Some Additional Coherence Data in the Inertial Subrange

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ABSTRACT

Some coherence measurements, taken at the Nässudden wind energy test site, for small separation distances at a height of 65–77 m, are analyzed in terms of a theoretical coherence model for isotropic turbulence, and for Davenport similarity.

The decay parameters for the horizontal wind components are found to be about the same for vertical and horizontal separation in the inertial subrange.

1. Introduction

Many practical and theoretical problems in atmospheric boundary layer research need information regarding the spatial structure of turbulence. Lateral and vertical correlations of wind speed have important applications to technical problems involving buildings, bridges and wind turbines. Load and fatigue calculations for large wind energy conversion systems require knowledge of simple statistical relations of the horizontal wind components at separated points on the blades.

However, lateral coherence measurements at higher levels (50–75 m) are sparse (Shiotani and Iwatani, 1979; Kristensen and Jensen, 1979; Eckman and Højstrup, 1985) and accordingly there is a great interest in testing the theoretical model proposed by Kristensen and Jensen (1979) and Irwin (1979). They show that when the separation distance is small compared to the height, only eddies in the inertial subrange part of the spectrum will contribute to the coherence, which can be expressed with a simple exponential function (Davenport similarity).

2. Site and instrumentation

The wind energy test site, Nässudden, is situated at the southwest coast of the island of Gotland in the Baltic. The area is flat and, in the main, sparsely covered with juniper bushes. The roughness length is estimated to $z_0 = 0.04$ m for the sector used in this investigation.

During August 1986 a special measuring campaign concerning coherence was performed. At the 145 m meteorological tower, a 9 m horizontal boom was fixed at a height of 65 m. When the wind was from a sector perpendicular to the boom, turbulence instruments were placed at both ends, as well as at the height of 77 m (hub height). The turbulence data were obtained using the MIUU turbulence instrument, a fast response wind-vane based hot-wire system, which measures the instantaneous values of temperature, wind direction and the three components of the wind (Högström et al., 1980; Högström, 1982). Data were sampled at 20 Hz in 30 min periods.

The dataset consists of 50 runs during stability conditions ranging from $z/L = -2.5$ to $z/L = 1.1$, where $L$ is the Monin–Obukhov length. The wind speed varied from 2.5 up to 14.1 m s$^{-1}$. However, the horizontal boom started to vibrate when the wind speed was higher than 8–9 m s$^{-1}$, creating false correlations for frequencies at about 1 Hz. Because of that, only coherence values up to $n = 0.1$ are used in this analysis. The wind direction varied ±20° around the direction perpendicular to the boom and thus the separation distance has to be corrected for that deviation. Figure 1 shows an example of spectra of the longitudinal and vertical wind components for the two laterally separated instruments. The agreement between measured spectra is good. They show an inertial subrange with the usual $-\frac{5}{3}$ slope and the relation $S_u(n)/S_w(n) = \frac{3}{5}$ is satisfied.

3. The models

The coherence is given by

$$\text{Coh}(n) = \frac{[\text{Co}(n) + Q^2(n)]/[S_1(n)S_2(n)]}{3}$$

(1)

where $n$ is the frequency, $Co$ the cospectral density, $Q$ the quadrature spectrum density, and $S_1$ and $S_2$ are the spectral densities of the velocity components at two points separated by the distance $D$.

As pointed out by Kristensen and Kirkegaard (1986), the result of coherence calculations depends on the degree of freedom ($f$) used in the spectral calculations. With a finite number of degrees of freedom there always will be an overestimation of the coherence. Kristensen and Kirkegaard also give an expression for correcting
the coherence when \( f \) is known. The correction decreases with increasing \( f \) and increasing coherence. In this investigation, \( f \) varies with frequency from \( f = 4 \) at \( n = 0.001 \) to \( f = 400 \) at \( n = 0.1 \), which gives some overestimation of the coherence at low frequencies.

Davenport (1961) proposed that coherence will only be a function of the ratio of separation distance \( D \) to wavelength \( \lambda \) in the longitudinal direction, and thus, with Taylor's hypothesis, the coherence can be written

\[
\text{Coh}(n) = F(nD/\bar{u})
\]

where \( \bar{u} \) = mean wind speed.

