Comparison of Rain-Proﬁling Methods from ARMAR Data in TOGA COARE with a View to a Possible Use with the TRMM Radar

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ABSTRACT
Rain measurements of the airborne Tropical Rainfall Measurement Mission (TRMM) radar simulator Airborne Rain-Mapping Radar in a typical event of Tropical Ocean and Global Atmosphere Coupled Ocean–Atmosphere Response Experiment are used to compare near-nadir rain-rate proﬁles retrieved from a set of deterministic attenuation-compensating algorithms. This set includes generic algorithms such as the Hitchens–Borden (HB) estimate and the surface reference (SR) methods constraining the total path-integrated attenuation (PIA), and two hybrid algorithms of which one uses the same principle as the “standard” TRMM radar-proﬁling algorithm. In absence of reference rain data for validating the retrievals, the study is based upon an intercomparison of the results and analyzes their features in relation with theoretical predictions. The rain data sequence is the same as used previously in a companion paper, which allowed the authors to get rain relations better adjusted to the observed rain system than those associated to a Marshall–Palmer (MP) drop size distribution. These two sets of rain relations are used here to study subsequent changes in the various rain retrievals. How comparing the results among the generic and/or the hybrid algorithms may help identify the physical assumptions and error sources in the various methods is pointed out.

With the MP relations, all methods provide almost similar retrievals in stratiform light rain. In convective heavy rain, the retrievals are largely scattered; erroneous HB-derived PIA estimates are responsible for a downward collapsing effect in the HB and the TRMM-like radar algorithms, which prevents them from recovering a credible cell-like structure. With the adjusted relations, the rain estimates from all methods are generally increased and in much better agreement. A mirror image algorithm performing PIA proﬁling from the surface is also exploited. The direct estimate of the surface backscatter coefﬁcient, obtainable below very light rain only, agrees with the value measured in clear air. The one-way PIA, immune to errors in the radar calibration and the rain relations, are found to be well correlated with those derived from SR algorithms over a 5-dB range, provided that a bulk correction factor involving these two error types be adjusted in the latter case.

1. Introduction
In the exploitation of the rain radar data of the Tropical Rainfall Measuring Mission (TRMM) (Simpson et al. 1988), a major task is to test and validate algorithms aimed at getting range-proﬁled rain rates. At the operating frequency (13.8 GHz), significant path-integrated attenuation (PIA) effects are unavoidable. During the last decade, numerous attenuation-compensating range-proﬁling algorithms were proposed and occasionally tested from numerical simulations and/or airborne rain radar data. Among the large number of works devoted to this purpose, Fujita (1989), Meneghini and Nakamura (1990), Meneghini et al. (1989), Meneghini et al. (1992), Kozu and Nakamura (1991), Kozu et al. (1991), Marzoug and Amayenc (1994), Iguchi and Meneghini (1994), Amayenc et al. (1996), Testud et al. (1996), Marécal et al. (1997), and Durden and Haddad (1998, manuscript submitted to J. Atmos. Oceanic Technol.), for example, discussed various single-frequency approaches for potential use with the TRMM radar.

For testing such methods, the Airborne Rain-Mapping Radar (ARMAR), usually installed on board the National Aeronautics and Space Administration (NASA) DC-8 aircraft, offers a unique opportunity because it is an excellent airborne simulator of the TRMM radar, which operates at K-band with downward-looking geometry and cross-track scanning (Durden et al. 1994a). Compared with the TRMM radar, however, ARMAR has a better range resolution (60 m instead of 250 m); a better cross-range resolution (the nadir footprint is about 800 m for a 12-km ﬂight altitude, instead of 4.2 km), preventing severe effects of nonuniform beamﬁlling from occurring; and a lower detection threshold (minimum detectable apparent reﬂectivity factor of 10 dBZ instead of 18 dBZ near the surface).

In this paper, data from ARMAR gathered during the
Tropical Ocean and Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE) over the western Pacific Ocean (Webster and Lukas 1992) are used to test several deterministic range-profiling algorithms in tropical convective rain. In absence of external sources of rainfall measurements from surface-based rain gauge networks or weather radars, for example, which could be used as reference data to validate directly the rain retrievals, the study is mainly oriented toward the intercomparison of the results from the different methods and the analysis of their sensitivities to various error sources. Airborne radars such as ARMAR are being used, in addition to other facilities, to provide reference rain data during field campaigns aimed at validating rain retrieval methods from the TRMM radar. In such a context, the data from TRMM and ARMAR will be exploited using similar approaches to define ultimately the best suitable one(s) for TRMM. Therefore, comparing the various rain retrieval methods has sound interest. It will be shown that such comparisons can actually help to identify advantages or deficiencies of some methods compared with others.

The involved ARMAR rain dataset is the same one used in Marecal et al. (1997), hereafter referred to as MTAKOV97. In this previous companion paper, a rain-profiling method based upon the global adjustment of a correction factor was defined and tested. This correction factor could be interpreted in terms of a bulk adjustment of the rain relations, referring to the Marshall–Palmer (1948) drop size distribution (DSD), used in the algorithm. It was shown that the adjusted relations were better tuned to the climatology of the observed rain system. Lending support to this result, a significant part of the comparison work made here deals with changes in rain retrievals from the various algorithms, subsequent to such changes in the reference rain (RR) relations.

Basically, the selected set of rain-profiling algorithms covers the major part of the existing “radar” methods. In addition to the standard $Z$–$R$ method ignoring attenuation correction, it is composed of generic attenuation-compensating algorithms, including the well-known approach of Hitschfeld and Bordan (1954, hereafter HB), which processes rain echoes only, and several surface reference (SR) algorithms involving additional processing of the surface echo to constrain the total path-integrated attenuation (PIA). The SR algorithms were described in Marzoug and Amayenc (1994) and Iguchi and Meneghini (1994, hereafter IM94). In addition, two hybrid algorithms, each one combining with previous methods, are examined. The first one, derived from IM94, is based on the same principle as one used in the “standard” rain-profiling algorithm selected for the TRMM radar; the second one is a new combination. The way the mirror-image (MI) return may be exploited to perform attenuation profiling versus range and provide direct a estimate of the surface backscatter coefficient ($\sigma^0$) below rain is also explored.

The paper is organized as follows. Information about the experiment and the ARMAR dataset used in this study is briefly recalled in section 2. The selected rain-profiling algorithms are outlined in section 3. Results of their application to ARMAR data are discussed in section 4. The conclusions and some prospects related to the exploitation of the TRMM radar data are given in section 5.

2. Experiment and data

The ARMAR data used were acquired on 6 February 1993 within a mesoscale convective system. The detailed characteristics of the rain system and of the ARMAR measurements were given in MTAKOV97. The considered rain sequence is the same one referred to as S1 in MTAKOV97. The 10-min time duration (1802:04–1812:20 UTC) corresponds to a leg of about 135 km along the rainband, according to the aircraft speed of 220 m s$^{-1}$ at 9.4-km flight altitude. To better understand the experimental results, it is useful to display again here, in Fig. 1 (similar to Fig. 3 in MTAKOV97), the vertical cross section of the apparent reflectivity factor $Z_m$ versus range (from the radar) for near-nadir observation with ARMAR. The picture is limited to the second half (1807:30–1812:20 UTC) of the rain sequence and involves 143 ray paths. The surface return near the 9.4-km range, conveniently expressed in the same unit (dBZ) as rain reflectivity, is followed by the mirrored rain echoes at larger ranges. The reference measurements of surface echoes outside the rain were acquired in clear conditions within a time sequence (1757:15–1802:02 UTC) adjacent to the rain sequence. The no-rain reference $\sigma^0$ measured at nadir viewing is $\sigma^0_0 = 12.9 \pm 0.4$ dB (Tani et al. 1995).

The total two-way PIA (in positive decibels) as estimated from the surface reference technique (Meneghini et al. 1983) and further used in the SR algorithms is shown in Fig. 2 (similar to Fig. 4 in MTAKOV97). The PIA estimate, obtained by forming the ratio of surface return powers outside and below rain, was computed by assuming that the ocean surface roughness is not modified by the surface wind and/or the raindrop impact (Moore et al. 1979; Atlas 1994; Durden et al. 1994b).

The data in Fig. 1 point out severe attenuation effects in convective rain between ray numbers 30 and 50 with a 33-dB PIA peak for ray number 40 (Fig. 2). This explains the strong decrease of $Z_m$ toward the surface (near 1809 UTC) below the 40-dBZ peak in the raincell core and the correlative weakening or even the disappearance of the mirrored echoes. For ray numbers greater than 60, the precipitation is stratiform with a bright band peaking at the 5-km range or 4.4-km height. Low attenuation effects little affect the reflectivity field. The isolated “blip” in surface return in Fig. 1, near ray number 72, is due to the overfly of a small island. As-
summing ocean background overestimates the reference \( \sigma_0 \) and increases the PIA artificially by about 8 dB.

