A Unified Formalism of Incoherent, Quasi-coherent, and Coherent Correlation Signals on Pulse-Pair Doppler Operation for a Cloud-Profiling Radar: Aiming for a Space Mission

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ABSTRACT

The correlation signals of the incoherent, quasi-coherent, and coherent backscatterings from clouds have been analytically calculated from a unified formalism based on the Eulerian description for a generic weather radar with pulse-pair function, especially aiming for a space-/airborne mission. The spectral broadening due to Doppler fading and wind shears is calculated from this formalism. Since the coupling of these two effects in the spectral broadening is substantial, a conventional estimation is found not to be adapted for the space-/airborne radar. The condition of pulse-pair interval to make data treatment of this operation correspond to that of the fast Fourier transform (FFT) operation is also derived. Further, comparing the derivation of the quasi-coherent scattering to that of the incoherent scattering within the unified formalism, as concerns the Fraunhofer region, it is proven that the former scattering can be represented in a similar form to the latter one, even for nonuniformly illuminated clouds, despite the difference in scattering properties. In addition, the reestimation of the volumetric reflectivity for the quasi-coherent scattering is performed. Finally, the Doppler correlation signals for the coherent backscattering and for the optical partial reflection from a keen boundary of cloud density are formulated.

1. Introduction

The demand for profiling the cloud distribution from space has been emerging in order to forecast the global warming on the base of global statistics provided by a space-/airborne cloud-profiling radar. Some missions require the accuracy of up to 0.1 m s$^{-1}$ for Doppler operation in the nadir direction. Pulse-pair Doppler operation at a high velocity such as 7 km s$^{-1}$ with a finite width of radiated beam may suffer from deterioration in coherence between paired pulses. To assess the feasibility of a spaceborne cloud profiling radar with a pulse-pair Doppler function, the diverse contribution to radar backscatterings not only from incoherent scattering due to each cloud particle, but also from quasi-coherent and coherent scattering due to turbulent effects at a high velocity, should be considered.

Conventionally, backscattering signals on pulse-pair Doppler operation have been calculated separately for incoherent and quasi-coherent scatterings from clouds. The former is calculated through the Lagrangian description as the sum of reflection from an ensemble of individual cloud particles. The latter is calculated through the Eulerian description from the fluctuation of either cloud particles transported by turbulence or turbulent air itself, which is more commonly called Bragg scattering from turbulence, reviewed in detail by Ishimaru (1978). For non-Doppler operation, Gossard and Strauch (1983) described a unified derivation of backscattering signals from clouds based on the Eulerian description, but with a small mathematical inconsistency found on the derivation of the incoherent scattering [Gossard and Strauch (1983), their Eq. (2.24)].

On derivations of the quasi-coherent backscattering, uniformly illuminated clouds were assumed in both Ishimaru (1978) and Gossard and Strauch (1983) for the theoretical explanation. In other words, spectral broadenings related to a radar geometrical configuration, such as due to wind shear or Doppler fading, were not stressed in these writings. In the experimental analysis of wind profilers, however, Wakasugi et al. (1987) adopted a formula of the spectral broadening due to wind shear, which had been originally derived for the incoherent scattering by Atlas et al. (1969), to the quasi-coherent scattering. In terms of rigor, this adoption is not so trivial for nonuniformly illuminated clouds, because on the derivation for the incoherent scattering, both the amplitude of electromagnetic waves and the density of clouds are functions of the absolute coordinates of a related point, as will be clearly shown in this
paper. On the contrary, for the quasi-coherent scattering, the former quantity is a function of the absolute coordinates of two correlated points, while the latter is a function of the relative coordinate of the two points. It is therefore necessary to legitimize and define the limit of this adoption by considering the quasi-coherent scattering for nonuniformly illuminated clouds.

Recently de Wolf et al. (2000) reconsidered simultaneous effects of the incoherent and quasi-coherent backscatterings from clouds on pulse-pair Doppler operation, aiming at unified formalism. They started a derivation of backscattering signals with the Lagrangian description, in which the total signal is represented through the discrete sum of the contribution from each particle by paying attention to its trajectory. Thereafter this formalism was converted to the Eulerian description by introducing a cloud density as a field quantity. This complicacy resulted in an implicit form of the analysis of correlational Doppler signals. Rather, their paper stressed reestimation of the backscattering cross sections of three kinds of scatterings (quasi-coherent, coherent, and incoherent scatterings) for intensity measurement to identify the dominant scattering mechanism.

The purpose of this paper is therefore to simultaneously calculate these three backscattering mechanisms on pulse-pair Doppler operation in the nadir direction for a space-/airborne radar through a unified formalism. Since a satellite moves at a high velocity, a finite width of radiated beam must be taken into account to study the effect associated with a radar geometrical configuration. The density of clouds will be treated as a time-dependent field quantity within the Eulerian description from the starting point. The derived formalism can also be applied to a generic weather radar with pulse-pair Doppler function. The paper is organized according to the following scheme. In section 2, the configuration of a space-/airborne Doppler radar and a general formalism are described. The detailed analyses for Doppler backscattering signals of the incoherent and quasi-coherent scatterings are derived in sections 3 and 4, respectively. Numerical reestimation of volumetric backscattering reflectivities of the quasi-coherent scattering is found in section 5. Section 6 is devoted to the analysis of the coherent backscattering from clouds and the partial reflection from a boundary of density. Section 7 is assigned to the summary and conclusion, followed by the appendix, in which the coupling of spectral broadenings between Doppler fading and vertical wind shears, and the incoherent backscattering from non-spherical cloud particles are derived. One important application of this formalism is accuracy evaluation in the radial velocity of cloud droplets, which will propose the feasibility of pulse pattern overcoming adverse effects on pulse-pair operation from space, as illustrated in S. Kobayashi et al. (2001, unpublished manuscript). However, consideration of this topic involves many factors beyond the purpose of this paper, which can be left to future research.
convolutional integral of signals from the cloud particles should be calculated. However, to simplify mathematical treatment, the following approximation can be adopted to pulse-pair Doppler operation in this formalism. Besides, it is pointed out that only the scattering from particles within the main lobe of the antenna in the Fraunhofer region will be considered later on.

A serious adverse effect on a spaceborne Doppler radar is the contamination of platform velocity into a target velocity due to a misdirected beam from the nadir direction. Even a small amount of misalignment of 0.1° causes contamination of order 10 m s⁻¹. Precise measurement of this contamination requires a large dynamical range of Doppler velocity that is inversely proportional to a pulse-repetition interval $T_{\text{pri}}$ (i.e., $\propto T_{\text{pri}}^{-1}$). On the other hand, the unambiguous range distance is proportional to a pulse-pair interval $T_{\text{pri}}$ (i.e., $\propto T_{\text{pri}}$). For a conventional pulse-pair operation of single polarization, $T_{\text{pri}}$ is identical to $T_{\text{pri}}$ so that simultaneous increase in both the dynamical ranges is impossible. A solution for this dilemma is the introduction of a polarization diversity method in which a pulse pair is constituted of two mutually ortho-polarized waves with a pulse pair interval $T_{\text{pri}}$ (Doviak and Sirmans 1973). The polarization diversity method has, of course, some inferior features to the conventional method. However, converting formalism from the former to the latter is so simple that the formalism in this paper will be proceeded on the base of the polarization diversity method.

