Numerical Simulation of Ice-Phase Convective Cloud Seeding

EH-R-Y Hsie, RICHARD D. FARLEY AND HAROLD D. ORVILLE

Institute of Atmospheric Sciences, South Dakota School of Mines and Technology, Rapid City 57701

(Manuscript received 25 October 1979, in final form 26 April 1980)

ABSTRACT

A two-dimensional time-dependent cloud model which covers a region 19.2 km x 19.2 km in the x and z directions with 200 m grid intervals, has been used to simulate silver iodide (AgI) seeding effects on strong convective clouds. The model is a set of conservation equations for momentum, energy and mass (air and water contents). One extra conservation equation is applied to trace the seeding agent which advects and diffuses along the flow field and interacts with the supercooled cloud fields. Contact and deposition nucleation are simulated. Only inertial impact and Brownian collection are considered as possible mechanisms for contact nucleation. Most of the AgI particles work as deposition or sorption nuclei in this study.

Three different soundings are tested. Most of the effort is used in testing sounding H1, from Miles City, Montana, 29 July 1975. Seeding at a different place (see H1/P1), at a different time (case H1/T1), and with different amounts of AgI (cases H1/M1 and H1/M2) are simulated. The effects of natural ice nuclei are also tested (cases H1/N2, H1/N3 and H1/S3). The model results show a "time window" for the icing process in natural (unseeded) cases. This time window is related to the concentration of the natural ice nuclei and the flow pattern. Concerning seeding effects, this study shows the potential to augment precipitation by AgI seeding. This potential is closely related to the dispersion of the proper amount of seeding agent into the proper region and at the proper time to obtain optimum effects.

1. Introduction

Cloud seeding simulations using a two-dimensional time-dependent convective cloud model are reported in this paper. Though there have been many elaborate laboratory experiments and field projects in the past three decades related to weather modification, the results of many tests have been inconclusive, particularly with respect to convective clouds. This is due primarily to the ephemeral nature of such clouds. They begin, grow and dissipate rapidly, so that the precipitation process has only a very limited time to be active. In addition, convective clouds may be primarily maritime or continental, warm or cool base. The ease with which precipitation forms depends on the cloud type. The warm-base maritime clouds invariably form rain by a coalescence process within 10–20 min of formation and within 3 km depth of cloud. On the other hand, the cool-base continental-type clouds often require 20–30 min and 6 km or so depth to form precipitation, mainly through an ice process.

The basic concept of precipitation augmentation for supercooled clouds is to produce ice crystals by inducing artificial freezing nuclei to the clouds. Under favorable conditions these ice crystals may grow big enough to fall out and melt as rain (Hess, 1974).

Simpson et al. (1965), Hosler et al. (1967) and Simpson and Wiggert (1969) simulated the effects of the release of latent heat due to artificial seeding in steady-state one-dimensional cumulus cloud models and found good correlations of the predicted and the observed cloud tops. However, the one-dimensional steady-state models have limited capability in simulating the microphysical processes (Warner, 1970), and the latent heat released after seeding may have been overestimated (Orville and Hubbard, 1975). In addition, these models fail to simulate the precipitation process adequately.

The potential use of multi-dimensional time-dependent cloud models to simulate seeding effects is of interest. Such a cloud model can capture the flow field and the distribution of precipitation in a more realistic way. Furthermore, it can trace the dispersion of the seeding agent and can be used to investigate the effects of cloud circulation on seeding.

Orville and Kopp (1974) used a two-dimensional time-dependent model (similar to the model described here) to simulate "idealized" cloud seeding experi-

---

1 Present affiliation: Department of Meteorology, Pennsylvania State University, University Park 16802.


ments. With different predetermined temperature thresholds related to light, moderate and heavy seeding, they required that all of the water substance carried above a corresponding temperature threshold be frozen. Updated results of that study showed a 0–18% increase in total precipitation for light seeding, a +8 to +60% increase for moderate seeding rates, and a -30 to +22% change in total precipitation for heavy seeding. The glaciation processes were too effective in these simulations and it is difficult to relate the implied amount of nucleant to reality.

Nelson (1979) tested hypotheses concerning the effect of AgI seeding on rainfall and hailfall from hail-bearing summer convective storms in a one-dimensional, time-dependent, detailed microphysical cumulus model which separated both the liquid and ice hydrometeors into 47 discrete size categories. The soundings which he used all produced severe hailstorms. The model results divided the initiation of hail embryos into two natural categories: 1) graupel hailstone embryos, involving cold base clouds in which the coalescence growth of liquid drops is insufficient and the dominant hail-embryo generation mechanism is the growth of small ice crystals from vapor; and 2) frozen-drop hailstone embryos involving warm-base clouds which generate significant numbers of millimeter sized drops (via the coalescence mechanism) which freeze at moderate degrees of supercooling. The model results suggested that the AgI particles can be used more efficiently for reducing the hailfall in frozen-drop embryo-type clouds than graupel embryo-type clouds. For graupel embryo-type clouds, both the rainfall and hailfall are increased by seeding. The interpretation of the responses of precipitation to the AgI seeding were uncertain, because the total amount of seeding agent used in the simulation was 10 kg of AgI. Also the model is one-dimensional, which makes the simulation of the precipitation process difficult. No precipitation over the sides of the updraft and no recycling of the precipitation are possible in this type of model.

Seeding of cold orographic clouds has been investigated extensively via numerical simulation (Young, 1974b,c; Plooster and Fukuta, 1975). Certain other work has concentrated on testing hail suppression concepts, not augmentation of precipitation (Young, 1977).

The model used in this paper is a two-dimensional time-dependent cloud model reported by Orville and Kopp (1977), with modifications. Improved microphysical interaction between ice and liquid phases is modeled, and one extra field is added to trace the seeding agent, which can advect and diffuse along the flow field into the clouds and interact with the hydrometeors.

In these simulations, the following three soundings have been used.

1) H1: A sounding for 0000 MST 29 July 1975, Miles City, Montana.
2) H3: A sounding for 2100 MST 21 July 1976, Miles City, Montana.
3) B3: A modified sounding for 1344 CDT 10 August 1973, St. Louis, Missouri.

These three soundings produce strong convective model clouds. Comparison with actual clouds was not done in this study but is being done in current studies. The initial impulse (3°C) was rather strong for actual clouds. The soundings were used to give us samples of cold base clouds in Montana and warmer base clouds in St. Louis. The H1 sounding did produce very light rainshowers clouds, as did the simulation; H3 produced smaller clouds than the simulation, and B3 resulted in clouds with moderate amounts of rain from clouds shallower than the simulated ones. Due to the limitation of computer time, most of our effort is concentrated on testing sounding H1. Thirteen cases have been run in this study: nine cases for sounding H1, two for H3 and two for B3. An updraft impulse seeding scheme was used in this simulation to put the seeding agent instantaneously into the updraft region under the cloud base (Dennis, 1977), as has been done in some operational projects. Ground generators or AgI pyrotechnic flares could be simulated, but have not been in this study.

The general results show the potential to augment precipitation by AgI seeding, but the initial seeding region and time of seeding are crucial to obtain optimum effects.

2. Characteristics of AgI and nucleation

A variety of substances have been tested as to their ice nucleating efficiency, such as silver iodide (AgI), lead iodide (PbI₂) and organic ice nucleants. Silver iodide was the first discovered, has been most widely used in field projects and most completely tested in laboratories, yet our knowledge is not complete enough to explain all aspects of its ice initiating behavior. Further investigation is still needed. For simplification, or due to lack of information, several assumptions related to nucleation are made.

a. Possible mechanisms for nucleation

The possible mechanisms by which AgI can produce the ice phase are as follows:

1) Deposition nucleation: by converting vapor to solid at ice supersaturation.
2) Contact freezing nucleation: by a crystallization process upon contact of an AgI particle with a supercooled droplet.

3) Sorption nucleation: by condensation of water on the nuclei surface, followed by crystallization at water supersaturation.

4) Immersion freezing nucleation: by activation of a particle which had been collected previously by a droplet.

Another possible mechanism is disruption nucleation which is not related to AgI seeding. For simplicity, contact and deposition nucleation will be the only modes of nucleation discussed in the following sections, although it is noted that, as modeled, what we refer to as deposition nucleation could also include sorption nucleation although it is not modeled per se. The nucleation curve we use to obtain the number of nuclei includes the deposition and sorption modes of nucleation.

b. Possible mechanisms for contact freezing nucleation

Young (1974a) describes inertial impact, Brownian motion, turbulent diffusion, and thermophoretic and diffusiophoretic collection as possible mechanisms for contact freezing nucleation. The phoretic collection rates are much less size-dependent and could possibly be one order of magnitude larger than the Brownian collection rate. However, due to the limitations of this model, we consider only the inertial impact and Brownian collection rates. This assumption may underestimate the contact freezing mechanism. But from results shown later (Table 5), most of the AgI particles in these simulations work as deposition nuclei. Consequently, omitting the phoretic processes will not influence the results significantly.

Brownian (\(S_{BW}\) and \(S_{BR}\)) and inertial impact (\(S_{IW}\) and \(S_{IR}\)) collection rates (due to cloud droplets and raindrops, respectively) can be calculated as follows (Fuchs, 1964; Chen and Orville, 1977).

1) Collection due to cloud droplets is

\[ S_{BW} = \frac{\Delta X_S}{\Delta t} = -4\pi D_S X_S N_w R_w, \quad (1) \]

\[ S_{IW} = \frac{\Delta X_S}{\Delta t} = -\pi R_w^2 X_S V_w F_{WS} N_w, \quad (2) \]

where \(X_S\) is the mixing ratio of the seeding agent, \(N_w\) the number concentration of cloud droplets, \(R_w\) and \(V_w\) the radius and terminal velocity of a cloud droplet (assumed to be 10 \(\mu\)m and 1 cm s\(^{-1}\), respectively, and used in the above two equations only); and \(F_{WS}\), the collection efficiency of cloud droplets for AgI particles, is assumed to be 10\(^{-4}\) (Fuchs, 1964; Pruppacher and Klett, 1978).

