. Change in vibration frequency versus height and time.

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<th>Mid-layer height (m)</th>
<th>$\Delta f$ (s)</th>
<th>Top of layer $f'$ (Hz)</th>
<th>$\Delta f'$ (Hz)</th>
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#### 4. Application

Although the SLWC measuring device has been already used operationally to collect data in winter orographic clouds, only a sample case particularly well suited for exposition is presented here. Analysis of other data collected will be reported later.

At 0010 GMT 8 February 1980 a stratocumulus deck estimated to be at about 1400 m elevation above ground was over Richmond, Utah (elevation 1386 m), the launch site for SLWC soundings. At that time a launch was made; the measured temperature and dew point are shown in Fig. 8. No evidence of precipitation could be seen falling from the clouds. As indicated by the temperature dew-point profiles, the strato-cumulus deck was the result of mixing at lower levels. Adiabatic values of liquid water would not be expected to exceed 0.3 g m$^{-3}$. In fact, due to mixing the actual SLWC would be expected to be somewhat lower.

In this example no vertical motion sample was made. However, for our present purpose, the balloon rise rate below cloud level is used for the airflow velocity and the vertical air motion is assumed zero. Therefore, we set $w = 4.6$ m s$^{-1}$. Values of $f'$ and the time between vibration-frequency samples are listed in Table 1 as a function of height.

The presence of supercooled liquid water is identified by the negative values of $\Delta f'$. The relatively strong positive changes in $f'$ above the cloud layer are due to sublimation. The amount of sublimation depends upon prior ice accumulation and the relative humidity of the ambient air. In this case the air above the cloud was very dry as shown in the sounding. In addition to these variations in $f'$ there is a very small temperature effect (0.053 Hz per °C reduction).

According to (15), the SLWC is 0.10 g m$^{-3}$ over a 204 m layer (694–767 mb) centered at 3228 m and 0.13 g m$^{-3}$ over a 205 m layer (676–658 mb) centered at 3432 m. Thus, the cloud thickness as determined by SLWC measurements is no more than 409 m, probably about 350 m, and the supercooled liquid water concentration is in the vicinity of 0.1 g m$^{-3}$.

Other soundings recently made show examples of much larger amounts of supercooled water and others show a virtual absence of supercooled water, especially in the presence of snowfall. However, many of these soundings will be described in a separate article, wherein the vertical distribution of supercooled water in winter orographic clouds will be described.

#### 5. Conclusion

We may conclude that a new method is available for measuring vertical profiles of supercooled liquid water concentrations, with the proviso that the airflow velocity relative to the instrument is available. At present this velocity is estimated from nearby and near-time observations by parachute dropsonde. (A new on-board vertical air motion measuring system is under development.)

The SLWC "cloudsonde" measuring system is particularly well suited for application to winter orographic clouds. Such measurements should lead to an improved understanding of cloud properties and their modification potential.

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**REFERENCES**