3. Selected algorithms and their characteristics

a. Generic attenuation-compensating algorithms

The standard \( Z-R \) method, using the apparent reflectivity \( Z_m \) to profile the rain rate \( R \) versus range via a prescribed \( Z-R \) relation, leads to an underestimation of \( R \) as PIA increases. Four generic algorithms correcting for PIA were used. Their mathematical principles were described in Marzoug and Amayenc (1994) and IM94. They are outlined below using a synthetic formulation along with comments on the characteristics, which are useful to interpret the experimental results.

All algorithms make use of a \( Z = aK^b \) relation where \( K \) is the attenuation coefficient in the radar equation to get the range profile of \( K \) (or \( Z \)). Then a unique \( R \) profile is indifferently obtained from a \( K = aR^b \) relation or a \( Z = eR^d \) relation, provided that the set of RR relations be self-consistent, that is, that each relation may be obtained by combining the other two. All coefficients are assumed to be range free with known exponents \( b, b, \) and \( d \). Further considered changes in the RR relations concern only the proportional coefficients \( a, a, \) and \( e \).

The general expression for the \( K \) estimate at range \( r \) is

\[
K(r) = [Z_m(r)/\alpha]^{1/b}[(fu)^{1/b} + \gamma S(r, r_s)]^{-1}, \tag{1}
\]

where \( \gamma = 0.46/b; \) here, \( u \) is the total two-way PIA factor \( A(r_0, r) \) between the rain-top range \( r_0 \) (i.e., the range of the first detected echo, in practice) and the surface range \( r_s; \) here, \( S(r, r_s) \) is defined as

\[
S(r, r_s) = \int_r^{r_s} [Z_m(s)/\alpha]^{1/b} ds \tag{2}
\]

and involves the apparent reflectivity factor \( Z_m(r) \); \( f = \delta a\delta C \) is a correction factor of scaling errors \( \delta a \) in \( \alpha \) (in
the Z–K relation) or δC in the radar calibration. The bias δα (δC) is a dimensionless (positive) multiplicative factor of the reference value. No correction in α and C implies $f = 1$.

The $Z(r)$ estimate, obtained from (1) via the $Z = αK^β$ relation, is

$$Z(r) = Z_m(r)δα[(fu)^{1/β} + γS(r, r_s)]^{−β}. \quad (3)$$

1) kZ Algorithm

The first algorithm (kZ), similar in essence to HBs approach, makes use in (1) and (3) of δα = 1, $f = 1$, and $u = A(r_0, r_s)$, which is estimated from integration of the radar equation for rain echoes according to

$$A(r_0, r_s) = [1 - f^{-1/β}γS(r_0, r_s)]^β \quad (4)$$

and used with $f = 1$, thereby defining the HB estimate of the total attenuation factor:

$$A_{kZ} = [1 - γS(r_0, r_s)]^{−1}. \quad (5)$$

The range-profiling from (1) or (3) is made downward from the top via $S(r_0, r)$, which is the only remaining term in the $Z$ and $K$ estimates after using (5) for $u$. The kZ algorithm identifies with the Z–R method as PIA goes to zero. The rain retrieval, however, very sensitive to $kZ$, algorithm identifies with the "measured" from the SE attenuation below rain. Here, $A_{kZ}$ is free of the storm structure at ranges less than $r_s$ (e.g., in the melting layer or ice region aloft). The kZS results are sensitive to "uncorrected" errors δα, δC, or δA (in $A_{kZ}$) but remain stable versus attenuation and converge toward the Z–R method for low PIA ($A_{kZ} \approx 1$). For nonphysical $A_{kZ}$ estimates ($A_{kZ} > 1$), not uncommon in practice, using $A_{kZ} = 1$ still provides a reliable rain estimate. Of course, changes in the RR provide different rain retrievals.

The second (kZA) and third (kZC) algorithms are similar, in essence, to the $α$ and $C$ adjustments (Meneghini and Nakamura 1990; IM94), respectively. They also make use of $u = A(r_0, r_s) = A_{kZ}$ but infer the correction factor $f$ from (4) according to

$$f = γS(r_0, r_s)(1 - A_{kZ}^{1/β})^{β}$$

and

$$f = [(1 - A_{kZ}^{1/β})(1 - A_{kZ}^{1/β})]^{β}. \quad (6)$$

The range-profiling from (1) and (3) is made partly from the top and the surface via $S(r_0, r_s)$ and $S(r, r_s)$, both appearing in the $Z$ and $K$ estimates. The kZA algorithm assumes $f = δC$ and $δα = 1$, while the kZC algorithm assumes $f = δα$ and $δC = 1$ in (1) and (3). The $K$ retrievals from kZA and from kZC are similar since they involve the product $f = δαδC$ only, while the $Z$ retrievals depend on the correction type and are related by $Z = fZC$ according to (3). For kZC, the $α$ adjustment (in the $Z–K$ relation) may be exploited in different ways to get the $R$ profile. IM94 proposed keeping either the $Z = eR^α$ relation or the $K = aK^β$ relation as constant and then performed experimental tests from airborne rain radar data at 35 GHz using the constant $Z–R$ relation hypothesis. In these approaches, the self-consistency of the three RR relations is kept by combining the adjusted $Z–K$ relation and the constant $Z–R$ (or $K–R$) relation to get the third one. A slightly more sophisticated approach, adjusting both $K–R$ and $Z–R$ in addition to $Z–K$, was defined in MTAKOV97. The associated $δα$ correction (in $K = aK^β$) and $δC$ correction (in $Z = eR^α$) are connected to the $δα$ correction (in $Z = aK^β$) by

$$δα = δα(1−k)(1−β); \quad δC = δα(1−α)(1−β). \quad (7)$$

As explained in detail in MTAKOV97, these corrections follow from an equivalent adjustment of $N_0$ (intercept parameter of the DSD), where the adjusted $N_0^{*}$ is related to the reference $N_0$ by $N_0^{*}/N_0 = f^{1/(1−β)}$ [cf. Eq. (10) in MTAKOV97]. The involved approach can be viewed as a "degraded" version of the $N_0$ adjustment method of Kozu and Nakamura (1991) in that it adjusts only the proportional coefficients (α, α, e) instead of the six coefficients (α, a, e, β, b, d) in the RR.

The C or $α$ adjustment is performed for every ray path. The kZA version involving a "full" adjustment of the three RR relations (hereafter referred to as kZA1) differs from the kZC version with constant Z–R (hereafter referred to as kZC2) used in IM94 or with constant $K–R$ (hereafter referred to as kZA3). The $Z_R$ retrieval does not depend on the kZA version since it involves a common $δα$ correction in the $Z–K$ relation only. This is not the case for the $R_s$ retrieval, which involves different $Z–R$ (or $K–R$) relations according to the version...
used for $kZa$. Finally, the $Z_{ac}$ and $R_{ac}$ retrievals verify the following (in)equalities, depending on the local correction factor $f_L$ given by (6):

$$Z_{a1} = Z_{a2} = Z_{a3} < Z_c \quad \text{and}$$

$$R_{a1} > R_{a3} = R_c > R_{a2} \quad \text{if } f_L < 1,$$  

(8a)

$$Z_{a1} = Z_{a2} = Z_{a3} = Z_c \quad \text{and}$$

$$R_{a1} = R_{a2} = R_{a3} = R_c \quad \text{if } f_L = 1, \quad \text{and}$$

(8b)

$$Z_c < Z_{a1} = Z_{a2} = Z_{a3} \quad \text{and}$$

$$R_{a1} < R_{a3} = R_c < R_{a2} \quad \text{if } f_L > 1.$$  

(8c)

Here, $R_{a1}$ and $R_c$ are identical since both estimates can be derived from the same $K$ estimate ($K_a = K_c$) via a constant $K$–$R$ relation for both cases. For exploiting ARMAR data, we used basically the $kZC$ and $kZa1$ (full adjustment of the RR relations) generic algorithms; the $kZa2$ algorithm (constant $Z$–$R$ relation) was occasionally used to explain some aspects of the results.

