Vertical Distribution of Aerosol Extinction Cross Section and Inference of Aerosol Imaginary Index in the Troposphere by Lidar Technique

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ABSTRACT

Vertical profiles of aerosol extinction and backscatter in the troposphere are obtained from multi-zenith angle lidar measurements. A direct slant path solution was found to be not possible due to horizontal inhomogeneity of the atmosphere. Regression analysis with respect to zenith angle for a layer integration of the angle-dependent lidar equation was thus employed to determine the optical thickness and aerosol extinction-to-backscatter ratio for defined atmospheric layers, and subsequently, cross-section profiles could be evaluated. Measurements were made with an elastic backscatter ruby lidar system with calibration by a standard target procedure. The results from 20 measurement cases are presented. For layer-aerosol optical thicknesses > 0.04, useful results were obtained, and corroboration by solar radiometer aerosol optical depth data was found. The mean mixed-layer aerosol extinction-to-backscatter ratio for the measurements was 19.5 sr with a standard deviation of 8.3 sr. With the use of an aerosol size distribution inverted from wavelength-dependent solar aerosol optical depth data, the measured extinction-to-backscatter ratio was compared to Mie theory calculations, and the imaginary index giving best agreement was determined. A maximum upper limit of 0.013 was indicated for the aerosol imaginary index, but the mean result was 0.003 for a real index of 1.52.

1. Introduction

In an attempt to understand the influence of particulates on radiative energy exchange within the earth-atmosphere system, a number of investigators (e.g., Herman and Browning, 1975; Braslau and Dave, 1973a,b; Yamamoto and Tanaka, 1972; Mitchell, 1971; Rasool and Schneider, 1971) have developed models for the transfer of radiation through the earth's atmosphere. All such calculations must be based on a model for atmospheric particulates. The model may be given in terms of the aerosol physical characteristics such as number density, size distribution and composition. In this case Mie scattering theory for spherical particles is used to generate the optical parameters on which radiative transfer solutions directly depend. Alternatively, an aerosol model may be defined by means of the fundamental parameters required for radiative transfer calculations—volume extinction cross section, single-scatter albedo and the scattering phase function.

Since the result of any radiative transfer solution depends on the characteristics of the aerosol model, the validity of predictions for atmospheric radiative energy exchange will be based on the accuracy with which the aerosol model corresponds to aerosol characteristics as actually found in the atmosphere. However, in relation to great variability of atmospheric particulates within the troposphere, relatively few measurements defining aerosol radiative characteristics have been made and, additionally, techniques for obtaining the optical parameters of atmospheric particulates have not been of conclusive validity (Toon and Pollack, 1976). In order to provide the extensive monitoring that is necessary to adequately model aerosol characteristics, remote sensing methods are the most useful. Direct remote sensing of aerosol optical parameters can be expected to be more tractable than remote measurement of aerosol physical characteristics which is necessarily less straightforward.

Since lidar is an optical remote sensing technique, measurement of aerosol optical parameters is an obvious application. However, there are a number of problems which must be overcome before useful quantitative data may be obtained from backscatter lidar returns. The elastic backscatter lidar signal is primarily related to the 180° combined molecular and aerosol backscatter cross section. Of itself, the 180° aerosol backscatter cross section is not a fundamental parameter for use in radiative transfer studies, nor is it well-related to other physical characteristics of the aerosol population. Even if the aerosol composition and size distribution were
known, the variation in shape for real aerosol particles will produce differing values of the 180° backscatter cross section. Additionally, there are difficulties associated with accurate measurement of the aerosol backscatter cross section. When the atmospheric optical extinction cross section is sufficiently great, such as in the lower troposphere, the variation with range of transmission through the atmosphere must be accounted for. Also, the calibration factor relating the return signal strength to the actual-scattering cross section must be known.

Lidar measurements which determine the aerosol extinction cross section with height are of greater utility than measurements of the 180° backscatter cross section. The extinction cross section is a fundamental parameter on which a radiative transfer model of the atmosphere may be based. Also, aerosol physical characteristics such as number density or mass concentration are more directly related to the extinction cross section than backscatter cross section. Knowledge of both the aerosol extinction cross section and backscatter cross section, in addition, can be applied for comparison to values calculated by Mie theory. Backscatter cross sections are sensitive to the imaginary (or absorption) index of scattering particles, and several previous studies (Waggoner et al., 1972; Grams et al., 1974) have applied backscatter measurements for inference of aerosol absorption index. As has been shown by studies such as those initially listed, the radiative effect of aerosol particles in the atmosphere is largely determined by the imaginary index of the particles. The influence of particle shape and nonhomogeneity on backscatter is also important and must be considered. However, since calculations for homogeneous distributions of spherical particles are widely applied to aerosol scattering and since particle index is not well known, relating scattering measurements to an effective refractive index is of interest.

In this paper we will describe a use of monostatic backscatter lidar for quantitative measurement of aerosol extinction and backscatter cross section within the troposphere. The results from a series of measurements will be reported. Also, in conjunction with aerosol size distributions obtained from inversion of solar extinction data, the extinction and backscatter cross-section values for a number of cases will be related to a refractive index for the aerosol population.

2. Solution procedure

Our basic experimental method is to make use of multi-zenith angle lidar returns for solution of aerosol scattering cross sections. The return signal \( V(z, \theta) \) for slant range lidar signals as a function of vertical height \( z \) and zenith angle \( \theta \) is given by

\[
V(z, \theta) = \frac{CE}{z^2 \sec^2 \theta} [\beta_p(z, \theta) + \beta_A(z)] T_p(z)^2 \sec^2 \theta T_A(z)^2 \sec^2 \theta, \tag{1}
\]

where \( \beta \) is the 180° volume backscatter cross section and \( T \) the effective vertical transmission for the given path. The subscripts \( p \) and \( R \) refer to the particulate and molecular components of the scattering and extinction, respectively. The term \( E \) is a relative measure of the transmitted pulse energy, and \( C \) is a calibration factor relating the received signal to the scattering cross sections and range terms.

As has been recognized by earlier lidar workers (Stanford, 1967; Hamilton, 1968) in the case of horizontally homogeneous particulate scattering, the vertical optical thickness can be found directly from slant range lidar measurements. This procedure is readily seen by taking the logarithm of the slant range lidar equation. The optical thickness to any height will then be given as the slope of \( \log[V(z, \theta) \sec^2 \theta/E] \) vs \(-2 \sec \theta\). Since the optical thickness could be thus found to all heights, the extinction cross section could then possibly be obtained by height differentiation.