In general, the scattered signal detected by the receiving antenna at the time $t_{\text{rec}}$ brings cloud information that has differences in both scattering time and position inside the clouds. This situation is illustrated in a bounce diagram in Fig. 2, where the abscissa and ordinate represent time and range distance, respectively. In Fig. 2, pulse pairs, which mutually ortho-polarize in the H and V directions, are transmitted with a pulse-pair interval $T_{\text{pri}}$. The leading edge of the first pulse of the H polarization is transmitted at time 0 from the antenna, and its trailing edge is transmitted at a later time $\tau_p$ equal to the pulse duration. The leading and trailing edges of the second pulse of the V polarization start at times $T_s$ and $T_s + \tau_p$, respectively. If a portion of the leading edge of the first pulse is scattered at a time $t$ by a cloud particle located at the position $ct$, then the reflected signal returns to the antenna at the time $t_{\text{rec}} = 2t$. When the clouds extend to some region about the position $ct$, the signal simultaneously received at the time $t_{\text{rec}} = 2t$ by the antenna includes cloud information across the distance $c\tau_p/2$ between point $A(t, ct)$ and point $B(t + \tau_p/2, c(t - \tau_p/2))$. If clouds exist around the line AB, the effect of cross-polarization reflection from the second pulse, which comes from clouds on the line CD, will work as apparent noise to the main signal, because these two signals are uncorrelated to each other. Unless the accuracy of Doppler velocity is considered, this apparent noise and the absorption by the clouds in the line CD play implicit roles in the formalism. This paper will therefore ignore the cross-signal effect for simplicity of the formalism. To approximate the scattering scheme from the line AB, we can introduce a scattered-range function of which the effective length is compressed to half the pulse length $c\tau_p/2$ in the following form for the H polarization:

$$A_{\text{scat}}^H(z, t) = \text{rect}[z; z_0(t), \sigma_z].$$

(2)

with a rectangle function defined by

$$\text{rect}[z; a, b] = \begin{cases} 1 & \text{for } |z - a| \leq b \\ 0 & \text{otherwise.} \end{cases}$$

(3)

In Eq. (2), the center and the width of the compressed pulse are denoted respectively by $z_0(t)$ and $\sigma_z$ through the pulse duration $\tau_p$ as follows:

$$z_0(t) = ct - c\tau_p/4 = z_0(t) - c\tau_p/4,$$

(4)

$$\sigma_z = c\tau_p/4.$$  

(5)

The notation of $z_0(t)$ is introduced for later use as

$$z_0(t) = ct = c t_{\text{rec}}/2.$$  

(6)

It is also noted that the scattering times for the leading and trailing edges of the pulse have a time lag of $\tau_p/2$. This is implicitly included in this approximation through the variable $z$ in Eq. (2) along the line AB in Fig. 2 for a fixed time $t$. The density at point B can be approximated as

$$n(x_b, t_s + \tau_p/2) \approx n(x_b - v\tau_p/2, t_s) = n(x_b, t_s).$$

(7)

In the last near equality, a small value of $|v\tau_p/2| < 1$ (cm) has been neglected in views of the local uniformity of clouds. In the same manner as Eq. (2), the scattered-range function for the V polarization can be defined

$$A_{\text{scat}}^V(z, \tau) = A_{\text{scat}}^H(z, \tau - T_s).$$

(8)

Finally, the application of Eq. (2) or (8) to Eq. (1) leads the received signal at the time $t_{\text{rec}} = 2t$ to the form

![Diagram of pulse-paired Doppler operation](Image)
respectively, have been defined as $A^{(h,v)}(x,v,t)$, with the superscripts $h$ and $v$ denoting the polarities of $H$ and $V$, respectively, have been defined as

$$A^{(h,v)}(x,v,t) = \frac{A_0}{z^2} \sqrt{\frac{\pi G_z G}{k^3}} \left( \frac{2J_0(klr/2z)}{klr/2z} \right)^2 A^{(h,v)}(z,t).$$

The traveling phase factor $\varphi(x)$ has been expanded for operation in the nadir direction with a small angle of beamwidth in the form of

$$\varphi(x) = 2(kz + kr^2/2z).$$

A reference of time coordinate is to be mentioned here. In the later sections as well as in this section, the time reference is taken at a time at which the leading edge of the first pulse of paired pulses is transmitted. However, when a series of paired pulses are considered, generalization can be made by transformation of

$$t \rightarrow t - n_{pr} T_{pr},$$

in which $T_{pr}$ is a pulse repetition interval and $n_{pr}$ is an arbitrary integer.

### b. Correlation of particle density

The consideration of pulse-pair Doppler operation in the view point of field theory (i.e., Eulerian description) requires us to formalize the time-dependent pair correlation of a local density function. The complete derivation of a time-independent pair correlation can be found in Landau and Lifshitz (1980) and Pathria (1972) for general particles. Thus, in this section, the derivation of a time-dependent pair correlation for general particles will be performed by expanding the method of Landau and Lifshitz (1980). Let $\hat{n}(x, t)$ be the local density function at a position $x$ and a time $t$ defined as

$$\hat{n}(x,t) = \sum_{i=1}^{N_p} \mu(x - r_i - v_i t),$$

with the Kronecker function of

$$\mu(x) = \begin{cases} 1 & \text{for } x = 0 \\ 0 & \text{otherwise}, \end{cases}$$

in which $r_i$ and $v_i$ are position and velocity vectors of some particle in the six-dimensional phase space at the time $t$ respectively. As long as shortly elapsed times of $t$ and $\tau$ are concerned, we can regard $v_i$ as a constant. The summation in Eq. (13) is taken over a unit volume and $N_p$ is the total number of particles in the unit volume.

The one-point correlation is therefore defined as

$$n_i(x, t) = \langle \hat{n}(x, t) \rangle = N_p \cdot \langle \mu(x - v_i t - r_i) \rangle,$$

where the last ensemble average is performed in the phase spaces of a particle 1. The pair correlation can be calculated to the form of the combination of the two-point correlation $n_{ij}(x, t; \xi, \tau)$ with the one-point correlation $n_i(x, t)$:

$$\langle \hat{n}(x, t) \hat{n}(\xi, \tau) \rangle = n_i(x, t; \xi, \tau) + \langle \delta(x - \xi + v_i (\tau - t)) \rangle n_i(x, t),$$

where the two-point correlation is defined in the form

$$n_{ij}(x, t; \xi, \tau) = N_p (N_p - 1) \langle \mu(x - v_i t - r_i) \mu(\xi - v_j \tau - r_j) \rangle_{ij}.$$

Further, the delta function in Eq. (15) has been introduced as a requirement of the normalization on integrating a configurational canonical distribution function over particles 1 and 2. Alternately, use of the density deviation $\delta n(x, t)$ from the locally averaged density $\bar{n}(x, t)$ rewrites the pair correlation of Eq. (15) to the form of

$$\langle \hat{n}(x, t) \cdot \hat{n}(\xi, \tau) \rangle = \langle \delta n(x, t) \delta n(\xi, \tau) \rangle_{\text{covet}} + n_i(x, t) n_j(\xi, \tau) + n_i(x, t) \langle \delta (\mathbf{r} - v_i T) \rangle_{\text{var}},$$

where new variables $r$ and $T$ have been defined as

$$\mathbf{r} = \xi - \mathbf{x}, \quad T = \tau - t.$$

In Eq. (17), the first term corresponds to the net pair correlation, the second term represents the contribution from the locally averaged density, and the third term represents the singularity related to a certainty of density existence at the center of the particle 1.

### c. Correlation of pulse-pair Doppler operation

The signal correlation $(H^*(t)V(\tau))$ of pulse-pair Doppler operation for arbitrary times $t$ and $\tau$ is formalized in this section. It is to be mentioned that $t$ and $\tau$ generally have no relation, and this point will be revisited at the end of this section. Denoted by $H(t)$ and $V(\tau)$ are electromagnetic waves (signals) that are orthogonally polarized in the $H$ and $V$ directions, respectively. These signals are received at times $2\tau$ and $2\tau - T$, after a shot of the first pulse as illustratively depicted in Fig. 2 for the special case of $\tau = t + T$. Since only the transmission in the nadir direction is considered, propagation schemes show no difference between the polarizations, as far as spherical cloud particles are concerned. On the other hand, the mathematical modification to nonspherical particles is straightforward for the incoherent scattering (see appendix). The result of that modification indicates that for the random orientation of nonspherical particles seen from the nadir direction, in which the equations of $\Delta k = 0$ and $\langle S_{\omega_0} \rangle = \langle S_{\omega_0} \rangle$ are satisfied, the formalism for spherical particles can also be applied to the nonspherical particles. However, for simplicity of
theoretical treatment, only the spherical particle will be considered in the rest of this paper.

