Point Probability Distributions of Frozen Soil*

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ABSTRACT

In some areas of the Pacific Northwest, frozen soils play a major role in surface runoff, soil erosion and sedimentation, but quantitative descriptions of the frequency and severity of soil frost are lacking.

The temporal characteristics of frozen soil in north-central Oregon were quantified by using a soil frost simulation model to calculate the presence or absence of frozen soil over a 30-year time period. The soil frost model correctly predicted the presence or absence of soil frost 80% of the time for a 5-year field calibration period. November through March simulation results for a 30-year time period showed that frozen soil occurred every year, and the soil was frozen an average of 57 days per year. The highest incidence of frozen soil occurred during January when the soil is frozen 67% of the time followed closely by February (53%) and December (51%). The number of freeze-thaw cycles other than diurnal cycles varied from 1 to 7 per year with an average of 3 per year for the simulation period.

1. Introduction

Soil frost or frozen soil occurs when the soil surface freezes and moisture migrates from the deeper soil layers to the freezing front and then freezes. Water infiltration rates in frozen soil are determined partly by the structure of the soil frost and the soil water content at the time of freezing (Storey, 1955). Trimble et al. (1958) proposed four terms to describe soil frost structure: granular, honeycomb, stalactite, and concrete. Concrete frost has a very low permeability and is the most common type of frost formed in bare agricultural soils (Storey, 1955; Trimble et al., 1958).

Soil frost strongly influences the hydrology in midlatitude climates characterized by low winter temperatures and transient shallow snow packs. When the soil freezes, infiltration rates decrease and runoff increases. In contrast to areas of permafrost, frozen soil in the midlatitude climates is transient and the soil may freeze and thaw several times each winter season. In addition, diurnal freeze-thaw cycles which involve only the top few millimeters of soil are a common phenomenon, especially on unprotected, tilled soils (Pikul and Allmaras, 1985). There is evidence to suggest that the number of freeze-thaw cycles may be a very important factor in reducing the shear strength and hence increasing the erodibility of the soil surface, especially in tilled soils with little vegetative cover (Formanek et al., 1984). In the Pacific Northwest, where there are dramatic differences in climate over relatively small geographic areas, quantitative descriptions of the frequency and severity of frozen soil are not available.

There is a strong association of frozen soils with increased runoff, flood producing runoff, erosion, and sediment production (Tigerman and Rosa, 1951; Storey, 1955; Johnson and McArthur, 1973). In parts of the Pacific Northwest east of the Cascade Mountains and in parts of the Intermountain West, frozen soils play a major role in runoff production, soil erosion and sedimentation (Yen and Molnau, 1982; Zuzel et al., 1982). Johnson and McArthur (1973) show that the most severe recent floods in the Northwest Interior occurred when the soil was frozen.

The frequency of freeze-thaw cycles has been described using air temperature data from established weather stations (Russell, 1943; Visher, 1945, 1946; Hershfield, 1974; Brown et al., 1984). This explicitly includes diurnal freeze-thaw cycles but produces no information on the persistence of soil frost after extended periods of below freezing air temperatures. Consequently, freeze-thaw cycles described by air temperature provide useful climatological data, but these procedures do not adequately describe whether the soil is frozen or not. Soil characteristics, the topographic variables of slope and aspect, and the influence of soil surface conditions, i.e., litter, crop residue, vegetation, and snow depth all influence soil freezing and thawing. The purpose of this research was to develop probability distributions of soil frost by using historical National Weather Service records and a soil frost model to simulate the presence or absence of frozen soil.

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Frozen soil was simulated using a soil frost model described by Cary (1982) because it is physically based and the only required inputs are maximum and minimum air temperature, snow depth and solar radiation. The model predicts net daily heat flux across the soil surface. The cumulative daily soil heat flux is then used to predict the presence or absence of soil frost. Negative values of cumulative soil heat indicate the presence of soil frost while positive values indicate an unfrozen soil. Cary (1982) gives the soil frost model as

\[ P = \sum_{j=1}^{J} G_n + G_u \]  

where \( P \) is the cumulative daily soil heat flux; \( J \) is the Julian day; \( G_n \) is the net heat flux per unit area across the soil surface during a 24-hour period; and \( G_u \) is the heat per unit area that flows upwards during a 24-hour period in winter.