2) Collection due to raindrops is similar to (1) and (2) but utilizes an assumed size distribution for the raindrops (Marshall and Palmer, 1948), i.e.,

\[ S_{BR} = \frac{\Delta X_S}{\Delta t} \]

\[ = -4\pi D_S X_S \int_0^\infty \frac{1}{2} D_R n_{OR} \exp(-\lambda_R D_R) dD_R \]

\[ = -2\pi D_S X_S n_{OR} \lambda_R^2, \quad (3) \]

\[ S_{IR} = \frac{\Delta X_S}{\Delta t} \]

\[ = -\pi a X_SE_RS \int_0^\infty \frac{1}{2} D_R^2 n_{OR} \exp(-\lambda_R D_R) dD_R \]

\[ = -\pi a X_SE_RS n_{OR} \Gamma(3 + b) \frac{4\lambda_R^{b+1}}{4\lambda_R^{b+1}} \quad (4) \]

where \(E_RS\), the collection efficiency of raindrops for AgI, is assumed to be 0.5 \(\times\) 10\(^{-4}\) (Fuchs, 1964). The diffusivity of the seeding agent is given by \(D_S = kT B\). With the Cunningham correction (Byers, 1965), the mobility \(B\) can be expressed as

\[ B = \frac{1 + (a'd/R_s)}{6\pi\eta R_s}, \]

where \(d\) is the mean free path, \(a' = 0.9\), \(R_s\) is the radius of the AgI particle (assumed to be 0.1 \(\mu\)m), and \(\eta\) is the dynamic viscosity of air.

c. Efficiency curve for contact and deposition nuclei

The experiments of Parungo (1973) show that contact freezing nucleation is operative at temperatures significantly warmer than for other possible nucleation mechanisms.

According to the theory of Cooper (1974),

\[ N_{da}(T_{da}) \approx N_{ea}(T_{ea}), \]

\[ T_{da} = 2.3 T_{ea}, \]

where \(N_{da}(T)[N_{ea}(T)]\) is the number of deposition [contact] nuclei active at the temperature \(T\); \(T_{da}\) and \(T_{ea}\) are expressed in degrees Celsius. This indicates that the efficiency curve for deposition and contact nuclei are different. Due to the lack of experimental data and for simplification, we use the same efficiency curve for all of the possible mechanisms for nucleation. The curve is based on generator output data fitted between -5 and -20\(^\circ\)C (Orville and Kopp, 1974):\(^3\)

\[ N_o(\Delta T) = \exp[-0.022(\Delta T)^2 + 0.88(\Delta T) - 3.8], \quad (5) \]
where $N_a(\Delta T)$ is the number of nuclei active at the supercooling $\Delta T$. The mode(s) of nucleation are not known for the data on which this curve is based, but most likely include deposition and sorption.

d. Assumptions

Some of the assumptions related to AgI nucleation are presented as follows:

(i) A monodisperse size distribution (radius = 0.1 $\mu$m) for AgI particles is used in calculating the collection rates.

(ii) One liquid drop can capture, at most, one active ice nucleus for contact freezing nucleation.

(iii) The collection rates of ice hydrometeors for AgI particles are neglected. The collection rates of ice hydrometeors for AgI particles are mainly due to photorec processes (Prodi, 1976).5

(iv) Photolytic deactivation is neglected. There are few studies about photolytic deactivation of AgI. Apparently, this deactivation depends on the components used to generate the AgI aerosol. The decay rates vary from a factor of 2 per hour to one to two orders of magnitude per hour (Super et al., 1975). Results of this study (Section 4b) show that the effects of seeding occur within 10 min, so we neglect photolytic deactivation.

(v) Due to the limitation of experimental data, we assume that all of the AgI particles are activated at temperatures -20°C and colder.

3. Cloud model

a. General description

This model is developed as a sequel to others, starting with the simulation of orographic cumulus (Orville, 1965) and is now mainly based on Chang (1977), Orville and Kopp (1977) and Chen and Orville (1980). It is a two-dimensional time-dependent cloud model with bulk water microphysics. The domain of the model is 19.2 km in both the $X$ and $Z$ directions with a 200 m grid interval.

The hydrodynamic and thermodynamic equations are similar to those of the hailstorm model of Orville and Kopp (1977). The hydrodynamic equations are those for a locally incompressible fluid with the extension to deep convection made by using a vorticity equation and a density-weighted streamfunction. Nonlinear eddy coefficients from Drake et al. (1974) are used in the calculation of turbulent diffusion.

Most of the microphysical equations and the parameterization technique are based on Wisner et al. (1972), Chang (1977) and Orville and Kopp (1977). Five classifications of water substance are considered: water vapor, cloud water, cloud ice, rain and precipitating ice. We assume that the terminal velocities of cloud water and cloud ice are insignificant and can be neglected compared with the velocities of air, rain and precipitating ice. The cloud ice crystals are assumed to be 10 $\mu$m radius spheres when the masses are required. The number concentration of cloud droplets is kept constant throughout the simulation, except in calculating the sink terms of seeding agent due to cloud droplets [Eqs. (1) and (2)]. The size distributions for rain and precipitating ice are hypothesized to be constant intercept exponential (i.e., Marshall-Palmer type) distributions. The ice phase is initiated by deposition nucleation, contact nucleation and raindrop freezing.

Certain additional microphysical processes are added and others are modified compared to Orville and Kopp (1977):

1) We add the aggregation process of cloud ice ($P_{IC}$) (Chang, 1977).

2) The interaction between raindrops and cloud ice is simulated based on Chang (1977).

3) The Bergeron-Findeisen process is modified in this simulation as described in a later section.

4) Cloud water and cloud ice may coexist in the region 0 to -40°C.

All of these processes are described in more detail in the next section.

b. Cloud physics

Autoconversion ($P_{RC}$)$^5$ and accretion ($P_{RA}$) are used to transform cloud water to rainwater. Accretion of cloud water ($P_{IAC}$), cloud ice ($P_{IC}$), and rain ($P_{IR}$) by precipitating ice are simulated. Aggregation of cloud ice ($P_{IC}$) is parameterized. The Bergeron-Findeisen process is used to transform cloud ice ($P_{IFC}$) and cloud water ($P_{IFW}$) to precipitating ice, as well as increase the amount of cloud ice ($P_{CW}$). Interactions between cloud ice and rain ($P_{RAC}$ and $P_{CAR}$) are included. "Shedding" ($P_{IAR}$) is simulated to transform cloud water to rainwater; it describes the process by which cloud water is collected by hailstones growing in a wet-growth mode (or by hailstones melting below the 0°C) and is shed to the rainwater field (Orville and Kopp, 1977). The cloud microphysical processes are demonstrated in Fig. 1.

1) Conservation equations

Four conservation equations are considered here:

$$\frac{\partial q}{\partial t} = -\nabla \cdot \mathbf{q} + \nabla \cdot \mathbf{K} \nabla q - P_R - P_I,$$  

$$\frac{\partial l_R}{\partial t} = -\nabla \cdot \mathbf{l}_R + \nabla \cdot \mathbf{K}_m \nabla l_R + P_R + \frac{1}{\rho} \frac{\partial}{\partial z} (V_l l_R \rho),$$

---

$^5$ The symbols refer to processes shown schematically in Fig. 1.

---

\[
\frac{\partial l_i}{\partial t} = -\nabla \cdot \nabla l_i + \nabla \cdot K_m \nabla l_i + P_i + \frac{1}{\rho} \frac{\partial}{\partial z} (U_l l_i \rho),
\]
\[
\frac{\partial X_S}{\partial t} = -\nabla \cdot \nabla X_S + \nabla \cdot K_a \nabla X_S + \text{"source"} + S_T,
\]
where \( q = l_{cw} + l_{cl} + r \); \( l_i, l_b, l_{cw}, l_{cl}, r \) and \( X_S \) are the mixing ratios for precipitating ice, rainwater, cloud water, cloud ice, water vapor and the seeding agent, respectively; \( P_i \) and \( P_b \) are the production terms for precipitating ice and rainwater; \( V_i \) and \( U_i \) are the mass-weighted mean terminal velocities for rainwater and precipitating ice, respectively. The last terms in (7) and (8) are the fallout terms. All of the first terms on the right-hand side are advection terms; the second terms are diffusion terms. The total sink for the seeding agent is given by
\[
S_T = S_{BW} + S_{IW} + S_{BR} + S_{IR} + S_{DEP},
\]
where \( S_{BW}, S_{IW}, S_{BR} \) and \( S_{IR} \) are calculated according to (1)–(4) and \( S_{DEP} \) represents the activated AgI particles which work as deposition nuclei. Source represents the seeding agent added at the time of seeding.

2) Production term for precipitating ice, \( P_i \)

If the temperature is greater than \( T_0 \) (273 K), melting of the precipitating ice \( (P_{IM}) \) will occur. The calculation is based on a heat balance concept. In the region \((T > T_0)\), all of the water collected is shed as rain.

If the temperature is below \( T_0 \), the following processes could occur:
\[
P_i = P_{ID} \text{ (or } P_{IW}) + P_{IC} + P_{IF} + P_{IC}, \quad (10)
\]
where \( P_{ID} \) is the dry growth rate, \( P_{IW} \) the wet growth rate (explained in more detail in the next subsection), \( P_{IC} \) is the rate at which rain interacts with cloud ice to form precipitating ice; \( P_{IF} \) is the rate at which raindrops freeze and at which cloud water is rimed and deposited on cloud ice to form precipitating ice; \( P_{IS} \) is the rate at which precipitating ice sublimes; and \( P_{IC} \) is the rate at which cloud ice aggregates to form precipitating ice.

(i) Dry growth \( (P_{ID}) \) or wet growth \( (P_{IW}) \). Precipitating ice grows by a dry- or wet-growth mode depending on which rate is smaller. If all of the liquid that is collected cannot be frozen, wet growth results and shedding could happen.
\[
\begin{cases}
P_{ID} = P_{I_{AW}} + P_{I_{AC}} + P_{I_{AR}} \text{ (dry growth)} \\
P_{IW} = f(T_c, \Delta r, l_{cl}) \text{ (wet growth)},
\end{cases}
\]
where \( P_{I_{AW}}, P_{I_{AC}} \) and \( P_{I_{AR}} \) are the rates at which hail accretes cloud water, cloud ice and rain, respectively. \( P_{IW} \) is the wet-growth rate calculated based on thermal equilibrium (Musil, 1970) and is a function of temperature, saturation water vapor mixing ratio difference between the stone and the environment, and the amount of cloud ice (Orville and Kopp, 1977). If wet growth is the proper mode for hail growth, the amount of rain actually frozen or shed is given by
\[
P_{I_{AR}} = P_{IW} - P_{I_{AW}} - P_{I_{AC}}.
\]
where $P'_{\text{IAC}}$ is the same as $P_{\text{IAC}}$ except that the collection efficiency of hail for cloud ice is set equal to 1 instead of using 0.1. If $P'_{\text{IAR}}$ is positive, some or all of the rain collected is frozen. If $P_{\text{IAR}}$ is negative, some of the cloud water collected by the precipitating ice is unable to freeze and is shed as rain.

