The Effect of Suspended Ice Crystals on Radiative Cooling

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ABSTRACT

Two periods of very low (below −40°C) surface temperature at Fairbanks, Alaska, were studied in detail as part of ice fog investigations during the 1961–1962 winter. The observed cooling rates from the snow surface up to 3000 m were too large to be satisfactorily explained by advection and/or by radiative heat losses from the air and from the snow surface. The excess is shown to be due to radiation from ice crystals suspended in air.

The ice crystals, formed by overall cooling of the air, act as heat sinks. It is proposed that heat flows from the air to the crystals and is radiated away. This process results in strong temperature gradients in the air immediately adjacent to the crystals. It may also account for the fact that humidity measurements show less than saturation values during occurrences of ice fog, light snowfall, or "diamond dust" crystal displays. The air temperature values used in determining humidity pertain to ambient air between the ice crystals, whereas the air in contact with crystals has a lower temperature and is saturated with respect to ice.

1. Introduction

As part of an investigation on ice fog in and around Fairbanks, Alaska, two prolonged, extreme (below −40°C) cold spells were studied in detail. The first one, from 15 to 29 December 1961, set an all time Fairbanks record for the month of December with 231 consecutive hours below −40°C, including an officially recorded minimum of −52°C (−61.6°F). The second cold spell lasted from 23 to 29 January 1962.

The mechanism of heat loss by radiation from the snow surface outlined by Wexler (1936) does not seem sufficient to explain the rapid cooling of the air in these cases. This is especially so because the cooling does not start at the surface and spread upward but occurs simultaneously at all levels up to 3 km. Advection must be considered as indicated by Wexler. However, in addition to advection, the radiative heat losses from ice crystals formed, and suspended, in the cooling air appear to play a major role. This will be demonstrated by outlining the physical history of the two cold spells and comparing observed and calculated cooling rates with, and without, allowance for radiation from suspended ice crystals.

The surface and upper air data used in this paper were obtained from the U. S. Weather Bureau station at Fairbanks International Airport, situated 6 km southwest of the city of Fairbanks (in the Tanana River Flats, 130 m above sea level).

It is not the purpose of this paper to discuss ice fog. However, since this study grew out of ice fog investiga-

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2 On leave from the Meteorological Institute, Oslo, Norway.

2. The period 14–30 December 1961

On 14 December high surface pressure started building up over Northern Alaska and during the following days gradually spread over the interior. Simultaneously, a low pressure center was situated over the Gulf of Alaska. This situation persisted until 29 December, when a new and stronger low pressure center moved into the Gulf and the high pressure system rapidly moved into Canada. During the whole period, 15 to 30 December, the high was centered north of Fairbanks, causing north-easterly winds at low levels.

Fig. 1 shows the temperature variations at different levels. Selected soundings are shown in Fig. 2; the strength and vertical extent of inversions are shown in

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Fig. 1. Temperature variations at selected heights, December 1961, from U. S. Weather Bureau rawinsonde data, Fairbanks.

Fig. 3, and the upper wind data for the period are summarized in Fig. 4.

Temperatures from a hill top (Birch Hill) 2.5 km northeast of the city and 200 meters above the flat, are available, but are not shown in Fig. 1. Only occasionally do they differ more than a few degrees from the free air temperatures at the 200-meter level. Thus it appears that a local strong surface inversion over the hill top does not form. This inference is supported by measurements made on micrometeorological towers during the 1962–63 winter. For example, during the cold period 7–9 January 1963 the temperature difference between 2 and 15 m above snow surface on top of Birch Hill rarely exceeded 1°C; during the same time period, similar measurements at the base of the hill, commonly showed differences of 4 to 5°C.

Even though surface winds were low, nearly always less than 2 m sec⁻¹, one could hardly consider the entire air mass to be stagnant. Upper level winds were variable (Fig. 4) and both the depth and strength of the inversions underwent considerable changes. The improved visibilities and higher temperatures between 18 and 19 December are, for instance, correlated to the higher

wind speeds at low levels, accompanied by a decreased ground inversion.