The correlation signal is represented in the form of

\[ R(\tau - t) = \langle H^*(t)V(\tau) \rangle = \int dx \, d\xi \langle \delta n(x, t) \beta^{(h, v)}(x, t) \delta n(\xi, \tau) \beta^{(v, h)}(\xi, \tau) \rangle. \]  

(19)

Here the notations of \( \beta^{(h, v)} \) have been introduced along with the illuminating function \( A^{(h, v)} \) of Eq. (10) as

\[ \beta^{(h, v)}(x, t) = A^{(h, v)}(x, t) \exp \left[ -j2k \left( z + \frac{x^2 + y^2}{2z} \right) \right] S(D) e^{-j(h,v)}. \]  

(20)

The correlation of pulse-pair operation can be thus calculated by substituting Eq. (13) into Eq. (19) in the same manner as in the last section, resulting in the form of:

\[ R(\tau - t) = \int dx \, d\xi \langle \delta n(x, t) \delta n(\xi, \tau) \rangle e^{i\tau(\beta^{(h, v)}(x, t) \beta^{(v, h)}(\xi, \tau))}. \]  

(21)

in which the subscript of 1 in \( v \) has been abbreviated for arbitrariness of particles. As a consequence, three kinds of scattering mechanisms on pulse-pair Doppler operation have been simultaneously derived from this formalism. The first term of Eq. (21) represents the quasi-coherent scattering from the net pair correlation of density of cloud particles, and the second represents the coherent scattering from the locally averaged density of clouds. Finally, the third term represents the incoherent scattering from each particle of clouds. The second term was often dropped in the derivations of the previous works (Gossard and Strauch 1983; de Wolf et al. 2000), even though they mentioned the existence of the coherent scattering. The double integral in the third term has been reduced to the single integral owing to the delta function of \( \delta[x - \xi + v(\tau - t)] \). An important remark is that the assumption of a frozen-in condition (Ishimaru 1978) has not yet been introduced in this step. However, the third term involves the expected dependence on time and velocity. This is reasonable because the third term reflects the kinetic property of each particle, which arises from the certainty of density existence at the center of one of the paired particles, as already mentioned.

### Table 1. Difference in scattering mechanisms between clouds and sea/ground surface.

<table>
<thead>
<tr>
<th>Mathematical origin</th>
<th>Clouds</th>
<th>Sea/ground surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>Singularity in pair correlation</td>
<td>Incoherent</td>
<td>None</td>
</tr>
<tr>
<td>Net pair correlation</td>
<td>Quasi-coherent (turbulence)</td>
<td>Coherent (averaged density)</td>
</tr>
<tr>
<td>One-point correlation</td>
<td>Incoherent (random surface)</td>
<td>Incoherent (averaged surface)</td>
</tr>
<tr>
<td>Specific direction</td>
<td>None</td>
<td>Normal to the averaged surface</td>
</tr>
</tbody>
</table>

It is valuable to mention the difference in scattering mechanism between clouds and sea surface as summarized in Table 1. Even though the incoherent scatterings are defined for both cases, the mathematical origins, and thus the physical properties, are substantially different. The incoherent scattering from clouds can be represented only by one-point functions due to the delta function, as shown in Eq. (21), while the incoherent scattering from sea/ground surface defined through a net-pair correlation is represented by two-point functions. Consequently, the incoherent scattering from sea/ground surface should be treated in a similar manner to the quasi-coherent scattering from clouds (i.e., turbulence effect) rather than the incoherent scattering from clouds. This confusion in definition is caused by the discreteness of cloud particles. The unified formalism, therefore, will not only give a generalized and systematic treatment of Doppler operation for cloud/rain particles, but will also give a guide for pulse-pair Doppler operation from a boundary surface such as a sea surface (S. Kobayashi et al. 2001, unpublished manuscript).

Before the introduction of the frozen-in condition, the incoherent term in Eq. (21) shall be analyzed in the next section to characterize and quantify pulse-pair Doppler operation for a space-/airborne cloud radar. In this section, the general formula of Eq. (21) has been derived for arbitrary times \( t \) and \( \tau \); however, in the usual pulse-pair experiment, the time \( \tau \) is chosen as

\[ \tau = t + T_s. \]  

(22)

Thus this relation will be used for all the later sections.

### 3. Incoherent scattering

The incoherent scattering was studied in detail in the past works, reviewed by Ishimaru (1978) and Doviak and Zrnic (1993). In this paper attention will be paid to pulse-pair Doppler operation with a finite width of beam from a platform moving at a high velocity. In addition, some problems in past works will be reconsidered. Substitution of Eq. (9) into the third term of Eq. (21) leads the incoherent term familiar to the weather radar scientist,
\[
R(T_x, t)_{\text{inc}} = \int dx n_i(x, t) \exp \{i \psi_i(0) - \psi_i(T_x, t)\}_{\text{initial}} \\
\times A^{(i)}(x, t) A^{(i)}(x + v T_x, t + T_x) \\
\times \langle |S(D)|^2 e^{i(kz - c(\omega x + v T_x))} \rangle_{n_i}.
\]  

Assuming that the initial phase distributes with a Gaussian profile of variance \( \sigma^2_\phi \) about zero mean, we can obtain

\[
\langle \exp \{i \psi_i(0) - \psi_i(T_x, t)\}_{\text{initial}} \rangle = e^{-\sigma^2_\phi^2/2}.
\]

which can be considered as unity as long as the initial phase is well controlled coherently. The traveling phase difference in Eq. (23) can be approximated:

\[
\varphi(x) = \varphi(x + v T_x) \\
= -2k v_x T_x \\
- 2k(2v_x T_x + v^2 T_x^2 + 2v_y T_x + v^2 T_y^2)/2z.
\]

Further, the particle velocity in Eq. (23) can be decomposed in the following forms:

\[
v_i(x, t) = v_{wi} - v_{pl} \quad (i = x, y),
\]

\[
v_i(x, t) = v_{wi} + v_p(D) - v_{pl}.
\]

In the expression of Eq. (26), the terminal falling velocity of a cloud particle of diameter \( D \) in the static air is denoted by \( v_p(D) \). The velocity \( v_p(x, t) \) of the cloud particle transported by airflow (wind) is assumed to be independent of its diameter, according to Amayenc et al. (1993), Sato et al. (1990), and de Wolf et al. (2000); hence, \( v_p \) is practically equal to the wind velocity. Finally, the velocity of the platform is designated by \( v_{pl}(t) \).

The two illuminating functions \( A^{(i)}(x, t) \) in Eq. (23), along with Eqs. (2) and (8), are related to each other in the form of

\[
A^{(i)}(x + v T_x, t + T_x) = A^{(i)}(x + v T_x, t)
\]

For a typical experiment, the range distance \( z_0 \) is much larger than the cell range distance:

\[ c \tau_r/2 \ll z_0(t) \quad \text{or} \quad z_0'(t). \]

Under the condition of Eq. (28), along with the local uniformity of clouds within the cell range, substitutions of Eqs. (24)–(27) into Eq. (23) with ignorance of the second-order terms of \( T_x \), yield the incoherent correlation signal to the form of

\[
R(T_x, t)_{\text{inc}} \approx |A_i|^2 \pi G G c \tau_r \frac{1}{k^2} \int dDN(D) |S(D)|^2 e^{-2k v_p(D) t} \\
\times \int dx dy \left[ 2 \frac{J_0(klr/2z_0)}{kr/2z_0} \right] \delta_n(x, t; z_0) \\
\times \langle \exp \{i[-2k(x - v_{pl} T_x) z_0^{-1}] \rangle \rangle_{v_{pl}, v_{pl}} \\
\times \langle \exp \{i[-2k(y - v_{pl} ) T_x z_0^{-1}] \rangle \rangle_{v_{pl}, v_{pl}},
\]

where the wind velocity \( v_w \) and the flight velocity \( v_{pl} \) have been represented by functions of transverse coordinates \( x \), and time \( t \) through the parameter \( z_0 = ct. \) Here \( N(D) \) is a size distribution function of diameter \( D \) of cloud particles, of which the integral over \( D \) is normalized as unity rather than the averaged density of the clouds:

\[
1 = \int dDN(D).
\]

The term concerning the ensemble average of the terminal falling velocity in Eq. (29) is now considered:

\[
I_p = \int dDN(D) |S(D)|^2 e^{-2k v_p(D) t}.
\]

