Transducer-Shadow Effects on Turbulence Spectra Measured by Sonic Anemometers

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ABSTRACT

We show that the horizontal turbulent velocity components measured by the common sonic anemometer array can suffer attenuation and crosstalk as a result of the flow blockage caused by the acoustic transducer assemblies. Using an analytical model of this “transducer-shadow effect”, flow-blockage data from test arrays, and a simple linear model of the fluctuating response, we show the nature of the distortion in the measured velocity spectra. We suggest that rather than correct for the shadow effect, which can be quite significant for horizontal velocity spectra and stress cospectra, it would be preferable to minimize it through design. There is encouraging evidence that the Kaijo-Denki transducer design produces much less shadow effect than the conventional (right circular cylinder) shape.

1. Introduction

Kaimal (1979) refers to the sonic anemometer as the “instrument of choice” among boundary layer physicists seeking fair-weather observations of atmospheric turbulence. Summarizing the reasons for this favored position, he points out that it has no moving parts to come into dynamic equilibrium with the flow, so its frequency response is limited only through the spatial filtering imposed by line averaging along the path. It responds linearly to wind velocity and is relatively free of contamination from other velocity components or temperature. Finally, he points out that as an absolute instrument, its calibration is established by its design parameters.

Although its operating principle is simple, a sonic anemometer is a complex instrument and understanding its response can be quite challenging. Kaimal (1979) surveys the progress that has been made in analyzing the effects of finite path length and path separation (Kaimal et al., 1968; Horst, 1973), sound-speed fluctuations (Friehe, 1976), leveling errors (Kaimal and Haugen, 1969), and transducer shadows. All but the last seem well understood.

The shadow effect refers to the velocity deficit in the transducer wakes along the acoustic paths. It seems to have been discussed first by Kaimal (1979), who presented some measurements of the resulting path-averaged velocity errors in a simple array. In his review paper, Wyngaard (1981a) pointed out that the shadow effect could be important in short-path arrays, and Kaimal and Gaynor (1983) show how it is being accounted for in the sonic anemometers at the Boulder Atmospheric Observatory. Hanafusa et al. (1982) and Coppin and Taylor (1983) discuss new, improved sonic anemometers but show that they also suffer from transducer-shadow effects.

Mestayer (1982) suggested that transducer-shadow effects could be the source of the spectral attenuation that he detected in sonic anemometer data. Hanafusa et al. (1982), on the other hand, on the basis of some brief field trials with two units mounted close together but with a 45-degree difference in orientation, found “no remarkable difference” in their output statistics (mean wind speed and direction and horizontal velocity variance). They concluded that “the difference causes no trouble in the use of those instruments in the field.” However, Coppin and Taylor (1983) were clearly concerned about shadow effects. They wrote that “deficiencies in horizontal directional response appear to be a major limitation in the field use of sonic anemometers.”

While there has been recent concern about the effects of transducer shadows on turbulence statistics measured by sonic anemometers, there has not been a definitive assessment of these effects. Unfortunately, it is difficult to study this problem experimentally, because the flow distortion in typical arrays can also influence the turbulence statistics (Wyngaard, 1981b), and it is not clear if the two sets of effects can be isolated. As a first step toward this assessment, therefore, we present a theoretical analysis of transducer-shadow ef-

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fects on turbulence spectra measured by the most common sonic array.

2. The transducer-shadow effect

Figures 1 and 2 (from Kaimal, 1979) show commercial three-axis sonic anemometer arrays. The unit shown in Fig. 1 is an EGG Model 198-3 used in the 1973 Minnesota experiments (Kaimal et al., 1976), while the unit in Fig. 2 is a Ball Brothers Model 125-197, 198 used at the Boulder Atmospheric Observatory. Each axis has two acoustic paths so that the transit time difference for sound pulses emitted from each end yields the path-averaged value of the wind velocity component in the path direction. The horizontal velocity components are obtained from the pair of horizontal axes, and the vertical axis provides the vertical velocity.

Figure 3 shows the field and commercial model of the short-path (10 cm) array developed by Coppin and Taylor (1983). It has the same basic geometry as the units in Figs. 1 and 2. However, this design uses only one transducer, serving alternately as a transmitter and a receiver, at each end of an axis. Figure 4 shows a Kajo-Denki unit (Hanafusa et al., 1982) of similar geometry.