On 25 December air started moving in from the south at higher levels, bringing in some high clouds, and weakening the inversion. When clouds appeared at medium altitudes on 27 December, the surface temperatures increased and visibility improved. However, the clouds rapidly dissolved and the surface temperature dropped to a record low for the month of December, —52°C on 28 and 29 December. The temperature at the 200-m

Fig. 4. Upper winds, December 1961, U. S. Weather Bureau rawinsonde data, Fairbanks. Isolines are drawn for 2, 5, 15, 25 and 35 m sec⁻¹. Wind directions are indicated by arrows, and dashed lines divide northerly from southerly flows. VAR = variable directions.
level (and at Birch Hill) approached the values of the flats on 27 December, thus creating an isothermal layer. The 200-m temperature started rising on 28 December, about 24 h before the increase in surface temperature. Not until the southerly flow penetrated to lower levels, accompanied by increasing cloudiness, did the ground inversion break up. Then the cold spell ended and temperatures at all levels rose rapidly.

3. The period 23–29 January 1962

The weather development was similar to the December case. On 23 January high surface pressure started building up over northern Alaska and slowly intensified. A stationary low was situated in the Gulf of Alaska and, on 26 January, a strong cyclone moved into the Gulf. By 28 January the cold spell ended.

Temperature variations at different levels during this cold spell are shown in Fig. 5. Temperatures at Birch Hill again closely followed those of the free air at the same level and are not shown. Most of the time the surface wind was less than 2 m sec\(^{-1}\). Selected soundings are shown in Fig. 6. Fig. 7 is a photograph taken 27 January from Birch Hill overlooking the southeastern Fairbanks area. The smoke plume from a power plant is clearly visible, drifting with the wind down the valley. The height to which it rises before spreading out horizontally is a function of the outlet temperature and velocity as well as of the lapse rate (negative) in the air. Thus, visual observation or photographs of smoke plumes do not, in themselves, provide reliable information on the increase of windspeed with altitude nor on the inversion.

4. Discussion of data

Wexler (1936 and 1941) has shown how extreme low surface temperatures can be explained theoretically by radiational cooling, and that temperatures at intermediate and higher levels will set a limit to the cooling of the surface air. Not before temperatures at these levels get sufficiently low can the extreme low surface temperatures be obtained. The radiative cooling of polar air during clear weather starts below and spreads upwards in successive stages. A strong surface inversion develops, but decreases in strength with time. Above the ground inversion an isothermal layer should develop and increase in thickness. But subsidence will have a tendency to lower the top of the isothermal layer and create a weaker inversion above the ground inversion. The data presented here agree with Wexler’s model; the ground inversion is strongest in the first parts of the cooling periods, and the effect of subsidence can be seen in Fig. 3.

Apart from the warming on 18 December, which can
partly be ascribed to advection, increased wind speed, and turbulent mixing, the observed patterns within the cooling periods seem on the whole to agree with Wexler's findings. However, the rapid rate of cooling of the entire air mass (up to 3 km) observed at the start of each period far exceeds that which can be explained solely by radiational cooling from the surface, and as indicated by Wexler (1936, p. 133) advection must also be considered.

From Fig. 1 and 4 it seems reasonable to attribute the cooling from 14 to 15 December to advection. A deep layer from ground to about 5 km was affected. The maximum cooling occurred at a height of 1200 m. Light snow was falling and the surface wind averaged 9 knots. On 15 December the wind speed decreased to below 5 knots and only then did a ground inversion develop.