However in practice the direct solution given above is of limited usefulness. Large errors can result both from horizontal inhomogeneity of backscatter and also from the signal noise associated with lidar return data. An example of multi-zenith angle lidar signals is shown in Fig. 1. The signals are plotted as a function of vertical height in a range and pulse energy normalized form \( P(z, \theta) \), where

\[
P(z, \theta) = \frac{V(z, \theta)z^2 \sec^2 \theta}{E}. \tag{2}
\]

The equipment with which these returns were acquired is described in a subsequent section. The effect of increased atmospheric attenuation at greater zenith angles is readily apparent. However, within the lower mixed layer, signal variation due to horizontal inhomogeneity is comparable to that due to the attenuation factor. Above the mixing layer, the atmosphere is more homogeneous, but the inherent signal-to-noise ratio of lidar return data decreases rapidly with range.

If the optical thickness to a given height is to be determined to an accuracy of 0.01, then changes in signal intensity due to increased attenuation at greater zenith angles must be determined to within a few percent. In practice it was found that at a given height within the mixing layer, a typical rms horizontal range variation of the backscatter cross section is 0.05–0.15. Also, for the lidar system being used in this investigation, the signal-to-noise ratio for tropospheric returns acquired from above the mixing layer could vary from 25 down to 2. Thus at any layer in the overlying atmosphere, large errors
result from a direct slant path determination of the optical thickness, and it follows that subsequent inference of extinction cross section by height differentiation is not possible.

Rather than a direct slant angle solution as given above, our approach is a procedure which applies atmospheric layer integration to multi-angle lidar return data. Layer averaging serves to ameliorate the effect of horizontal inhomogeneity and signal noise. In addition, the solution to be described effectively includes constraints such as the necessary increase of optical thickness with height.

A backscatter lidar signal is a function of the volume backscatter cross section and the two-way path transmittance. The path transmission \( T(z) \) may be expressed in terms of the backscatter cross section and a ratio of the backscatter to extinction cross section as

\[
T(z) = \exp(-\tau(z)) = \exp \left[ -\int_{z_0}^{z} S \beta(z')dz' \right],
\]

where

\[
S = \frac{\sigma/\beta}{\rho(180')} = \frac{4\pi}{\rho(180')},
\]

and where \( \sigma \) is the volume extinction cross section, \( \tau \) the optical thickness and \( \rho(\theta) \) the normalized scattering phase function. The above expression for \( T(z) \) when differentiated may be used to eliminate \( \beta(z) \) from the lidar return equation. The resulting differential equation can be integrated to give an equation relating signal attenuation to an integration of the received return. A procedure of this type was first presented by Hitschfeld and Bordan (1954) for the radar return equation, and Fernald et al. (1972) have analyzed vertical lidar return data by an integrated equation that included a separation of particulate and molecular scattering terms.

By the procedure outlined above, a transmission form of the slant angle lidar equation (1) integrated over a layer from \( z_1 \) to \( z \) may be derived (see the Appendix). The result is

\[
T_+(z)T_+(z) = T_+(z_1)T_+(z_1) - \gamma \int_{z_1}^{z} S_p (z') C T_R(z')^{\gamma} dz',
\]

where \( \gamma = 2 \sec \theta \) and \( x = S_p/S_R \). The term \( S_p \) is the extinction-to-backscatter ratio for the particulate scattering only and \( S_R \) is the extinction-to-backscatter ratio for molecular scattering.

In order to apply the above equation, the assumption is made that the particulate extinction-to-backscatter ratio is effectively constant through a layer allowing \( S_p \) to be taken outside the integral. Since the molecular terms in Eq. (4) are obtainable from knowledge of the atmospheric temperature and pressure structure, there are four quantities left to be determined; \( C, S_p, \) and the particulate transmission terms \( T_R(z) \) and \( T_R(z_1) \). In this study, the calibration constant \( C \) was found through an independent measurement procedure to be described later. The transmission to the bottom of the layer, \( T_R(z_1) \), is unity at the surface and obtainable for subsequent layers from the derived transmission of lower layers. The two unknowns to be found are thus the particulate extinction-to-backscatter ratio \( S_p \) and the transmission to the top of the layer \( T_R(z) \). With return data from only a single angle, either \( T_R(z) \) or \( S_p \) must be known for a solution to be possible. Fernald et al. (1972) obtained a solution from vertical data by assuming \( S_p \) to be constant throughout the atmosphere and by estimating \( T_R(z) \) at the top of the troposphere through an iterative procedure based on solar transmission data.

For return data at several angles, Eq. (4) represents a nonlinear system of equations with two unknowns. The resulting set of observation equations may be solved for optimum values of \( T_R(z) \) and \( S_p \). No additional information other than the system calibration constant is required. The layer transmission is determined from information directly available in the multi-angle data.

Two assumptions are made in applying the solution procedure. First, as stated above, it is assumed that the particulate extinction-to-backscatter cross-section ratio is constant through the atmospheric layer. It is also assumed that the effective vertical transmission is homogeneous for all slant paths through the layer, which is to say that

\[
\int_{z_1}^{z} S_p (z', \gamma) dz'/\gamma
\]

is the same for all paths. Since \( S_p \) is assumed constant, it is thus implied that the average backscatter cross section for each path through the layer is constant.

The advantage for application of Eq. (4) over the direct slant path calculation previously described is in part that the averaged backscatter cross section for each slant path through a layer is assumed constant rather than assuming a constant backscatter cross section at each level. Since eddy-scale inhomogeneity is the primary concern for lidar slant path measurements, quantitative analysis of multi-angle lidar signals is improved by averaging over sufficiently thick layers.

The validity of a constant particulate extinction-to-backscatter ratio will depend on the selection of atmospheric layers. The extinction-to-backscatter ratio is a function of the shape, composition and size distribution of the particulates in a volume. As long as the relative shape of the size distribution and type of particulates does not change, \( S_p \) will be independent of location. Most of the atmosphere's particulate load is found in the boundary layer near
the earth's surface. The prevalent mixing of the boundary layer would be expected to produce uniformity of the aerosol population. Above the boundary layer, there is a background aerosol population in the middle and upper troposphere. Within the boundary layer and within the overlying troposphere, aircraft and balloon sampling studies (Blifford, 1970; Reagan et al., 1977) have found that size distributions are generally similar for differing heights. It thus may be expected that the particulate extinction-to-backscatter ratio will not vary significantly within defined atmospheric layers. Exceptions would be when the relative humidity within a layer is such as to produce variable condensational growth of particulates, or when local surface point sources of particulates are important. However, in general for mixed, dry atmospheric conditions, both uniformity of aerosol population characteristics and horizontal homogeneity of layer integrated particulate-scattering parameters may be expected to be valid.