The $kZa$ and $kZC$ algorithms are stable versus attenuation but are sensitive to “uncorrected” errors ($\delta a$ for $kZC$ and $\delta C$ for $kZa$) and especially to errors $\delta a$, in $A_{zm}$ for low PIA. For nonphysical $A_{zm}$ estimate ($A_{zm} > 1$), (6) yields a meaningless negative $f_L$, and the rain retrieval fails. If $f_L = 1$, which implies $A_{lz} = A_{zm}$, it can be verified from (1)–(6) that the common rain retrievals from $kZaC$ according to (8b) also identify with those from $kZS$ and to those from $kZ$ (if it does not diverge for low SNR). Self-consistent changes in the RR relations [i.e., according to (7) with constant $\delta a$], as those further considered (cf. section 4), modify the local correction factor $f_L$. Thus, they also modify the rain retrievals from $kZaC$ except $kZa1$, which restores a unique set of “adjusted” rain relations by definition.

In MTAKOV97, a global $\alpha$ or $C$ adjustment (or GA method) instead of a path-by-path local adjustment was also proposed and tested. It relies on the use of a bulk correction factor $f_g$ [cf. Eq. (7) in MTAKOV97], which is determined by minimizing in least squares sense, over a set of $N$ contiguous ray paths, the difference between the HB estimate (5) and the SR estimate ($A_{zm}$) of the total PIA factor. The bulk correction factor $f_g$ (i.e., the mean bias in the $\delta a\delta C$ product) may be interpreted in terms of an offset in the radar calibration (with $\delta C = f_g$) or a self-consistent bulk adjustment of the RR relations [with $\delta a = f_g$ and $\delta a, \delta C$ from (7)], depending on additional hypotheses (MTAKOV97). Given a set of RR relations using global adjustment (same $f_g$ for all paths), instead of local adjustment (path-dependent adjusted $f_L$), modifies the rain retrievals from $kZaC$. However, it is worthwhile to note that for the further considered case of self-consistent modifications of the RR relations deriving from a bulk correction $f_g$ (cf. section 4a), the rain retrieval from $kZS$ with such modified RR relations (and no additional adjustment by definition) is similar to those that would be obtained from $kZa1$ with a global adjustment of the genuine RR relations (i.e., the GA method by MTAKOV97). In other words, the RR are already adjusted for $kZS$ while they are adjusted in the same manner by the GA method. For exploiting ARMAR data, the only local adjustment was used, while the global adjustment was not.

### b. Hybrid algorithms

The idea of hybrid algorithm relies on the use of a weighted combination of two generic algorithms aimed at optimizing the advantage/drawback balance of both components. For example, a suitable combination should allow a shift from the $kZ$ algorithm for low PIA to an SR algorithm for high PIA. IM94 first proposed such a hybrid approach combining $kZ$ and $kZa$ (or $kZC$) with local $\alpha$ (or $C$) adjustment. Also, they tested the $\alpha$-adjustment case implying the constant $Z$–$R$ relation hypothesis ($kZa2$) from airborne rain radar data at 35 GHz. This hybrid algorithm, quoted below $kZH_\alpha$, was used to define the standard (2A–25) range-profiling algorithm for the TRMM radar (Iguchi et al. 1997). Another hybrid algorithm, combining $kZ$ and $kZS$ and referred to below as $kZH$, is also proposed.

#### 1) $kZH_\alpha$ algorithm

The solution for $Z_{H_\alpha}$ retrieved from the $kZH_\alpha$ algorithm defined by IM94 is given by

$$Z_{H_\alpha}(r) = Z_{zm}(r)[1 - e_H g S(r_o, r)]^{-\beta},$$  

(9)

with a hybrid correction factor $e_H$, defined as

$$e_H = 1 + \omega_1 e - 1 = \omega_1 e + (1 - \omega_1),$$  

(10)

where $e = e_L$ is the local correction factor for the “generic” $kZa$ algorithm, related to $f_L$ from (6) by

$$e_L = f_L^{-1/\beta}.$$  

(11)

Here, $Z_{H_\alpha}$ from (9) yields the $kZ$ ($kZa$) solution for $e_H = 1$ ($e_H = e_L$), as also given by (3) with $f = \delta a = 1$ ($f = \delta a = f_L$). For the $C$ adjustment case, $Z_{HC} = Z_{zm} e_H^{\gamma}$ (IM94). The attenuation-dependent weighting factor $\omega_1$, defined as

$$\omega_1 = \gamma S(r_o, r) = 1 - A_{lz}^{1/\beta},$$  

for $\gamma S(r_o, r) \leq 1$ (12)

and

$$\omega_1 = 1,$$  

for $\gamma S(r_o, r) > 1,$ (13)

involves the HB estimate for the total PIA factor $A_{lz}$ as given by (5).

For low (large) PIA, $A_{lz} \approx 1$ (0), then $\omega_1 \approx 0$ (1), thus $e_H \approx 1$ ($e_L$), and $kZH_\alpha$ identifies asymptotically with $kZ$ ($kZa$). The hybrid correction factor $e_H$ is neither associated to $kZ$ ($e = 1$) nor to $kZa$ ($e = e_L$) but is a weighted linear combination of them in the unlimited “hybridization range.” Therefore, $kZH_\alpha$ operates a smooth transition from $kZ$ to $kZa$ results as PIA increases, thus preventing erroneous retrievals from (or
even the failure of) $kZa$, which may result from errors in the SE attenuation estimate ($A_{aw}$) for low PIA, as well as the divergence or collapsing of $kZ$ for large PIA. The result, however, is sensitive to a bias in $A_{aw}$ via $\omega_s$. Changing the RR modifies $A_{aw}$, then $\omega_s$ from (12), and thus $e_j$. The subsequent change in $e_j$ from (10) modifies the rain retrieval from $kZH\alpha$.

Different variants of the $kZH\alpha$ algorithm could be used in relation with the three $\alpha$ adjustment types ($i = 1$, 2, 3) for $kZa$, discussed in section 3a. To preserve the coherency with the formulation used in IM94 and in the TRMM-like radar algorithm (Iguchi et al. 1997), as well as to limit the number of outputs at reasonable size, the ARMAR data were exploited with the $kZH\alpha$ algorithm variant involving the constant $Z-R$ relation hypothesis (i.e., $kZa2$).

2) $kZH$ Algorithm

The proposed $kZH$ algorithm is a hybrid combination of the $kZ$ and $kZS$ algorithms. The hybridization is performed directly on the total PIA factor $u = A(r_o, r_i)$ according to
\[
u_i = \omega_z u_i + (1 - \omega_z) u_{iZS},\]

where $u_i = A_i$ given by (5) and $u_{iZS} = A_{iw}$ (see section 3a).

The $K(Z_i)$ retrieval is provided by (1) [(3)] using $u = u_{i1}$, $f = 1$, and $\delta = 1$. The attenuation-dependent weighting factor $\omega_z$ is defined as
\[
\omega_z = 1, \quad \text{for } dBu_{iZ} \leq dBu_o,\]

\[
\omega_z = (dBu_{iZ} - dBu_o)/(dBu_i - dBu_o), \text{ for } dBu_o < dBu_{iZ} \leq dBu_i,\]

\[
\omega_z = 0, \quad \text{for } dBu_{iZ} > dBu_i,\]

where $dBu_{iZ} = -10 \log A_{iZ} -10 \log A_{iw}$ is $A_{iZ}(A_{iw})$ expressed in positive decibels, and $dBu_o (dBu_i)$ is a prescribed low (high) attenuation threshold in decibels.

The hybrid algorithm $kZH$ identifies with $kZ$ ($kZS$) for low (large) PIA below $dBu_{iZ}$ (above $dBu_{iZ}$), while it is a weighted combination of them within the limited hybridization range [$dBu_{iZ}$] with some sensitivity to a bias in the $dBu_{iZ}$ (i.e., $A_{iZ}$) estimate via $\omega_z$. Like $kZH\alpha$, $kZH$ prevents erroneous retrievals from $kZS$ if large error occurs in the SE attenuation estimate ($A_{aw}$) for low PIA, as well as the divergence or collapsing of $kZ$ for large PIA. In the latter case (large PIA), an added advantage is to keep the benefit by integrating from the surface via $kZS$. The rain retrievals from $kZ$ and $kZS$, thus from $kZH$, change with the RR. Since $kZH$, like its “components” $kZ$ and $kZS$, does not involve any adjustment of the RR relations, the best conditions for application is to use climatologically tuned RR relations if they are available.

c. Mirror image algorithm

The theory of the MI return in rain, which involves complex scattering mechanisms from rain and surface, was first studied under simplifying assumptions by Meneghini and Atlas (1989). Few MI algorithms, restricted to nadir viewing, were defined and occasionally tested from airborne radar measurements (Meneghini and Nakamura 1988; Liao et al. 1995). The MI algorithm used here is derived from Meneghini and Atlas (1986) and allows direct range-profiling of the PIA from the surface for a nadir path. However, it is formulated in terms of apparent reflectivity factor (in dBZ) of the direct ($Z_D$) and mirror-reflected ($Z_M$) rain echoes, rather than ratio of received powers, for the same rain volume. Given the one-way PIA (in positive decibels) over the altitude range $[0, H]$

\[
A_i(H) = \int_0^H K(s) \, ds,\]

a convenient form of the MI algorithm, is given by
\[
(Z_D - Z_M) = T_{MI} + 4A_i,\]

where $x_j = H/H_i, H_i$ is the aircraft flight altitude, $H$ is the echo altitude (i.e., the altitude of the $j$th range gate above the surface), $a = k/\sigma^0, \sigma^0$ is the surface backscatter coefficient below rain (or $\sigma^0_s$), $k = 11\Gamma^2/\theta^2, \theta_\phi$ is the one-way 3-dB radar beamwidth, $\Gamma^2$ is the Fresnel reflectivity for smooth surface, and $C_{MI} = -10 \log F^4$ is a constant term.