An interesting case where the temperature fell simultaneously with the pressure occurred in February 1963, when a cold upper trough moved in and became stationary. From 2 to 3 February the temperature fell from −19°C to −38°C. From then on the minimum temperature gradually decreased to −43°C on 5 February. The pressure (reduced to mean sea level) fell from 1030 mb on 2 February to 1004 mb on 5 February. Clear sky prevailed and “diamond dust” ice crystals were observed during the rapid cooling. The advection of cold, dry air aloft was apparently the main cause of the cooling.

5. Net radiation

With less than 4 hours of sunshine at the end of December, and the sun barely above the horizon, the effect of incoming solar radiation can safely be neglected. The effect of the ice fog on the infrared radiation is difficult to estimate. Ice fog rarely exceeds 30 m in thickness, and contains a low water vapor density because of the overall low temperature. It should have little effect on the net outgoing radiation heat flux; this is especially so because the top of the inversion, which generally lies well above the ice fog, has the maximum water vapor content. However, within the ice fog itself, radiation heat exchange will weaken the inversion. In time the ice fog may become isothermal, or, if conditions persist, a layer with negative lapse rate may form near the ground. Thus, the presence of ice fog should decrease the rate of surface cooling. This shows in the temperature and radiation data in the first 29 m as presented by Robinson et al. (1955). Their discussion refers to an

\[ Q = 0.105 + 0.00187T, \]

where the units of \( Q \) are ly min\(^{-1}\) and \( T \) is temperature (C). Hoinkes (1961), using radiometer and temperature data from Little America finds figures in good agreement with Wexler.

No reliable radiometer data are available from the periods investigated. We are mainly interested in correct orders of magnitude, so Elsasser's radiation chart (2nd revised edition) was used to simplify the calculations (Haltiner and Martin, 1957; Elsasser, 1942). A linear pressure reduction has been used throughout in place of the former square-root reduction, in accordance with newer findings (Elsasser, 1960).

Table 1 gives the calculated fluxes from the chart as compared to the mean values from Wexler's formula and from Hoinkes's data.

On 14 December and 24 January light snow was falling, but so light that the vertical visibility was unlimited. The correction to the calculated fluxes should be negligible when we consider the very low concentration of the snow particles. As there was no inversion on 14 December and only a rather weak one on 24 January, it is reasonable that the values calculated from Elsasser's chart are higher than the mean values taken from observations by Wexler and Hoinkes. With the strong inversion on 19 December the agreement between values is better. The use of Elsasser's radiation chart seems justified. The measurements of water vapor are inaccurate at low temperatures, but the main effect of the water vapor comes from the warmer layers aloft where the values are more trustworthy. Although the chart may give slightly high values it at least will give the correct order of magnitude.

When comparing the calculated fluxes (Elsasser) with those derived from observations (Wexler, Hoinkes) it is necessary to keep in mind the wide scattering of the values obtained from the radiometers. For example, at −38°C Wexler (1941) observed variations from 0.03 to 0.08 ly min\(^{-1}\). Also, the calculated values are based on temperature measurements from a standard instrument shelter which, under these conditions, may be several degrees higher than the temperature of the snow surface.

\[ ^{1} \text{Many terms exist to describe the appearance of small crystals which form in clear air (see \textit{Glossary of Meteorology}, 1959, p. 164 and p. 293; American Meteorological Society, Boston, Mass.)}. \text{The terms “diamond dust” or “ice crystals” are used here and the occurrence of such an event is conveniently lumped under the term “ice crystal display.” Such occurrences indicate existence of ice in the air which may go unrecorded in the meteorological records. The crystals produce optical effects, such as “sun dogs,” halos, etc., which are generally included under “halo phenomena” (Neuberger, 1951).} \]
or even of a low-placed radiometer. This may contribute to the difference between calculated and empirical values in Table 1, because Wexler used the air temperature measured near the radiometer and Hoinke used the temperature of the radiometer itself.