(ii) Interaction between rain and cloud ice, $P_{\text{IA}}$. We assume that all of the raindrops which interact with the ice crystals are frozen, i.e.,

$$P_{\text{IA}} = P_{\text{IAC}} + P_{\text{CAR}},$$

where $P_{\text{IAC}}[P_{\text{CAR}}]$ is the rate at which rain (cloud ice) accretes cloud ice (rain) to form precipitating ice. The rates are based on the geometric sweepout concept (Chang, 1977), i.e.,

$$P_{\text{IAC}} = \frac{E_{\text{RC}} m_0 \Delta t_{\text{cl}} \Gamma(3.8)}{4 \lambda_r^{3.8}},$$

$$P_{\text{CAR}} = \frac{E_{\text{RC}} m_0 \Delta t_{\text{cl}} \rho_w \Gamma(6.8)}{24 m_1 \lambda_r^{6.8}},$$

where $m_1$ is the mass of a 10 $\mu$m radius ice crystal, and $\Gamma$ denotes the gamma function. Eq. (13) represents a slow transformation of cloud ice to precipitating ice and is a sink for cloud ice, while (14) is a rapid transformation of rain to precipitating ice and is a sink for rain.

(iii) Raindrop freezing and the riming and depositional growth of cloud ice to form precipitating ice, $P_{\text{IF}}$. The formula for raindrop freezing ($P_{\text{IFR}}$) is from Wisner et al. (1972). The Bergeron-Findeisen process in Orville and Kopp (1977) is modified. Here we deplete both cloud water ($P_{\text{IFW}}$) and cloud ice ($P_{\text{IFC}}$). So $P_{\text{IF}}$ can be expressed as

$$P_{\text{IF}} = P_{\text{IFR}} + P_{\text{IFW}} + P_{\text{IFC}},$$

where $P_{\text{IFR}}$ is the rate at which raindrops are frozen to form hail embryos. $P_{\text{IFW}}$ and $P_{\text{IFC}}$ are the rates at which cloud water and cloud ice transform to precipitating ice by deposition and riming based on the growth of a 50 $\mu$m radius ice crystal. The determination of how much of the cloud ice is transformed to precipitating ice and how much remains in the cloud ice field is somewhat artificially set as being based on the growth time needed for an ice crystal to grow from 40 to 50 $\mu$m radius. From Koenig (1971), the depositional growth rate of a single crystal is

$$\frac{dm}{dt} = a_1 m^{a_2}$$

and

$$\int dt = \frac{1}{a_1 m^{a_2}}.$$

The time needed for a crystal to grow from 40 to 50 $\mu$m is then given by

$$\Delta t_1 = \frac{1}{a_1 (1 - a_2)} [m_{40}^{1-a_2} - m_{50}^{1-a_2}].$$

where $a_1$ and $a_2$ are temperature-dependent parameters and $m_{40}$ and $m_{50}$ are the masses of 40 and 50 $\mu$m size ice crystals.

The portion of the cloud ice mixing ratio, which is transformed to precipitating ice, is calculated as

$$l_{\text{IC50}} = l_{\text{CI}} \frac{\Delta t}{\Delta t_{\text{f}}},$$

where $\Delta t_{\text{f}}$ is the time step used for numerical integration of the cloud physics processes. The ratio $\Delta t/\Delta t_{\text{f}}$ varies from 0.5 to 10% and depends on temperature and the time step required for numerical stability. $l_{\text{IC50}}$ is calculated only to find the total number of ice crystals used to compute the rate at which cloud ice and cloud water transform to precipitating ice. Thus a separate conservation equation for $l_{\text{IC50}}$ is not required. The number concentration of these somewhat fictitious 50 $\mu$m size ice crystals is computed as

$$N_{\text{IC50}} = \frac{l_{\text{IC50}}}{m_{\text{IC50}}}.$$

Thus $P_{\text{IFW}}$ can be expressed as

$$P_{\text{IFW}} = N_{\text{IC50}}(a_1 m_{\text{IC50}} + \rho C \pi R_{\text{IC50}}^{3} V_{\text{IC50}} E_{\text{C}}),$$

where $R_{\text{IC50}}$ is the radius of an ice crystal (assumed to be 50 $\mu$m), $V_{\text{IC50}}$ the terminal velocity of a 50 $\mu$m size ice crystal, and $E_{\text{C}}$ the collection efficiency. Then

$$P_{\text{IFC}} = l_{\text{CI}}/\Delta t = l_{\text{IC50}}/\Delta t,$$

is the sink term for cloud ice.

(iv) Aggregation of ice crystals, $P_{\text{IC}}$. The aggregation rate of ice crystals is parameterized according to concepts originally proposed by Kessler (1969), but with some modification (Chang, 1977), i.e.,

$$P_{\text{IC}} = a(l_{\text{CI}} - l_{\text{IO}}),$$

where $a$ is a parameter which is temperature-dependent and $l_{\text{IO}}$ a threshold amount required for aggregation to occur. In this study, we set $l_{\text{IO}} = 0$; i.e., when cloud ice is produced, aggregation happens at once.

(v) Sublimation term, $P_{\text{IS}}$. If hail is advected out of the cloud, sublimation will take place at temperature $T < T_{0}$ (273 K). The rate is proportional to the vapor mixing ratio difference between the hailstone surface and its environment.

(vi) Final form of $P_{\text{I}}$.

- For dry growth [from Eqs. (10), (11), (13)–(19)]:

$$P_{\text{I}} = [(P_{\text{IAR}} + P_{\text{IAC}} + P_{\text{IAR}}) + (P_{\text{IAC}} + P_{\text{CAR}}) + (P_{\text{IFR}} + P_{\text{IFW}} + P_{\text{IFC}}) + P_{\text{IC}} + P_{\text{IS}}] \times (1 - \delta) + \delta P_{\text{IM}}.$$

- For wet growth [from Eqs. (10), (12), (13)–(19)]:

$$P_{\text{I}} = [P_{\text{IAR}} + (P_{\text{IAC}} + P_{\text{CAR}}) + (P_{\text{IFR}} + P_{\text{IFW}} + P_{\text{IFC}}) + P_{\text{IC}} + P_{\text{IS}}] (1 - \delta) + \delta P_{\text{IM}}.$$
where
\[
\delta = \begin{cases} 
0, & T \leq T_0 \\
1, & T > T_0 
\end{cases}
\]

All of the formulas, except those for \( P_{RAC}, P_{CAR}, P_{IFW}, P_{IFC} \) and \( P_{IC} \), shown above can be found in Orville and Kopp (1977).

3) Production Term for Rainwater, \( P_R \)

The production term for rain follows Orville and Kopp (1977):
\[
P_R = P_{RC} + P_{RA} + P_{RE} - P_{LAR} \quad \text{(or } P_{LAR} - P_{CAR} \text{)}
- P_{IFR} - \delta P_{IM},
\]
where \( P_{RC} \) is the rate at which small droplets coalesce to form raindrops, \( P_{RA} \) the rate at which raindrops accrete cloud water and \( P_{RE} \) the rate at which raindrops evaporate.

4) Seeding Effects

The seeding material is put into the domain instantaneously under the cloud base, a method used on some operational cloud seeding projects. It will interact with the cloud, as follows:

(i) Interaction with rainwater. When a supercooled raindrop captures an active ice nucleus, contact nucleation occurs. The number concentration of active ice nuclei \( N_{ab}(\Delta T) \) captured by raindrops at a supercooling \( \Delta T \) is calculated from Eq. (5) as
\[
N_{ab}(\Delta T) = \left\{ -S_{ER} + S_{IR} \right\} \left[ \frac{N_a(\Delta T)}{N_a(20^\circ C)} \right] \Delta T m_s^{-1},
\]
where \( N_a(\Delta T)/N_a(20^\circ C) \) is the fraction of the AgI particles activated at a supercooling of \( \Delta T \), and \( m_s \left( = 2.38 \times 10^{-14} \text{ g} \right) \) is the mass of an AgI particle (assumed to be 0.1 \( \mu \text{m} \) radius). Because we assume one liquid drop can capture at most one active ice nucleus, the ratio of \( N_{ab}(\Delta T) \) to the total number of raindrops indicates the fraction of rainwater which is transferred to precipitating ice due to the seeding agent at a supercooling \( \Delta T \). The Marshall-Palmer distribution of raindrops tends to overestimate the total number of raindrops. Therefore we set a lower limit of 0.01 cm diameter for the size distribution of raindrops when we integrate the Marshall-Palmer exponential function. The total number of raindrops \( N_R \) with this lower limit imposed is
\[
N_R = \frac{n_{AR}}{\lambda_R} \exp(-0.01\lambda_R).
\]
From (23) and (24), the transformation rate from rainwater to precipitating ice due to seeding is
\[
P_{IESR} = l_R \frac{N_{ab}(\Delta T)}{\Delta T N_R}.
\]

(ii) Interaction with cloud water.

- Contact freezing nucleation: Following the same procedure outlined in the previous section, \( P_{CSWC} \), the rate at which cloud water transforms to cloud ice due to contact nucleation may be written as
\[
P_{CSWC} = l_{CW} \frac{N_{aw}(\Delta T)}{\Delta t N_w},
\]
where \( N_{aw}(\Delta T) \) is the counterpart of \( N_{ab}(\Delta T) \) for cloud water and is defined by
\[
N_{aw}(\Delta T) = -S_{BW} + S_{IW} \left[ \frac{N_a(\Delta T)}{N_a(20^\circ C)} \right] \Delta tm_s^{-1}.
\]

- Deposition nucleation: In the cloudy environment, we assume the relative humidity (RH) is 100% with respect to a weighted-mean saturation mixing ratio, weighted with respect to the amounts of cloud water and cloud ice. If RH is less than 100%, the cloud will evaporate instantaneously to supply the vapor needed to keep RH = 100%. Under this assumption, the activation of deposition nuclei depletes the cloud water in the cloudy environment. As was noted earlier, deposition nucleation, as modeled, also includes sorption nucleation.