The correction term $T_{MI}(x_j, \sigma^0, \Gamma^2, \theta_\phi)$ must be removed from $(Z_D - Z_M)$ at every altitude $H_j$ to estimate the four-way PIA, $4A_i$. The constant term $C_{MI}$ can be derived from theory (using $\Gamma^2 = 0.65$ or $C_{MI} = 4$ dB) or estimated from the data at low height in light rain where $x \ll 1$, $ax^2 \ll 1$, and $A_i \approx 0$, which allows the calculation of $C_{MI} = (Z_D - Z_M)$.

The $T_{MI}$ computation requires a knowledge of the surface roughness. Meneghini and Atlas (1986) defined several approximation cases governed by the term
\[
T_j = ax^2 = \frac{11\Gamma^2(H_j/H_0)^2}{(\sigma^0\theta_\phi^2)},\]

in (20), which depends on $\sigma^0$ and the experiment conditions via $H_0$ and $\theta_\phi$. In the most general case $(0.1 < T_j < 10) or H_j \approx L$, where $L$ is the beam footprint diameter at nadir), no approximation is possible. In the “close” case ($T_j > 0.1$ or $H_j < L$), the MI algorithm is $\sigma^0$ free. In the “far” case ($T_j > 10$ or $H_j > L$), $\sigma^0$ below rain can be estimated directly if the MI returns are detected up to the rain-top altitude ($j = n$) by combining (19) and (20) and the radar equation for the SE to eliminate $A_i$, which yields
\[ \sigma_v = (Z_D - Z_M)_a + 2Z_v - u(x_v) \]  
with \[ u_v = C_{MI} + 10 \log k + 20 \log[x_v/(1 + x_v)] + 2C_s, \]  
where \( x_v = H_v/H_0, \) \( Z \) is the “apparent” surface reflectivity (in dBZ), and \( C_s \) is the radar calibration constant for surface measurements. Here, \( A_v \) profiling is further performed from (19) using the \( \sigma_v^0 \) estimate in \( T_{MI}, \) the approximation cases for the ARMAR measurement conditions are examined in section 4.

In the present paper, the MI algorithm is primarily used to get one-way PIA estimates, \( A_{MI}, \) which are immune to errors in the radar calibration and the rain relations; then they are compared to one-way PIA estimates, \( A_{SR}, \) derived from the SR approach involving \( A_{in}. \) For this purpose, a generalized integral form of the \( kZS \) algorithm (Marzoug and Amayenc 1994; Amayenc et al. 1996) was used, providing the following estimate of the two-way PIA factor over \([0, H_1]:\)

\[ A_j = [1 + (A_{in}f)^{-1/2}gS(H_j)]^{-\beta}, \]

where \( S(H_j) \) is given by (2) with \( H_j = (r_j - r) \) for a nadir path, \( \gamma = 0.46/\beta, \) and \( f = \deltaa\deltaC \) is the correction factor from (6), which has to be known. Note that the genuine \( kZS \) algorithm assumes \( f = 1 \) (cf. section 3a). The one-way PIA (in positive decibels) derived from (24) and further compared with \( A_{MI,1} \) is

\[ A_{SR,1}(H_j) = -10 \log A_j^{1/2} \]

\[ = -5 \log[1 + (A_{in}f)^{-1/2}gS(H_j)]^{-\beta}, \]

which implies the use of a reference \( Z = \alpha K^\beta \) relation.

The MI algorithm used here does not allow calculations of the height profiles of the rain rate. Only path-averaged rain rates over \([0, H_1]\) can be obtained according to \( RPAV_j = (KPAV_j/a)^{1/3}, \) where \( KPAV_j = A_{SR,1}/H_j \) is the path-averaged attenuation coefficient and \( a \) and \( b \) are the coefficients of the \( K = ar^b \) relation. In fact, differing the PIA to different altitudes could, in principle, provide a height profile of \( K, \) then \( R. \) This procedure was tested, but it failed because the derived \( K(H_j) \) was found to be too noisy to provide reliable \( R(H_j) \) estimates.

4. Rain retrieval results

a. Selected sets of reference rain relations

Two sets of RR relations, fitting to a rain-rate range of about 5–100 mm h\(^{-1}\), were selected for the application of the algorithms. Set 1, given by

\[ Z = 4.43 \times 10^4 K^{1.356}, \]  
\[ K = 0.0230 R^{1.100}, \]  
\[ Z = 265.5 R^{1.614}, \]

with \( Z \) in mm\(^6\) m\(^{-3}\), \( K \) in dB km\(^{-1}\), and \( R \) in mm h\(^{-1}\), was obtained in MTAKOV97 assuming exponentially shaped DSD with an intercept parameter \( N_0 = 8 \times 10^6 \) m\(^{-4}\) (Marshall and Palmer 1948). The RR relations (25)–(28) of set 1 are hereafter referred to as the MP relations.

Set 2 was derived in MTAKOV97 from results of the GA method (see section 3a) applied to the same ARMAR data (with set 1 used as reference), which yielded an adjusted bulk correction factor \( f_B = (\deltaa) = 0.51. \) Therefore, it was obtained from set 1 after corrections \( \deltaa = f_B = 0.51 \) in (26), \( \deltaa = 0.698 \) in (27), and \( \deltae = 0.313 \) in (28) derived from (7). Set 2, given by

\[ Z = 2.26 \times 10^4 K^{1.356}, \]  
\[ K = 0.0161 R^{1.100}, \]  
\[ Z = 83.1 R^{1.614}, \]

was found to be in excellent agreement with the set of rain relations derived from in situ DSD data of a 2DP-S probe aboard the Electra National Center for Atmospheric Research aircraft in the same rain system (MTAKOV97). As an example, the \( Z \) values from the 2DP-derived \( Z\)–\( K \) relation, and from (31), differ by less than 0.5 dB for \( 0.01 < K < 0.5 \) dB km\(^{-1}\) (or \( 0.7 < R < 124 \) mm h\(^{-1}\)), according to (29). The closeness to unity of the bulk correction factor (\( f_B = 0.985 \)) obtained when the 2DP-derived relations are used as reference in the GA method, confirmed that set 2 is well “adapted” to the climatology of the observed rainy system, though likely better representative of convective rain than stratiform precipitation. The RR (29)–(31) of set 2 are hereafter referred to as the \textit{adjusted relations}. 

b. Generic and hybrid algorithms

The rain-profiling algorithms tested with ARMAR data and their main characteristics are summarized in Table 1. Note that \( KZas3 \) algorithm (\( \alpha \) adjustment and constant \( K\)–\( R \) relation) and the GA method by MTAKOV97 (RR relations adjusted with a bulk correction factor \( f_B \)) are not mentioned. However, these algorithms are implicitly tested considering that the \( R \) retrievals of \( KZas3 \) and \( KZC \) are identical [see Eq. (8)], and the \( R \) retrievals of the GA method identify with those of \( KZS \) with the adjusted relations (see section 3a). The algorithms were applied to ARMAR data for various incidence angles corresponding to the cross-track scanned \(( \pm 20^\circ \) off nadir) of the radar beam. However, the results presented here are focused on rain retrieval near nadir, involving 303 ray paths for the entire rain sequence. The hybrid algorithm \( kZH \) was run with an arbitrary low (high) attenuation threshold \( dB_{L0} = 1 \) dB (dB\( _{H0} = 3 \) dB) of the hybridation range. A total two-way PIA of 1 dB (3 dB) corresponds, for example, to a constant rain rate of about 4 mm h\(^{-1}\) (10 mm h\(^{-1}\)) over a 5-km rain depth. The low threshold, below which \( KZH \) identifies with \( KZ \), exceeds the uncertainty in the SE-derived total PIA due to the observed standard error (0.4 dB) in the reference \( \sigma_v^0 \) estimate in clear air, while
ignoring possible bias in the surface roughness below rain. The high threshold, above which kZH identifies with kZS, is large enough to get a low relative uncertainty in the SE-derived total PIA, and thus a reliable use of kZS.