The nonlinear set of equations represented by Eq. (4) may be solved for optimum values of $S_p$ and $T_p(z)$ by linear expansion and development of normal equations. Equal statistical weight was applied to return data from each angle. Errors for $S_p$ and $T_p(z)$ can be computed statistically as part of the multi-angle lidar return analysis. The nonlinear regression and the error analysis procedure is reported by Spinhirne (1977). The estimated errors will reflect degradation of results due to inhomogeneity of atmospheric scattering parameters and also from return signal measurement errors. A modeling study was undertaken to examine the effect of a non-constant extinction-to-backscatter ratio within a solution layer and also systematic zenith angle variation of the effective vertical optical thickness. It was found that although significant errors could result in the solution, the magnitude of the resulting error was reflected in the error calculations.

Once the particulate extinction-to-backscatter ratio has been determined for a layer, Eq. (4) may then be applied in order to determine $T_p(z)$ for all heights $z$ within the layer. The vertical profile of particulate extinction cross section can then be found by differentiating $T_p(z)$. In order to be statistically consistent, $T_p(z)$ must be calculated from all available angular measurements. Also, the same weighting of measurements as was used in the integrated solution for the entire layer must be applied. Consistent weighting will not necessarily result if $T_p(z)$ profiles are calculated separately for each angle using Eq. (4) and the average then found. An appropriate procedure is given by Spinhirne (1977).

In order to apply the above solution procedure to the entire atmosphere, lidar return data for all angles would have to be obtained throughout the atmosphere. In practice, due to limitations imposed by the lidar sensitivity, dynamic range and transmitter-receiver geometry, there will be both a minimum and a maximum range for the lidar signal acquisition. For the actual experimental application of Eq. (4), additional procedures were thus required in order to extend results throughout the atmosphere. In the case of the return data near the ground, data will not be available below the height $R_0/\sec \theta_z$, where $R_0$ is the minimum range for signal acquisition. An iterative procedure was therefore employed for heights lower than $R_0$ in order that the multi-angle solution could be applied to the initial atmosphere layer close to the surface. The iteration involved estimation and successive recalculations of the $T(z)$ term in Eq. (4) for $z = R_0$ until convergence was obtained. All data available below $R_0$ from slant zenith angles was used in the iteration with the return from the largest zenith angle being extrapolated from the minimum received height to the surface. Since the lack of return data near the surface could affect experimental uncertainties, the iteration procedure was also accounted for in the experimental error.

In the upper troposphere and stratosphere, particulate-scattering cross sections are small in comparison to the scattering cross sections within the boundary layer. Measurements of upper atmosphere cross sections are thus difficult from several aspects. First, the total particulate optical thickness involved is small and the variance of multi-angle returns resulting from particulate attenuation will be minor. Also, since the particulate cross sections may be only a few percent of the molecular cross sections, the system calibration $C$ must be accurately known in order to differentiate particulate and molecular contributions. In the case of multi-angle lidar measurements obtained from the surface, the layer ranges involved lead to small signal-to-noise ratios and compound the difficulty in measuring the small upper atmosphere cross sections.

The application of the multi-angle technique to the upper levels of the atmosphere was thus dependent on the signal-to-noise ratio of the return data which in turn was a function of the lidar system and of the number of signal returns that were averaged. For the lidar system that was applied in this investigation, the maximum range for which return data could be usefully acquired was such that data were obtained throughout the troposphere only in near vertical directions. The resulting lack of angular return data for larger zenith angles, coupled with a relatively low signal-to-noise ratio, did not permit application of the multi-angle solution above the boundary layer. Errors which resulted from the regression analysis of Eq. (4) were too large to provide useful values of the extinction-to-backscatter ratio. The multi-angle solution was therefore applied only within the lower mixed layer of the
troposphere. In order to obtain the volume extinction cross-section values in the upper troposphere, the assumption had to be made that the extinction-to-backscatter ratio for the upper troposphere was the same as found for the mixed layer and then Eq. (4) was applied with the scattering ratio known. Application of the resulting upper troposphere cross-section profiles must be made with consideration of the assumption involved.

3. Experimental measurements

The principal experimental requirement for successful application of the multi-angle lidar solution was adequate measurement accuracy. There are three experimental parameters involved: the return signal strength \( P(z, \theta) \), the relative transmitted pulse energy \( E \) and the system calibration constant \( C \). The basis of the slant angle measurement procedure is the increased propagation attenuation for greater zenith angles. Since the variation of the signal due to the attenuation change is of the order of several percent or less, measurements of \( P(z, \theta) \) and \( E \) must be made with corresponding accuracy. Also, in order to correctly differentiate molecular and particulate scattering, the value of the calibration constant must be precisely known.

a. Lidar

Lidar measurements were made with a 1 J ruby-laser system. The lidar receiver consisted of an 8-inch cassegrain telescope and RCA 7265 photomultiplier tube detector. The dynamic range of return signals was limited by a 1 m separation of the transmitter and receiver. With a 4 mrad field acceptance angle, the transmitted laser pulse was fully overlapped with the receiver field of view at 850 m range. A filter limited the receiver wavelength bandwidth to 1.4 nm. The lidar transmitter and receiver were situated on an alti-azimuth mount and could be pointed with 0.1° accuracy.

Signals were acquired by an automated data acquisition system consisting of a signal compression amplifier, A/D converter and digital tape unit. The signal compression amplifier (Spinhrine and Reagan, 1976) maintained accurate signal acquisition over the full dynamic range of the lidar return signal. Only 6-bit A/D conversion was employed, but due to adequate signal compression and the inherent noise of the lidar return signal, little practical benefit would have been gained by greater conversion accuracy. All information required for signal processing, including a precise relative measurement of the transmitted pulse energy (Reagan et al., 1976), was digitally recorded. Approximately 10 returns were averaged for each angle of a slant path measurement, and Fig. 1 is an example of the resulting signals. The maximum effective range for signal acquisition was 17 km.

b. Calibration

A calibration of the lidar system was required in order to relate the atmospheric backscatter cross section to the magnitude of the return signal. One possible procedure would be to assume that the ratio of particulate-to-molecular backscatter cross section at some level in the atmosphere is known. For measurements within the troposphere, however, an independent determination of the calibration is desirable in order to avoid possible errors inherent in such an assumption.