As Figs. 1–4 reveal, in these arrays the scale of the transducers and their mounts is not small compared with the path length. Thus, one expects that the flow blockage near the path ends could cause this path-av-

![Image of a sonic array](image_url)

**Fig. 1.** The three-component sonic array used in the 1973 Minnesota experiments. Photo courtesy J. C. Kaimal.
eraged wind velocity component to be somewhat less
than its value in the free stream.

Kaimal (1979) demonstrated this transducer-shadow
effect through wind tunnel and atmospheric tests. We
present his results in Fig. 5, which shows the axial ve-
locity attenuation in a single-axis unit as a function of
the inclination $\theta$ of the flow vector to the acoustic path.
The attenuation is greatest when the flow is along the
path, decreases as $\theta$ increases, and also decreases as
$L/d$ (path length/transducer diameter) increases.

Figure 6 introduces velocity attenuation data at
$L/d = 10$ measured in single-axis test arrays by Han-
afusa et al. (1982) and Coppin and Taylor (1983). While
the three sets of results are qualitatively similar, there
are substantial differences; in particular, the Hanafusa
et al. results essentially indicate no attenuation for flow
angles greater than about 30 degrees, while the Kaimal and Coppin–Taylor tests show significant attenuation until 60–70 degrees. Figure 6 also includes a response curve for the field and commercial model of the Coppin–Taylor anemometer. This curve was provided by Dr. Coppin, who indicated that the improvement in response is due to transducer redesign.

The differences in directional response shown in Fig. 6 are evidently due to differences in transducer geometry. Both the Kaimal (1979) and Coppin–Taylor (1983) tests used transducers in the shape of a right circular cylinder (Figs. 1–3). Hanafusa et al. used a specially shaped Kajjo–Denki transducer (Fig. 4) designed to minimize the shadow effect (Mitsuta, 1974); Fig. 6 shows that its response is indeed much improved. The smaller difference between the Coppin–Taylor (1983) and Kaimal (1979) results could be due to their design differences. The Kaimal test array had two
acoustic paths, one slightly above the other; hence, there were a pair of transducers at each end, as in the arrays of Figs. 1 and 2. The Coppin–Taylor test array had coincident paths, with only one transducer at each end, as in their field and commercial model shown in Fig. 3.

The dependence of the velocity error on \(L/d\) indicated by Kaimal’s data in Fig. 5 seems plausible physically. One can easily show that if, for fixed flow angle \(\theta\) and geometrically similar transducers, the axial length of the transducer wake scales with transducer diameter \(d\), then the axial velocity error averaged over the path length \(L\) increases linearly with \(d/L\). Figure 7, a plot of the velocity attenuation at \(\theta = 0\) versus \(d/L\), shows that this is a reasonable approximation to both the Kaimal and Hanafusa et al. test data. Coppin and Taylor showed results for only one \(d/L\) value.

Figures 6 and 7 suggest that the dependence of the velocity attenuation on inclination angle \(\theta\) is more sensitive to the design of the transducer than is its dependence on \(L/d\). We will present calculations for two varieties of transducers: right circular cylinders, used in the arrays of Figs. 1–3, which we will call BAO type; and streamlined transducers (Fig. 4), which we will call Kajo–Denki type. In order to make our analytical calculations tractable, we will use simple parameterizations of their directional response:

\[
\begin{align*}
U_L^{m}/U_L &= C + (1 - C) \sin \theta & \text{(BAO-type),} \\
U_L^{m}/U_L &= 1 - (1 - C)e^{-a \sin \theta} & \text{(Kajo–Denki-type).}
\end{align*}
\]

Here the superscript \(m\) means measured value. In each case \(C\) is the response at \(\theta = 0\). Figure 6 shows that (1a) with \(C = 0.75\) is a fairly good fit to the Coppin–Taylor (1983) data; a slightly smaller \(C\) would fit the Kaimal data. Equation (1b), with \(C = 0.74\) and \(a = 11.5\), fits the Hanafusa et al. data very well.