First, let us consider the results obtained with the MP relations (set 1). Figures 3 and 4 illustrate the Z retrievals from the four generic algorithms (kZ, kZS, kZC, and kZα1) and the two hybrid algorithms, kZHα and kZH. As mentioned earlier (sections 3a and 3b), kZα1 performs a full adjustment of the three RR relations, while kZHα implies the constant Z-R relation hypothesis (kZα2). For each algorithm, contour plots of the retrieved Z is displayed in a nadir/along-track 2D frame limited to the second half of the rain sequence (1807:30–1812:20 UTC), similar to that shown in Fig. 1 for Zα. Examples of rain-rate profiles retrieved from the same set of algorithms are shown in Fig. 5 for a path in convective rain through the raincell core (ray number 40) with an SE-derived PIA of 24.7 dB and in Fig. 6 for a path in stratiform rain (ray number 110) with an SE-derived PIA of 3.3 dB. In both cases, the R retrievals from the Z-R method [using Zα in (28)] and from the generic algorithm kZα2 are also plotted for comparison. The distribution of the local correction factors fα involved in the kZα/C algorithms versus the total SE-derived PIA is shown in Fig. 7 for the entire rain sequence. For PIA < 5 dB, the fαs are scattered between 0 and 1.5 reflecting uncertainties in the PIA estimates; then, for PIA > 10 dB, they stabilize near about 0.5, close to the bulk adjustment factor fα found in MTA-KOV97.

In stratiform light or nearly stratiform moderate precipitation for ray numbers less than 30 or greater than 60 all algorithms provide Z retrievals reasonably close from each other, as seen in Figs. 3 and 4. However, for some ray numbers between 10 and 25, and near 87, the kZα and kZC algorithms fail because the SE-derived PIA factor has nonphysical value (Aα > 1). For those cases, kZHα provides adequate kZ-like retrievals. Besides, the local spike observed near ray number 72 for the generic SR algorithms kZS and kZC, and to a lesser extent for kZα, are spurious effects due the SE-derived total PIA overestimation associated to island overfly alluded to earlier with respect to Fig. 1 (see section 2). This spike does not appear in the kZ results, which ignore the SE-derived PIA; it is eliminated in the kZHα and kZH results fitting to the kZ results in light rain. Also, the R retrievals from all algorithms are comparable, as seen in Fig. 6, for ray number 110. The artificially high rain rates near the 5-km range are due to the bright band. Therefore, the R retrievals should be considered for ranges beyond 5.4 km only, that is, in the rain region extending up to 4-km height from the surface. Below the bright band, the attenuation-compensating algorithms yield mean rain rates ranging from 6.5 to 9 mm h⁻¹, slightly in excess of the Z-R method estimate (≈5 mm h⁻¹).

In convective rain (for ray numbers between 25 and 55, typically) where the PIA is large, the Z retrievals may largely differ. The Z retrieval from kZ in Fig. 3 collapses downward with a maximum effect for the largest PIA near ray path 40 (cf. Fig. 2). Conversely, all other algorithms recover a cell-like structure with different peak magnitudes, except kZHα, which also points out a tendency for collapsing downward. The ranking by order of magnitude of the Z retrievals for the generic algorithms (Zc > ZkZS > Zα > ZkZ) agrees with that predicted by IM94 for fL < 1 (i.e., e > 1 with their notations). For the R retrievals, the ranking is modified
Fig. 3. Reflectivity factors (in dBZ) as a function of range from the radar, retrieved from the generic algorithms $kZ$ (upper panel), $kZS$ (middle panel), and $kZC$ (lower panel). The time frame and the ray path numbers are the same as given in Fig. 1. The results are obtained using the MP relations. Paths where the $kZC$ retrieval is not possible are left blank.

and agrees with that predicted by (8) for $kZa1$, $kZC$, and $kZa2$, as seen in Fig. 6 for ray number 40. For the same case, the $R$ retrievals from $kZH$ and $kZS$ are similar, while the $R$ retrieval from $kZH\alpha$ is in between those of its components ($kZ$ and $kZa2$) and points out a collapsing effect versus range similar to that seen for $kZ$.

The retrieved path-averaged rain rates over a 4-km rain depth from the surface (i.e., beyond 5.4-km range from the radar) are spread between 35 and 65 mm h$^{-1}$ for $kZa2$, $kZS$, $kZC$, $kZa1$, and $kZH$, while it is about 21, 13, and 7 mm h$^{-1}$ for $kZH\alpha$, $kZ$, and $ZR$, respectively. The downward collapsing effect in $kZH\alpha$, $kZ$, and $ZR$
increases the discrepancies with other retrievals near the surface.

Why the $kZH\alpha$ results do not fit to the $kZ\alpha 2$ results for large $\text{PIA}_R$, as expected by definition, has to be explained. In fact, it was found that the HB estimate $A_{kZ}$ from (5) defining the weighting factor $\omega_i$ from (12), which, in turn, governs the hybrid correction factor $\epsilon_H$ in (10), is generally much greater than $A_{\text{PIA}_S}$. In other words, the total $\text{PIA}$ derived from rain echoes (or $\text{PIA}_R = -10 \log A_{kZ}$) is much smaller than the SE-derived $\text{PIA}$ [or $\text{PIA}_S (\text{dB}) = -10 \log A_{\text{PIA}_S}$]. As an example, for ray number 40 (Fig. 6), $f_L = 0.58$ (or $\epsilon_L = 1.486$) in $kZ\alpha 2$, $A_{kZ} = 0.225$ (or $\text{PIA}_R = 6.5 \text{ dB}$), $A_{\text{PIA}_S} = 3.39 \times 10^{-3}$ (or $\text{PIA}_S = 24.7 \text{ dB}$), and $\omega_i = 0.662$. The sub-
sequent $e_H = 1.322$ is sufficiently different from $e_L = 1.486$ to keep significant $kZ$ versus $kZa2$ weighting in the $kZH\alpha$ results. Note that $A_{s2} \gg A_{in}$ (or $\text{PIA}_R \ll \text{PIA}_S$) also explains why the $kZ$ results collapse when compared to the SR algorithms’ results. Using $A_{in}$ instead of $A_{s2}$ in (12) would yield $\omega_1 = 0.984$, along with $e_H = 1.478$, which is close to $e_L = 1.486$; therefore $kZa2$-like results from $kZH\alpha$ are observed as expected. In this case, however, a spurious $A_{in}$ estimate (for low PIA) like that encountered near ray number 72 (during island overfly) could not be eliminated. This points out some deficiency of the hybrid algorithm $kZH\alpha$ for large PIA in heavy rain, which may come from largely erroneous $A_{s2}$ (or $\text{PIA}_R$) estimates. In the present results, this deficiency is a consequence of an ill-adapted $\alpha$ coefficient of the $Z = aK^p$ MP relation. For light rain, such an
Fig. 7. Distribution of the local correction factor $f_L$ as a function of the total PIA derived from the surface echo near nadir using the MP relations for all paths of the entire rain sequence (1802:04–1812:20 UTC).

effect is not penalizing since the path attenuation is weak, and $kZ\alpha_a$ is heavily weighted toward its $kZ$ component anyway. For ray number 110 (Fig. 6), for example, the same kind of computation as above provides $f_L = 0.73$ (or $e_L = 1.257$), $A_{iz} = 0.568$ (or $\pi a = 2.5$ dB), $A_{im} = 0.46$ (or $\pi a = 3.3$ dB), and $\omega_i = 0.338$. The subsequent $e_H = 1.09$, close to 1, implies $kZ$-like results from $kZH\alpha$, as observed.

For the hybrid algorithm $kZH$, the effective hybridization range of the $kZ$–$kZs$ pair is limited to moderate PIA (here, from $dBu0 = 1$ dB to $dBu1 = 3$ dB). Therefore, the $kZH$ results are not too penalized by a bias in the $A_{iz}$ estimate (and correlatively in $dBu = \pi a$), which is also involved in the weighting function $\omega_i$ from (15) to (17) and in the hybrid PIA estimate from (14). Hence, $kZH$ results, though likely affected for moderate PIA, fit to those of $kZ$ for low PIA (below $dBu0$) and to those of $kZs$ for large PIA (above $dBu1$), as seen in Figs. 3–6.