The number of AgI particles active as deposition nuclei at a supercooling \( \Delta T \) is computed as
\[
N_{ab}(\Delta T) = X_S \left[ \frac{N_a(\Delta T)}{N_a(20^\circ C)} \right] m_s^{-1}.
\]

Seeding enhances the Bergeron-Findeisen process through deposition nucleation, and will transform the cloud water to cloud ice. The rate \( P_{CSWD} \) is
\[
P_{CSWD} = N_{ab}(\Delta T) a_i m_s^g.
\]

5) Transformation Between Cloud Water and Cloud Ice

If the cloudy atmosphere is colder than \(-40^\circ C\), homogeneous nucleation will occur naturally. Saunders' (1957) isobaric freezing is used to calculate the temperature change. Between 0 and \(-40^\circ C\), cloud water and cloud ice can coexist. The transformation between these two hydrometeor fields is based on contact and deposition nucleation with natural and artificial ice nuclei, and depositional growth of cloud ice (Bergeron-Findeisen process). Saunders' equation can be used to give the temperature change due to freezing all of the cloud water. In this model, we take part of the temperature change due to total freezing, that portion being based on the ratio of the mass transformation between 0 and \(-40^\circ C\) to the total cloud water existing in the same region, in order to prevent overestimating the heat released. If the temperature is higher than 0\(^\circ\)C, the cloud ice melts back to cloud water.
6) NATURAL ICE NUCLEI

No separate conservation equation is solved for natural ice nuclei. The number concentration of active natural ice nuclei is given by Fletcher (1962), and is exponential in form, i.e.,

\[ N_n(\Delta T) = n_0 \exp(\beta_1 \Delta T) \]

where \( \Delta T \) is the supercooling and \( n_0 \) and \( \beta_1 \) are parameters with values of \( \beta_1 \) ranging between 0.4 and 0.8. The value \( \beta_1 = 0.5 \) is used for the standard case; \( n_0 \) can vary by several orders of magnitude. The typical value \( n_0 = 10^{-8} \text{ L}^{-1} \) is used in this study.

All of the natural ice nuclei are assumed to work as depositional nuclei in initiating the ice phase in this model. The rate at which cloud water transforms to cloud ice is

\[ P_{CNWD} = \frac{N_n(\Delta T)}{1000 \rho} (a_{1} m_n^{2a}), \]

where \( m_n (= 1.05 \times 10^{-15} \text{ g}) \) is the mass of a natural ice nucleus.

c. Initial and boundary conditions

Radiosonde sounding data of temperature, humidity and pressure are used as the input data. The horizontal wind in the direction of motion of the storm is reduced by a factor of 5 [see discussion in Orville and Kopp (1977)]. To initiate the convection, temperature and water vapor impulses are put in the domain with maximum excesses of 3°C and 2 g kg\(^{-1}\), respectively. The model has been tested with three different soundings. Soundings H1 and H3 are from Miles City, Montana, in the High Plains region; B3 is from St. Louis, Missouri. Observations in the northern High Plains have shown that there is no significant coalescence growth of large liquid drops, and that precipitation is initiated through the Bergeron-Findeisen process (Dye et al., 1974). Because of this, we eliminate the autoconversion process, which transforms cloud water to rainwater, in running soundings H1 and H3. Soundings B3 is modified by putting a stable layer in the upper parts of the sounding in order to suppress the development of the cloud. Without modification, this sounding will produce a model cloud with a top close to 13 km.

The boundary conditions developed by Orville and Kopp (1977) are applied. These conditions allow heat and vapor to diffuse into and precipitation to fall out through the lower boundary. Cloud shadow effects are applied at the lower boundary. Lateral boundaries are open. Clouds and seeding agents, as well as other variates, can pass through the lateral boundary.

d. Numerical techniques

The NCAR library subroutine PWSCRT is used to solve the Poisson-type equation. The Crowley (1968)-Leith second-order advection scheme is used to calculate the advection terms except for the seeding agent.

Because this advection scheme does not conserve mass exactly, a finite-element advection scheme developed by Rainer Bleck (unpublished manuscript, NCAR; Chen and Orville, 1977) is applied to calculate the advection term for the seeding agent. We have tested this scheme in a smaller (6 km \( \times \) 13 km), two-dimensional warm cloud model with closed boundary conditions in a pure advection calculation. The results of this test were satisfactory concerning the mass conservation. The minor deviation from conservation of the seeding agent mass in this study (shown later) is mainly due to the advection through the lateral boundaries, but is negligible.

4. Results

a. General description

Most of the seeded cases are seeded under cloud base in the region of the strongest updraft when the cloud top reaches the \(-10^\circ\text{C}\) level. The initial distribution of the seeding material, which forms a rectangular block, has maximum values in the center (13 grid points for case H1/M1, 3 grid points for all other seeded cases), and decreases exponentially outward (2 grid points). The maximum values are \(6.25 \times 10^{-10} \text{ g g}^{-1}\) for case H1/M2 and \(2.5 \times 10^{-9} \text{ g g}^{-1}\) for all other seeded cases. This yields a total of \(\sim 400 \text{ g km}^{-1}\) (the kilometer is the unit distance in the y direction, the model having x-z dimensions). Conceptually, an instantaneous volume source is released below cloud base.

Cases H1/P1, H1/T1, H1/M1 and H1/M2 are used to test the effects of seeding in a different place, at a later time, and with different amounts of seeding agent (both more and less), respectively. Soundings H3 and B3 are used to test the effects of seeding a more severe convective cloud and a cloud in which the autoconversion process is efficient, respectively. For this last case (B3), the cloud base is lower and much warmer, which allows the precipitation to be initiated by the coalescence process instead of by the ice phase as in the cooler, high-base clouds of the High Plains. Table 1 summarizes the differences in all of the cases which have been run in this study. Heights and temperatures of cloud tops and bases at the time of seeding are listed in Table 2.

The model cloud forms at 11.25 min for sounding H1. The cloud develops very rapidly. The maximum updraft of 22.4 m s\(^{-1}\) occurs near 22.5 min. The cloud moves to the right rather quickly and arrives at the right-hand lateral boundary at 26 min. The updraft core passes the right-hand lateral boundary at 42 min. The cloud base is near 3 km (+5°C); all heights are above ground level. The cloud top is near 14 km
Table 1. Cases run in this study.

<table>
<thead>
<tr>
<th>Cases*</th>
<th>Total amount of seeding agents (g km⁻¹)</th>
<th>Initial seeding time (min)</th>
<th>β₁†</th>
<th>Initial seeding region</th>
<th>With or without autoconversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1/N1</td>
<td>0</td>
<td>—</td>
<td>0.5</td>
<td>—</td>
<td>w/o</td>
</tr>
<tr>
<td>H1/N2</td>
<td>0</td>
<td>—</td>
<td>0.6</td>
<td>—</td>
<td>w/o</td>
</tr>
<tr>
<td>H1/N3</td>
<td>0</td>
<td>—</td>
<td>0.8</td>
<td>—</td>
<td>w/o</td>
</tr>
<tr>
<td>H1/S1</td>
<td>397</td>
<td>14</td>
<td>0.5</td>
<td>Fig. 8a</td>
<td>w/o</td>
</tr>
<tr>
<td>H1/S3</td>
<td>397</td>
<td>14</td>
<td>0.8</td>
<td>Fig. 8a</td>
<td>w/o</td>
</tr>
<tr>
<td>H1/P1</td>
<td>420</td>
<td>14</td>
<td>0.5</td>
<td>Fig. 8b**</td>
<td>w/o</td>
</tr>
<tr>
<td>H1/T1</td>
<td>397</td>
<td>17</td>
<td>0.5</td>
<td>Fig. 8a + 3 min</td>
<td>w/o</td>
</tr>
<tr>
<td>H1/M1</td>
<td>1440</td>
<td>14</td>
<td>0.5</td>
<td>Fig. 8c</td>
<td>w/o</td>
</tr>
<tr>
<td>H1/M2</td>
<td>99.3</td>
<td>14</td>
<td>0.5</td>
<td>Fig. 8c</td>
<td>w/o</td>
</tr>
<tr>
<td>H3/N1</td>
<td>0</td>
<td>—</td>
<td>0.5</td>
<td>—</td>
<td>w/o</td>
</tr>
<tr>
<td>H3/S1</td>
<td>427</td>
<td>14</td>
<td>0.5</td>
<td>Fig. 19</td>
<td>w/o</td>
</tr>
<tr>
<td>B3/N1</td>
<td>0</td>
<td>—</td>
<td>0.5</td>
<td>—</td>
<td>w</td>
</tr>
<tr>
<td>B3/S1</td>
<td>495</td>
<td>15</td>
<td>0.5</td>
<td>Fig. 26</td>
<td>w</td>
</tr>
</tbody>
</table>

* The first two characters represent the sounding used; the last two characters represent the case run.
** Underlining indicates the difference from one case to the standard one.
† β₁ is the parameter used in Fletcher’s equation: \( N_s (ΔT) = n_s \exp(β_1 ΔT) \).

See text for further details on these cases.

(−65°C). Precipitation first reaches the ground at 38 min, but the precipitating process is not very efficient. In an attempt to exclude the uncertain effect of the boundaries, we compare all of the results before 51 min for the cases run with sounding H1. The maximum rain accumulated on the ground is just 1.18 mm at 51 min. The hailfall is small. The cloud outlines at 21, 27 and 33 min are shown in Figs. 2a–2c. The amount of rainfall and hailfall is negligible compared to the other two cases (H3/N1, B3/N1), as is shown in Figs. 3a and 3b.

The cloud begins to form at 7.75 min for sounding H3, which is a more unstable and more moist sounding than H1. The cloud outlines of case H3/N1 at 21, 27 and 30 min are presented in Figs. 4a–4c. The maximum updraft (36.1 m s⁻¹) occurs near 34 min. At later times, new clouds grow from the rear (left) and feed into the main cell, enhancing the convection. The sloping updraft and a gust front are obvious in this case (H3/N1). The cloud top eventually rises to near 14 km (−65°C). The cloud base is near 2 km (+10°C). Quite a bit of precipitation falls on the ground (Figs. 3a and 3b).