3. Effects on velocity spectra

Figure 8 shows the geometry of the horizontal axes of the arrays in Figs. 1–4. Let the two sonic axes, denoted with subscripts \(A\) and \(B\), be located symmetrically with respect to the \(x_1\) axis, and let the angle between these axes be \(2\alpha\). Let the angle between the wind vector and the \(x_2\) axis be \(\gamma\). Since each sonic axis measures the projection of the wind on its path vector, in the absence of measurement error the outputs would be

\[
\begin{align*}
U_A &= U_1 \cos \alpha - U_2 \sin \alpha, \\
U_B &= U_1 \cos \alpha + U_2 \sin \alpha.
\end{align*}
\]

Thus, we would recover the wind components from

\[
\begin{align*}
U_1 &= (U_A + U_B)/(2 \cos \alpha), \\
U_2 &= (U_B - U_A)/(2 \sin \alpha).
\end{align*}
\]

The measured velocity components are subject to the transducer-shadow effects, which we can now incorporate through (1).

a. BAO-type transducers

We consider first an array fitted with BAO-type transducers; that is, ones of right circular cylinder shape, as in the arrays of Figs. 1–3. Using (1a), we write the actual response of the \(A\) and \(B\) axes as

\[
\begin{align*}
U_A^{m} &= U_A[C + (1 - C) \sin \theta_A], \\
U_B^{m} &= U_B[C + (1 - C) \sin \theta_B].
\end{align*}
\]
Then from (2) through (7) and the geometry of Fig. 8 we have

\[ U_1^m = U_1 \left( \frac{f_A + f_B}{2} \right) + U_2 \tan \alpha \left( \frac{f_B - f_A}{2} \right), \quad (8) \]
\[ U_2^m = U_1 \cot \alpha \left( \frac{f_B - f_A}{2} \right) + U_2 \left( \frac{f_A + f_B}{2} \right), \quad (9) \]

where the shadow functions can be written as

\[ \left( \frac{f_A + f_B}{2} \right) = C + (1 - C) \sin \gamma \sin \alpha, \quad (10) \]
\[ \left( \frac{f_B - f_A}{2} \right) = (1 - C) \cos \gamma \cos \alpha. \quad (11) \]

In the absence of shadow effects \( f_A = f_B = 1 \), and (8) and (9) yield \( U_1^m = U_1, U_2^m = U_2 \). With shadow effects, however, we see that the measured horizontal velocity components suffer attenuation and crosstalk (mixing of the two components).

We can write immediately the expression for vertical velocity measured with BAO-type transducers:

\[ U_3^m = U_3 [C + (1 - C) \sin \theta]. \quad (12) \]

Here \( \theta \) is the angle between the wind vector and the vertical axis.

Having our basic results (8), (9), and (12), we can now examine transducer-shadow effects on turbulence spectra. We will begin with the horizontal velocity components, for which we will do two calculations. The first uses coordinates aligned with the sonic array, and the second uses the more conventional mean-wind coordinates. Finally, we will briefly discuss the vertical velocity spectrum and stress cospectra.

1) Horizontal Velocity Spectra in Anemometer Coordinates

We use coordinates aligned with the axes of symmetry of the sonic anemometer, as shown in Fig. 8. To reveal the effects on turbulence, we begin by decomposing the velocity field into mean and fluctuating components,

\[ U_i = \bar{U}_i + u_i, \quad i = 1, 2, 3, \quad (13) \]

with the lower case denoting the fluctuations. We assume that the rms velocity fluctuations are small compared to the mean, so that we can derive a linearized pair of equations for the turbulent components. After some algebra, (8) and (9) yield

\[ u_1^m = c_1 u_1 + c_2 u_2, \quad (14) \]
\[ u_2^m = d_1 u_1 + d_2 u_2. \quad (15) \]

Here the coefficients are

\[ c_1 = C + (1 - C) \sin \alpha (1 + 2 \cos^2 \gamma) \sin \gamma, \quad (16) \]
\[ c_2 = (1 - C) \sin \alpha \cos \gamma (1 + 2 \sin^2 \gamma), \quad (17) \]
\[ d_1 = 2(1 - C) \cos \alpha \cot 2\alpha \cos^3 \gamma, \quad (18) \]
\[ d_2 = C - 2(1 - C) \cos \alpha \cot 2\alpha \sin^3 \gamma, \quad (19) \]

where \( \gamma \) is the angle of the mean wind vector; i.e.,

\[ \tan \gamma = -\bar{U}_1/\bar{U}_2. \quad (20) \]

The linear equations (14) and (15) express the attenuation and crosstalk of horizontal turbulent velocity components, referred to the symmetry axes of the sonic array (Fig. 8). They yield simple expressions for the measured spectra:

\[ \Phi_1^m = c_1^2 \Phi_1 + c_2^2 \Phi_2 + 2c_1c_2 \Phi_{12}, \quad (21) \]
\[ \Phi_2^m = d_1^2 \Phi_1 + d_2^2 \Phi_2 + 2d_1d_2 \Phi_{12}. \quad (22) \]

Here \( \Phi_1 \) is the spectrum of \( u_1 \).