6. Cooling rates

Table 2 shows calculated cooling rates using the radiation chart by Elsasser. In two cases the radiation chart by Yamamoto (1953) has also been used as a check. Yamamoto’s chart is especially practical for calculating cooling rates at specific levels. The effect of carbon dioxide was neglected in the latter case, as its effect on the temperature changes is relatively small near the ground. Yamamoto finds his chart to give somewhat higher cooling rates than the chart by Elsasser (Yamamoto, 1953). Yamamoto’s method refers to a specific level, whereas Elsasser’s radiation chart refers to a layer of air. The calculated and observed cooling rates agree reasonably well on 19 December, at least they are of the same order of magnitude. This is not the case on 14 December and 24 January.

The initial rapid cooling occurs not only at the surface but at higher levels as well. This is not in agreement with the cooling model of Wexler where the cooling starts at the surface and gradually spreads upwards. At first sight the explanation for this would be advection of colder air. Undoubtedly advection plays an important part. However, another effect mentioned by Wexler but not seriously considered, is the effect of radiation from ice surfaces in the very light snowfall or in ice crystal displays accompanying the cooling of the air. In most cases this effect appears to be of primary importance.

7. Effect of ice crystals in the air

Both periods started after or at the end of a light snowfall which ended in an ice crystal display. The record for Fairbanks, 1959 to 1963, shows that all the cold spells, when the surface temperature dropped below -34.4°C (=-30°F) started during a snowfall or an ice crystal display. During observations made from 1961 through 1964 rapid coolings were not observed without associated formation of ice crystals in the air. Crystal displays during cooling were observed on a hill 600 m above the flats as well as in the flats. There were 12 such cases during these 4 winters each of which showed the rapid rate of cooling. This relationship is also noted in the data compiled by Byers (1940) for the Fairbanks winters 1936-1938. His data show 10 similar cold spells, 6 of which started with snowfall or snowflurries reported. In two cases thin “stratus” clouds were reported. At the low temperatures involved, they must have been pure ice clouds from which ice crystals would precipitate out without being reported. Often this type of low “cloud” has no definite base or top and, according to our usage, would be referred to as a crystal display. Only in one case were no clouds or snowflurries reported. Ice crystal displays were not noted in the 1936-1938 data, yet may well have occurred without affecting unlimited vertical visibility. In general, the occurrence of crystal displays, or light snowfall associated with sudden onset of cold spells seems to be the rule for interior Alaska, not the exception, even in the extreme case of no advection.

The formation and growth of ice crystals from vapor will release latent heat to the air; on the other hand, an ice crystal radiates essentially like a black body in the infrared. It receives radiation from the ground as well.
as from water vapor and carbon dioxide. Water vapor densities calculated from observed relative humidity values during the rapid coolings are shown in Table 3. Very light snow was falling on 14 December and ice crystals were reported on 15 December and 24 January. The values in Table 4 were obtained as follows:

1) The radiative heat loss was found by using Eissler's radiation chart, including the heat flux resulting from ice crystals treated as black bodies placed at the level concerned. For simplicity the ice crystals are assumed to be plates of radius 25 microns, and with a concentration of 1 per cm² at any time.¹

2) The heat gain per cm³ is the heat of sublimation times the water vapor density differences calculated in Table 3.

3) The calculated rate of cooling (Table 4) is obtained from the formula:

\[ \frac{dT}{dt} = \frac{dQ}{E_p m_s + E_i m_i} dt. \]

Where \( dQ = \) net loss of heat, \( dt = 12 \) hours, \( E_p = \) specific heat of dry air, \( E_i = \) specific heat of ice and \( m_s \) and \( m_i = \) mass of air and of ice crystal in one cm³ of air. Even if \( m_i \) is not well known, the term \( E_i m_i \) being several orders of magnitudes smaller than \( E_p m_s \), can safely be neglected.

The heat loss by radiation is proportional to the concentration and to the square of the radius of the crystals. Even if the size is smaller, say 10 μ, and the concentration is assumed to be one crystal per cm³, the radiative heat excess exceeds the heat added. The latter assumption of size and concentration, representing an extreme case

¹ These values are consistent with observations by M. Kumai, K. Francis and the authors during typical ice crystal displays in the Fairbanks area.