Sounding B3 is a very moist sounding. A moist layer (RH = 100%) exists between 525 and 473 mb. The first model cloud, which is a stratus, forms at 2.5 min near 5 km. A convective cloud forms below (with cloud base of 1 km) the stratus near 5 min. The coalescence process by which small droplets form large drops is very efficient. The first radar echo is produced by rain, as opposed to case H1/N1 and H3/N1 in which radar echoes are produced by precipitating ice. The maximum updraft is 22.6 m s⁻¹ near 18.5 min. The cloud base is at −1 km (+20°C); the cloup top is near 10 km (−35°C). This case produces the largest amount of rainfall (Fig. 3a) but only minor hailfall (Fig. 3b). The cloud outlines of case B3/N1 at 21, 27 and 33 min are shown in Figs. 5a–5c.

b. Transfer and sinks of seeding agent

All of the seeding cases simulate seeding under the cloud. The seeding material is advected into the cloud. A typical example (case H1/S1) is shown in Figs. 6 and 7. Figs. 6a and 6b show that the seeding agent advects into the cloud very rapidly and has its effect in the cold region of the cloud (mainly, −5 to −15°C). In Fig. 6d, the seeding material seems to be concentrated in the lower parts of the cloud. However, Fig. 7 shows that the total seeding agent remaining in the domain is less than 3%. Most of the seeding agent has already been used. Table 3 shows the percentage of seeding agent captured due

Table 2. Cloud parameters at seeding time.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Cloud base (km)</th>
<th>Cloud-base temperature (°C)</th>
<th>Cloud top (km)</th>
<th>Cloud-top temperature (°C)</th>
<th>Updraft at the center of seeding region (m s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1/S1</td>
<td>3.3</td>
<td>+4</td>
<td>5.0</td>
<td>−10</td>
<td>13.0</td>
</tr>
<tr>
<td>H1/S3</td>
<td>3.3</td>
<td>+4</td>
<td>5.0</td>
<td>−10</td>
<td>13.0</td>
</tr>
<tr>
<td>H1/P1</td>
<td>3.3</td>
<td>+4</td>
<td>5.0</td>
<td>−10</td>
<td>7.5</td>
</tr>
<tr>
<td>H1/T1</td>
<td>3.6</td>
<td>+2</td>
<td>6.6</td>
<td>−20</td>
<td>17.5</td>
</tr>
<tr>
<td>H1/M1</td>
<td>3.3</td>
<td>+4</td>
<td>5.0</td>
<td>−10</td>
<td>13.0</td>
</tr>
<tr>
<td>H1/M2</td>
<td>3.3</td>
<td>+4</td>
<td>5.0</td>
<td>−10</td>
<td>13.0</td>
</tr>
<tr>
<td>H3/S1</td>
<td>2.2</td>
<td>+8</td>
<td>5.4</td>
<td>−15</td>
<td>16.0</td>
</tr>
<tr>
<td>B3/S1</td>
<td>1.1</td>
<td>+20</td>
<td>6.2</td>
<td>−10</td>
<td>4.5</td>
</tr>
</tbody>
</table>
to each possible mechanism after 51 min of simulation (for all of the seeded cases).

The symbols used in Table 3 are explained as follows:

**SBW:** percentage of seeding agent captured by cloud droplets due to the Brownian motion

**SIW:** percentage of seeding agent captured by cloud droplets due to the inertial impacts

**STW:** total sink due to cloud droplets

**SBR:** percentage of seeding agent captured by raindrops due to the Brownian motion

**SIR:** percentage of seeding agent captured by raindrops due to the inertial impacts

**STR:** total sink due to raindrops

**STD:** total sink due to deposition nucleation.

Table 3 shows clearly that most of the seeding agent acts as deposition nuclei. The contact nuclei are captured primarily by cloud droplets, mainly through the Brownian motion mechanism, although this amounts to less than 0.5%. In case B3/S1, 26.2% of the seeding agent still remains in the domain after 51 min of simulation. This is because the updraft in the seeded region is not as strong as in the other cases. The seeding agent does not advect into the cloud as efficiently as in the other cases. At later times, part of the seeding agent transfers to a downdraft area and moves downward. Therefore in this case (B3/S1), the seeding agent does not work as effectively as in the other cases.

Conservation of the seeding agent for case H1/S1 is good. In the first 12 min after seeding (14–26 min), the mass increases slightly (<0.01%). After 26 min of simulation, the mass decreases (<1.2%); this is due to advection through the lateral boundaries.

c. Cases run with sounding H1

1) H1/N1 and H1/S1

The initial seeding region is shown in Fig. 8a. The initial seeding conditions and parameters of the cloud at the time of seeding for case H1/S1 are listed in Tables 1 and 2. The seeding does not induce much dynamic change within 10 min after seeding. Six minutes after seeding, the kinetic energy increases 0.67%. Figs. 9a–9c show the general cloud outlines at 21, 27 and 33 min for case H1/S1. Comparing Figs. 2a and 9a, the main difference is the

Fig. 2. Numerical simulation of cloud and precipitation at (a) 21 min, (b) 27 min and (c) 33 min for case H1/N1. Cloud areas (100% relative humidity) are outlined by a solid line. Streamlines are dashed lines. Small solid circles and asterisks indicate rain and precipitating ice greater than 1 g kg⁻¹, respectively, and the S's indicate cloud ice greater than 0.1 g kg⁻¹. The 0, −20 and −40°C levels are indicated on the right-hand side of (a).
Fig. 3. Distributions of accumulated (a) rainfall and (b) hailfall on the ground at 51 min. The dashed line is the result for H1/N1, the solid line for H3/N1, and the dotted line for B3/N1.

The early formation of the ice phase in the supercooled region of the cloud for the seeded case. Figs. 2a–2c exhibits a time window within which little icing occurs. Comparing Fig. 9b to Fig. 2b, the seeded case already has quite a bit of cloud ice and precipitating ice existing in the updraft core. Figs. 9c and 2c show the difference in the height of the center of the precipitating ice, the seeded case having a lower center of mass (at ~6 km vs 7 km in the unseeded case H1/N1). The early formation of cloud ice leads to the early formation of precipitating ice.

Fig. 4. Numerical simulation of cloud and precipitation at (a) 21 min, (b) 27 min and (c) 30 min for case H3/N1. Otherwise as in Fig. 2.
Below the precipitating ice, there is abundant cloud water available for riming and accretional growth.

The differences between H1/S1 and H1/N1 for various accumulated production terms for precipitating ice versus time are shown in Fig. 10. Differences in Bigg's freezing ($P_{IR}$) are insignificant and are not included. Fig. 10 shows that the difference in depositional and riming growth to produce precipitating ice ($P_{IFW} + P_{IPC}$) is small (compared to the other terms plotted) in the later stages. The positive values for $P_{IFW} + P_{IPC}$ from 20 to 31 min are clearly due to the seeding which initiates the riming process on ice crystals earlier than it occurs in the natural case. Similarly, the increase in aggregation of cloud ice ($P_{IC}$) is due to the increase in cloud ice content created by seeding. Note that this increase is delayed ~2 min from the increase noted for $P_{IFW} + P_{IPC}$ and that it becomes nearly constant when the natural case develops similar cloud ice contents (~30 min). The initial increase in accretional growth of precipitating ice ($P_{IM}$) is due to increased precipitating ice contents produced by $P_{IFW} + P_{IPC}$ and $P_{IC}$. The dramatic increases in this rate and the interaction between rain and cloud ice ($P_{IR}$) must await increases in the rain content as shown in Fig. 11 (~30 min). Only that portion of the rain content which is recycled into the cold regions is available to these processes. As Fig. 11 clearly shows, the main difference in production of rain between the natural and seeded cases is due to the melting of precipitating ice ($P_{IM}$). Accretional growth of rain ($P_{RA}$) is increased less dramatically and the shedding of rain during wet growth of precipitating ice ($P_{IR}$) shows almost no change.

Figs. 12a–12d show the precipitating ice content distribution for the seeded and unseeded cases, respectively, at 24 and 30 min. A broader distribution and larger amounts of precipitating ice are produced by seeding. From Figs. 12c and 12d, the center of the precipitating ice mass is lower in the seeded case (5 km vs 7 km for seeded versus unseeded cases). The difference in location leads to earlier melting of the precipitating ice and "shedding" by collection of cloud water to form rain. Total accumulated rainfall on the ground at 51 min is 1.5 kt km$^{-1}$ for the natural case and 7.0 kt km$^{-1}$ for the seeded case, a near fourfold increase in rainfall, although note that these are very small amounts of rainfall (Table 6).

2) H1/N2, H1/N3 and H1/S3

The differences between cases H1/N2, H1/N3 and H1/N1 reflect the influence of number concentra-
Fig. 6. The distributions of seeding agent (solid) for case H1/S1 at (a) 15 min, (b) 18 min, (c) 21 min and (d) 24 min, respectively. Cloud areas (100% relative humidity) are outlined by a dotted line. Streamlines are dashed lines. The maximum seeding agent values are (a) $6.79 \times 10^{-10}$ g g$^{-1}$, (b) $2.06 \times 10^{-10}$ g g$^{-1}$, (c) $2.81 \times 10^{-11}$ g g$^{-1}$ and (d) $5.32 \times 10^{-12}$ g g$^{-1}$, respectively.

Fig. 7. The percentage of seeding agent in the domain versus time for case H1/S1.

The parameter $\beta_1$ in Fletcher's equations are 0.5, 0.6 and 0.8 for cases H1/N1, H1/N2 and H1/N3, respectively. The number concentration of active natural ice nuclei at $-20^\circ$C, $N_n(-20^\circ$C), with different values of $\beta_1$ are listed in Table 4.