Since both the squared scattering amplitude \( |S(D)|^2 \) and the Doppler phase term \( e^{-2k v_p(D) t} \), are interrelated, neither the resultant amplitude \( |I_p| \) nor the phase \( \arg (I_p) \) can correspond to the physical observable. However, taking the pulse-pair interval \( T_s \) so as to satisfy the condition of

\[ 2k v_p(D) T_s \ll 1, \]

the convergence of the integral over \( D \) is fast enough to yield an approximated relation up to the second order:

\[
\int dDN(D) |S(D)|^2 \exp[-2k v_p(D) T_s] \\
= \langle |S(D)|^2 \rangle_D \exp[-2k^2 T_s^2 \sigma^2_{v_p}],
\]

where the weighted average and variances of the terminal falling velocity are defined as

\[
Z_N = \langle |S(D)|^2 \rangle_D = \int dDN(D) |S(D)|^2 \\
\bar{v}_p = \frac{1}{Z_N} \int dDN(D) |S(D)|^2 v_p(D) \\
\bar{v}^2_p = \frac{1}{Z_N} \int dDN(D) |S(D)|^2 v^2_p(D) \\
\sigma^2_{v_p} = \bar{v}^2_p - \langle v^2_p \rangle_p.
\]

These definitions correspond to those in the fast Fourier transform (FFT) method (Atlas et al. 1973; Frisch et al. 1995; Sato et al. 1990; Wakasugi et al. 1987), except that the conventional definition involves the density \( n \) of clouds in the size-distribution function. If Eq. (32) is not satisfied, then, alternatively, it would be assumed that the distribution of

\[
N(D) |S(D)|^2 \left( \frac{dv_p}{dD} \right)
\]
is Gaussian of \( v_i \) to obtain the form of Eq. (33). If the terms of the scattering amplitude and the Doppler phase in Eq. (31) were independently ensemble averaged with respect to particle diameter, in a manner based on the assumption in Sachidananda et al. (1986, 1989), then the ensemble average of the terminal falling velocity in the Doppler-phase term would be the simple geometrical average weighted by only the size distribution function, thus leading to misinterpretation of data.

To proceed with the analysis, it is supposed that each component of the wind velocity \( \mathbf{v}_w \) obeys a Gaussian distribution, the ensemble averages of which are given by

\[
\langle v_{wx}(x_i, t; z_0) \rangle = v_{wx} + \kappa_w x_i,
\]
\[
\langle v_{wy}(x_i, t; z_0) \rangle = v_{wy} + \kappa_w y_i,
\]
\[
\langle v_{wz}(x_i, t; z_0) \rangle = v_{wz} + \kappa_w y_i + \kappa_w y_i,
\]

where \( v_{wx}(i = x, y, z) \) are constants, and \( \kappa_w(i, j = x, y, z) \) are wind shear constants. In this formalism only the linear coefficients of wind shears will be considered for the theoretical purpose. Further, the corresponding variances of the wind velocity \( \mathbf{v}_w \) are assumed to be position-independent constants:

\[
\sigma^2_{v_w} = \text{const} \quad (i = x, y, z). \tag{37}
\]

On the other hand, the satellite velocity \( \mathbf{v}_p \) is supposed to satisfy a stable trajectory along only the \( x \) axis with no fluctuation:

\[
\langle v_{px} \rangle = v_{px} = \text{const} \quad \langle v_{py} \rangle = \langle v_{pz} \rangle = 0. \tag{38}
\]

The assumptions of Eqs. (32) and (36)–(38), along with a constant local density \( n \), simplify Eq. (29) to the form

\[
R(T_i; t)_{\text{inc}} = |A_o|^2 \pi G G_0 n k^2 \left( |S(D)|^2 \right)_{\text{inc}} (v_{\text{pf}}/2) \rho_{\text{comb}}(T_i; t)
\times e^{-2i T_i^o z_0} e^{-2i T_i^o \ln z_0} e^{-2i \pi z_0 T_i^o}, \tag{39}
\]

In the course of derivation, the terms related to \( \sigma^2_{v_w} \) and \( \sigma^2_{v_p} \) have been omitted because these terms are smaller by the order of \( (v/\zeta)^2 \approx (\text{angle of beamwidth})^2 \). The normalized correlation \( \rho_{\text{comb}}(T_i; t) \) in Eq. (39), depending on a radar geometrical configuration, is defined by

\[
\rho_{\text{comb}}(T_i; t) = \int dx \int dy \left[ \frac{J_0(klr/2z_0)}{klr/2z_0} \right]^4
\times \exp(-2j(klr/2z_0))
\times \exp(-j2k(x v_x + \kappa_w x + \kappa_w y) T_i)
\times \exp(-j2k(y v_y + \kappa_w y) T_i z_0^{-1})
\times \exp(2ky(x v_x + \kappa_w x) T_i z_0^{-1})
\]  
\[
\exp(2ky(y v_y + \kappa_w y) T_i z_0^{-1})
\]

in which the averaged effective wind velocity has been introduced as

\[
\mathbf{v}_e = \mathbf{v}_w - \mathbf{v}_p = \text{const}
\]
\[
\mathbf{v}_e = \mathbf{v}_w = \text{const} \quad (i = y, z). \tag{41}
\]

The Airy-pattern function in Eq. (40) can be approximated as an exponential function, as often adopted in the radar engineering:

\[
\left[ \frac{2J_1(klr/2z_0)}{krlr/2z_0} \right]^2 \approx \exp\left(-\frac{r^2}{2\sigma^2_{\text{comb}}} \right). \tag{42}
\]

Numerically the transverse beamwidth \( \sigma_y \) can be evaluated as

\[
\sigma^2_y = \frac{\theta_y^2 \theta_y^2}{2\pi^2 \ln 2} \tag{43}
\]

through the angle of beamwidth

\[
\theta_y = 2\pi/kl. \tag{44}
\]

The form of Eq. (42) can be shown to be equivalent to that of Probert-Jones (Atlas et al. 1969).

From this stage there are two ways to proceed. One conventional way is to calculate the power spectrum along an isodop (i.e., a contour line of Doppler frequency). However, since the isodop is not a simple linear function in the presence of the wind shears, this calculation is not tractable. Thus, the other way of the direct integration of Eq. (40) should be taken. The calculation described in the appendix yields the normalized correlation function of

\[
\rho_{\text{comb}}(T_i; t) = \exp(-2j\mathbf{v}_e T_i) \exp(-2k^2\sigma^2_{\text{comb}} T_i^o), \tag{45}
\]

where the spectral broadening \( \sigma^2_{\text{comb}} \) is represented through half the angle of beamwidth \( \theta_y = \theta_y/2 = \pi/kl \). The Airy-pattern function in Eq. (40) can be approximated as

\[
\frac{\theta_y^2 \theta_y^2}{2\pi^2 \ln 2} \approx \left[ \left( \mathbf{v}_e + \kappa_w z_0 \right)^2 + \left( \mathbf{v}_e + \kappa_w z_0 \right)^2 \right]
+ \frac{\theta_y^2 \theta_y^2}{2\pi^2} \left( \kappa_w x + \kappa_w y \right)^2. \tag{46}
\]

The last term of Eq. (46) reflects the perturbational effect of the horizontal wind shears as a higher-order contribution. Equation (46) is a generalized and combined form of Sloss and Atlas (1968) and Atlas et al. (1969) for the case of a circular symmetric illumination \( \phi_0 = \phi_0 \).