An independent calibration was accomplished by a closed-loop comparison to the magnitude of an integrated return pulse from a flat target of known diffuse reflectance. The standard target calibration was similar to that described by Hall and Ageno (1970), but with automated signal acquisition and precise measurement of all parameters involved. In order to obtain the integrated target return, an attenuating filter of transmittance \( T_f \) is placed in the optical path of the receiver, and the resistive load \( R_L \) of the detector was replaced by a capacitative load with a time constant \( t \) much greater than the lasing pulse period. The integrated target transient was acquired by the same data system as was used for the atmospheric return, and the resulting signal was fitted to determine \( t \) and \( V_T \), the peak integration voltage. The calibration constant \( C \) as defined in Eq. (1) can be shown to be given by

\[
C = t(R_L/R_T)(V_T/E)(\text{\pi}cr^2/2pT_T T_f \cos \alpha),
\]

(5)
where $R_I$ is the resistance associated with $t$, $r_T$ the range to the target, $\rho$ the diffuse target reflectance and $c$ the speed of light. The target was placed so that the atmospheric transmission to the target, $T_T$, and the angular reflection factor $\cos \alpha$ were sufficiently known. The other parameters in Eq. (5) may be measured independently or determined from the target return data. The attenuating filter transmission $T_F$ was measured by redundant laboratory methods. An estimate of error indicated that $C$ could be measured within 4% accuracy by the above technique. The most important sources of measurement uncertainty were the error in determination of $T_I$ and estimated variance for the responsivity of the receiving photomultiplier tube.

c. Solar radiometer

In order to test the validity of the lidar-derived profiles of particulate extinction cross section, the lidar measurements were made in conjunction with solar radiometer measurements of total particulate optical depth. Shaw et al. (1973) have described the apparatus and procedures that were used to obtain the solar radiometer total optical depth measurements. King and Byrne (1976) describe the method that was employed to obtain the wavelength-dependent particulate optical depth from the total optical depth measurements.

A particulate optical depth for the entire atmosphere had to be derived directly from the lidar measurements in order to provide a value for comparison to the solar radiometer results. Lidar return data was acquired and analyzed only within the troposphere to a height of 16.5 km. The optical thickness to this level was determined from the lidar data as previously described. A value of the total particulate optical depth was produced by adding an estimate of the particulate optical thickness above 16.5 km to the lidar derived value. A value of $0.002 \pm 0.0015$ was employed as an appropriate upper atmosphere thickness for periods during which the measurements of this investigation were made. As will be noted, this estimated particulate thickness for above 16.5 km was small in comparison to both the radiometer and lidar measurement uncertainties.

The comparison between the lidar and solar radiometer derived optical depth measurements was independent of aerosol physical characteristics. If homogeneous Mie scattering is assumed to apply for the aerosol population, another comparison between the lidar and solar radiometer measurements was possible. That is that the wavelength dependence of the particulate optical depth may be related to the measured particulate extinction-to-backscatter ratio. As will be discussed, this comparison is dependent on, and thus may be related to, the effective refractive index of the aerosol population.

4. Profile results

A series of the multi-angle lidar measurements were acquired in order to test the applicability of the layer-integrated solution procedure. The measurements were made from an installation located on the edge of Tucson, and returns were acquired from a direction out over the Tucson Valley. All of the measurements to be reported were made in the early evening after sunset. Daylight measurements could not be used due to excessive signal noise from background radiation. Except for several cases from Fall 1974, the measurements to be reported here were acquired in Fall 1975 and Spring 1976. The availability of corroborative solar transmission data was the main limitation to the frequency of observations.

Although the results from multi-angle measurements would be expected to improve as the number of angles and signal returns increased, possible temporal variation of the atmosphere restricted the desirable time period for acquiring data. In practice, measurements were made at ten angles with nine returns averaged for each angle, the exception being the vertical zenith angle for which four times that number were averaged. Between 30 to 45 min were required to complete the measurements. The angles were chosen at equal secant increments with the largest zenith angle being that for $\sec \theta_x = 5$. For angles with $\sec \theta_x > 5$ the horizontal range involved becomes excessive and, in addition, the 0.1° uncertainty in the lidar elevation angle would have become significant.

The accuracy to which particulate extinction cross sections could be determined by the multi-angle method was dependent on several factors, the most important of which was the degree of existing inhomogeneity of the atmosphere. Another important factor was the magnitude of the particulate optical depth. For a greater particulate optical thickness, scattering parameters could be more accurately measured. The vertical extent of the particulate mixing was also important. When a significant fraction of the particulate optical thickness was below 850 m, the lowest height for which returns for all angles were acquired, measurements became necessarily inaccurate.

Table 1 presents a summary of results for 20 multi-angle measurement cases. The aerosol extinction-to-backscatter ratio is the value computed within the boundary or mixed layer. The height of the mixed layer was determined from visual inspection of the lidar return profiles. The value of optical depth listed is that derived directly from the lidar measurements. Representative volume extinction cross-section profiles are shown for three of the cases in Figs. 2, 3 and 4. The molecular and aerosol cross section within the lower troposphere are plotted. The
TABLE 1. Results of lidar slant path measurements.