8. Crystal temperatures, humidity measurements and cooling mechanism

In making the above calculations the ice crystals were assumed to have the same temperature as the (surrounding) air. However, this is not strictly correct because as the ice crystals cool by radiation they will be cooler than the air, and strong temperature gradients will form in the air immediately adjacent. Although the differences are small they are of special interest in that they may explain the fact that during light snowfalls and ice crystal displays we generally measure less than 100% relative humidity with respect to ice. These measurements pertain to the air between the ice crystals, whereas the temperature of the air in contact with

It is clear from Figs. 2 and 6 that the observed cooling rates entered in the tables are not exceptional. For example, the temperature at 1025 m decreased 5.3°C between 0200 and 1400 on 24 Jan. and 2.8°C between 0200 and 1400 on 25 Jan.

Table 3. Water vapor densities. (Relative humidity and air temperature values taken from radiosonde records.)

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Height (m)</th>
<th>Temp. C</th>
<th>Observed relative humidity (Over water)</th>
<th>Observed relative humidity (Over ice)</th>
<th>Water vapor density (g m⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 Dec</td>
<td>1400</td>
<td>915</td>
<td>-24.5</td>
<td>66</td>
<td>84</td>
<td>0.486</td>
</tr>
<tr>
<td>15 Dec</td>
<td>0200</td>
<td>915</td>
<td>-27.3</td>
<td>63</td>
<td>82</td>
<td>0.365</td>
</tr>
<tr>
<td>16 Dec</td>
<td>0200</td>
<td>95</td>
<td>-22.1</td>
<td>59</td>
<td>73</td>
<td>0.532</td>
</tr>
<tr>
<td>16 Dec</td>
<td>1400</td>
<td>95</td>
<td>-25.0</td>
<td>43</td>
<td>55</td>
<td>0.303</td>
</tr>
<tr>
<td>24 Jan</td>
<td>1400</td>
<td>1025</td>
<td>-26.8</td>
<td>64</td>
<td>83</td>
<td>0.386</td>
</tr>
<tr>
<td>25 Jan</td>
<td>0200</td>
<td>1025</td>
<td>-28.7</td>
<td>58</td>
<td>77</td>
<td>0.296</td>
</tr>
</tbody>
</table>

Table 4. Heat budget and cooling rates per cm³ air at the levels given in Table 3.

<table>
<thead>
<tr>
<th>Date and time 1961–1962</th>
<th>Height (m)</th>
<th>Heat exchanges (cal X 10⁻⁵ per cm³)</th>
<th>Cooling (C) Calculated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Heat loss by radiation</td>
<td>Heat gained by</td>
<td>Net heat</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>liberation of latent heat</td>
<td>loss</td>
</tr>
<tr>
<td>14 Dec 1400 to 15 Dec 0200</td>
<td>915</td>
<td>10.25</td>
<td>0.82</td>
<td>9.7</td>
</tr>
<tr>
<td>16 Dec 0200 to 16 Dec 1400</td>
<td>95</td>
<td>21.50</td>
<td>1.55</td>
<td>19.55</td>
</tr>
<tr>
<td>24 Jan 1400 to 25 Jan 0200</td>
<td>1025</td>
<td>8.16</td>
<td>0.61</td>
<td>9.55</td>
</tr>
</tbody>
</table>

* This value was measured at the 1025-m level, the average for the layer 1025 to 1248 m was 2.7°C (see Table 2).
the ice crystal may be low enough to be saturated. In the examples cited in Table 3 the temperature difference between the measured air temperature and the temperature corresponding to saturation with respect to ice and water is close to 2°C and 3°C, respectively. (The range of values is 1.8°C to 2.6°C and 4.7°C to 6.1°C, excluding the dates for 16 December 1400 where the values were 6.1°C and 12.3°C, respectively. In the latter case there is reason to suspect that the crystals, if present, were evaporating.)