However, Eq. (46) involves more information than the generalization, that is, the coupling of the spectral broadenings between the Doppler fading due to \( \mathbf{v}_e \), \( \mathbf{v}_e \), and the vertical wind shears \( \kappa_w x, \kappa_w y \). This coupling effect can be explained as a natural consequence within the formalism as follows. Ignoring the wind shears in Eq. (40), the isodop can be represented as the line of

\[
\omega = 2k \left( \frac{x v_x + y v_y}{z_0} + \mathbf{v}_e + \mathbf{v}_e \right). \tag{47}
\]

For this equation, when \( \mathbf{v}_e = 0 \) is assumed, as sche-
in which the normalized total correlation function is represented by

$$
\rho_{\text{inc}}(T_s ; t) = e^{-2k_s^2 T_s \sigma_{\text{inc}}} e^{-2k_t^2 T_s \sigma_{\text{inc}}} e^{-2k_z^2 T_s \sigma_{\text{inc}}} e^{-2k_0^2 T_s \sigma_{\text{inc}}}.
$$

(49)

The part except for $\rho_{\text{inc}}(T_s ; t)$ in Eq. (48) can be reduced to the weather radar equation (Doviak and Zrnic 1993; Gossard and Strauch 1983) by introducing the volumetric reflectivity $\eta$:

$$
\eta = 4\pi n(|S(D)|^2)_{\text{D}}.
$$

(50)

As earlier described, the Doppler fading is one of the dominant factors of decorrelation for a spaceborne Doppler radar. A convenient approximated relation can be derived to obtain a good coherence between consecutive paired pulses in the form of

$$
v_{ap} T_s < 0.36l.
$$

(51)

It indicates that the platform must move at most about one-third times the distance of the diameter of the antenna between the shots of the pulses. As an example, for a 95-GHz Doppler radar moving at a velocity of 7 km s$^{-1}$ with an antenna diameter of 2 m, the inequality of Eq. (51) is found to be violated for the coherence time of $T_s \approx 100$ μs, which is comparable to a typical pulse repetition interval of a Doppler radar. To increase the coherence time, it is necessary to increase the diameter of the antenna.

4. Quasi-coherent scattering

The general description of quasi-coherent scattering from cloud particles in turbulence is found in Ishimaru (1978) and Gossard and Strauch (1983), both of which paid attention to uniformly illuminated clouds. In this section, the correlation function of the quasi-coherent scattering on pulse-pair Doppler operation is formalized for nonuniformly illuminated clouds. To continue analysis beyond Eq. (21), a locally frozen-in condition shall be introduced. It assumes that the system is constituted of cloud particles of a uniform diameter $D$. Hence, the variances of wind velocity and airplane velocity, as well as those of the falling velocity due to the distribution of particle diameter, are taken into account at a later stage in a similar manner to Ishimaru (1978). Furthermore, an isotropic and locally homogeneous turbulence, on which Kolmogorov theory is satisfied, is assumed. Then the modification on the corresponding power spectrum, as described in Ishimaru (1978, appendix C-12) and Doviak and Zrnic (1993, section 10-1), allows us to consider the existence of the correlation function $B_r(r - v T_s)$ for the net pair correlation $(\delta n(x,t)) (\delta n(\xi, t + T_s))$, as far as wavenumbers in the inertial subrange and dissipation range are concerned. Along with these conditions, the correlation for the quasi-coherent scattering can be written from the first term of Eq. (21):

![Fig. 3. Schematic figure of the effect of Doppler fading, which, in the Fraunhofer approximation, can be regarded as an apparent vertical wind shear.](image-url)
\[ R(T_s; t)_{\text{qcoh}} = \int \int \, \text{d}x \, d\xi (\delta n(x, t) \delta n(\xi, t) + T_s))_{x=vT_s} \]
\[ \times \beta^2(x, t) \beta(\xi, t + T_s) \]
\[ = \int \int \, \text{d}x \, d\xi B_j(r - vT_s) \beta^2(x, t) \]
\[ \times \beta(\xi, t + T_s). \quad (52) \]

Let \( \Phi_{nd}(K) \) be the power spectrum of the correlation function \( B_j(r) \); then Eq. (52) is rewritten
\[ R(T_s; t)_{\text{qcoh}} = \int \int \, \text{d}x \, d\xi \int \text{d}K \Phi_{nd}(K) \exp[jK(r - vT_s)] \]
\[ \times A^{(b)}(x, t)A^{(b)}(\xi, t)|S(D)|^2 \]
\[ \times \exp[-j\phi(x, t + T_s) - \phi(\xi, t)] \]. \quad (53)

where the initial phase term has been omitted for simplicity, and the relation of \( A^{(b)}(\xi, t + T_s) = A^{(b)}(\xi, t) \) from Eq. (27) has been used. From Eqs. (52) and (53), it is seen that adoption of the broadenings due to wind shear and Doppler fading, which were originally derived for the incoherent scattering, to the quasi-coherent scattering is not trivial, as mentioned in the introduction, because the correlation \( B_j(r - vT_s) \) is a function of the relative coordinate of \( r \), while the illuminating functions \( A^{(b)} \) and the traveling phases \( \phi \) are those of the absolute coordinates of \( x, \xi \).

To consider the effect of nonuniformly scattered clouds, the new coordinates should be introduced:
\[ r = \xi - x \quad R = (\xi + x)/2. \quad (54) \]
The traveling-phase term in Eq. (53) can be approximated within the illuminated volume:
\[ \phi(\xi, t + T_s) - \phi(x, t) \approx 2k[r_\| + (R, r_\perp + R, r_\perp)/z_0] \]
\[ = 2kR \cdot r, \quad (55) \]
in which the position dependent wavenumber \( k(R) \) has been introduced by ignoring the second order terms as
\[ k(R) = kR/z_0 + kR/z_0 \hat{\xi} + k \hat{r}. \quad (56) \]

For further simplification of calculation, the scattered-range function \( A^{(b)}_{nd}(z, t) \) in the illuminating function \( A^{(b)}(x, t) \) is approximated with an exponential function of
\[ A_{nd}(z, t) = \exp[-(z - z_0^2(t))/4\sigma_z^2]. \quad (57) \]
in which the factor of 4 in the denominator has been introduced, instead of 2, to satisfy the relation of
\[ |A_{nd}(z, t)|^2 = \text{rect}[z; z_0^2(t), \sigma_z]. \quad (58) \]
This difference in the factor will give only slight change in a normalization factor. Substitution of Eqs. (54) and (55) into Eq. (53) results in variable-separated integrals over \( r \) and \( R \). A simple calculation shows that the integral over \( r \) can be approximated by the delta function of
\[ (2\pi)^3 \delta(K - 2k(R)) \]
under the following conditions: for the integral over \( r_\| \)
\[ cT_s \gg \lambda/4\pi, \quad (60) \]
and for the integrals over \( r_\perp \) and \( r_\perp \)
\[ 2k_\perp \gg 2^{-1/2}\sigma^{-1} \quad (i = x, y). \quad (61) \]

The former condition is satisfied for usual operation of a cloud radar. The latter condition can be shown to eventually comprise that of the Fraunhofer approximation:
\[ z_0 \gg k(l/2)^2/2. \quad (62) \]

Therefore, as far as the scattering from the Fraunhofer region is concerned, the approximation to the delta function of Eq. (59) has been legitimized, giving the correlation signal of the quasi-coherent scattering to the form of
\[ R(T_s; t)_{\text{qcoh}} = |A_0|^2 \frac{\pi G_r G_t z_0^2}{k^2} \int \, \text{d}R \exp[-(R - z_0^2(t))/2\sigma_z^2] \]
\[ \times \exp\left[-\frac{R^2 + R_\perp^2}{2(\sigma_r/\sqrt{2})^2}\right] \]
\[ \times \frac{R_\perp^{-4} (2\pi)^3 \Phi_{nd}(|2k(R)| |S(D)|^2 \]
\[ \times \exp[-j2k(R)vT_s]. \quad (63) \]

Since the isotropic turbulence is assumed, we can replace the 3D spectrum \( \Phi_{nd}(K) \) in vector \( K \) with the corresponding scalar counterpart \( \Phi_{nd}(K) \) in scalar \( K \), according to Ishimaru (1978) and de Wolf et al. (2000). Further, the ensemble averages of \( D, v_\perp, \) and \( v_\perp, \) along with the approximation of \( |2k(R)| = 2k, \) lead Eq. (63) to the final form of
\[ R(T_s; t)_{\text{qcoh}} = |A_0|^2 \frac{\pi G_r G_t z_0^2}{k^2} \int \, \text{d}x \, \text{d}y \exp[-x^2 + y^2]/2\sigma^2 \]
\[ \times (2\pi)^3 \Phi_{nd}(2k) \int \, \text{d}DN(D)|S(D)|^2 \]
\[ \times e^{-j2k(y)\hat{y}r_\perp} \langle e^{i[2(kv_\perp - y\hat{y})r_\perp]} \rangle_{v_\perp, y\hat{y}} \]
\[ \times \langle e^{i[2(kv_\perp - y\hat{y})r_\perp]} \rangle_{v_\perp, y\hat{y}}, \quad (64) \]

where \( R \) has been replaced with \( x \). This form corresponds to that of the incoherent scattering defined by Eq. (29). Hence, it is concluded under the condition of Eqs. (60) and (62) that all the results of the incoherent scattering can be applied to the quasi-coherent scattering on pulse-pair Doppler operation for nonuniformly illuminated clouds, by replacing the cloud density \( n \) with the power spectrum \( (2\pi)^3 \Phi_{nd}(2k) \).