Let us examine \( \Phi_{12} \). It is simplest to do this by expressing \( u_1 \) and \( u_2 \) in terms of components measured in mean-wind coordinates (Fig. 8), denoting these with a tilde:

\[ u_1 = \tilde{u}_1 \sin \gamma + \tilde{u}_2 \cos \gamma, \quad (23) \]
\[ u_2 = -\tilde{u}_1 \cos \gamma + \tilde{u}_2 \sin \gamma. \quad (24) \]

Thus, we have

\[ \Phi_{12} = \cos \gamma \sin \gamma (\tilde{u}_2 - \tilde{\phi}_1) + \tilde{\Phi}_{12} (\sin^2 \gamma - \cos^2 \gamma). \quad (25) \]

Consider first \( \tilde{\Phi}_{12} \), the cospectrum of \( \tilde{u}_1 \) and \( \tilde{u}_2 \). In an idealized (i.e., horizontally homogeneous, barotropic, quasi-steady) boundary layer, \( \tilde{\Phi}_{12} \) (and its integral, \( \tilde{u}_1 \tilde{u}_2 \)) differs from zero only due to Coriolis effects. Considering these effects in the context of the Reynolds equation for \( \tilde{u}_1 \tilde{u}_2 \) (Wyngaard, 1982) suggests that \( \tilde{u}_1 \tilde{u}_2 \) should be negative (in the Northern Hemisphere). Model calculations (e.g., Wyngaard et al., 1974) confirm this but also suggest that it is small in the sense that its correlation coefficient \( r = \tilde{u}_1 \tilde{u}_2/(\tilde{u}_1^2 \tilde{u}_2^2)^{1/2} \) does not exceed about 0.2 in magnitude. The AMTEX observations (Lenschow et al., 1980) show that \( r \) is very small in the convective PBL, averaging about 0.01 in magnitude. Hence, we will take \( r \) to be zero and therefore assume that the second term on the right side of (25) vanishes.

The first term on the right side of (25) involves the difference between \( \tilde{\Phi}_1 \) and \( \tilde{\phi}_1 \). According to Panofsky and Dutton (1984), their integrals, \( \tilde{u}_1^2 \) and \( \tilde{u}_2^2 \), are equal under convective conditions and \( \tilde{u}_1^2 \) is about 20% smaller than \( \tilde{u}_2^2 \) under neutral conditions. Thus, the contribution of the first term in (25) is also small, and we will take \( \tilde{\Phi}_{12} \) to be zero.

We have evaluated (21) and (22) for typical parameter ranges, assuming \( \tilde{\Phi}_{12} = 0 \), and present the results in Fig. 9. We used \( 2\alpha = 90 \) and 120 degrees, and used \( C = 0.95, 0.85, \) and 0.75, corresponding (Fig. 7) to \( L/d = \) of about 60, 20, and 10, respectively. We evaluated the crosstalk terms by arbitrarily assuming that the \( u_1 \) and \( u_2 \) spectra were equal, which is not critical
to the results since the crosstalk terms in (21) and (22) are at most only a few percent of the leading terms.

2) **HORIZONTAL VELOCITY SPECTRA IN MEAN-WIND COORDINATES**

Turbulent velocities are more often referred to coordinates aligned with the mean-wind vector. We can easily rewrite our results (14) and (15) in these coordinates by rotating our axes through an angle $90^\circ - \gamma$ (Fig. 8), where $\gamma$ is the mean-wind direction. The result is

$$\tilde{u}_1 = p_1 \tilde{u}_1 + p_2 \tilde{u}_2,$$

$$\tilde{u}_2 = q_1 \tilde{u}_1 + q_2 \tilde{u}_2,$$  

where we use the carat to denote quantities measured in mean-wind coordinates. The $p, q$ coefficients are

$$p_1 = c_1 \sin^2 \gamma - (c_2 + d_1) \sin \gamma \cos \gamma + d_2 \cos^2 \gamma,$$  

$$p_2 = c_2 \sin^2 \gamma + (c_1 - d_2) \sin \gamma \cos \gamma - d_1 \cos^2 \gamma,$$  

