The Frequency Distribution of Lagos/Ikeja Wind Gusts

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ABSTRACT

In a majority of cases, the frequency distribution of maximum values of meteorological elements is either normal or skewed positively, although good agreement with the normal distribution is not generally found. Several types of extreme-value distributions are available. Gumbel's 1954 distribution, the most widely tested and used, appears as a straight line when plotted on extreme-value probability paper and extrapolation is simple.

Daily maxima of homogeneous wind data for Lagos (Ikeja) (06°35'N, 03°20'E) for 1948–1962 have provided 4033 observations. These were first plotted as an actual frequency distribution, smoothed, and then plotted on extreme-value probability paper. The reduced variate \( y \) is related to the frequency \( p \) by the relation \( y = \ln(-\ln p) \). The straight line Gumbel distribution, the equation of which is \( z = u + (1/u)y \), where \( u \) is the mode and \( 1/u \), the slope of the line, is related to the standard deviation of the sample population. Since these data fell along a straight line, Gumbel's distribution provided a good fit. The larger the number of observations the more closely Gumbel's theory tends to apply.

1. Objective

In the process of climatological statistical analysis of the raw data of various meteorological elements accumulated by the Nigerian Meteorological Service, investigators have been faced with quite a variety of extreme-value distributions from which to choose. These include the logarithmic normal, Pearson's Type III, Jenkinson's (1955) distribution and the Fisher-Tippett Type I distribution already widely applied by Gumbel. With respect, especially, to the frequency distribution of maximum values, previous workers have recognized the skewness of the distribution and the lack, at times complete lack, of fit of the normal distribution to the observations. Since the form of the distribution is generally not known initially, a guess is usually made as to the type to be adopted. It should be remembered, however, that while two different distributions may yield very satisfactory fits to the data, the tails of the distributions can be significantly different. Statistically, however, this may present no problems at times as the part of the distribution generally of interest is well away from that part where observations provide information about the overall shape.

Gumbel (1954) has proved beyond doubt that for initial distributions of the exponential type, the distribution of maximum values will follow the asymptotic theory. In support of this, Boyd and Kendall (1956), who have compared some of the different statistical approaches to the subject, state that "the different methods of predicting extreme values that have been examined give widely differing results. None of the methods is completely satisfactory for small samples, but it appears that there is little reason for preferring the methods of Thom or Jenkinson to the more straightforward method developed by Gumbel."

Difficulties similar to those encountered by Whittingham (1960), who showed that the Gumbel distribution is a good fit to the daily maximum wind gusts observed at Brisbane from 1938–1953, were faced by the author in his work. Thus, it was felt that an effort should be made to see whether or not Whittingham's findings would hold for the tropics by studying for a fairly long period the distribution of daily maximum wind gusts at a tropical station, having a type of climate where quiet winds normally prevail but which is occasionally visited by the violent winds associated with large-scale mature storms.

2. Gumbel standard skewed distribution

The wind record for Lagos/Ikeja (06°35'N, 03°20'E) obtained from Meteorological Office pressure-tube anemograph (Mark II) charts yielded a sample population of 4033 daily wind gust maxima in knots for the unbroken years 1948–1962 inclusive. Prior to this period the anemometer was situated at Apapa, the old site of the Lagos airport. The record for Lagos which excludes the Apapa data is therefore perfectly homogenous with respect to site and instrumentation. The solid line in Fig. 1 shows the actual frequency distribution of the 4033 daily maximum gusts. The mode at 14 kt is very pronounced, as are, to a lesser extent, the secondary minima which occur at 13, 15, 21, 26 and 31 kt. The statistical procedure (similarly adopted by Whittingham) which involves casting half the odd knot frequencies to the one above was then employed, and
this resulted in the smoothed relative frequency (dashed) curve of Fig. 1. The asymmetrical nature of the distribution, i.e., its marked positive skewness, is immediately apparent.

In Fig. 2 the above distribution is plotted on extreme-value probability paper. Owing to the large number of points, it was necessary to plot the more congested regions as vertical bars with the number of points alongside. For example, at 14 kt, occupying the frequency from 35.83 to 45.15%, there is a vertical bar representing 377 points. The plotting positions of the points are given by \( m(n+1) \) where \( m \), the rank of an individual gust in ascending order ranging from 1 up to \( N \), is the total number of observations.

The reduced variate \( y \) has a vertical scale alongside that of frequency, and is linear. The reduced variate is related to the frequency probability \( p \) by the relation

\[
y = -\ln(-\ln p).
\]

Gumbel’s extreme-value theory shows that the plot (on extreme-value paper) of a series of observed values \( x \), in this case the daily maximum wind gust, should approximate to a straight line. This is an appealing feature of his distribution in that there is no difficulty with regard to extrapolation. The equation to the Gumbel line is

\[
x = u + (1/a)y,
\]

where \( u \) is the modal value of the distribution, and \( 1/a \), the slope, \( dx/dy \), of the Gumbel line, is related to the standard deviation of the sample, and is thus a measure of dispersion. These parameters \( u \) and \( 1/a \) may be estimated from

\[
1/a = \sqrt{6S/\pi} = 0.7797S, \tag{1}
\]

\[
u = \bar{x} - \gamma/d = \bar{x} - 0.4505S, \tag{2}
\]

where \( \bar{x} \) is the mean value of the sample, \( S \) the standard deviation, and \( \gamma (=0.5772) \) Euler’s constant. The Gumbel line in Fig. 2 has the equation \( x = 13.19 + 4.73y \).