At the end of this section, it is worth mentioning that
the results of this section can also be applied to the quasi-coherent scattering from nonuniformly illuminated air turbulence through the replacement of \( |S(D)|^2 \) \( (2\pi)^2 \Phi_n(2k) \), with \( 2\pi k^4 \Phi_n(2k) \), where \( \Phi_n(2k) \) is the power spectrum of dielectric-coefficient (refractive index) fluctuation. Note that the scattering from the turbulent air does not include terms related to the particle falling velocity \( u(D) \).

5. Reestimation of volumetric backscattering reflectivities

De Wolf et al. (2000) recently reestimated volumetric backscattering reflectivities for the quasi-coherent scattering from cloud particles through the relation between the structure constant \( C_n^2 \) and the variance \( \langle \delta n^2 \rangle \) of cloud density fluctuation at the origin by modifying a formula of Ishimaru (1978) defined for the variance of refractive index fluctuation:

\[
C_n^2 = 1.91 \langle \delta n^2 \rangle L_0^{-2/3},
\]

where \( L_0 \) is the outer scale length of a turbulence. For numerical estimation, de Wolf et al. assumed the relation of \( \langle \delta n^2 \rangle = \pi^2 \) with the averaged density \( \bar{n} \). The value of \( \langle \delta n^2 \rangle \) is strictly defined at the origin of the net pair correlation through the two-point correlation function \( n_p(x, \xi) \) in the form of

\[
\langle \delta n^2 \rangle = n_p(0, 0) - \pi^2.
\]

Needless to say, the two-point correlation of \( n_p(0, 0) \) is of the order of \( \pi^2 \). However, it is not so evident that the value of \( \langle \delta n^2 \rangle \) has the order of \( \pi^2 \). To explore this problem within the formalism of this paper, Eq. (15) is integrated over a finite small volume \( V > 1/\bar{n} \) in the vicinity of origin with the condition of \( t = \tau \), obtaining the approximated relation of

\[
\langle \delta n^2 \rangle = n_p(0, 0) - \pi^2 - \langle (\Delta N)^2 \rangle - \frac{\bar{N}^2}{\pi^2},
\]

in which \( \langle (\Delta N)^2 \rangle \) and \( \bar{N} \) represent the variance and the average of particle numbers in the small volume \( V \), respectively. If a turbulence is strong enough to give the relation \( \langle (\Delta N)^2 \rangle = N^2 \), which is possible for the small volume, then Eq. (67) approximately gives the relation

\[
\langle \delta n^2 \rangle \approx \pi^2.
\]

Hence, the assumption by de Wolf et al. (2000) has been concluded to be plausible in some extremely strong turbulence.

To estimate the magnitude of the quasi-coherent scattering from cloud particles for more realistic situations, the following information on liquid water content (LWC) and diameters of cloud particles are used:

\[
\rho_w = 10^3 \text{ g m}^{-3}, \quad D = 1 -100 \mu m,
\]

\[
\text{LWC} = 10^{-4} -10^{-1} \text{ g m}^{-3}.
\]

In this range of diameter, scattering can be approximated by the Rayleigh approximation for the GHz band of less than 100 GHz. Then the volumetric backscattering reflectivity of the quasi-coherent scattering is represented as a function of the LWC and the volume \( V_d \) of cloud particle of diameter \( D \) in the form of

\[
\eta_{qcoh} = 4\pi |S(D)|^2 (2\pi)^2 \Phi_n(2k)
\]

\[
= \frac{9V_d^2}{4\pi^4} \left| \frac{\epsilon_q - 1}{\epsilon_q + 2} \right| (2\pi)^3 0.063 L_0^{-2/3} (2k)^{-11/3} \times \left( \frac{\text{LWC}}{\rho_w V_d} \right)^2.
\]

The choice of the outer scale \( L_0 \) of inertial subrange may spread in some range depending on authors in the previous papers, for instance, \( L_0 = 20 -130 \text{ m} \) (Ishimaru 1978) or 25 m (de Wolf et al. 2000). Thus, \( L_0 = 50 \text{ m} \) is chosen for the later calculation. The calculated result is plotted in Fig. 4. The curve labeled with \( \eta_{qcoh\_max} \) is the maximum value under the extremely strong turbulence defined by Eq. (68), which is given for LWC = \( 10^{-1} \text{ g m}^{-3} \). However, theoretically, the quasi-coherent scatterings from cloud particles can exist in any region below the curve of \( \eta_{qcoh\_max} \) for the same condition. In the figure, the volumetric reflectivity of incoherent scattering from the clouds of \(-30 \text{ dBZ} \) and that of a typical quasi-coherent scattering from turbulent air are also plotted. It is seen from Fig. 4 that the quasi-coherent scattering from cloud particles may be important in the GHz bands in the case of an extremely strong turbulence.

![Fig. 4. Volumetric backscattering reflectivity vs frequency: \( \eta_{qcoh\_max} \) is the estimated maximum value of the quasi-coherent scattering from cloud particles for an extremely strong turbulence; \( \eta_{inc\_30} \) is the incoherent volumetric reflectivity from clouds of \(-30 \text{ dBZ} \); and \( \eta_{qair} \) is a typical value of the quasi-coherent scattering from turbulent air.](image-url)
6. Coherent backscattering from clouds and partial reflection from a boundary

The coherent backscattering on pulse-pair Doppler operation will be considered in this section, in the course of which Doppler signals due to the partial reflection on a keen boundary of cloud density will also be derived. The derivation in this section will assume an extremely idealized boundary shape, such as a keen and flat one. In this sense, the formalism is rather closer to mathematical physics than radar science, especially for the GHz band. However, the previous treatments of the coherent backscattering (Gossard and Strauch 1983; de Wolf et al. 2000) were not so sufficient, causing some confusion with the partial reflection on a boundary. Thus formalism in this section aims to characterize the correlation signal of coherent backscattering on pulse-pair operation. The locally frozen-in condition is again adapted to the averaged density, obtaining the relation on the one-point correlation:

\[ n_j(\xi, t + T_s) = n_j(\xi - \nu T_s, t), \]

(71)

where the definition of Eq. (26) is applied to the velocity \( \nu \). Substitution of Eq. (71) into the coherent term in Eq. (21) yields

\[ R_{\text{coh}}(T_s; t) \]

\[ = \int dx n_j(x, t) A^{*0}(x, t) S^* (D) \exp \left\{ j2k \left[ z + \frac{x^2 + y^2}{2z} \right] \right\} \]

\[ \times \int dx n_j(x, t) A^{*0}(x, t) S(D) \]

\[ \times \exp \left\{ -j2k \left[ z + \frac{x^2 + y^2}{2z} + 2xv_x T_s + 2yv_y T_s \right] \right\} \]

\[ \times e^{-j2kzT_s}, \]

(72)

where the replacement of \( \xi - \nu T_s \) with \( x \) in the second integral has been made.