The cloud outlines at 24 min for cases H1/N1, H1/N2 and H1/N3 are shown in Figs. 13a–13c. The

<table>
<thead>
<tr>
<th>Cases</th>
<th>SBW*</th>
<th>SW*</th>
<th>STW*</th>
<th>SBR*</th>
<th>SIR*</th>
<th>STR*</th>
<th>STD*</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1/S1</td>
<td>0.2285</td>
<td>0.0022</td>
<td>0.2308</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>98.6</td>
</tr>
<tr>
<td>H1/S3</td>
<td>0.2285</td>
<td>0.0022</td>
<td>0.2307</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>98.6</td>
</tr>
<tr>
<td>H1/P1</td>
<td>0.1236</td>
<td>0.0012</td>
<td>0.1248</td>
<td>0.0000</td>
<td>0.0011</td>
<td>0.0011</td>
<td>92.7</td>
</tr>
<tr>
<td>H1/T1</td>
<td>0.1164</td>
<td>0.0011</td>
<td>0.1175</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>99.8</td>
</tr>
<tr>
<td>H1/M1</td>
<td>0.1947</td>
<td>0.0019</td>
<td>0.1966</td>
<td>0.0000</td>
<td>0.0004</td>
<td>0.0004</td>
<td>96.2</td>
</tr>
<tr>
<td>H1/M2</td>
<td>0.3226</td>
<td>0.0023</td>
<td>0.2349</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>99.3</td>
</tr>
<tr>
<td>H3/S1</td>
<td>0.3319</td>
<td>0.0033</td>
<td>0.3332</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>99.6</td>
</tr>
<tr>
<td>B3/S1</td>
<td>0.3476</td>
<td>0.0036</td>
<td>0.3512</td>
<td>0.0002</td>
<td>0.0786</td>
<td>0.0784</td>
<td>91.5</td>
</tr>
</tbody>
</table>

* The meaning of each term is explained in the text.
Fig. 8. The distribution of the seeding agent at the time the seeding agent is introduced for (a) H1/S1, (b) H1/P1 and (c) H1/M1, all at 14 min. Cloud areas (100% relative humidity) are outlined by a dotted line. Streamlines are dashed lines.

The icing process shows large differences in these three cases.

Case H1/S3 is run with the same initial seeding conditions as case H1/S1 except using $\beta_1 = 0.8$ instead of 0.5 (Table 1). The total accumulated rainfall on the ground at 51 min shows a twofold increase [with respect to case H1/N3 (Table 6)]. Compared with the increase between case H1/S1 and H1/N1, case H1/S3 is less effective. If we consider another parameter ($n_o$) in Fletcher's equation, which can vary by several orders of magnitude, the results could display an even larger difference. This result shows that the effectiveness of seeding is affected significantly by the number concentration of natural ice nuclei.

3) H1/P1

Compared to case H1/S1, the initial seeding region for case H1/P1 is 600 m lower and 1200 m to the left (Fig. 8b). More time is required to advect the seeding agent into the supercooled region of the cloud. Fig. 14 shows the precipitating ice content.
distribution at 24 and 30 min for case H1/P1. Figs. 14a, 12a and 12b show that the seeding does affect the production of precipitating ice, but the effects are concentrated on both sides of the cloud. In the middle of the cloud, case H1/P1 does not produce enough precipitating ice to accrete liquid hydrometeors efficiently. Fig. 15 presents the differences in the accumulated production term for precipitating ice accretion of cloud water, cloud ice and rainwater and the melting of precipitating ice between H1/N1 and H1/P1. Comparing Figs. 10, 11 and 15, both the accumulated accretion of precipitating ice for cloud ice, cloud water and rain, and the accumulated melting of precipitating ice are smaller than in case H1/S1. This means that the precipitating ice is not produced quickly enough to accrete the liquid hydrometeors which exist in the cloud. The total accumulated rainfall on the ground at 51 min for case H1/P1 shows less than a onefold increase compared to the natural case H1/N1 (Table 6). Compared to the results for H1/S1, this is a significant difference. Case H1/P1 compared against the standard seeding case H1/S1 shows the importance of creating precipitating ice at the proper location.

4) H1/T1

Case H1/T1 is used to test the effects of time of release of the seeding agent by seeding at a later time; that is, 3 min later than in case H1/S1. At the time of seeding, the cloud top has already reached the -20°C level (Tables 1 and 2). The updraft in the seeding region is quite strong (17.5 m s⁻¹ at the center of the seeding region). The airflow advects the seeding agent into the supercooled region very effectively. Fig. 16 shows the percentage of the seeding agent which remains in the domain for cases H1/P1 and H1/T1. Figs. 7 and 16 show that case H1/T1 uses the seeding agent more effectively (within 5 min). The precipitating ice fields at 24 and 30 min are shown in Fig. 17. Fig. 17 compared with Fig. 14 shows that case H1/P1 has more precipitating ice existing within the domain than case H1/T1, but that case H1/T1 has more precipitating ice existing in the central part of the cloud, which is a favorable region for growth. This broader and larger distribution of the precipitating ice field for H1/T1 compared to H1/P1 is due to the transport of the seeding material. Case H1/T1 is seeded directly under cloud base in the strongest updraft region. The seeding agent spreads along the updraft core and produces more precipitating ice in the central region of the cloud. After 6 min (at 30 min of simulation time), the precipitating ice fields distribute in the same pat-
tern and with almost the same amounts as shown in Figs. 17b and 14b. This indicates that the precipitating ice grows more rapidly in case H1/T1 than H1/P1, since H1/P1 was seeded 3 min earlier. The total amount of rain and precipitating ice within the whole domain, at 24 and 30 min for cases H1/T1 and H1/P1, are listed in Table 5.

The total rainfall at 51 min for case H1/T1 compared to the standard case (H1/N1), shows an increase by a factor of 2.7. The precipitation process is more efficient for case H1/T1 than case H1/P1, which is due to the more effective transport of the seeding agent into a favorable region. However, compared with case H1/S1, we find that the rainfall is decreased by seeding in a different location or at a later time. This is apparently due to the fact that the natural precipitation processes have reached a more mature stage in these cases and are less susceptible to seeding.

5) H1/M1 and H1/M2

Case H1/M1 is seeded at the same time and height as case H1/S1, but with a broader initial seeding region (Fig. 8c) and a larger amount of seeding agent. Case H1/M2 is run with the same initial seeding conditions as H1/S1 except that the total amount of seeding agent is reduced. The cloud development
for case H1/M1 is very similar to case H1/S1. Figs. 18a and 18b show the precipitating ice content distribution at 24 and 30 min. Compared with Figs. 12a and 12c, the patterns are similar to each other for cases H1/S1 and H1/M1. The total accumulated rainfall on the ground shows a fourfold increase compared to case H1/N1. Although the total seeding agent at the initial seeding time was increased by a

---

**Fig. 13.** Numerical simulation of cloud and precipitation at 24 min for (a) case H1/N1, (b) case H1/N2 and (c) case H1/N3. Cloud areas (100% relative humidity) are outlined by a solid line. Streamlines are dashed lines. Small solid circles and asterisks indicate rain and precipitating ice greater than 1 g kg⁻¹, respectively, and the S's indicate cloud ice greater than 0.1 g kg⁻¹.
factor of 3.6 (relative to case H1/S1), the production of precipitating ice is not increased. The total rainfall on the ground is increased only slightly compared with case H1/S1. This is because these model clouds have a well-organized circulation which results in most of the seeding material tending to concentrate in discrete zones within the cloud, which seems to agree with observations (Linkletter and Warburton, 1977). Four hundred grams of seeding agent are enough to glaciate these discrete zones which then spread throughout the cloud. Adding more seeding agent does not seem to increase rainfall proportionately in the model cases.

Case H1/M2 is seeded with one-fourth of the seed-
ing agent amount which is used in case H1/S1. The cloud outlines and hydrometeor content distributions are similar to case H1/P1. The total accumulated rainfall on the ground shows an 87% increase compared to case H1/N1.

d. Cases run with sounding H3-H3/N1 and H3/S1

Only two cases are run with sounding (H3)—seeded (H3/S1) versus unseeded (H3/N1). The initial seeding region is shown in Fig. 19 when the cloud top reaches the −15°C level. The cloud outlines for case H3/S1 are shown in Figs. 20a–20c at 21, 27 and 30 min. Figs. 20a and 4a show the earlier formation of cloud ice in the supercooled region of the cloud for the seeded case. Figs. 20b and 4b show a broader distribution of precipitating ice for the seeded case. Figs. 20c and 4c show that rain is formed earlier in the seeded case. These comparisons indicate that the effect of seeding is to initiate the precipitation processes sooner.

The differences between H3/S1 and H3/N1 for the various production terms for rain and precipitating ice which are affected by seeding are shown in Figs. 21 and 22, respectively. Fig. 21 shows that the melting of precipitating ice \( P_{IM} \) is increased initially but that it eventually shows little change. Increases in accretional growth of rain \( P_{RA} \) must await the formation of rain through melting but ends up showing the most significant increase. Fig. 22 indicates the seeding causes increases in the mechanisms which generate precipitating ice \( P_{IPW} + P_{IFC} \) and \( P_{IR} \) soon after seeding, \( P_{IA} \) induced changes occurring later). The increased amounts of precipitating ice cause a slight increase in the accretional growth of precipitating \( P_{IA} \) ice initially but later switches to a decrease relative to the natural case. This is due to the fact that some of the cloud water available for growth of the precipitating ice in the natural case has been depleted by the generation mechanisms in the seeded case.

Though the formation of rain is increased, the total accumulated rainfall on the ground at 51 min has decreased slightly relative to the unseeded case. This is because most of the excess rain in the seeded case transforms to precipitating ice via interactions with cloud ice \( P_{IA} \), and falls out as hail. The precipitating ice content distribution for cases H3/N1 and H3/S1 at 24 and 30 min are shown in Fig. 23. Larger amounts of precipitating ice are found in the seeded case, but they are distributed at the same height as the unseeded case (as opposed to the comparisons between cases H1/S1 and H1/N1, which show a different distribution in height). Part of the
precipitating ice is recycled by the strong updraft, and remains inside the cloud. In this situation, the precipitating ice has a better chance to grow large and fall out as hail. The rainfall decreases, but the hailfall increases (Table 6). The total precipitation increases a small amount (55.5 kt km⁻¹ for the seeded case versus 55.3 kt km⁻¹ for the unseeded case). Fig. 24 shows the total accumulated difference of rainfall and hailfall on the ground versus time between cases H3/N1 and H3/S1. A maximum at 40 min in Fig. 24 is due to the seeding effect. But after 40 min, the difference approaches zero. This means that the natural precipitation process begins to have its effect. Finally, the difference in the total accumulated rainfall on the ground does not change much. The seeding initiates the precipitation process earlier but does not augment the precipitation. Fig. 25 shows the differences in the distribution of rainfall and hailfall at the ground between the seeded and unseeded case. The positive and negative areas are nearly equal for the rainfall distribution. Thus the total amount of precipitation does not change much, but is redistributed. More tests are needed to give better understanding.

e. Cases run with sounding B3-B3/N1 and B3/S1

The initial seeding region for case B3/S1 is shown in Fig. 26. The precipitation process is different from the cases run with soundings H1 and H3. The warm-cloud precipitating process is very important in these two cases (B3/N1 and B3/S1). Autoconversion ($P_{RA}$) serves to initiate the accretion process ($P_{RA}$). The biggest production term for rain is $P_{RA}$ for case B3/N1 as opposed to the other unseeded cases in which the melting of precipitating ice is the biggest source of rain. Fig. 27 shows the cloud outlines for case B3/S1 at 21, 27 and 33 min. Comparing Fig. 27a with Fig. 5a, the main difference is in the early appearance of precipitating ice which comes mainly from the freezing of raindrops. Fig. 27b shows a broader distribution for ice hydrometeors, compared with Fig. 5b.