$$q_1 = -c_2 \cos^2 \gamma + (c_1 - d_2) \sin \gamma \cos \gamma + d_1 \sin^2 \gamma,$$  

$$q_2 = c_1 \cos^2 \gamma + (c_2 + d_1) \sin \gamma \cos \gamma + d_2 \sin^2 \gamma,$$  

and the $c, d$ coefficients are defined in (16)–(19). The spectral equations are (taking $\tilde{C}_{012} = 0$)

$$\tilde{\Phi}_1 = p_1 \tilde{\Phi}_1 + p_2 \tilde{\Phi}_2,$$  

$$\tilde{\Phi}_2 = q_1 \tilde{\Phi}_1 + q_2 \tilde{\Phi}_2.$$  

We evaluated the spectral equations (32) and (33) for the same parameter values used earlier and present the results in Fig. 10. We again evaluated the crosstalk terms by assuming the streamwise and lateral velocity spectra are equal. As before, however, crosstalk has minor influence on the results.

3) **THE VERTICAL VELOCITY SPECTRUM**

If we start from Eq. (12) for the vertical velocity response, express $\sin \theta$ in terms of the velocity components, and then let the velocity field have mean and fluctuating parts, we obtain an equation for the measured vertical velocity. We find that the error terms in this equation are third order in the fluctuating velocity field, however, so that they are negligible in the linear approximation we are using here. Thus, we can take
the measured vertical velocity spectrum to be error-
free.

4) STRESS COSPECTRA

We can briefly illustrate the transducer-shadow ef-
effects on stress. We adopt mean-wind coordinates and
denote the cospectra of the horizontal stress com-
ponents $\mathbf{u}_3$ by $C_{13}$ and $C_{22}$, respectively. 
Since the $u_3$ component is unaffected, we have im-
mediately from (26) and (27)
\[
\begin{align*}
\hat{C}_{13} &= p_1 \hat{C}_{13} + p_2 \hat{C}_{22}, \\
\hat{C}_{22} &= q_1 \hat{C}_{13} + q_2 \hat{C}_{22}.
\end{align*}
\]

Thus, the measured stress cospectra also suffer atten-
uation and crosstalk.

The lateral stress $\mathbf{u}_2 \mathbf{u}_5$, while negligible in the surface 
layer, can be large in mid-PBL. Unfortunately, however, $\mathbf{u}_2 \mathbf{u}_5$ appears to bear no simple relation to $\mathbf{u}_3 \mathbf{u}_5$, so that we cannot make any general statements about the be-
behavior of (34) and (35). Instead, we simply show in 
Table 1 the values of the $p$, $q$ coefficients for 90° and 
120° arrays with BAO-type transducers for $L/d = 10$ 
and 20.

With no wake effects, $p_1 = q_2 = 1.0$, $p_2 = q_1 = 0$; we 
see from Table 1 that the wake effect can cause signif-
ificant departures from these values. In the surface layer,

\[
\begin{align*}
\gamma \text{, DEGREES}
\end{align*}
\]

\[
\begin{align*}
\hat{c}_1 &= f_+ S_+ \sin \gamma \cos \gamma - S_- \tan \alpha \cos^2 \gamma, \\
\hat{c}_2 &= f_- \tan \alpha + S_+ \sin^2 \gamma - S_- \cos \gamma \sin \gamma \tan \alpha, \\
\hat{d}_1 &= f_- \cot \alpha - S_+ \cos^2 \gamma + S_- \cos \gamma \sin \gamma \cot \alpha, \\
\hat{d}_2 &= f_+ \cos \gamma \sin \gamma S_+ + S_- \sin^2 \gamma \cot \alpha,
\end{align*}
\]

where the lateral stress $\mathbf{u}_2 \mathbf{u}_5$ is normally negligible, un-
der worst-case conditions ($L/d = 10$, 90° array) $\mathbf{u}_3 \mathbf{u}_5$
can be underestimated by about 20%.

b. Kajo–Denki-type transducers

The analysis framework of the previous section car-
ries over for Kajo–Denki-type transducers, but the 
crosstalk and attenuation coefficients change. We will 
briefly discuss the nature of the spectral response this 
produces.