At first the wind shears are ignored for the calculation of Eq. (72), and later this assumption will be rationalized from the result. For clouds of a uniformly distributed density \( n \), the second integral in Eq. (72), denoted by \( I_2 \), can be calculated under the condition of Eq. (28) along with the scattered-range function of Eq. (57). The stationary phase approximation on integrations over \( x \) and \( y \) yields

\[ I_2 = \frac{A_0}{z_0} \sqrt{\frac{\pi G G}{k^2} S(D) \frac{4\pi r_T}{2} e^{-4\pi r_T^2} e^{-j2kz_0} e^{-j2kT_s} e^{-j2kz_0} T_s} \]

\[ \times \exp \left\{ -\left( v_x T_s \right)^2 + \left( v_y T_s \right)^2 \right\} \cdot 2\sigma^2 \]

\[ \times \exp \left\{ -j\frac{k}{z_0} \left( (v_x T_s)^2 + (v_y T_s)^2 \right) \right\}. \]

(73)

The arguments in the last two exponentials should be omitted in the same manner as in the derivation of Eq. (29). Thus, it is concluded that the effect of Doppler fadings can be ignored in Eq. (73), which in turn means that the competitive effect of the vertical wind shears as described in section 3 can also be ignored, leading to the legitimacy of ignoring wind shears as a first approximation. The first integral in Eq. (72), on the other side, is obtained by setting \( \nu = 0 \) in Eq. (73). Finally, Eq. (72) can be asymptotically calculated, followed by the ensemble averages of particle diameter \( D \) and the wind velocities under the condition of Eq. (32) in the form of

\[ R_{\text{coh}}(T_s; t) \]

\[ = \pi^3 \frac{P G G}{4k^2z_0^2} \langle |S(D)|^2 \rangle_{n^2} \left( \frac{cT_s}{2} \right)^3 e^{-2\pi r_T^2} \rho_{\text{coh}}(T_s) \]

along with the normalized correlation function of

\[ \rho_{\text{coh}}(T_s) = e^{-j2kz_0 + 2kT_s} e^{-2\pi r_T^2} \rho_{\text{coh}}(T_s) \]

(75)

where the notations are defined already in Eq. (34). Relevantly, this signal does not involve the spectral broadening due to the Doppler fading nor the vertical wind shears. It is also noted that the amplitude is proportional to the square of density rather than the density itself, and inversely proportional to the square of the range distance \( z_0 \) in common to the other scattering regimes. The necessary condition to observe this signal is therefore found to be \( 8\pi T_s^2 \ll 2 \) or

\[ cT_s < \lambda, \]

(76)

which, however, cannot be satisfied for usual cloud-profiling radars in the GHz band. Equation (76), from a different view, suggests that a thin layer of clouds congregated within a narrow distance less than the wavelength may give a finite contribution to \( R_{\text{coh}}(T_s; t) \), while this situation again seems impossible to observe in the GHz band. A remaining possibility for observing \( R_{\text{coh}}(T_s; t) \) may be in a short time as a transient process after a radar pulse injects on a keen and flat boundary of cloud density. Assuming the keen and flat boundary as a step function, then during the short time \( \Delta t \ll \lambda/c \), the signal can be roughly calculated by setting \( n_j(x, t) = nc\Delta t/2\sigma(z - z_0) \) in Eq. (72). A small calculation yields

\[ R_{\text{coh}}(T_s; t) \approx \pi^3 \frac{P G G}{4k^2z_0^2} \langle |S(D)|^2 \rangle_{n^2} \rho_{\text{coh}}(T_s) \]

(77)

where the approximation of \( \Delta t \approx \lambda/c \) has been used.

On the keen boundary of a step function, optical partial reflection can also be expected. The correlation signal of the partial reflection can be formally derived from Eq. (72) by replacing \( n_j(x, t) \) with \( \delta(z - z_0) \), and \( |S(D)|^2 \) with \( \pi^3 n^2 \langle |S_{\text{coh}}(D)|^2 \rangle_{n^2} \langle |S_{\text{coh}}(D)|^2 \rangle_{n^2} /4\pi k^2 \). The last replacement of \( |S(D)|^2 \) has been obtained from a theory of the effective refractive index of clouds (Oguchi 1973). Represented by \( S_{\text{coh}}(D) \) is the forward-scattering amplitude of cloud particles of diameter \( D \), and the en-
semble average of the diameter $D$ has been taken on this stage because the partial reflection is attributed from the macroscopic optical property. The result of the calculation is written in the form of
\[
R_{\text{part}}(T_s; t) = \frac{\pi^2 G G}{16 k^2 z_0^2} (S_{\text{rad}}(D))_{\text{inc}} n^2 \rho_{\text{part}}(T_s), \quad (78)
\]
with the normalized correlation function
\[
\rho_{\text{part}}(T_s) = e^{-2i\pi T s} e^{-2i\pi T s} e^{-2i\pi T s}. \quad (79)
\]
In the derivation of Eq. (79), the averaged falling velocity of the boundary has been roughly approximated as the geometrical average of the terminal falling velocity of cloud particles in the form [cf. Eq. (34)]
\[
\overline{v}_f = \int dDN(D) v_f(D) \quad \overline{v}_f' = \int dDN(D) v_f'(D)
\]
\[
\sigma_{v_f}^2 = \overline{v}_f^2 - \overline{v}_f'^2. \quad (80)
\]
Although the transient coherent backscattering and the partial reflection are written in similar forms, with competitive magnitudes in the Rayleigh scattering regime, these two processes must be distinguished in view of the following two points. First, the transient coherent backscattering is body scattering attributed from the backscattering cross sections of cloud particles; hence the averaged falling velocity in Eq. (75) is weighted by the square of the scattering amplitude and the size distribution function. On the other hand, the optical partial reflection is surface scattering through the consistency of a boundary condition, which is caused by the macroscopic difference in refractive indices represented by the forward-scattering cross sections; hence the averaged falling velocity in Eq. (79) is simply represented by the geometrical one of Eq. (80). Second, the partial reflection occurs during all the time the pulse passes over the boundary, while the transient coherent scattering is restricted in the short time $\Delta t$ after the pulse injection. To estimate the order of the transient coherent backscattering or the partial reflection to the incoherent scattering, the ratio of Eq. (77) to Eq. (48) is taken:
\[
R_{\text{coh}} / R_{\text{inc}} \approx \frac{n A^4}{\theta_c^2 c T_p}, \quad (81)
\]
It means that the magnitude of the transient coherent backscattering, and hence the partial reflection, can be larger than that of the incoherent scattering by several orders for the GHzz radar. As a conclusion, in a situation in which a transient coherent backscattering occurs in the Rayleigh scattering regime, the same order of optical partial reflection simultaneously occurs. Since the former process endures in the very short time $\Delta t$, it is practically impossible to distinguish the transient coherent backscattering from the partial reflection.

For the more rigorous consideration, it is necessary to perform an explicit time-dependent analysis along with the difference in forward-scattering amplitudes of complex values across a keen boundary, because the details of scattering schemes on this kind of interface strongly depend on the relative position between the pulse and interface, which is beyond the scope of the formalism in this paper.

For a boundary surface with randomness comparable to radar wavelength, it may be unnecessary to consider the coherent backscattering. The partial reflection itself is classified into coherent and incoherent scatterings. The coherent partial reflection corresponds to the formalism in this section. The incoherent partial reflection that is attributed from the net pair correlation of a surface function should be derived from a generalized Kirchhoff theory (e.g., Beckmann and Spizzichino 1963) as a totally different regime, in which Doppler fading and vertical wind shears may take an important role.

7. Summary and conclusions
The incoherent, quasi-coherent, and coherent backscatterings from clouds have been considered analytically for pulse-pair Doppler operation on a space-airborne radar. The correlation signals of the three scattering mechanisms have been simultaneously derived from a unified formalism based on a time-dependent Eulerian description, which simplifies and clarifies the physics of pulse-pair Doppler operation. The results are summarized as follows.

For the incoherent scattering, the coherence time due to Doppler fading reaches nearly 100 $\mu$s, which can be considered as a dominant factor in the deterioration of coherence on a spaceborne Doppler radar. It requires a short time interval $T_s$ of paired pulses for efficient operation.

The coupling between the spectral broadening due to the effective horizontal wind velocity, including a flight velocity and that due to vertical wind shears has been derived in the form of Eq. (46). This coupling effect is of importance for an air/spaceborne radar, which can be either constructive or destructive, depending on the sign of the vertical wind shear constants $\kappa_u$ and $\kappa_v$, or, equivalently, depending on the flight direction.

The condition of the pulse-pair interval in Eq. (32) has been specified so that the measured Doppler velocity should have the relevant physical observables identical to those of the FFT method.