For easing the overall comparison of $R$ retrievals, while avoiding a large number of plots, the mean retrieved rain rates $\langle R \rangle$ for every algorithm, computed from all range gates (about 2700) within the 0.5–1-km altitude slab over the entire rain sequence, are listed in Table 2. The mean rain rate $\langle R \rangle$ is contributed by low to high rain rates involved in the sample and is representative of a large-scale ($\approx 100$ km) spatial average of the rainfall near the surface. For $kZ\alpha$ and $kZC$, the (common) points where the retrievals fail are not considered. In Table 2, the results quoted $R_{MP}$ refer to the use of MP relations, as discussed to this point. Those quoted $R_{adj}$ refer to use of the adjusted relations and are discussed later. The linear correlation coefficient, the rms deviation between the two estimates, as well as the mean bias of $\langle R_{adj} \rangle$ relative to $\langle R_{MP} \rangle$, are also listed. Besides, the bulk comparison of $R$ retrievals is further illustrated by some selected scatterplots of retrieved point rain rates ($R_{MP}$ and/or $R_{adj}$) over the same data sample for different algorithms and/or RR relations.

Results for the MP relations case, in Table 2, point out the weak $\langle R_{adj} \rangle$ for $Z-R$ in the absence of PIA correction. The $\langle R_{MP} \rangle$ retrieved from $kZ$ is slightly greater than that retrieved from $Z-R$, owing to PIA correction but is still smaller (by a factor of 1.5–4) than those retrieved from the generic SR algorithms because of the $kZ$ collapsing. This is illustrated by the scatterplot in Fig. 8a, which compares the point rain rates retrieved from $kZ$ and $kZs$ in the 0.5–1-km altitude slab. For the generic SR algorithms, $\langle R_{MP} \rangle$ is about 2 times larger for $kZ\alpha1$ than for $kZs$, with $kZC$ in between, and 2.5 times greater for $kZ\alpha2$ than for $kZ\alpha1$. For the hybrid algorithm $kZH\alpha$, $\langle R_{MP} \rangle$ is halfway between the $kZ$ and $kZs$ results due to the aforementioned impact of erroneous $A_{iz}$ weighting for large PIA. For the hybrid algorithm $kZH$, $\langle R_{MP} \rangle$ is closer to the $kZs$ than to the $kZ$ retrieval and exceeds $\langle R_{adj} \rangle$ from $kZ\alpha1$ by about 30%. The scatterplot in Fig. 9a, comparing the point rain rates retrieved from $kZH\alpha$ and $kZH$ in the 0.5–1-km altitude slab, points out the collapsing tendency of $kZH\alpha$ for large rain rates.

Table 2. Mean rain rate retrieved near nadir from the various algorithms within the 0.5–1-km altitude slab for the entire rain sequence using the MP relations $\langle R_{MP} \rangle$ or the adjusted relations $\langle R_{adj} \rangle$. Here, $\Delta R$, $B$, and $\rho$ are the rms deviation, the mean bias, and the linear correlation coefficient between the two estimates, respectively. The number in parentheses for $\Delta R$ ($B$) is the relative rms (bias) in percent.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$\langle R_{adj} \rangle$ (mm h$^{-1}$)</th>
<th>$\langle R_{MP} \rangle$ (mm h$^{-1}$)</th>
<th>$\Delta R$ [mm h$^{-1}$ (%)]</th>
<th>$B$ [mm h$^{-1}$ (%)]</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z-R</td>
<td>3.9</td>
<td>8.1</td>
<td>5.3 (105.4)</td>
<td>4.2 (105.4)</td>
<td>1.00</td>
</tr>
<tr>
<td>$kZ$</td>
<td>5.6</td>
<td>22.4</td>
<td>35.6 (663.5)</td>
<td>16.8 (302.6)</td>
<td>0.48</td>
</tr>
<tr>
<td>$kZs$</td>
<td>11.9</td>
<td>21.4</td>
<td>14.0 (81.4)</td>
<td>9.5 (80.6)</td>
<td>1.00</td>
</tr>
<tr>
<td>$kZH\alpha$</td>
<td>14.1</td>
<td>19.0</td>
<td>7.2 (36.6)</td>
<td>4.9 (34.9)</td>
<td>1.00</td>
</tr>
<tr>
<td>$kZ\alpha1$</td>
<td>20.9</td>
<td>20.9</td>
<td>0.1 (0.2)</td>
<td>0.0 (0.2)</td>
<td>1.00</td>
</tr>
<tr>
<td>$kZ\alpha2$</td>
<td>8.7</td>
<td>27.8</td>
<td>28.3 (230.1)</td>
<td>19.1 (219.5)</td>
<td>1.00</td>
</tr>
<tr>
<td>$kZH\alpha$</td>
<td>7.1</td>
<td>27.3</td>
<td>36.6 (460.0)</td>
<td>20.2 (284.2)</td>
<td>0.76</td>
</tr>
<tr>
<td>$kZH$</td>
<td>10.6</td>
<td>21.4</td>
<td>18.1 (713.5)</td>
<td>10.8 (101.5)</td>
<td>0.88</td>
</tr>
</tbody>
</table>

* Average of all algorithms except Z-R.
Let us now analyze the algorithms results using the adjusted relations (set 2) instead of the MP relations (set 1). Results in Table 2 show an increase in the retrieved \( \bar{R}_{\text{adj}} \) by a factor of 2–4, except for \( \bar{kZC} \) (increased by a factor of 1.35) and for \( \bar{kZa1} \), which is unchanged as expected (see section 3a). The new distribution of the local correction factors \( f_L \) for the adjusted relations may be inferred from that shown in Fig. 7 for the MP relations, by dividing each value by the global correction factor \( f_B = 0.51 \). Therefore, the new \( f_L \)'s are scattered from 0 to 3 (instead of 1.5) for \( \text{PIA} < 5 \) dB, then stabilize near about 1 (instead of 0.5) for \( \text{PIA} < 10 \) dB. An increase in the \( f_L \)'s explains the increase in the \( \langle R_{\text{adj}} \rangle \) from \( \bar{kZC} \) and \( \bar{kZa2} \). Besides, the closeness of \( f_L \)'s to unity, implying \( A_{\bar{K}} = A_{\bar{K}_{\text{adj}}} \) for large \( \text{PIA} \), explains that the \( kZ \) retrieval fits much better to the generic SR algorithms retrievals and that the \( kZH \alpha \), \( kZa2 \), and \( kZ \) retrievals are now much closer to each other since the downward collapsing effect for \( kZ \) and \( kZH \alpha \) is suppressed most of the time (cf. section 3). More precisely, change in the RR relations from the adjusted to the MP case modifies \( \langle R \rangle \) by a multiplicative factor for \( \bar{ZR} \), \( \bar{kZS} \), \( \bar{kZC} \), and \( \bar{kZa} \) \((\rho = 1 \text{ and } \Delta R \approx B)\) without a large spread, while \( \langle R \rangle \) from \( kZ \), \( kZH \alpha \), and \( kZH \) is modified in a more complex way, pointing out a lower correlation. Among the algorithms involving adjustable parameters, with the exception of \( kZa1 \), \( kZC \) (involving the constant \( K-R \) relation, like \( kZa3 \)) shows the smallest sensitivity.
to change from the MP to the adjusted relations, while $kZ\alpha_2$ and $kZH\alpha$ (involving a constant $Z-R$ relation) have larger sensitivities to this change. This is likely a consequence of the fact that the $K-R$ relation is known to be less sensitive to DSD variability than the $Z-R$ relation (Atlas and Ulbrich 1977). The rain profiles retrieved for the ray path number 40 (convective rain) using the adjusted relations are shown in Fig. 10 and may be compared to results of Fig. 5 referring to the MP case.

The changes in the point rain rates retrievals associated to the use of the adjusted relations is illustrated in Fig. 8b (Fig. 8c) for $kZ$ ($kZS$) and in Fig. 8d for $kZ$ versus $kZS$, which has to be compared with Fig. 8a when referring to the MP case. For $kZ$ (Fig. 8b), the retrievals are increased by a factor of 4 (or more) and poorly correlated ($\rho = 0.48$, cf. Table 2). For $kZS$ (Fig. 8c), the retrievals are increased by a stable factor of about 2. The $kZ$ and $kZS$ retrievals are in better agreement for the adjusted case (Fig. 8d) than for the MP case (Fig. 8a). From the MP to the adjusted case, the correlation coefficient $\rho$ changes from 0.5 to 0.58, while the mean bias (referring to $kZS$) changes from $-53.1\%$ to $+4.5\%$. Some retrievals located largely out of the bisectrix still come from an isolated collapsing or divergence tendency in $kZ$.