Net heat flux, \( G_n \), across the soil surface can be expressed as

\[ G_n = \frac{\lambda}{\pi} \left[ \frac{(T_h - T_{lw})d_w}{d_w} - \frac{(T_h - T_{lc})d_c}{d_c} \right] \]

where \( \lambda \) is the soil thermal conductivity; \( T_{lw} \) is the soil surface low temperature; \( T_h \) is the soil surface high temperature; \( T_{lc} \) is the soil surface low temperature which occurs the following morning; \( d_w \) and \( d_c \) are the damping depths for the warming and cooling periods, respectively; and \( t_{lw} \) and \( t_{lc} \) represent the hours of warming and cooling, respectively.

Using detailed field datasets, Cary developed empirical relationships between available weather station measurements and the variables in Eq. (2). These relationships are

\[ T_h = T_{max} + C \]

where \( T_{max} \) is the maximum air temperature and \( C \) is a factor which accounts for the effects of solar radiation on soil temperature and is approximated by

\[ C = D(S_D/S_D)\text{1/2} \]

where \( D \) is an empirical site specific constant (calibra-

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### Table 1. Topographic characteristics of sites near Moro, Oregon used to verify the frost prediction model.

<table>
<thead>
<tr>
<th>Site and winter number</th>
<th>Elevation (m)</th>
<th>Year</th>
<th>Slope (percent)</th>
<th>Aspect (deg)</th>
<th>Frost prediction* (percent)</th>
<th>Distance from Moro weather station (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>561</td>
<td>1979/80</td>
<td>14</td>
<td>S40W</td>
<td>83</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>567</td>
<td>1980/81</td>
<td>11</td>
<td>S61W</td>
<td>82</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>539</td>
<td>1981/82</td>
<td>24</td>
<td>N65E</td>
<td>91</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>539</td>
<td>1982/83</td>
<td>25</td>
<td>N65E</td>
<td>63</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>539</td>
<td>1983/84</td>
<td>24</td>
<td>N65E</td>
<td>80</td>
<td>2</td>
</tr>
</tbody>
</table>

* Percent of days between 1 November and 31 March in which frost occurrence was correctly predicted using a site constant of 1.0 and an initial soil heat of 4190 kJ m\(^{-2}\).
tion parameter); $S_i$ is observed solar radiation; and $S_{i0}$ is potential solar radiation.

$T_{iw}$ and $T_{lc}$ are calculated as

$$T_{iw} = T_i + 0.5$$  \hspace{1cm} (5)

$$T_{lc} = T_{iw} + 0.5$$  \hspace{1cm} (6)

where $T_i$ is the daily minimum air temperature for the current day and $T_{iw}$ is the daily minimum temperature for the next day.

The soil warming period is calculated from the relationship:

$$t_w = C - 1 + 2 \cos(J/57.3) + Y + t_0$$  \hspace{1cm} (7)

where $t_0$ is the hours of daylight on day $J$ and $Y$ is a factor whose value depends on where the maximum/minimum air temperatures fall with respect to 0°C. Values of the variable $Y$ are calculated from

$$Y = 0 \quad \text{for} \quad T_h \geq 0$$  \hspace{1cm} (8)

$$Y = -[1 + \ln(1 - T_h)] / 12 \quad \text{for} \quad T_h < 0.$$  \hspace{1cm} (9)

$$Y = \ln(T_h - T_{lw}) / 12 \quad \text{for} \quad T_h > 0 \quad \text{and} \quad T_{lw} > 0.$$  \hspace{1cm} (10)

Damping depth for the warming and cooling periods is calculated from the functions:

$$d = 15 T_a / (T_a - 1)$$  \hspace{1cm} (11)

where $T_a$ is an average soil temperature calculated from:

$$T_a = -0.13 \bar{T}^2 + 0.2 \bar{T} - 0.8$$  \hspace{1cm} (12)

where

$$\bar{T} = 0.7(T_h - T_{lw}) + T_{iw}$$  \hspace{1cm} (13)

for the warming period and

$$\bar{T} = 0.3(T_h - T_{lw}) + T_{lc}$$  \hspace{1cm} (14)

for the cooling period.