The correlation signal of the quasi-coherent scattering from cloud particles has been calculated for nonuniformly illuminated clouds. Under the condition of Eqs. (60) and (62), it has been proven that the form of the correlation signal derived in the incoherent scattering from clouds, including the spectral broadenings, can be also applied to the quasi-coherent scattering, even for the inhomogeneous illumination, by replacing the cloud density $n$ with the power spectrum $(2\pi)^4 d\Phi_d(2k)$. That in turn means that no bimodal spectrum can be observed for simultaneous observation of the quasi-coherent scattering from clouds with the incoherent scattering from clouds,
contrary to a conjecture by de Wolf et al. (2000), under the assumption that a particle velocity transported by a turbulence is independent of the diameter of the particle. Further, it is noted that the same expansion of the theory can be applied to the quasi-coherent scattering from turbulent air itself, in which no terms related to the terminal falling velocity are included; hence, this can be attributed to the bimodal spectrum as well known.

The reestimation of backscattering volumetric reflectivities for the quasi-coherent scattering from cloud particles has also been performed. The relation \(|n_2| = n^2\) assumed by de Wolf et al. (2000) has been derived in the more rigorous sense in the formalism of this paper. The result, that the quasi-coherent scattering from cloud particles can be important for the GHz band of less than 100 GHz in the case of an extremely strong turbulence, is in accordance with previous works (Gossard 1979; Gossard and Strauch 1983; de Wolf et al. 2000).

Finally, the coherent backscattering on Doppler operation has been considered. This signal can be theoretically observed as a transient process in a very short time just after a pulse injects on a keen and flat boundary of density. The expected Doppler signal is written in the form of Eq. (77), which has a similar form to the optical partial reflection simultaneously occurring on the keen boundary, represented by Eq. (78). Although these two processes are of competitive magnitudes in the Rayleigh scattering regime, the former process endures only in the short time \(\approx \lambda/c\), while the latter endurance during the entire time the incident pulse passes over the boundary. Hence the former process is merged into a very beginning part of the latter process, and practically cannot be distinguished. Further, the former process is body scattering even though it occurs near the surface, and hence its averaged Doppler velocity is weighted by the square of a scattering amplitude, given by Eq. (34). The partial reflection, however, is caused by surface scattering, and hence its averaged Doppler velocity is simple geometrical, given by Eq. (80).

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APPENDIX

Coupling of Spectral Broadening between Doppler Fading and Vertical Wind Shears

For a space-/airborne radar, the coupling of the spectral broadening due to Doppler fading and wind shears becomes more important as the flight velocity increases. The derivation of this effect is started with Eqs. (40) and (42). A small calculation yields

\[
\rho_{\text{comb}}(T_z; t) = C_{\text{comb}} \int dx\, dy\, e^{-j2kz_0 T_z} \times \exp \left\{- \left( \frac{x^2}{\sigma_x^2} + j2g_{k}\kappa_{xy} + \frac{y^2}{\sigma_y^2} \right) + j2g_{k}(\bar{u}_x + \kappa_{xy} z_0)x \right. \\
\left. + j2g_{k}(\bar{u}_y + \kappa_{xy} z_0)y \right\}, \tag{A1}
\]

in which \(C_{\text{comb}}\) is the normalizing constant, and the constants of \(\kappa\) and \(g_k\) are defined as follows:

\[
\kappa = \kappa_{xy} + \kappa_{yy}, \quad g_k = \frac{kT_z}{z_0}. \tag{A2}
\]

Here the infinite limit of integration is chosen because the two-way gain function decays exponentially. However, the direct integration of Eq. (A1) is intractable unless the cross term of \(j2g_{k}\kappa_{xy}\) is removed in the argument of the exponential, which can be achieved by rotating the \(x-y\) axes by 45°. The result is written in the form of

\[
\rho_{\text{comb}}(T_z; t) = C_{\text{comb}}' \int d\tilde{x}'\, dy' \times \exp \left\{- \left( \frac{1}{\sigma_{\tilde{x}}^2} + jg_{k}\kappa \right)\tilde{x}'^2 - \frac{1}{\sigma_{\tilde{y}}^2} - jg_{k}\kappa \right\}\tilde{y}'^2 \\
- j\sqrt{2}g_{k}\{(u_\| + u_\perp)\tilde{x}' + (u_\perp - u_\|)\tilde{y}'\}, \tag{A3}
\]

where the normalizing constant \(C_{\text{comb}}'\) includes the phase factor \(e^{-j2k\kappa z_0}\). Furthermore, new velocities \(u_\|\) and \(u_\perp\) have been defined as

\[
u_\| = \bar{u}_x + \kappa_{xy} z_0, \quad u_\perp = \bar{u}_y + \kappa_{xy} z_0. \tag{A4}
\]

The orders of \(jg_{k}\kappa_{xy}^2\) and \(jg_{k}\kappa_{yy}^2\) in Eq. (A3) can be estimated for a typical space-/airborne radar as follows:

\[
g_{k}\kappa_{xy}^2 \approx g_{k}\kappa_{yy} \approx \frac{kT_z}{z_0} \kappa_{xy}^2 \approx 10^{-5} - 10^{-2}. \tag{A5}
\]

It enables us to approximate Eq. (A3) to the form
\[ \rho_{\text{comb}}(T; t) = C_{\text{comb}}^n \int dx' \, dy' \times \left[ 1 - jg_s \kappa(x'^2 - y'^2) - \frac{g_s^2 \kappa^2}{2}(x'^4 + 2x'^2y'^2 + y'^4) \right] \times \exp\left[ -\frac{1}{\sigma_r^2} x'^2 - j\sqrt{2}g_s(u_x + u_y)x' \right] \times \exp\left[ -\frac{1}{\sigma_r^2} y'^2 - j\sqrt{2}g_s(u_x - u_y)y' \right]. \]  

Hence, a lengthy calculation yields, by noting \( g_s \kappa \sigma_r^2 = g_s \kappa x'^2 \ll 1 \),

\[ \rho_{\text{comb}}(T; t) = \exp[-2k^2 \sigma_{\text{comb}}^2(T)]. \]

in which the spectral broadening \( \sigma_{\text{comb}}^2 \) can be calculated in the form equivalent to Eq. (A5):

\[ \sigma_{\text{comb}}^2 = \frac{7.53 \left\{ (\nu_x + \kappa_x z_0)^2 + (\nu_y + \kappa_y z_0)^2 \right\}}{2k^2l^2} + \frac{7.53 \zeta_0^2(\kappa_x + \kappa_y)^2}{2k^4l^4}. \]  

a. Incoherent backscattering from nonspherical cloud particles

The modification to nonspherical particles results in the form of

\[ R(T; t)_{\text{inc}} \approx |A_0|^2 \pi G G \frac{1}{k^2} \frac{c \tau_x}{\zeta_0} \frac{1}{2} e^{-i2\Delta k\Delta z e^{-2\sigma(\Delta k)}}, \]

\[ \times \langle S_{\text{bh}}(s)S_v(\alpha) e^{-i2\nu_0(s; \alpha; T)} \rangle \int dx \, dy \left[ 2J_0(kl/2z_0) \right], \]

\[ \times n_f(x, T, \nu_0) k^{(l-2k_0)}(\nu_0,-\nu_0+\nu_0z_0) \nu_{\text{inc}}, \]

\[ \times \langle \exp[j[-2k(\nu_x - \nu_{\text{inc}})T_s z_0^{-1}]) \nu_{\text{inc}}, \]

\[ \times \langle \exp[j[-2k(\nu_y - \nu_{\text{inc}})T_s z_0^{-1}]) \nu_{\text{inc}}, \]

which is the counterpart to Eq. (29). Here \( \sigma \) is as defined in Eq. (5); \( \Delta k = k^{(\alpha)} - k^{(0)} \) has been defined through the propagation wavenumbers \( k^{(0)} \) and \( k^{(0)} \) for H and V polarizations, respectively; \( \Delta z_0 \) is a thickness of clouds; and \( S_{\text{bh}}, S_v \) are diagonal elements of a backscattering amplitude matrix as a function of geometrical parameter \( \alpha \), such as diameter, shape, orientation, etc. When the orientations of nonspherical particles are completely random viewed from the nadir direction, \( \Delta k \approx 0 \) and \( \langle S_{\text{bh}}(s) \rangle \approx \langle S_v(\alpha) \rangle \) can be reasonably satisfied and returned to the form of Eq. (29).

REFERENCES