The spectral response equations (21) and (22) now 
have coefficients

\[
\begin{align*}
f_+ &= (f_A + f_B)/2, \\
f_- &= (f_B - f_A)/2, \\
S_+ &= \left( \frac{\partial f_+}{\partial \theta_B} + \frac{\partial f_+}{\partial \theta_A} \right)/2.
\end{align*}
\]
Table 1. Array response with BAO-type transducers.

<table>
<thead>
<tr>
<th>$\gamma$ (°)</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$q_1$</th>
<th>$q_2$</th>
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</thead>
<tbody>
<tr>
<td>90</td>
<td>0.93</td>
<td>0°*</td>
<td>0°*</td>
<td>0.75</td>
<td>0.96</td>
<td>0°*</td>
<td>0°*</td>
<td>0.85</td>
</tr>
<tr>
<td>80</td>
<td>0.91</td>
<td>0.12</td>
<td>0.029</td>
<td>0.77</td>
<td>0.95</td>
<td>0.071</td>
<td>0.017</td>
<td>0.86</td>
</tr>
<tr>
<td>70</td>
<td>0.88</td>
<td>0.21</td>
<td>0.046</td>
<td>0.83</td>
<td>0.93</td>
<td>0.13</td>
<td>0.028</td>
<td>0.90</td>
</tr>
<tr>
<td>60</td>
<td>0.83</td>
<td>0.27</td>
<td>0.044</td>
<td>0.90</td>
<td>0.90</td>
<td>0.16</td>
<td>0.027</td>
<td>0.94</td>
</tr>
<tr>
<td>50</td>
<td>0.77</td>
<td>0.27</td>
<td>0.020</td>
<td>0.97</td>
<td>0.86</td>
<td>0.16</td>
<td>0.012</td>
<td>0.98</td>
</tr>
</tbody>
</table>

90° array

<table>
<thead>
<tr>
<th>120° array</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
</tr>
<tr>
<td>80</td>
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<tr>
<td>70</td>
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<tr>
<td>60</td>
</tr>
<tr>
<td>50</td>
</tr>
</tbody>
</table>

* Less than $10^{-3}$.

$$S_{-} = \left( \frac{\partial f_B}{\partial \theta_B} - \frac{\partial f_A}{\partial \theta_A} \right) / 2.$$

We did calculations for an array with $L/d = 20$, using our shadow-effect parameterization (1b) with $C = 0.83$ and $a = 15.5$, which gives an excellent fit to the Hanafusa et al. test data.

For brevity we show in Fig. 11 only the results for the mean-wind coordinates. Note that the response is quite different from that for the same array fitted with

![Graphs showing spectral response](image-url)
BAO-type transducers. The absence of significant wake effects for inclination angles greater than about 30° with Kaijo–Denki transducers (Fig. 6) gives essentially perfect response for wind directions near the axis of symmetry of the array. It is difficult to interpret the details of the response in a simple way, however, because of the complex interactions of the wake effects, the array geometry, the crossstalk and attenuation, and the coordinate rotation. We simply note that the streamwise and lateral responses are quite different in general, and in practice (with high turbulence levels) are probably not as peaked as indicated by our linearized calculations in Fig. 11.

The stress response is again given by (34) and (35). Table 2 shows the p, q coefficients for Kaijo–Denki-type transducers with \( L/d = 20 \) (i.e., \( C = 0.83 \) and \( a = 15.5 \)). Note by comparison with Table 1 that the stress response is generally improved over that for BAO-type transducers. Note that for extreme flow angles, however, there can be considerable stress crosstalk.

4. Discussion

The shadow-induced errors indicated by our illustrative results in Figs. 9–11, and Tables 1 and 2, could be detrimental in research applications. For example, researchers often use the appearance of a 4/3 ratio between spectra of lateral (\( u_2 \) or \( u_3 \)) and longitudinal (\( u_1 \)) velocity components in the inertial subrange as an indicator of local isotropy. Figure 10 shows, however, that for the typical 120° array with \( L/d = 20 \) and BAO-type transducers uncorrected for wake effects, the \( u_1 \) spectrum is attenuated by a factor ranging from 0.96 to 0.83 as the wind direction varies from 90° to 50°, while the \( u_2 \) spectrum is modified by a factor ranging from 0.87 to 1.05. The \( u_3 \) spectrum is unaffected, to our level of approximation. Hence, the measured ratio of uncorrected \( u_2 \) and \( u_1 \) spectra in a locally isotropic field would vary from about 1.2 to 1.7, while the measured \( u_3 \) and \( u_1 \) spectra would have a ratio varying from 1.4 to 1.6. Figure 10 shows that the effects are even stronger in a 90° array.