The differences between B3/S1 and B3/N1 for the various accumulated production terms are shown in Figs. 28 and 29. Fig. 28 clearly shows that interaction between rain and cloud ice ($P_{IA}$) is dramatically increased in the seeded case. Note that the influence of seeding on $P_{IA}$ is immediately apparent since this case has an efficient autoconversion process. Notice also that the increase in $P_{IA}$ leaves less rain avail-

---

**Fig. 20.** Numerical simulation of cloud and precipitation at (a) 21 min, (b) 27 min and (c) 30 min for case H3/S1. Cloud areas (100% relative humidity) are outlined by a solid line. Streamlines are dashed lines. Small solid circles and asterisks indicate rain and precipitating ice $> 1$ g kg⁻¹, respectively, and the $S$'s indicate cloud ice $> 0.1$ g kg⁻¹.
able for the probabilistic freezing process ($P_{fhr}$) which is decreased in the seeded case, and similarly less rain is available for accretional growth ($P_{dr}$). As is indicated in Fig. 29, the melting of precipitating ice shows the largest increase due to seeding. The shedding of water during the wet growth of precipitating ice ($P_{fhr}$) also shows a significant increase in the seeded case. There is, in fact, much more wet growth in the B3 cases than in any of the H1 and H3 cases. The decrease in accretional growth of rain ($P_{fhr}$) throughout most of the history of the seeded case is due to the fact that $P_{fhr}$ has significantly reduced the amount of rain in the cold region of the cloud.

The total rainfall on the ground at 51 min increases 28%. Though the percentage increase is smaller than the standard seeded case run with sounding H1, the total amount of increase is larger (23.3 kt km$^{-1}$ increase for cases B3/S1 versus 5.8 kt km$^{-1}$ increase for case H1/M1). The difference in the distribution of rain at the ground at 51 min is shown in Fig. 30. Not all of the area within the domain shows an increase in rainfall. The reason is not very clear, but is possibly due to circulation changes after seeding. The hailfall increases in the seeded case (Table 6). The hail embryos are produced from frozen raindrops through the interaction with cloud ice ($P_{IA}$, Fig. 28). This process ($P_{IA}$) transforms rainwater to precipitating ice very efficiently:

5. Summary and conclusions

A two-dimensional time-dependent cloud model has been used to test the effects of seeding on

strong convective clouds. All of the treated cases are seeded in the subcloud layer. A conservation equation is applied to trace the seeding agent which disperses along the airflow. Improvements over Orville and Kopp (1977) have been made in the microphysical processes for the transformations between liquid and ice hydrometeors. In addition to those cloud characteristics reported by Orville and Kopp (1977), these results also capture some important observed features of convective clouds.

1) Cloud ice initially forms in peripheral regions of the updraft core at the onset of rapid glaciation.

2) A time window for cloud glaciation is depicted by this model (Figs. 2a–2c).

<table>
<thead>
<tr>
<th>Cases</th>
<th>Rainfall†</th>
<th>Hailfall†</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1/N1</td>
<td>1.5</td>
<td>$1.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>H1/N3</td>
<td>2.3 (59%)*</td>
<td>$6.4 \times 10^{-5}$</td>
</tr>
<tr>
<td>H1/S1</td>
<td>7.0 (374%)</td>
<td>$9.8 \times 10^{-5}$</td>
</tr>
<tr>
<td>H1/S3</td>
<td>7.0 (201%)**</td>
<td>$1.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>H1/M1</td>
<td>7.3 (399%)</td>
<td>$1.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>H1/M2</td>
<td>2.7 (87%)</td>
<td>$1.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>H1/P1</td>
<td>2.4 (62%)</td>
<td>$1.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>H1/T1</td>
<td>4.1 (177%)</td>
<td>$4.7 \times 10^{-4}$</td>
</tr>
<tr>
<td>H3/N1</td>
<td>54.3</td>
<td>1.0</td>
</tr>
<tr>
<td>H3/S1</td>
<td>53.1 (−2%)</td>
<td>2.4 (143%)</td>
</tr>
<tr>
<td>B3/N1</td>
<td>83.6</td>
<td>$2.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>B3/S1</td>
<td>106.9 (28%)</td>
<td>1.0</td>
</tr>
</tbody>
</table>

* The numbers in the parentheses are the percentage increase or decrease compared to the standard unseeded cases.
** The percentage increase in rainfall for case H1/S3 is compared with case H1/N3.
† Units are kt km$^{-1}$. 
3) The circulation of the cloud influences the dispersion of the seeding agent significantly (Section 4c).

4) The seeding material is consumed in 10–15 min, mostly acting as deposition (and sorption) nuclei.

The total rainfall and hailfall at 51 min for each case are listed in Table 6. For clouds with inefficient precipitation processes, the model results indicate that the seeding could increase precipitation several fold, although total amounts are small (<10 kt km\(^{-1}\)). Moreover, the response of precipitation to seeding is closely related to the transport of the seeding agent. The model cloud produced by sounding H1 has one primary cell with vertical drafts. The crucial point for precipitation augmentation is to produce enough precipitating ice in the upper portion of the updraft core, the most favorable region for hail growth. All of the seeded cases seem to induce maximum precipitating ice in the peripheral regions of the clouds. This is due to the effect of the circulation in the cloud. Thus, the heavy seeding case (H1/M1) does not produce significant differences compared to the moderate seeding case (H1/S1).
For case H1/P1, the seeding position is not optimum. Most of the seeding agent is not dispersed into the updraft core. The precipitating ice content in the updraft core region is almost the same as in the natural case H1/N1. The life stage of the cloud is also an important factor. The more mature the cloud is, the less beneficial seeding will be (case H1/T1). Also, the concentration of natural ice nuclei affects the seeding results (cases H1/N3 and H1/S3). The more natural nuclei, the less effective seeding is. For the light seeding case (H1/M2), seeding does not create enough precipitating ice to give optimum results.

For the cloud with moderately efficient precipitation processes and strong convection (H3/N1), seeding does not increase the precipitation significantly, but does redistribute the precipitation. Due to the sloping updraft, part of the precipitating ice can fall into the updraft core and can be recycled by the strong updraft. This kind of cloud seems less susceptible to seeding and the hailfall could be increased after seeding.

The hail embryos come from different sources ($P_{IA}$) for the cloud with an active autoconversion process and lower cloud base (B3/N1) compared to cases H1/N1 and H3/N1. Though the seeding increases both the rainfall and hailfall, more tests are needed.

There are several deficiencies in all of these simulations:

1) The flow pattern cannot be captured exactly in a two-dimensional cloud model. This will affect the dispersion of the seeding agent. Our techniques are easily adaptable to three-dimensional models and can be tested in such models. However, 10–100 times more computer power is required for each case run.

2) The Marshall-Palmer type distribution for precipitating ice tends to melt the larger hailstones more efficiently than the small hailstones (Orville and Kopp, 1977). Consequently, the hailfall and the seeding effects on hailfall are suspect. Corrections are being made to the model hail melt equation.

3) The cloud produced by sounding H1 moves across the domain and out through the side boundary.
leading to uncertain effects induced by the lateral boundaries.

4) The initial seeding time and region for cases H3/S1 and B3/S1 are probably not optimum. Changing the time and location of seeding on these cases might lead to further insight.

The results of this simulation have shown the capability of this model to increase our understanding of the precipitating processes and their modifica-

---

Fig. 27. Numerical simulation of cloud and precipitation at (a) 21 min, (b) 27 min and (c) 33 min for case B3/S1. Otherwise as in Fig. 2.

Fig. 28. The differences in the production terms for precipitating ice (hail) as a function of time between cases B3/S1 and B3/N1. A positive value means an excess for the seeded case. See text for a description of the mnemonic codes.

Fig. 29. As in Fig. 28 except for precipitating rain.
tion due to seeding. Further improvements of the model are possible. Our recommendations are as follows:

1) To separate the nucleation efficiency curves for contact and deposition nucleation (for both natural and AgI ice nuclei).

2) To include phoretic processes (because this simulation underestimates the contact freezing nucleation).

3) To improve ice crystal aggregation (the rate of aggregation of cloud ice seems to be overestimated with the threshold \( l_{0l} \) set to zero).

4) To assign a distribution function for cloud ice (because the depositional growth rate of cloud ice using Köneig's scheme depends on the size of the ice crystal; a distribution function for cloud ice seems more plausible than the monodisperse size distribution used in this study).

The feasibility of a two-dimensional time-dependent cloud model to study cloud seeding effects is clearly displayed by this study. The seeded cases run in this simulation are presented in Fig. 31, a schematic showing the number of cases run and the changes in precipitation. Only a few points are filled in. For further understanding, more tests with different soundings and other seeding methods (cloud top and ground based AgI seeding and seeding with dry ice) are needed.

**Acknowledgments.** We thank Dr. Arnett S. Dennis for his review and suggestions concerning the work. Also we thank Mr. Mel Flanagan who drafted several of the figures, and Mrs. Joie Robinson who prepared the manuscript.

This research was sponsored by the Water and Power Resources Service, U.S. Department of the Interior, under Contracts 5-07-DR-12100 and 8-07-83-V0009. Acknowledgment is made to the National Center for Atmospheric Research, which is sponsored by the National Science Foundation, for most of the computer time used in this research, and to the Computer Center, South Dakota School of Mines and Technology, for the computer time used in the initial stages of this research.