From the mode and slope, the value of the variate can be found. This corresponds to a value of daily maximum

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**Table 1. Probabilities for daily gust data.**

<table>
<thead>
<tr>
<th>( p ) [per cent]</th>
<th>( y )</th>
<th>( x )</th>
<th>( T ) [years]</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>2.25</td>
<td>23.78</td>
<td>10</td>
</tr>
<tr>
<td>95</td>
<td>2.97</td>
<td>27.16</td>
<td>20</td>
</tr>
<tr>
<td>98</td>
<td>3.90</td>
<td>31.53</td>
<td>50</td>
</tr>
<tr>
<td>99</td>
<td>4.60</td>
<td>34.82</td>
<td>100</td>
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</tbody>
</table>
gust which on the average will be equalled or exceeded once in $T$ years (the return period). Alternatively, the maximum daily $T$-year gust can be calculated using the relation $T = \frac{1}{(1-p)}$ where $p$ is the frequency of occurrence. Table 1 shows the results obtained for Lagos (Ikeja).

Data on annual maximum values of wind for 14 years, 1946-1961, excluding 1952, have similarly been subjected to Gumbel’s analysis and the results plotted on extreme-value probability paper as shown in Fig. 3. Values of $y$, $x$, and $T$ at various probability levels are shown in Table 2.

Gumbel’s method of testing the goodness of fit of the observations to the Gumbel line involves drawing control curves on either side of the line, as shown in the figure. The half-width of the 68% confidence band is $1.14x$ (slope of Gumbel’s line) for the highest observed and the extrapolated extremes. The fit is considered good if two-thirds of the observed points lie within the control curves.

3. Discussion and results

It is clear that the Gumbel distribution is very simple to use and to interpret with regard to the statistics of extreme values.

In Section 2, secondary minima were said to have

<table>
<thead>
<tr>
<th>$p$ (per cent)</th>
<th>$y$</th>
<th>$x$</th>
<th>$T$ (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>2.25</td>
<td>54.0</td>
<td>10</td>
</tr>
<tr>
<td>95</td>
<td>2.97</td>
<td>57.5</td>
<td>20</td>
</tr>
<tr>
<td>98</td>
<td>3.90</td>
<td>63.0</td>
<td>50</td>
</tr>
<tr>
<td>99</td>
<td>4.60</td>
<td>67.0</td>
<td>100</td>
</tr>
</tbody>
</table>
occurred at the odd numbers of knots of daily maximum winds.

Whittingham (1960), using data with mph units, states that "such a feature would not be in evidence today due to the practice of converting into knots." He goes further to attribute this phenomenon to "the fact that the Dines chart is ruled into even 2-mph divisions" and that such a frequent occurrence of odd values "indicates a personal failing common to all observers."

This analysis accords with Whittingham's observations but fails to indicate any basis for his conclusions, although we have used identical analysis procedures.

The only point where there seems to be similarity in the results is on the occurrence of secondary minima which in our case is very marked at 13, 15, 21 and 31 kt (i.e., integers 1, 3, and 5).

As can be seen from Fig. 1, the mode occurs at around 14 kt, with the mean maximum gust at 15.9 kt; the standard deviation is 6.1 kt.

Climatological investigations presently being undertaken by the Nigerian Meteorological Service seem to indicate that a 10–15 yr record yields a fairly reliable analysis, and will give a reasonably good estimate of the mode. The uncertainty arises mainly in the value of the slope since the effect of an error of 1 kt, for example, in the mode causes a 1-kt error in the calculated extremes for all return periods. On the other hand, a 1-kt error in the slope will cause an error of about 2.3 kt in the 10-yr extreme, and about 3 kt in the 20-yr extreme.

A few of Nigerian stations now have Dines anemometer records for periods of up to 20 yr. Investigations have already begun to fit Gumbel's distribution to these data.

The highest recorded daily gust in Lagos area is 53 kt, resulting from a thunderstorm. It is well to remember
that our analysis had disregarded the agency causing
the strong wind gust data used in this analysis. Not
every case has been examined with particular reference
to the associated weather conditions and it would
therefore be risky to make any firm statement. Experi-
ence at Lagos (Ikeja) indicates that wind gusts at least
greater than the mean are invariably associated with
moderate or severe thundersqualls.

Gumbel’s distribution fits quite well both the daily
maximum and annual maximum gusts for this station,
the value of the slope in each case being almost the
same.

The Gumbel distribution for a given station can
therefore be defined in terms of the modal gust and the
slope of the Gumbel line, where the modal value may
be thought of as being a kind of average, and the slope
a measure of dispersion, closely related to the standard
deviation.

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