Davenport also suggested an exponential form of the coherence function

\[
\text{Coh}(n) = \exp(-anD/\bar{u}).
\]

Several investigations have confirmed Eq. (3) and determined the decay parameter \( a \), especially for vertical separations (i.e., Davenport, 1967; Pielke and Panofsky, 1970; Perry et al., 1978; Soucy et al., 1982). However, some measurements (Roepelwski et al., 1973; Iwatani, 1977; Kristensen et al., 1981) show that Eq. (3) is only an approximation, which can be used for practical applications.

A model assuming homogeneous, stationary and isotropic turbulence has been derived by Kristensen and Jensen (1979) and Irwin (1979) for lateral separation. Later in 1983 Kristensen et al. expanded the theory to vertical separation and no wind shear, allowing small deviations from isotropy. The models relate the behavior for the coherence function to the ratio \( D/L \), where \( L \) is the integral scale of turbulence.

As—at least in neutral air—\( L \) is proportional to \( z \), the result can be summarized in the following way. When \( D/z \ll 1 \), only eddies in the inertial subrange contribute to the coherence, which can be expressed with Eq. (3). If, however, the separation distance and the height are of the same order (\( D/z \approx 1 \)) the coherence is not a monotonical function any more and will not approach unity when \( n \to 0 \). There are no eddies large enough to make the coherence unity. The vertical component is more sensitive to an increase in the ratio \( D/z \) than the horizontal ones.

4. Results

Measured coherence for lateral separation are shown in Figs. 2 and 3 for the \( u \)- and \( v \)-components for near-neutral stratification with \( D/z = 0.15 \). The solid lines are given by Eq. (3) with the decay parameter values \( a_{u}^{*} = 14 \) and \( a_{v}^{*} = 7 \), which are the same as given by Kristensen et al. (1981). The coherence measurements for \( u \) and \( v \) taken during stable and unstable conditions give the same decay parameters, a behavior which is expected in the inertial subrange. An example is shown in Fig. 4 for the \( v \)-component in stable air.
The coherence for the vertical component exhibits much more scatter, but the measurements can be approximated with an exponential function with the decay parameter $a_z^{w} = 10$ for unstable and near-neutral stratification. However, in stable air the integral scale $L$ is of the same order as $D$ and the coherence is no longer a monotonically decreasing function. In Fig. 5, three curves for different stabilities and $D/L$ values are plotted as functions of $nD/u$. The peak values are displaced towards higher ($nD/u$) values as stability and hence $D/L$ increases. The general behavior is similar to that of the theoretical curves obtained by Kristensen and Jensen (1979). The positions of the maxima are about the same, but the coherence values are higher ($\approx 0.9$) for all three cases. This could reflect the low number of degrees of freedom in this part of the frequency range.

For the vertical separation the instrumental setup at Näsudden gives $D/z = 0.17$. Figures 6 and 7 show coherence measurements of the $u$- and $w$-component in near-neutral air together with the relations obtained with the decay parameters $a_z^{u} = 14$ and $a_z^{w} = 5$.

For the lateral component, measurements give $a_z^{v} = 8$. The decay parameter for the horizontal wind components agree with the values referred to in Panofsky and Dutton (1984). Again there is no variation in decay parameters with stability except for the vertical component in very stable air, where the coherence curves resemble the behavior found for lateral separation.
5. Conclusions

Coherence measurements taken at small separation distances \((D/z \ll 1)\) give some support to the theoretical model for isotropic turbulence proposed by Kristensen and Jensen (1979). The coherence functions of the horizontal wind components can thus be approximated with an exponential expression for all stability conditions, and the decay parameters in the inertial subrange region agree with the finding of others:

\[ a_y^u = a_z^u = 14 \]

and

\[ a_y^v \approx a_z^v = 7-8. \]

Hence lateral and vertical coherence for the horizontal components are equally strong in the inertial subrange.

The decay parameters fitted to measurements of the \(w\)-component in unstable and near neutral air, \(a_y^w = 10\) and \(a_z^w = 5\), are not the same as found in other investigations. However, the relative order of the decay constants for lateral separation \(a_y^u < a_y^w < a_y^v\) and \(a_z^w < a_z^v < a_z^u\) for vertical separation agrees with the calculations in Kristensen et al. (1983) for \(D/L \ll 1\). For stable stratification the coherence function for the vertical component cannot be expressed with an exponential function.

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REFERENCES