Mestayer (1982) has critically surveyed the evidence for the 4/3 ratio between \( u_2 \) and \( u_1 \) spectra in the surface layer. His review indicates that the situation is not yet clear; some researchers (e.g., Kaimal et al., 1972) report finding the 4/3 ratio from sonic anemometer measurements, while others (e.g., Busch, 1973) using hot wires report its onset at frequencies well beyond the range of the sonic anemometer. This situation is complicated by the response problems of hot-wire anemometers which, while having excellent spatial and temporal resolution, suffer from parasitic sensitivities not yet completely understood in atmospheric applications. Mestayer concludes that there are serious doubts about the validity of the 4/3 ratios obtained with sonic anemometers, however, and suggests that they could suffer from response problems due to the transducer wakes.

Our results indicate that the fidelity of measured 4/3 ratios depends very much on \( L/d \), the transducer type, and the alignment of the array with respect to the wind. The Kansas results (e.g., Kaimal et al., 1972) were obtained with Kaijo–Denki transducers in \( L/d = 20 \) arrays frequently realigned with the changing mean wind direction (J. C. Kaimal, personal communication, 1984), and Fig. 11 indicates their response was satisfactory. On the other hand, Fig. 10 shows that arrays with uncorrected, BAO-type transducers could give unreliable spectral ratios.

As another example, researchers often use the inertial subrange spectral level to infer the dissipation rate of turbulent kinetic energy, using the Kolmogorov expression:

\[
\Phi_i = \alpha \epsilon^{2/3} \kappa^{-5/3}, \quad i = 1, 2, \text{ or } 3.
\]

Here \( \alpha \) is a constant, \( \kappa \) is wavenumber, and \( \epsilon \) is the dissipation rate. Since from (44) \( \epsilon \) is proportional to the 1.5 power of inertial range spectral level, it follows that the spectral errors in Figs. 9 and 10 cause correspondingly larger errors in dissipation rate. In the worst case (dissipation found from the \( u_2 \) spectrum, using a 90° array with \( L/d = 10 \) and uncorrected, BAO-type transducers) the dissipation rate is underestimated by nearly 60%. On the other hand, dissipation inferred from the \( u_3 \) spectrum would be error free.

If transducer technology permits, minimizing their shadow effects through optimum design seems preferable to compensating for the effects in the subsequent data analysis because the inevitable uncertainties in the shadow data, and the influence of turbulence on the transducer wakes, render these corrections somewhat uncertain. It is possible, for example, that the axial velocity attenuation characteristics of a given array are somewhat different in the turbulent atmosphere and in low-turbulence wind tunnels.

<table>
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<th>Table 2. Array response with Kaijo–Denki-type transducers.</th>
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| \(| \gamma | p_2 | p_3 | q_1 | q_2 | \)
| (°)           |     |     |     |     |     |
| 90° array     |     |     |     |     |     |
| 90           | 1.00 | 0*  | 0*  | 1.00 |
| 80           | 1.00 | 0.011| 0*  | 0.99 |
| 70           | 0.99 | 0.11 | 0.0041| 0.95 |
| 60           | 0.94 | 0.45 | 0.015 | 0.88 |
| 50           | 0.85 | 0.42 | 0.013 | 0.96 |
| 120° array   |     |     |     |     |     |
| 90           | 1.00 | 0*  | 0*  | 1.00 |
| 80           | 1.00 | 0*  | 0*  | 1.00 |
| 70           | 1.00 | 0.0039| 0*  | 1.00 |
| 60           | 1.00 | 0.049| 0*  | 1.00 |
| 50           | 0.97 | 0.31 | 0.005 | 1.054 |

* Less than 10⁻³.
Finally, we emphasize that data from any sonic anemometer are subject to the combined effects of the transducer wakes and the flow distortion caused by the bulk of the array itself. This “probe-induced” flow distortion can be particularly detrimental to stress measurements (Wyngaard, 1981b) but has yet to be studied in detail for sonic anemometers; however, there are disturbing indications (e.g., Kondo and Sato, 1982) that it can be substantial. Until these flow distortion effects are studied, it will be difficult to confirm the transducer-shadow effect experimentally. Nonetheless, in view of our results, we believe it merits the attention of manufacturers and users of sonic anemometry.

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REFERENCES