The upward heat flux, $G_u$ was calculated as

$$G_u = (A - 0.2 T_a) \sin[(J + 80)/57.3]$$  \hspace{1cm} (15)

where $A$ is an empirical constant.

For snow on the ground and maximum or minimum air temperatures greater than 0°C, soil surface temperatures are calculated from:

$$T_h, T_{lc}, T_{iw} = 0$$  \hspace{1cm} (16)

and for snow on the ground and maximum or minimum air temperatures less than 0°C, soil surface temperatures are calculated as

$$T_h, T_{lc}, T_{iw} = 5T/(I + 5)$$  \hspace{1cm} (17)

where $T$ is the daily maximum or minimum air temperature (°C) and $I$ is the snow depth (cm).

In this study, the empirical relationships given by Cary were used without modification and the constant $A$ in Eq. 15 was assigned a value of 1.5. It was necessary to choose a model that includes radiation effects because the field calibration sites were located on steep slopes with various aspects (Table 1). Since solar radiation is not a readily available variable, a procedure was developed to predict daily values of solar radiation from potential solar radiation and daily temperature range. The model used was that of Cengiz et al. (1981):

$$S_i = 49.03 + 0.10 S_{i0} - 7.26 T_r + 0.06 S_{i0} T_r$$  \hspace{1cm} (18)

where $S_i$ is the estimated daily solar radiation reaching the soil surface; $S_{i0}$ is the potential solar radiation reaching the surface and $T_r$ is the daily temperature range. Daily potential solar radiation at the surface was calculated from

$$S_{i0} = \frac{60}{r^2} \left[ \cos \delta \cos \theta (\sin S_i - \sin S_r) 12 / \pi \right. \hspace{1cm}$$

$$+ \sin \delta \sin \theta t_0 12 / \pi \right] [0.29 \cos \theta + 0.52]$$  \hspace{1cm} (19)

where $J_0$ is the solar constant (1.35 kW m⁻²); $\theta$ is the latitude; $\delta$ is the solar declination; $S_r$ is time of sunrise; $S_i$ is time of sunset and $r$ is the radius vector of the sun.

Solar declination can be approximated from

$$\delta = \arcsin\left(0.39785 \sin[4.868961 + 0.17203 J] \right.$$  \hspace{1cm} (20)

$$+ 0.033446 \sin(6.224111 + 0.017202 J)) \right]$$

and the radius vector, sunrise, sunset, and daylength are calculated from

$$r = 1 - 0.0167 \cos[0.017203 (J - 3)]$$  \hspace{1cm} (21)

$$s_r = \arccos(\tan \theta \tan \delta)$$  \hspace{1cm} (22)

$$S_s = \arccos(-\tan \theta \tan \delta)$$  \hspace{1cm} (23)

$$t_0 = (S_s - S_r)/15.$$  \hspace{1cm} (24)

Values of $S_i$ and $S_{i0}$ can now be used in Eq. 4 to calculate $C$ and $t_0$ used in Eq. 7 to calculate the soil warming period. A procedure to correct the predicted radiation and daylength for slope and aspect using the algorithm described by Swift (1976) was also included.

The model requires an initial value of soil heat [$P$, Eq. (1)] and an empirical, site-specific constant [$D$, Eq. (4)] which accounts for the effects of cover and other soil physical properties (Cary, 1982). An initial soil heat of 4190 kJ m⁻² was estimated using the average soil temperature and heat capacity in the top 30 cm of soil (Cary, 1982). The site constant was obtained by maximizing the number of days the model correctly predicted the presence or absence of soil frost. Soil frost was simulated during 1 November–31 March for each of the winters listed in Table 1.

Soil frost for a 30 year period was simulated using National Weather Service records for Moro, Oregon. Daily maximum air temperature, minimum air temperature and snow on the ground for November through March, 1948–78, were used in the simulation. The field verified site constant and initial soil heat were
Table 2. Some climatic characteristics related to soil frost during the 1 November and 31 March winter period at Moro, Oregon.