<table>
<thead>
<tr>
<th>Date</th>
<th>Time (MST)</th>
<th>$S_p$ for mixed layer (sr)</th>
<th>$\rho(180^\circ)$</th>
<th>Height of mixed layer (km)</th>
<th>Total aerosol optical depth</th>
<th>Optical depth of mixed layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 Nov</td>
<td>1820</td>
<td>26.8 ± 9.8</td>
<td>0.47</td>
<td>3.21</td>
<td>0.066 ± 0.028</td>
<td>0.057</td>
</tr>
<tr>
<td>20 Nov</td>
<td>1810</td>
<td>29.4 ± 4.8</td>
<td>0.43</td>
<td>2.52</td>
<td>0.061 ± 0.010</td>
<td>0.048</td>
</tr>
<tr>
<td>21 Nov</td>
<td>1835</td>
<td>38.6 ± 15.9</td>
<td>0.33</td>
<td>2.01</td>
<td>0.067 ± 0.019</td>
<td>0.047</td>
</tr>
<tr>
<td>2 Dec</td>
<td>1910</td>
<td>46.4 ± 13.7</td>
<td>0.27</td>
<td>2.07</td>
<td>0.058 ± 0.013</td>
<td>0.049</td>
</tr>
<tr>
<td>4 Aug</td>
<td>2225</td>
<td>17.7 ± 2.9</td>
<td>0.71</td>
<td>5.91</td>
<td>0.118 ± 0.014</td>
<td>0.109</td>
</tr>
<tr>
<td>7 Aug</td>
<td>2108</td>
<td>22.0 ± 3.4</td>
<td>0.57</td>
<td>4.68</td>
<td>0.132 ± 0.021</td>
<td>0.114</td>
</tr>
<tr>
<td>10 Sep</td>
<td>2115</td>
<td>16.8 ± 7.5</td>
<td>0.75</td>
<td>3.45</td>
<td>0.039 ± 0.019†</td>
<td>0.031</td>
</tr>
<tr>
<td>23 Oct</td>
<td>1740</td>
<td>19.5 ± 6.2</td>
<td>0.64</td>
<td>2.70</td>
<td>0.180 ± 0.032</td>
<td>0.156</td>
</tr>
<tr>
<td>13 Nov</td>
<td>1925</td>
<td>17.3 ± 10.3</td>
<td>0.73</td>
<td>1.80</td>
<td>0.047 ± 0.019†</td>
<td>0.032</td>
</tr>
<tr>
<td>4 Dec</td>
<td>2335</td>
<td>23.6 ± 9.1</td>
<td>0.53</td>
<td>2.49</td>
<td>0.043 ± 0.017†</td>
<td>0.032</td>
</tr>
<tr>
<td>8 Dec</td>
<td>1840</td>
<td>29.6 ± 16.1</td>
<td>0.42</td>
<td>1.80</td>
<td>0.068 ± 0.018†</td>
<td>0.057</td>
</tr>
<tr>
<td>16 Jan</td>
<td>1845</td>
<td>20.8 ± 12.8</td>
<td>0.60</td>
<td>0.78</td>
<td>0.041 ± 0.014</td>
<td>0.028</td>
</tr>
<tr>
<td>25 Feb</td>
<td>2050</td>
<td>25.1 ± 10.3</td>
<td>0.50</td>
<td>3.81</td>
<td>0.051 ± 0.019</td>
<td>0.041</td>
</tr>
<tr>
<td>9 Mar</td>
<td>1945</td>
<td>14.9 ± 4.3</td>
<td>0.84</td>
<td>2.88</td>
<td>0.047 ± 0.011†</td>
<td>0.034</td>
</tr>
<tr>
<td>13 Mar</td>
<td>2020</td>
<td>17.0 ± 5.5</td>
<td>0.74</td>
<td>1.68</td>
<td>0.037 ± 0.009</td>
<td>0.027</td>
</tr>
<tr>
<td>22 Mar</td>
<td>2150</td>
<td>13.8 ± 5.7</td>
<td>0.91</td>
<td>4.56</td>
<td>0.068 ± 0.018†</td>
<td>0.056</td>
</tr>
<tr>
<td>26 Mar</td>
<td>1945</td>
<td>16.9 ± 5.6</td>
<td>0.74</td>
<td>2.70</td>
<td>0.058 ± 0.013</td>
<td>0.047</td>
</tr>
<tr>
<td>28 Apr</td>
<td>1945</td>
<td>21.8 ± 3.5</td>
<td>0.58</td>
<td>4.83</td>
<td>0.111 ± 0.014</td>
<td>0.101</td>
</tr>
<tr>
<td>11 May</td>
<td>2000</td>
<td>15.7 ± 4.0</td>
<td>0.80</td>
<td>3.59</td>
<td>0.072 ± 0.017</td>
<td>0.061</td>
</tr>
<tr>
<td>26 May</td>
<td>2100</td>
<td>18.0 ± 2.8</td>
<td>0.70</td>
<td>3.84</td>
<td>0.107 ± 0.015</td>
<td>0.098</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>19.5*</td>
<td>0.64*</td>
<td>3.05</td>
<td>0.073</td>
<td>0.061</td>
</tr>
</tbody>
</table>

* Weighted average.
† Cirrus.

heights for all graphs are referenced from a ground level of 732 m above sea level.

The 20 November extinction coefficient profile of Fig. 2 was derived from the measurements as partially shown in Fig. 1. Representative error bars are shown at selected height levels. The error bars reflect both the measurement uncertainty, primarily that which resulted for the extinction-to-backscatter ratio, and also the inherent fluctuation of the particulate scattering parameters from atmospheric inhomogeneity. For the measurement of 20 November, there was a well-defined mixed layer that, relative to the other measurement cases, had low horizontal inhomogeneity. The mixed-layer optical thickness and the height of the mixed layer was somewhat less than the average for the measurements. The estimated relative error for the extinction-to-back-scatter ratio was 16% and for the extinction cross section within the mixed layer, 18–20%.

The data for 7 August had a greater degree of horizontal inhomogeneity than for 20 November. However, as seen from Fig. 3 the relative measurement uncertainty is similar. The increased optical thickness for 7 August, 0.114 rather than 0.048, resulted in a more accurate solution.

Even with a very pronounced horizontal inhomogeneity, a solution was possible. The variance of the results however, will reflect the level of inhomogeneity. The measurement for 23 October, shown in Fig. 4, was made shortly after the passage of a dry cold front, and the level of aerosol loading was a result of high wind speeds associated with the front. Turbulence produced by the winds resulted in a visible degree of aerosol inhomogeneity. Even though the thickness of the mixed layer was 0.180, the relative error for the extinction cross-section measurement was also large.

As seen from Table 1, a number of cases had

Fig. 2. The lower troposphere extinction cross section for 20 November. The particulate and molecular cross sections are shown separately. The vertical height is referenced from the surface which was 723 m above sea level. The particulate profile is the layer-integrated multi-angle solution corresponding to the signals shown in Fig. 1. Representative error bars are shown every kilometer for the estimated uncertainty of the particulate profile.
greater error for the measurement results than the days previously discussed. These results were obtained primarily on days for which there was a small particulate optical thickness, and the particulate loading was distributed close to the surface. As example, for 21 November, the mixed-layer optical thickness was similar to that for 20 November. However, for 21 November, there was a strong increase of particulate concentration toward the surface. Since the major fraction of the particulate optical depth was below 1 km, the solution accuracy was low. In another example (10 September) the mixed-layer optical thickness was only 0.031, and even though the particulate optical thickness was distributed mainly above 1 km, the low total thickness value resulted in a 45% uncertainty for the extinction-to-backscatter ratio measurement.

In general, however, it was found that the scattering cross section results were usefully accurate for those cases where the solution layer optical thickness exceeded 0.04. In cases where the mixed layer was sufficiently thick such as 4 August, a multangle analysis with several solution layers was possible. Within the calculated error limits, the results agreed with that obtained for the single solution layer.

Lidar and solar radiometer values for the total aerosol optical depth are compared in Table 2. As may be seen the lidar and radiometer values agreed within calculated error limits except for those days when the radiometer optical depth measurement was \( \pm 0.035 \). In these cases the lidar tended to give larger values. The assumptions which were involved in determination of the total atmospheric optical depth from the lidar measurements would become increasingly important for small optical depths.

As mentioned, for the above mixed-layer troposphere it was necessary to assume the same extinction-to-backscatter ratio that was obtained for the mixed layer. In most all cases, the above mixed layer optical thickness was determined to be near 0.01. If the total optical depth was only several times greater, the above assumption when inaccurate, could produce a significant systematic error of the lidar total optical depth value. Inaccuracy of the assumed stratospheric optical thickness should not have been a problem. During the period in which the measurements reported here were made, there was no major volcanic perturbation of the stratosphere (McCormick et al., 1978). The stratospheric optical thickness was thus small in relation to the error estimate of the total optical-depth measurement.