---

**APPENDIX**

**List of Symbols**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>constant in empirical formula for ( V )</td>
<td>2115</td>
<td>cm(^{1-\phi} ) s(^{-1} )</td>
</tr>
<tr>
<td>( a' )</td>
<td>parameter in Cunningham correction</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>( a_3 )</td>
<td>parameter in Bergeron process</td>
<td></td>
<td>kg(^{-0.5} ) s(^{-1} )</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
<td>Value</td>
<td>Units</td>
</tr>
<tr>
<td>---------</td>
<td>-----------------------------------------------------------------------------------------------</td>
<td>--------</td>
<td>----------------</td>
</tr>
<tr>
<td>(a_d)</td>
<td>parameter in Bergeron process</td>
<td>0.8</td>
<td>s g (^{-1})</td>
</tr>
<tr>
<td>(b)</td>
<td>constant in empirical formula for (V_i)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td>mobility of the seeding agent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>mean free path of air molecules</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>(D_R)</td>
<td>diameter of raindrop</td>
<td></td>
<td>cm</td>
</tr>
<tr>
<td>(D_S)</td>
<td>diffusion coefficient of seeding agent</td>
<td></td>
<td>cm(^2) s (^{-1})</td>
</tr>
<tr>
<td>(E_{CW})</td>
<td>collection efficiency of ice crystals for cloud liquid</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(E_{RC})</td>
<td>collection efficiency of rain for cloud ice</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(E_{RS})</td>
<td>collection efficiency of raindrops for Ag(I) particles</td>
<td>0.5 \times 10(^{-4})</td>
<td></td>
</tr>
<tr>
<td>(E_{WS})</td>
<td>collection efficiency of cloud droplets for Ag(I) particles</td>
<td>10(^{-4})</td>
<td></td>
</tr>
<tr>
<td>(k)</td>
<td>Boltzmann's constant</td>
<td>1.38 \times 10(^{-23})</td>
<td>J K (^{-1})</td>
</tr>
<tr>
<td>(K_m)</td>
<td>momentum eddy coefficient</td>
<td></td>
<td>m(^2) s (^{-1})</td>
</tr>
<tr>
<td>(K_h)</td>
<td>heat eddy coefficient</td>
<td></td>
<td>m(^2) s (^{-1})</td>
</tr>
<tr>
<td>(l_{CW})</td>
<td>mixing ratio of cloud water</td>
<td></td>
<td>g g (^{-1})</td>
</tr>
<tr>
<td>(l_{CI})</td>
<td>mixing ratio of cloud ice</td>
<td></td>
<td>g g (^{-1})</td>
</tr>
<tr>
<td>(l_{CI50})</td>
<td>mixing ratio of cloud ice which is transformed to precipitating ice by depositional and riming growth</td>
<td></td>
<td>g g (^{-1})</td>
</tr>
<tr>
<td>(l_i)</td>
<td>mixing ratio of precipitating ice</td>
<td>0</td>
<td>g g (^{-1})</td>
</tr>
<tr>
<td>(l_{OR})</td>
<td>delay of aggregation of cloud ice</td>
<td></td>
<td>g g (^{-1})</td>
</tr>
<tr>
<td>(l_b)</td>
<td>mixing ratio of rain</td>
<td></td>
<td>g g (^{-1})</td>
</tr>
<tr>
<td>(m_{1})</td>
<td>mass of a 10 (\mu) m size ice crystal</td>
<td>3.84 \times 10(^{-9})</td>
<td>g</td>
</tr>
<tr>
<td>(m_{140})</td>
<td>mass of a 40 (\mu) m size ice crystal</td>
<td>2.46 \times 10(^{-7})</td>
<td>g</td>
</tr>
<tr>
<td>(m_{150})</td>
<td>mass of a 50 (\mu) m size ice crystal</td>
<td>4.80 \times 10(^{-7})</td>
<td>g</td>
</tr>
<tr>
<td>(m_n)</td>
<td>mass of a natural ice nucleus</td>
<td>1.05 \times 10(^{-15})</td>
<td>g</td>
</tr>
<tr>
<td>(m_S)</td>
<td>mass of an Ag(I) particle</td>
<td>2.38 \times 10(^{-14})</td>
<td>g</td>
</tr>
<tr>
<td>(n_0)</td>
<td>parameter in Fletcher's equation</td>
<td>10(^{-5})</td>
<td>L (^{-1})</td>
</tr>
<tr>
<td>(n_{OR})</td>
<td>number of raindrops per unit diameter</td>
<td>0.08</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(N_a)</td>
<td>number of active Ag(I) nuclei</td>
<td></td>
<td>L (^{-1})</td>
</tr>
<tr>
<td>(N_{AD})</td>
<td>number of active Ag(I) nuclei which work as deposition nuclei</td>
<td></td>
<td>g (^{-1})</td>
</tr>
<tr>
<td>(N_{AR})</td>
<td>number of active Ag(I) nuclei captured by raindrops</td>
<td></td>
<td>g (^{-1})</td>
</tr>
<tr>
<td>(N_{AW})</td>
<td>number of active Ag(I) nuclei captured by cloud droplets</td>
<td></td>
<td>g (^{-1})</td>
</tr>
<tr>
<td>(N_{Cn})</td>
<td>number of active contact ice nuclei</td>
<td></td>
<td>g (^{-1})</td>
</tr>
<tr>
<td>(N_{kn})</td>
<td>number of active deposition ice nuclei</td>
<td></td>
<td>g (^{-1})</td>
</tr>
<tr>
<td>(N_{JO50})</td>
<td>number of hypothetical 50 (\mu) m size ice crystal</td>
<td></td>
<td>g (^{-1})</td>
</tr>
<tr>
<td>(N_{kn})</td>
<td>number of active natural ice nuclei</td>
<td></td>
<td>L (^{-1})</td>
</tr>
<tr>
<td>(N_w)</td>
<td>number of cloud droplets</td>
<td>1000</td>
<td>cm(^{-3})</td>
</tr>
<tr>
<td>(P_{CAR})</td>
<td>rate cloud ice accretes rain to form precipitating ice</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{CSWC})</td>
<td>rate cloud water transforms to cloud ice due to contact freezing with Ag(I) particle</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{CNWD})</td>
<td>rate of depositional growth of natural ice nuclei</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{CSWD})</td>
<td>rate of depositional growth of Ag(I) particles</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IA})</td>
<td>rate precipitating ice is produced</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IAC})</td>
<td>rate precipitating ice interacts with cloud ice to form precipitating ice</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IAC})</td>
<td>rate precipitating ice accretes cloud ice in wet growth</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IAR})</td>
<td>rate precipitating ice accretes cloud ice in dry growth</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IAR})</td>
<td>rate precipitating ice accretes rain in dry growth</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IAR})</td>
<td>rate rain actually frozen or shed in wet growth</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IAW})</td>
<td>rate precipitating ice accretes cloud water</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IC})</td>
<td>rate of aggregation</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{ID})</td>
<td>rate of dry growth</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IF})</td>
<td>rate rain and cloud liquid freeze</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IFC})</td>
<td>rate cloud ice grows to form precipitating ice</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IFR})</td>
<td>rate rain is freezing</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IFW})</td>
<td>rate cloud water is freezing</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>(P_{IM})</td>
<td>rate precipitating ice melts</td>
<td></td>
<td>g g (^{-1}) s (^{-1})</td>
</tr>
<tr>
<td>Notation</td>
<td>Description</td>
<td>Value</td>
<td>Units</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>$P_{IS}$</td>
<td>rate sublimation of precipitating ice</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$P_{IW}$</td>
<td>rate of wet growth</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$P_{R}$</td>
<td>rate rain is produced</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$P_{RA}$</td>
<td>rate rain accretes cloud water</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$P_{RAC}$</td>
<td>rate rain accretes cloud ice to form precipitating ice (contribution is from cloud ice content)</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$P_{RC}$</td>
<td>rate of autoconversion</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$P_{RE}$</td>
<td>rate rain evaporates</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$q$</td>
<td>mixing ratio of water vapor, cloud water, and cloud ice</td>
<td>$\mu g$</td>
<td>$\mu g$</td>
</tr>
<tr>
<td>$R_{R50}$</td>
<td>radius of hypothetical ice crystal</td>
<td>50</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$R_{S}$</td>
<td>radius of Agl particle</td>
<td>0.1</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$R_{W}$</td>
<td>radius of cloud droplet</td>
<td>10</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$S_{BR}$</td>
<td>sink of Agl due to rainwater by Brownian collection</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$S_{BW}$</td>
<td>sink of Agl due to cloud water by Brownian collection</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$S_{REP}$</td>
<td>sink of Agl due to deposition nucleation</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$S_{R}$</td>
<td>sink of Agl due to inertial impact collection</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$S_{IW}$</td>
<td>sink of Agl due to cloud water by inertial impact collection</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$S_T$</td>
<td>total sink of seeding agent</td>
<td>$g$</td>
<td>$g \cdot s^{-1}$</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>time step for numerical integration</td>
<td>$s$</td>
<td>$s$</td>
</tr>
<tr>
<td>$\Delta t_1$</td>
<td>time needed for ice crystal growth from 40 to 50 $\mu m$</td>
<td></td>
<td>$K$</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
<td></td>
<td>$C$</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>supercooling</td>
<td></td>
<td>$m s^{-1}$</td>
</tr>
<tr>
<td>$U_1$</td>
<td>terminal velocity of precipitating ice</td>
<td></td>
<td>$m s^{-1}$</td>
</tr>
<tr>
<td>$V$</td>
<td>velocity of air</td>
<td></td>
<td>$m s^{-1}$</td>
</tr>
<tr>
<td>$V_{R50}$</td>
<td>terminal velocity of hypothetical ice crystal</td>
<td>1</td>
<td>$m s^{-1}$</td>
</tr>
<tr>
<td>$V_t$</td>
<td>terminal velocity of rain</td>
<td></td>
<td>$m s^{-1}$</td>
</tr>
<tr>
<td>$X_S$</td>
<td>mixing ratio of seeding agent</td>
<td></td>
<td>$g^{-1}$</td>
</tr>
<tr>
<td>$\beta_t$</td>
<td>parameter in Fletcher's equation</td>
<td></td>
<td>$c^{-1}$</td>
</tr>
<tr>
<td>$\lambda_t$</td>
<td>parameter in rain distribution</td>
<td></td>
<td>$cm^{-1}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>dynamic viscosity of air</td>
<td></td>
<td>$cm g^{-1} s^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of air</td>
<td></td>
<td>$g cm^{-3}$</td>
</tr>
</tbody>
</table>

REFERENCES