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Precipitation (mm)</td>
<td>185</td>
<td>169</td>
<td>211</td>
<td>210</td>
<td>218</td>
<td>180</td>
</tr>
<tr>
<td>Average air temperature (°C)</td>
<td>2.05</td>
<td>0.87</td>
<td>2.21</td>
<td>2.06</td>
<td>2.14</td>
<td>3.07</td>
</tr>
<tr>
<td>Average snow depth (cm)</td>
<td>10.13</td>
<td>6.62</td>
<td>3.30</td>
<td>11.72</td>
<td>4.00</td>
<td>6.39</td>
</tr>
<tr>
<td>Days with snow on ground</td>
<td>24</td>
<td>32</td>
<td>20</td>
<td>32</td>
<td>8</td>
<td>31</td>
</tr>
</tbody>
</table>

* Average value for days with snow on the ground.

not varied from year to year. Model output was used to characterize and quantify soil frost at the Moro site using standard methods of hydrologic frequency analysis (Linsley et al., 1975; Haan, 1977). Plotting positions for these data were calculated using the relationship suggested by Weibull (1939).

3. Results and discussion

An analysis of 406 observed daily solar radiation values versus radiation values predicted by Eq. (16) yielded a correlation coefficient of 0.81. Cengiz et al. (1981) reported a correlation coefficient of 0.87 using Eq. (16) versus observed solar radiation.

For the 1980–84 calibration period, a site constant of 1.00 and an initial soil heat of 4190 kJ m⁻² produced the maximum number of correct predictions for the presence or absence of soil frost when compared to the field observations. The percentage of correct predictions for each year is shown in Table 1. Both the site constant and initial soil heat are within the range of values suggested by Cary (1982). Overall, the model correctly predicted the presence or absence of soil frost 80% of the time for the 5-year calibration period. The differences between the 1980–84 test years and the 1948–78 mean values for precipitation, average air temperature, snow on the ground, and number of days with snow cover are listed in Table 2.

The 1982/83 winter had the lowest percentage of days with correctly predicted soil frost. This winter also had the least snow depth and days with snow on the ground (Tables 1 and 2). During the winter of 1982–83 the soil froze deeply but also thawed rapidly during clear, warm weather. The low accuracy of prediction during the 1982–83 winter appears to be a consequence of the sluggish response of the frost model to rapidly warming conditions.

An example of the simulation model output and the presence or absence of soil frost for 1983/84 is shown in Fig. 2. Values of soil heat less than zero indicate the presence of soil frost, while values greater than zero indicate a thawed soil condition. It is not possible with this model to directly infer frost depths from soil heat, although, in general, lower soil heat values indicate deeper frost depths, and increases in soil heat while the soil is frozen indicate surface thawing.

Model output for the 30-year weather record at Moro, Oregon (Table 2) suggests that frozen soils occur every winter at this site. The calculations apply to a level surface so that differences in slope and aspect are not considered. The soil was frozen from 6 to 116 days per year with an average of 57 days per year. Freeze–thaw cycles varied from 1 to 7 cycles per year. A freeze–thaw cycle as defined for this study is a day when the soil heat changes from positive to negative. The exceedance probability of freeze–thaw cycles is plotted in Fig. 3. We modeled the number of freeze–thaw cycles per year with a gamma distribution having a scale parameter of 2.591 and a shape parameter of 0.925. The parameters were estimated from the model derived data using the method of maximum likelihood. The fitted gamma distribution and the estimated parameters are shown in Fig. 3. Correlation analysis of the model derived probabilities and the theoretical probabilities produced a correlation coefficient of 0.991 with a standard error of 0.04.