Disagreement between the lidar and solar radiometer measurements could also be related to possible temporal variation of the aerosol optical depth. The radiometer optical depth values were determined by the Langley plot method, which involves measuring the solar intensity as a function of zenith angle. The Langley plot measurements were made in the period from noon to sunset, and the lidar measurements were made some hours afterward. Variation of the particulate optical depth would directly affect the lidar-radiometer comparison. However, since

![Table 2. Comparison of lidar, radiometer optical depth.](image)

<table>
<thead>
<tr>
<th>Date</th>
<th>Radiometer optical depth</th>
<th>Lidar optical depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>23 Oct</td>
<td>0.171 ± 0.023</td>
<td>0.180 ± 0.032</td>
</tr>
<tr>
<td>7 Aug</td>
<td>0.163 ± 0.010</td>
<td>0.132 ± 0.021</td>
</tr>
<tr>
<td>26 May</td>
<td>0.104 ± 0.015</td>
<td>0.107 ± 0.015</td>
</tr>
<tr>
<td>28 Apr</td>
<td>0.093 ± 0.033</td>
<td>0.111 ± 0.014</td>
</tr>
<tr>
<td>22 Mar</td>
<td>0.081 ± 0.008</td>
<td>0.068 ± 0.018</td>
</tr>
<tr>
<td>7 Nov</td>
<td>0.071 ± 0.012</td>
<td>0.066 ± 0.028</td>
</tr>
<tr>
<td>26 Mar</td>
<td>0.061 ± 0.013</td>
<td>0.058 ± 0.013</td>
</tr>
<tr>
<td>20 Nov</td>
<td>0.057 ± 0.010</td>
<td>0.061 ± 0.010</td>
</tr>
<tr>
<td>21 Nov</td>
<td>0.056 ± 0.014</td>
<td>0.067 ± 0.019</td>
</tr>
<tr>
<td>13 Mar</td>
<td>0.043 ± 0.002</td>
<td>0.037 ± 0.009</td>
</tr>
<tr>
<td>13 Nov</td>
<td>0.038 ± 0.002</td>
<td>0.047 ± 0.019</td>
</tr>
<tr>
<td>25 Feb</td>
<td>0.034 ± 0.005</td>
<td>0.051 ± 0.019</td>
</tr>
<tr>
<td>16 Jan</td>
<td>0.022 ± 0.003</td>
<td>0.041 ± 0.014</td>
</tr>
<tr>
<td>4 Dec</td>
<td>0.012 ± 0.002</td>
<td>0.043 ± 0.017</td>
</tr>
</tbody>
</table>
the Langley plot method of determining total optical depth requires temporal homogeneity, variation of aerosol optical depth with time could typically be expected to be small. There were cases, though, when the aerosol optical depth varied by a factor of 3 or 4 from one day to the next. For small optical depths, the errors which would arise in the radiometer measurement due to change in the particulate depth were difficult to identify.

In some cases, the complete profile through the upper troposphere was not available. Although the lidar-radiometer measurements were made on days with no visibly apparent clouds, as seen in Table 2, thin cirrus clouds were detected by the lidar for about one-third of the measurements. The only other indication of the cirrus was the occasional presence of a lunar halo. In such cases, the lidar profile was extrapolated beyond the cloud base, using an average profile obtained from the other measurements.

An average aerosol-extinction cross section was compiled from the measurements made. The lidar measurements were not acquired on a systematic basis but were spread throughout the year and over a range of atmospheric conditions. The average aerosol extinction cross section is shown in Fig. 5, and the values at 0.3 km height increments are listed in Table 3. The bars indicated in Fig. 5 represent twice the standard deviation of the measurements, and do not reflect the propagated measurement errors. A widely used measurement of aerosol extinction cross section has been that given by Elterman (1968). Although the difference in location and time does not permit a direct comparison, it may be noted that for the upper troposphere, the aerosol extinction cross section values given here are approximately one-fourth the magnitude of the values listed by Elterman for 0.7 μm. In terms of the optical thickness above 4 km the difference would be 0.02. Since the computed lidar optical depth values tended to be somewhat larger than the corresponding solar radiometer values, a larger aerosol extinction cross section for the upper troposphere would have resulted in a more marked disagreement.

In addition to the extinction cross section, the aerosol backscatter cross section was obtained from the measurements. The average molecular and total backscatter cross sections for the lidar measurements are shown in Fig. 6. A large number of measurements for the ratio of total-to-molecular backscatter cross section (called the backscatter ratio)
have been made with stratospheric lidar (Russell et al., 1976). Such measurements are normalized to an assumed molecular scattering at some level of the upper troposphere or stratosphere, and attenuation, being small, is neglected. The stratospheric measurements when extended into the upper troposphere indicate a minimum backscatter ratio of less than 1.05. The minimum of the average backscatter ratio found here was somewhat larger (1.1 ± .07).

5. Scattering comparison

It is known that tropospheric aerosol populations, in general, are collections of particles of differing size, shape and composition. Although some particles may be homogeneous liquid drops, others may be amorphous solid particles of irregular shape and composition. However, the standard practice of aerosol radiative calculations has been to employ Mie scattering theory of homogeneous spherical particles. For measurements of aerosol scattering parameters in the atmosphere, it is therefore of interest whether self-consistency of the measurements can be related through Mie calculations, and if so, what effective particle refractive index gives the best description of scattering parameters.

In this study measurements were made of the wavelength-dependent aerosol optical depth and of the aerosol backscatter and extinction cross sections at one wavelength. These measurements could be related in the following manner. The initial procedure was to obtain the columnar aerosol size distribution by a Mie-theory-based inversion of the radiometer optical depth data. Using the size distribution, the extinction-to-backscatter ratio was then computed for varying particle refractive index, and the statistical range of agreement with the lidar measured extinction-to-backscatter ratio could be determined. The extinction-to-backscatter ratio was employed in order to normalize to the columnar size distribution. The size distribution was an average for the entire atmosphere whereas the extinction-to-backscatter ratio was determined for the mixed layer. However, in most cases the optical thickness of the atmosphere above the mixed layer was a small fraction of the mixed-layer thickness, and the size distribution could thus be correlated with the mixed-layer lidar measurement.