Let us follow this point one step further by looking at the temperature distribution in the air around the crystals. The first question that comes to mind is: “How far from the crystal is the air temperature different from ambient, i.e., from the air between the crystals?” For simplicity, assume spherical crystals with diameter of 25 μ, and concentration 1 crystal per cm³. In keeping with the values in Table 3 consider the relative humidity of the air to be about 80% relative to ice. Assume the surface temperature of the crystal to be 2°C lower than the ambient air temperature and that the air in contact with it is saturated with respect to ice. For convenience we use temperatures of 240K (−33°C) for the crystal surface, and 242K (−31°C) for the ambient air; this gives a relative humidity of 81% for the ambient air. Assume further, that the air temperature gradient produced by the cooling crystals extends for such a short distance that heat transfer by convection is negligible, as a first approximation, and consider conduction alone. Then we can equate the outgoing heat flux by radiation from the crystal, to the incoming heat by conduction from the air,

$$F = aT^4 = k \frac{dT}{dn},$$  

(1)

where $F$ is the heat flux, ly sec⁻¹,

- $a$ is the Stefan–Boltzmann constant $= 1.36 \times 10^{-8}$ ly sec⁻¹ K⁻⁴,
- $T$ is degrees Absolute,

$k$ is the thermal conductivity of air $= 57 \times 10^{-6}$ cal sec cm⁻¹ C⁻¹,

d$t/ dn$ is the thermal gradient normal to the surface C cm⁻¹.

Since we have assumed the temperature difference between the ice surface and the ambient air to be $dT = 2°C$, we can solve Eq. (1) for $dn$

$$dn = \frac{kdT}{aT^4} = \frac{57 \times 10^{-6}}{4.5 \times 10^{-3}} \times 2 = 0.025 \text{ cm} = 250 \mu.$$  

(2)

Within this short distance from the crystal surface, the computed gradient (80°C cm⁻¹) is about 10⁵ greater than normal atmospheric lapse rates. The above assumptions imply a simple linear gradient; however, it is more likely that an exponential decrease of temperature would prevail from the crystal into the air. This would make the gradient even steeper near the crystal and would restrict it to within about 100 μ from the crystal surface. Thus under the assumed conditions, the calculated value of 250 μ represents an extreme maximum distance for the temperature anomaly caused by the crystal to extend into the surrounding air. Two other factors contribute to insure this, they are:

1) the possibility of some convective heat transfer as the crystal falls through the air, and

2) the liberation of latent heat of sublimation by the crystal if it is growing.

An order of magnitude for the amount of latent heat transfer may be obtained by assuming a growth rate for our spherical ice crystal whose initial volume and mass are $7.9 \times 10^{-9}$ cm³ and $7.25 \times 10^{-9}$ gm, respectively. If, as it falls through the air, this crystal adds 10 per cent to its mass it will liberate

$$[678 \text{ cal gm}^{-1}] \times [0.725 \times 10^{-9} \text{ gm}] = 4.92 \times 10^{-7} \text{ cal},$$

which when spread over the surface area (1.96 $\times 10^{-5}$ cm²) amounts to $2.5 \times 10^{-2}$ ly. If this 10 per cent growth occurs in an hour the resultant heat flux would be $6.9 \times 10^{-6}$ ly sec⁻¹, if it occurs in 1 minute the heat flux would be $4.2 \times 10^{-4}$ ly sec⁻¹. A faster growth rate would be very unlikely because of the low vapor density at the temperatures involved. In these cases the radiative heat flux is orders of magnitude greater than the flux due to liberation of latent heat. Thus, the crystals, even when growing, have lower temperature than the surrounding air which is cooled as they fall through. Spatial relations between the crystals and the localized air temperature gradients produced by them are shown in Fig. 8.

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REFERENCES