The distribution of predicted soil frost by months is shown in Fig. 4 as a ratio of total frozen to total non-frozen days for the 1948–78 period. The soil frost model output shows that historically, January has the highest incidence of soil frost; the soil was frozen 67% of the time. Also, model output indicates that the soil was frozen more than 50% of the time in both December and February. Our field data during the calibration period failed to show the presence of soil frost during the month of March, but examination of mean daily

![Fig. 2. Soil frost simulation model output and measured frost for Moro, Oregon, 1983/84. Modeled soil heat values < zero predict the presence of soil frost.](image)
air temperatures for the 30-year period revealed that mean daily air temperatures were less than 0°C 8% of the time. This demonstrates that frozen soils can occur in March. In general, the simulation model appears to thaw the soil too slowly during the spring warming period (Fig. 2) and the distribution of soil frost during March should be interpreted with caution.

A large range of from 6 to 116 days per year of frozen soil can be expected at Moro, Oregon. The cumulative frequency distribution derived from the model and the fitted theoretical distribution of the number of frozen soil days per year is plotted in Fig. 5. The model derived frequencies were approximated with a normal distribution. We chose the normal distribution because the model derived data appears to be normal in the first 4 central moments, i.e.; the coefficient of skewness is not different from zero. The parameters of the fitted normal distribution are also shown in Fig. 5.

In order to test the goodness of fit of the theoretical distribution, we performed a correlation analysis using the exceedance probabilities derived from model output and the theoretical exceedance probabilities. This test yielded a correlation coefficient of 0.976. We also applied the Kolmogorov–Smirnov test and were not able to reject the hypothesis of a normal distribution. An obvious difference in the distributions exists in the lower tail because the normal distribution is unbounded while the model derived data has a lower bound of 6. From these analyses we concluded that the normal distribution can with the indicated parameters (Fig. 5), be used to represent the simulation model distribution on the number of frozen soil days per year.

Recurrence interval is defined as the average number of years within which a given event will be equalled or exceeded (Linsley et al., 1975). It is the reciprocal of the exceedance probability. A tabulation of selected recurrence intervals derived from the fitted distribution for days of frozen soil per year is presented in Table 3. The 2-year recurrence interval (50% exceedance probability) is consistent with the 5 years of field data used for model verification. During the 1980–84 period the 30-year average of 57 days of frozen soil was exceeded only during the 1983/84 winter.

A knowledge of the distribution of the number of days per year of soil frost will be useful not only in runoff and sediment prediction but also in planning any activity, such as construction excavation, which cannot be readily accomplished while the ground is frozen. Runoff and sediment production depend on the infiltration rate of the soil in nearly all predictive

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**Fig. 3.** Cumulative frequency and fitted gamma distribution of the number of freeze–thaw cycles per year at Moro, Oregon, 1948–78.

**Fig. 4.** Monthly distribution of frozen soil expressed as a ratio of days with frozen soil to number of days in the month for 1948–78.

**Fig. 5.** Cumulative frequency and fitted normal distribution of the number of frozen soil days per year at Moro, Oregon, 1948–78.
models. If the infiltration rate is adjusted to reflect frozen or unfrozen soil conditions, it should result in more accurate runoff and sediment prediction. The procedures shown here can also be used to develop joint probability distributions of rain on snow with frozen or unfrozen soil and rain on frozen or unfrozen soil in the absence of snow cover. Historical climatological data can be used as input to the soil frost model and screening algorithms, such as described by Zuzel and Greenwalt (1985), used to delineate the antecedent conditions.

4. Summary and conclusions

The temporal characteristics of frozen soil in the Moro, Oregon area were quantified by using 30 years of weather records as input to a soil frost simulation model. November through March simulation results indicated that frozen soils occur each year. The number of freeze–thaw cycles varied from 1 to 7 per year with an average of 3 per year and the number of days the soil was frozen ranged from 6 to 116 with a mean of 57 days per year. A knowledge of the number of days of frozen soil and the monthly distribution of frozen soil is useful in runoff and erosion prediction and for planning any activity which cannot be readily accomplished while the soil is frozen. The number of freeze–thaw cycles per year is a major factor in reducing the shear strength of the soil surface and increasing soil erodibility. Freeze–thaw cycle distributions can be used as a planning tool for structures and roadbeds in areas where frost heaving is a potential problem. The procedures used in this report can be used in any area where a reasonably complete historical weather record is available to develop the required probability distributions.

REFERENCES