The imaginary component of the particle index is of particular interest. Scattering calculations show that, in general, the dependence of the backscatter cross section on the imaginary component of the particulate refractive index is large. In comparison, the dependence of the extinction cross section on the particle index is minor. As mentioned previously, other studies have attempted to apply backscatter measurements in order to determine particulate imaginary index. Direct measurement of particulate imaginary index is difficult. For this reason it has been thought useful to examine the imaginary index indirectly by measurement of the backward scattering. The validity of the approach depends on the deviation of actual aerosol scattering from that modeled by Mie calculations. Nonsphericity has been found by laboratory experiments to have the same effect on backscatter cross sections as would an increased imaginary index (Pinnick et al., 1976; Holland and Gagne, 1970). In both cases the backscatter is decreased, as could be expected since particle backscattering is primarily related to surface wave scattering. The inference is thus that backscattering measurements would give an upper limit for the imaginary index.

The sensitivity of the extinction-to-backscatter ratio to the imaginary index will depend on the nature of the size distribution. Scattering calculations show that at the lidar wavelength of 0.6943 μm, the dependence of the backscatter cross section on imaginary index is predominately for particles ≥ 1.0 μm radius. For particles of smaller radius, the dependence on the real index is more pronounced. Thus the relative sensitivity of the extinction-to-backscatter ratio to real or imaginary index will depend on the size range of particles which dominate the aerosol scattering.

The procedure for inverting a columnar size distribution from wavelength-dependent optical depth measurements has been described in detail by King et al. (1978). A direct Phillips-Twomey inversion was employed, and results were independent of any analytic expression for the size distribution. As discussed by King et al., the inversion results are insensitive to the assumed particle refractive index. The inversion results have shown a widely varying shape for the size distribution from case to case.

Comparison between the lidar-measured extinction-to-backscatter ratio and the size distribution calculation is shown graphically for 26 May in Fig. 7. An estimate of the solution accuracy was provided by the inversion procedure, and from which, an error range for the calculated extinction-to-backscatter cross section could be obtained. In Fig. 7 the calculated extinction-to-backscatter ratio is plotted as a function of the imaginary component k for several real indices. The lidar measured extinction-to-backscatter ratio is also shown. The error limits represent the estimates for a single standard deviation of the quantities.

From data and calculations such as shown in Fig. 7, a best agreement value of k could be determined. The results from six cases are presented in Table 4 for each of four values of real index. The acquired solar radiometer data were not sufficiently accurate on all days for the size distribution inversion procedure to be workable, and as a result, inverted size distributions were available for only six of the
days on which lidar measurements were analyzed. In all of these cases the lidar measurement fell within a range predicted from the size distribution inversion.

An alternate procedure to the size distribution inversion, which didn’t require highly accurate optical depth data, was also attempted. A Junge size distribution integrated from 0.02 to 10 μm radius was assumed, and the effective slope parameter \( v \) was found from the optical depth data (Shaw et al., 1973). However, the values of imaginary index inferred with the Junge size distributions were found to be generally in disagreement with the values that resulted with the inverted size distributions. In addition, for some days the lidar measurement could not be justified through comparison with calculations based on the effective \( v \) parameter. Thus, the refractive index comparison was not useful without an inverted size distribution.

By a statistical procedure, an estimated uncertainty range for an effective imaginary index could be arrived at. The estimated uncertainty range for the real index of 1.52 is listed in Table 4. Also listed is the error for the extinction-to-backscatter ratio and the percentage of the laser wavelength extinction cross section due to particles < 1 μm radius. As previously mentioned, the influence of the imaginary index on the extinction-to-backscatter ratio is much greater for particles > 1 μm radius than for those < 1 μm. Thus when a larger percentage of the extinction cross section is due to particles < 1 μm radius, the range of values for \( k \) which fall within the error limits of the measured extinction-to-backscatter ratio will be greater. Also the variation of \( k \) for different real particle index will be larger when the scattering is primarily due to particles < 1 μm radius. When both the scattering by large particles was significant and the uncertainty of the extinction-to-backscatter ratio was small, such as for 26 May, the uncertainty range for \( k \) was less than for other cases.

A separate means for inferring the effective real index was not available. However, the variation of \( k \) for the differing values of real index was comparable to the estimated 1.52 uncertainty range. Of the values of real index for which calculations were made, the median lidar and size distribution result did not agree in most cases only for \( n = 1.45 \). The comparison in that case was within the uncertainty range for \( k = 0 \), however.

Due to the lack of precision for the extinction-to-backscatter ratio and to the variation of real index, a significant range of complex index values were possible for each case. However, the comparison indicated a maximum upper limit for the imaginary index of 0.015 regardless of the real index. In some cases the upper limit was significantly less. For a real index of \( n = 1.52 \), \( k = 0.003 \) was the average of the median values.

The range of ambiguity for other measurements of complex index, such as that by De Luisi et al. (1976), has been similar to that given here. The use of the extinction-to-backscatter ratio determined by lidar measurements has several advantages compared to other backscattering methods for inferring imaginary index. The lidar measurements are averaged through the mixed atmospheric layer rather

---

**Table 4. Aerosol imaginary index results.**

<table>
<thead>
<tr>
<th>Date</th>
<th>( S_p ) (st)</th>
<th>( N = 1.45 )</th>
<th>( N = 1.50 )</th>
<th>( N = 1.52 )</th>
<th>( N = 1.54 )</th>
<th>Percent of extinction cross section due to particulates &lt;1 μm radius</th>
<th>Uncertainty range of ( k ) for ( m = 1.52 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 Aug</td>
<td>22.0 ± 3.4</td>
<td>0.0000</td>
<td>0.0003</td>
<td>0.0027</td>
<td>0.0052</td>
<td>93.9</td>
<td>0.0000 - 0.0059</td>
</tr>
<tr>
<td>23 Oct</td>
<td>19.5 ± 6.2</td>
<td>0.0000</td>
<td>0.0029</td>
<td>0.0055</td>
<td>0.0087</td>
<td>51.6</td>
<td>0.0021 - 0.0099</td>
</tr>
<tr>
<td>13 Nov</td>
<td>17.3 ± 10.3</td>
<td>0.0000</td>
<td>0.0009</td>
<td>0.0038</td>
<td>0.0074</td>
<td>38.4</td>
<td>0.0000 - 0.0095</td>
</tr>
<tr>
<td>13 Mar</td>
<td>17.0 ± 5.5</td>
<td>0.0000</td>
<td>0.0003</td>
<td>0.0017</td>
<td>0.0046</td>
<td>64.0</td>
<td>0.0000 - 0.0048</td>
</tr>
<tr>
<td>26 Mar</td>
<td>16.9 ± 5.6</td>
<td>0.0000</td>
<td>0.0002</td>
<td>0.0012</td>
<td>0.0038</td>
<td>73.3</td>
<td>0.0000 - 0.0062</td>
</tr>
<tr>
<td>26 May</td>
<td>18.0 ± 2.8</td>
<td>0.0005</td>
<td>0.0008</td>
<td>0.0017</td>
<td>0.0034</td>
<td>36.7</td>
<td>0.0008 - 0.0023</td>
</tr>
</tbody>
</table>
than being an in-situ measurement. In addition, since
the ratio of cross sections is determined, the measu-
rement does not have to be related absolutely to
the particle number density. The values of particle
imaginary index as determined from backscatter
measurements, however, need to be properly inter-
preted. The values as given here are those which
give best agreement with Mie calculations when a
single index of refraction is assumed for all particle
sizes. The possible influence of nonsphericity was
previously mentioned. Gillespie et al. (1978) have
presented an example which considers addition of a
minor fraction of strongly absorbing small particles
to a distribution of weakly absorbing particles. Their
computed cross sections would indicate indeter-
minate results if the single-scatter albedo for the
nonhomogeneous distribution was to be found from
a measured backscatter and single index of refrac-
tion calculations. The applicability of single index
Mie calculations in other more general cases of non-
homogeneous distributions must be considered.