Similar results, but for the $kZH\alpha-kZH$ couple of hybrid algorithms, are shown in Fig. 9b (Fig. 9c) for $kZH\alpha$ ($kZH$), and in Fig. 9d for $kZH\alpha$ versus $kZH$, which has to be compared with Fig. 9a referring to the MP case. The retrievals from $kZH\alpha$ (Fig. 9b) or from $kZH$ (Fig.
Fig. 10. Same as Fig. 5 but with the adjusted relations. The new local correction factor is \( f_L = 1.14 \).

9c), using either the MP relations or the adjusted relations, are fairly correlated (cf. \( \rho = 0.76 \) for \( kZ\alpha \) and \( \rho = 0.88 \) for \( kZH \) in Table 2). The correlation coefficient between the \( kZH \) and \( kZ\alpha \) retrievals changes from \( \rho = 0.69 \) for the MP case (Fig. 8a) to \( \rho = 0.73 \) for the adjusted case (Fig. 8d), while the mean bias (referring to \( kZH \)), changing from \(-33\% \) to \(+27.8\% \), points out a persisting difference \((\pm 30\%) \) in the rain rates retrieved from \( kZH \) and \( kZ\alpha \).

Finally, results of Table 2 indicate that all algorithms, except \( Z-R \), provide \( \tilde{R}_{adj} \) ranging from \(-17\% \) below to \(+21\% \) above the ensemble mean \((22.9 \text{ mm h}^{-1})\) for different \( s_0 \) values. The close (far) case holds below \( H_j = 160 \) to \( 320 \) m (beyond \( 1600-3200 \) m) for \( s_0 = 13-7 \) dB. The maximum altitude range observed for the detectable MI returns with ARMAR is \( 300-5000 \) m. Below \( 300 \) m, the rain echoes cannot be processed because of the contamination by the surface echo via sidelobe effects. The altitude of \( 5000 \) m is the uppermost limit. Above \( 3000 \) m, the MI returns are usually scarce, except in very light rain. Thus, the close and far cases almost never hold, especially for large \( s_0 \); therefore, \( (19)-(20) \) have to be used without approximation. Figure 11 shows the variation with height of the correction term \( T_{MI} \) computed from (20) using \( G_2 = 0.63 \) and the same set of \( s_0 \) values. The \( s_0 \) dependency in \( T_{MI} \) is absent in the close case, then increases with height and the surface roughness and becomes linear versus height in the far case. As an example, for \( H_j = 1000 \) m \( T_{MI} \) increases from 6 to 10 dB when \( \sigma^0 \) decreases from 13 to 7 dB. Thus, the \( T_{MI} \) correction, which has to be removed to get the \( 4A_j \) estimate in (19), may be a large fraction of the \((Z_d - Z_M)\) signal.

The Fresnel surface reflectivity \( \Gamma^2 \) was determined using data as close as possible to the surface \((H_j = 300 \) m\) in light rain for 30 ray paths where \( R \), estimated from the \( Z-R \) method, did not exceed \( 5 \) mm h\(^{-1}\). A fast-converging iterative process was used to adjust also the \( a(\Gamma^2) \) correction term in the estimation of \( C_{HI} \) from (19).
to (20). The mean data-derived value $\Gamma^2 \exp = 0.85$ is larger than $\Gamma^2 = 0.63$, as expected from theory. Thus, $C_{\text{MI}}$ in (19) is 1.41 dB instead of 4 dB. Such large $\Gamma^2$ estimates were also reported by Meneghini and Kozu (1990) from similar airborne radar measurements over ocean at 10 and 35 GHz. This may result from some contamination of the $(Z_D - Z_M)$ signal next to the surface by sidelobes and/or multiple scattering effects.

A crude direct estimation of $\sigma_n^2$ (below rain) was attempted by selecting a few paths in very light rain where MI returns could be detected up to $H_p = 4.5$ km, considered as the rain top where the far approximation holds (see section 3c). Since 4.5 km is the altitude of the brightband peak, the $(Z_D - Z_M)_n$ signal was corrected by linear extrapolation between 4 km (just below the bright band) and 4.5 km, while assuming a constant attenuation rate. For these few paths, $u_n = 167.2$ dB (with $\Gamma^2 = 0.85$) from (23), and the mean signal is $(Z_D - Z_M)_n + 2Z_p = 180 \pm 1$ dB. The subsequent $\sigma_n^2 = 12.8 \pm 1$ dB, estimated from (22), is in excellent agreement with $\sigma_{\text{NR}}^2 = 12.9 \pm 0.4$ dB measured in clear air. Since the weak involved rain rates are not expected to have a significant impact on the ocean surface roughness, the closeness of the two estimates satisfies the conditions of the analysis.

The comparison of the $A_{\text{MI},j}$ from (19) to (20) and the $A_{\text{SR},j}$, computed from (25) using the $\alpha$ and $\beta$ coefficients of the MP $Z-K$ relation (26) was made by means of least squares linear regression. The comparison of path-averaged rain rates, obtainable from $A_{\text{MI},j}$ and $A_{\text{SR},j}$, via a common $K = aR^\beta$ relation (cf. section 3c), is not shown since it does not give more information. The analysis involved measurements for all range gates where MI returns were detected along the 143 ray paths of the dataset. The results obtained with $\Gamma^2 = 0.63$ and $\sigma_n^2 = \sigma_{\text{NR}}^2 = 12.9$ dB in $A_{\text{MI},j}$, and with $f = 1$ in $A_{\text{SR},j}$ extend over a dynamic range of about 5 dB. About 31% of the 7896 data points corresponding to nonphysical “negative” $A_{\text{MI},j}$ estimates and those mainly located at low level below about $H_p \approx 800$ m were eliminated. Therefore, using $\Gamma^2 = 0.63$ induces a bulk negative bias in the $A_{\text{MI},j}$ estimates. For the remaining positive estimates, the regression provides

$$A_{\text{MI},j} = 1.30A_{\text{SR},j} - 0.22$$

$$\rho = 0.87, \text{STD} = 0.33 \text{dB},$$

which indicates that $A_{\text{MI},j}$ exceeds $A_{\text{SR},j}$ by 30% on average.

Using $\Gamma^2 \exp = 0.85$ instead 0.63, with all other parameters unchanged, reduces the relative number of nonphysical negative $A_{\text{MI},j}$ estimates to 5% instead of 31%. However, the regression line still indicates that $A_{\text{MI},j} > A_{\text{SR},j}$. In fact, several parameters can modify the regression results. We just mentioned the effect of modifying $\Gamma^2$. Changing $\sigma_n^2$ also modifies the results for both $A_{\text{MI},j}$ and $A_{\text{SR},j}$, while changing $f$ modifies $A_{\text{SR},j}$ only. It was verified that an arbitrary bulk change in $\sigma_n^2$ by +1 dB (-1 dB), while keeping $f = 1$, improves (degrades) the correlation between the two PIA estimates. Finally, the “best” results were obtained using $\Gamma^2 \exp = 0.85$, $\sigma_n^2 = \sigma_{\text{NR}}^2 = 12.9$ dB, and $f = 0.5$, which is the value of the bulk adjustment factor derived from (7) in MTAKOV’97. Note that taking $f = 0.5$ is equivalent to directly using the adjusted $Z-K$ relation in the $A_{\text{SR},j}$ estimate (cf. section 4a). The results, restricted to positive $A_{\text{MI},j}$ estimates, are plotted in Fig. 12. The regression provides

$$A_{\text{MI},j} = 0.94A_{\text{SR},j} + 0.04$$

$$\rho = 0.87, \text{STD} = 0.35 \text{dB},$$

which indicates a bulk difference between the two estimates of only 6%.

It is worth noting that the best correlation results are obtained using the data-derived values of the key parameters: $\Gamma^2 = 0.85$, $\sigma_n^2 = \sigma_{\text{NR}}^2 = 12.9$ dB, and $f = 0.5$. This suggests that the MI algorithm results, immune to errors in the radar calibration and the rain relations, can actually help to validate the bulk correction factor $f = 0$ involved in the PIA estimates of the SR algorithms.