6. Conclusion

In this investigation we have examined the appli-
cation of multi-angle lidar measurements for quan-
titative determination of aerosol extinction and
backscatter cross sections in the troposphere. As a
result of atmospheric inhomogeneity, a direct solu-
tion of the multi-angle measurements at individual
height levels was not found to be feasible. An
analysis procedure was thus developed which em-
ployed a layer-integrated form of the angle-depen-
dent lidar equation. The assumptions that were re-
quired in order to apply the integrated equation were
that the aerosol extinction-to-backscatter ratio was
constant within a defined layer and that the effective
vertical transmission through the layer was the same
for all slant paths. By regression analysis with
respect to zenith angle, the optical thickness and
the aerosol extinction-to-backscatter ratio for a layer
could be determined and, subsequently, the scatter-
ing cross sections calculated.

When applied within the lower mixed layer of
the atmosphere, the layer-averaged multi-angle
analysis procedure proved to be workable under all
conditions for which measurements were attempted.
However, a layer-aerosol optical thickness of \( \sim 0.04 \)
was in general required in order to obtain usefully
accurate results. Limitations of the lidar system
used in this investigation did not permit direct
application of the multi-angle analysis in the upper
troposphere, but with the assumption that the aerosol
extinction-to-backscatter ratio was the same as that
found for the lower troposphere, the aerosol cross
sections throughout the troposphere could be
determined. A value for the total atmospheric
aerosol optical depth was derived directly from the
lidar measurement and was compared to the optical
depth obtained from a solar radiometer measure-
ment. The values were found to agree within the
estimated error limits except for cases where the
radiometer aerosol optical depth was less than 0.035.
The measurements reported here were made under
arid conditions with relatively low aerosol optical
depth. Since greater optical depths would be found
in most other areas (Flowers et al., 1969), adequate
results from similar measurement in other locations
could be expected on that basis. The limitations
that would be imposed by greater humidity has yet
to be determined.

The measurement procedure required acquisition
of lidar signals within an accuracy of a few percent,
and a relative calibration procedure which involved
analysis of signals from a known target was em-
ployed. The results attained would indicate that
measurements could be made with the necessary
accuracy and that routine quantitative measure-
ments by lidar are feasible.

The aerosol extinction-to-backscatter ratio within
the mixed layer was found to vary widely. The
weighted average was 19.5 sr, but the standard devi-
ation of the measurements was 8.3 sr. In terms of
the normalized phase function \( p(180^\circ) \), the values
are \( 0.64 \pm 0.27 \). For those days on which an optical
depth inversion for the aerosol size distribution
was possible, the lidar extinction-to-backscatter
ratio agreed with values calculated for realistic
particle refractive index. Such agreement was not
attained when a procedure which assumed the shape
of the size distribution was employed. The complex
index values which gave the best fit between the
inverted size distribution and the lidar measure-
ment were determined. The main limitations of these
measurements as a method for inferring the upper
limit of the effective aerosol imaginary index is that
the appropriate real index was not determined and
that in most cases the estimated uncertainty of the
measurements would permit a significant range of
results. However, for the measurements that were
made, the mean result was an imaginary index of
0.003 for a real index of 1.52. If the possible un-
certainty range of the comparison was considered,
a maximum upper limit of 0.015 would be indicated
for the imaginary index.

Acknowledgment. The research reported in this
paper has been supported by the Atmospheric Sci-
ences Section of the National Science Foundation
under Grant ATM 75-15551A01.

APPENDIX

Derivation of Angle-Dependent Integrated
Lidar Equation

As in Eq. (3) the particulate transmission term
\( T_p(z) \), can be expressed as

\[
T_p(z) = \exp \left[ -\gamma \int_0^z S_p \beta_p(z')dz' \right]
\]  
(6)
or in differentiated form as
\[ dT_\beta(z)/dz = T_\beta(z)\left[-\gamma S_\beta S_\beta(z)\right]. \tag{7} \]
Solving (7) for \( \beta_\gamma(z) \) and substituting into (1) yields
\[ dT_\beta(z)/dz = -\gamma S_\beta S_\beta(z)T_\beta(z) = -\gamma S_\beta P(z,\gamma)/CT_\beta. \tag{8} \]
Multiplying by the integrating factor
\[ \exp\left[-\gamma S_\beta\int \beta_\gamma(z')dz'\right] \]
and combining the derivative, Eq. (8) may be expressed as
\[ d/l/dz\left[T_\beta \exp[-\gamma S_\beta\int \beta_\gamma(z')dz'] \right] = \left[-\gamma S_\beta P(z,\gamma)/CT_\beta \right] \exp[-\gamma S_\beta\int \beta_\gamma(z')dz'] \tag{9} \]
For molecular scattering the ratio of extinction to backscatter cross section is
\[ S_R = \sigma_R/\beta_R = 8\pi/3 \tag{10} \]
(\( \beta \) is not defined as normalized over a 4\( \pi \) solid angle) so that
\[ \exp\left[-\gamma S_\beta\int \beta_\gamma(z')dz'\right] = T_\beta^R, \tag{11} \]
where \( x = 3S_\beta/8\pi \), and from (9)
\[ d/dz[T_\beta T_\beta^R] = \left[-\gamma S_\beta P(z,\gamma)/CT_\beta \right] T_\beta^R - 1. \tag{12} \]
Integrating (12) over a layer from \( z_1 \) to \( z \) gives
\[ T_\beta(z)T_\beta^R(z) - T_\beta(z)T_\beta^R(z_1) = -\gamma \int_{z_1}^{z} \left[S_\beta P(z',\gamma)/CT_\beta(z')\right]^{\alpha z-\nu}dz', \tag{13} \]
as the angle-dependent lidar equation integrated over a layer from \( z_1 \) to \( z \).

REFERENCES