5. Conclusions and prospects

Near-nadir rain data gathered by the airborne TRMM radar simulator (ARMAR) within a typical event of TOGA COARE over ocean were used to study several deterministic algorithms aimed at providing range-profiled rain rates. Except the standard $Z-R$ method, ignoring attenuation correction, the attenuation-compensating algorithms include 1) the $K-Z$ algorithm, which is similar to the Hitschfeld–Bordan (HB) approach; 2) a set of generic SR algorithms performing rain profiling
from the surface (kZS algorithm), a path-by-path local adjustment of the reference rain relations involved in the analysis (kZa1 and kZa2 algorithms), or a local adjustment of the radar calibration (kZC algorithm); and 3) two hybrid algorithms, kZHa (a weighted combination of kZ and kZa2), which is based upon the same principle as the standard profiling algorithm of the TRMM radar and kZH (a weighted combination of kZ and kZS), which is a new approach. The selected generic algorithms are closely representative of almost all known “radar” methods, except the SR algorithms of Kozu and Nakamura (1991) and Fujita (1989), also adjusting the rain relations but with other constraints. Many hybrid algorithms, however, differing from the selected ones, could be elaborated due to a largely open choice for the combined algorithm pairs and/or the weighting functions (IM94). Lending support on previous related works (Marzoug and Amayenc 1994; IM94; MTAKOV97), the algorithm’s characteristics preparing the interpretation of the experimental results were outlined.

The dataset, including light stratiform rain and heavy convective rain with large PIA effects, is the same as one used in a previous companion paper (MTAKOV97), which provided us with a set of adjusted relations better climatologically tuned to the observed rain system than those associated to a Marshall–Palmer (MP) DSD. The MP relations and the adjusted relations were used here to study subsequent changes in the rain retrievals from the involved algorithms. In the absence of independent rain estimates to validate directly the retrievals, the present study was focused on the intercomparison of the results and the analysis of the observed features in relation with theoretical predictions. The intercomparison deals with retrieved rain structures in the nadir/along-track 2D frame, typical rain profiles recovered in light and heavy rain, and large-scale (~135 km) mean rain rates (R) retrieved near the surface in the 0.5–1-km altitude slab. The main results are hereafter summarized.

With the MP relations case in light stratiform rain, all attenuation-compensating algorithms provide almost similar retrievals, slightly in excess of the Z–R retrieval. However, the local adjustment in the generic SR algorithms kZai/C fails in the case where the large relative uncertainty in low PIA estimates produce nonphysical A_{im} (>1) estimates. Besides, the hybrid algorithms are able to correct for spurious results obtained with the SR algorithms when the SE-derived attenuation is largely erroneous, for example, when overflying a small island while assuming oceanic background. For convective rain with large PIA effects, the retrieved rain profile results are spread over a large range of values. The collapse of the retrievals from the Z–R method is expected due to the lack of attenuation correction. The generic SR algorithms and the kZH hybrid algorithm provide cell-like structure (with different peak magnitudes) as expected in region of large SE-derived PIAs. Conversely, the kZ generic algorithm and the kZHa hybrid algorithm have downward collapsing behavior, which produces a drastic discrepancy with other retrievals near the surface. The kZH collapsing is predictable from the large overestimation of the total PIA factor in the HB estimate (A_{hb}) with respect to the SE-derived estimate (A_{im}), that is, the PIA is underestimated in the HB approach as a consequence of a large value of the α coefficient in the MP Z–K relation. The more surprising kZHa collapsing results from the same cause and its correlative impact on the weighting function used in the algorithm; this thereby reveals a potential PIA-dependent deficiency of the TRMM-like radar algorithm in such a case. The kZH algorithm may be weakly penalized by such a defect, but for moderate PIA only. Excluding Z–R retrieval, the large-scale mean rain rates (R_{mp}) retrieved near the surface are within a ratio of 1–4, depending on the considered couple of algorithms; they are spread from 50% below to 85% above the ensemble mean of the (R_{mp})'s.

For the adjusted relations case, the collapsing effect in kZ and kZHa for large PIA is not observed in general
because $A_{m}$ and $A_{w}$ are much closer to each other. The rain retrievals from the attenuation-compensating algorithms, including those from $kZ$, are more comparable. They are significantly increased with respect to the MP case except for $kZa1$, which is unchanged because the adjusted relations are derived from the MP ones while using the same adjustment scheme as in this algorithm. Excluding $Z-R$ again, the associated mean rain rates near the surface ($R_{av}$) are within a ratio of 1–1.45 and are spread from 17% below to 21% above the mean of the ($R_{av}$)’s, which is itself two times larger than the mean of the ($R_{stn}$)’s. For the $Z-R$ method, ($R_{av}$) is increased by a factor of 2 compared with ($R_{stn}$) but is still 2.5 times smaller than the ensemble mean; this points out the importance of the attenuation corrections in the estimation of the large-scale mean rainfall. The improved agreement between the retrievals confirms that the adjusted relations are better tuned to the observed rain regime than the MP relations.

The best profiling method(s) cannot be easily extracted from the present results. However, which algorithms seem more reasonable relative to uncertainties in the RR relations can be briefly discussed. Adjusted relations in $kZa1$ must be close to microphysical measurements (MTAKOV97), which provides some evidence of superiority compared with other approaches. Also, $kZC$ is moderately sensitive to the initial relations. The other algorithms are more likely to be in error since the derived rain rates in heavy rain are significantly lower when using the MP relations. In fact, intercomparing the retrievals from the generic and/or hybrid algorithms may actually help to identify the physical assumptions upon which the various methods are built and identify error sources. A striking example, shown here, is that observing a systematic downward collapsing of $kZ$ and $kZH\alpha$ with respect to other retrievals is a signature of a well-identified defect in the reference rain relations.

For the TRMM case, however, effects other than those specifically studied here have to be taken into account. For example, how nonuniform beamfilling effects, almost absent with ARMAR, modify the obtained conclusions via the associated changes in the PIA estimate (Amayenc et al. 1993; Amayenc and Tani 1997) has to be evaluated. The correction of nonuniform beamfilling effects in the TRMM radar hybrid algorithm (Kozu and Iguchi 1996, 1997) may change its performances relative to the above-mentioned potential deficiency. Also, the assumption of range-free coefficients in the rain relations may bias the retrieval, especially in stratiform rain where a nonattenuating ice layer is present aloft. This drawback may be shaped with a $kZS$ algorithm, which performs rain profiling from the surface. The $kZH$ hybrid algorithm, identifying with $kZS$ for a large PIA, also benefits by this advantage. However, the absence of any correction for the rain relations in $kZS$ and $kZH$ suggests preferably using them with climatologically tuned relations. The TRMM-like hybrid algorithm $kZH\alpha$ (using $kZa2$ with an implicit constant $Z-R$ relation) performs such a type of correction but may suffer from a severe PIA-related deficiency in the case of ill-adapted reference rain relations. This drawback may likely be overcome by different independent or combined ways such as 1) the use of alternative weighting functions $\omega 1$, as alluded to in IM94, so that the $kZ$ contribution can be more strongly damped at high attenuations and 2) the introduction of additional information in $\omega 1$ from the SE-derived PIA according to its estimated reliability. Fortunately, such a flexibility exists in the TRMM radar-profiling algorithm (Iguchi et al. 1997) to perform relevant tests. Also, other hybrid schemes, involving $kZa1$ (with a full correction of the three rain relations) instead of $kZa2$, for example, could be investigated. Finally, information on the proper reference rain relations to be used a priori could be derived from the global-adjustment method (MTAKOV97) and/or the $N_r$-adjustment method (Kozu and Nakamura 1991) provided that error in the radar calibration is small.

A mirror image algorithm providing PIA profiling at nadir viewing was also tested. Limitations inherent to airborne measurements conditions were pointed out. The main results are as follows.

1) The experimentally derived Fresnel surface reflectivity $\Gamma^2 = 0.85$ exceeds the theoretical value (0.63), most likely because of the sidelobes and/or multiple scattering effects close to the surface.

2) The direct estimate of $\sigma _r^1$, obtainable below very light rain only, is in excellent agreement with $\sigma _{SR}^1$ measured in clear air.

3) The one-way PIA’s (from the surface), as estimated from the MI algorithm (immune to the radar calibration and the rain relations) and from a “generalized” integral form of the SR algorithm $kZS$ depending on the adjustment of the bulk correction factor $f = \delta a \delta C$, are well correlated over a 5-dB dynamic range. The best correlation, pointing out a 6% difference between the two estimates (in dB), is obtained while using data-derived values of the key parameters: $\Gamma^2 = 0.85$, $\sigma _{SR}^0 = 12.9$ dB, and $f = 0.5$. This opens an attractive way of exploiting the MI algorithm results to validate the PIA estimates of the SR algorithms via the control of $f$.

The application conditions of the MI algorithm seem better for spaceborne rather than airborne geometry (Meneghini and Nakamura 1988; Liao et al. 1995). As a matter of fact, for the spaceborne case the lower beam divergence improves the matching of the direct and mirror-reflected rain volumes, and the use of close approximation over an extended altitude range should weaken the sensitivity of PIA-profiling results to the surface roughness. However, new limitations may appear as a consequence of a higher detection threshold and of increased nonuniform beamfilling effects. Relevant tests have to be conducted with real data from the TRMM radar.
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