Evaluation of a New Polarimetrically Based Z–R Relation

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ABSTRACT

A new polarimetrically based (or, pol-based) Z–R relation of the form \( Z = aR^{1.5} \) is described and evaluated where the multiplicative coefficient \( a \) is continuously adjusted as the drop size distribution evolves in space and time. The methodology is based on previous studies involving estimation of the normalized gamma drop size distribution parameters (DSD) using radar measurements of \( Z_a, Z_r, \) and \( K_{dp} \). In moderate-to-intense rainfall, the retrieval of the DSD parameters are formulated to account for the effects of drop oscillations using the “effective” \( \beta \) concept where the axis ratio \( (r) \) versus \( D \) relation is assumed to be linear and of the form \( r = 1 - \beta D \) in the underlying raindrop shape model. Rayleigh scattering with analytic approximations are used to show that the \( \beta \) estimator in Gorgucci et al. (2000) based on \( Z_a, Z_r, \) and \( K_{dp} \) is of the correct form. The changes in the effective \( \beta \) in a storm cell is studied as the cell evolves from the growth phase to the mature phase (with microburst and rain rates of around 100–120 mm h\(^{-1}\)). The systematic shift in \( \beta \) with increasing rain rates in this cell is shown to be consistent with the collisional probability model results of Beard and Johnson (1984). For evaluation of the pol-based Z–R relation, six storm events from the Tropical Rainfall Measuring Mission–Large-Scale Biosphere–Atmosphere (TRMM–LBA) experiment and Texas and Florida Underflights Experiment-B (TEFLUN-B) are analyzed using radar data from the NCAR–S-band Polarimetric (SPOL) radar and a network of gauges specially deployed for these two campaigns. For storm total accumulation, the new pol-based Z–R algorithm gives a normalized bias of 6% (radar overestimate) and normalized standard error of 20%. The corresponding values for a conventional Z–R relation (after stratiform/convective separation) are \(-18\%\) and \(24\%\). The pol-based Z–R method continuously “tracks” the drop size distribution and so no classification of rain types is necessary.

1. Introduction

For a single-polarized radar there is only one measure (reflectivity) that can be used to estimate rain rate using a Z–R relation. Improvements are possible if the rain type can be classified as stratiform or convective before applying different Z–R relations for each type (e.g., Steiner and Smith 1998; Tokay and Short 1996). It is well known that different Z–R relations must be used for maritime as opposed to continental-type rain events. In any case, systematic biases in the radar retrieval of rainfall cannot generally be avoided unless the Z–R relation is “calibrated” using rain gauges. The literature dealing with these issues is large and it is not the intent here to dwell on Z–R relations. Rather, our intention is to show how dual-polarized radar data can be used to derive a Z–R relation that is continuously varying in space and time as the storm microphysics evolves without the need for classifying the rain types a priori. This is based on retrieving the drop size distribution (DSD) parameters of a normalized gamma model using radar measurements of \( Z_a, Z_r, \) and \( K_{dp} \) as described in several recent articles (Gorgucci et al. 2002; Bringi et al. 2002, 2003). A significant advance has been the inclusion of the effects of drop oscillations in moderate-to-intense rain rates in the retrieval of the DSD parameters. The DSD retrieval of the “generalized” intercept parameter \( (N_v) \) and the shape parameter \( (\mu) \) are then used to estimate the multiplicative coefficient of the Z–R relation. It is implicit in our formulation that accounting for spatial and temporal changes in the DSD is an important factor in improving the accuracy of radar estimates of rain rate, which is supported by other studies (e.g., Carbone and Nelson 1978; Ryzhkov and Zrnić 1996; Brands et al. 2002). For such improvements to be realized, it is assumed that the radar resolution volume is uniformly filled and below the melting level, which puts an upper limit on the useful maximum range to around 100 km for a typical 1° beam. At long ranges, systematic changes in the DSD can occur from the elevated height of the radar resolution volume to the surface, for example, due to evaporation or accretion (Zawadzki 1984), but are not considered here.

Polarimetric techniques for estimating the rain rate use information about the size, shape, and orientation (or, canting angle) distribution of the raindrops (together
with their fall speed). In essence, the important polarimetric radar measureables are differential reflectivity ($Z_{dr}$) and specific differential phase ($K_{dp}$) in addition to the conventional radar reflectivity factor at horizontal polarization ($Z_{hh}$). A thorough explanation of polarimetric theory is available in Bringi and Chandrasekar (2001). This paper develops a polarimetrically based (or, pol-based) Z–R relation of the form $Z = aR^{b}$ where the coefficient $a$ is continuously estimated mainly from the polarimetric measureables $Z_{dr}$ and $K_{dp}$ along with $Z_{hh}$. This is in contrast to directly developing a rain rate algorithm based on $Z_{dr}$ and $K_{dp}$, for example, of the form $R(Z_{dr}, K_{dp}) = aK_{dp}^{b}Z_{dr}^{c}$.  The pol-based Z–R method appears to be more robust with respect to “noisy” measurement fluctuations in $Z_{dr}$ and $K_{dp}$, which occur at low rain rates or in stratiform rain events. However, it does depend on the system radar constant (or, radar system gain) whereas both $Z_{dr}$ and $K_{dp}$ are independent of the system gain (note that $Z_{dr}$ will depend on the differential system gain between H and V polarization channels, but this is easier to calibrate compared to the absolute system gain). It will be shown that the pol-based Z–R method offers all the advantages of radar polarimetry, but still keeps the form that most radar meteorologists are accustomed to. In addition, there is no need to separate stratiform and convective rain types before applying the pol-based Z–R method. A different method of developing a polarimetric-tuned Z–R relation is given in Ryzhkov et al. (1997) using only $Z_{dr}$ and $K_{dp}$ data. Brandes et al. (2003) also propose a DSD-based rain-rate estimator using only $Z_{dr}$ and $Z_{hh}$ data.

This paper is organized as follows: first, a background section is included to highlight the manner in which the effects of drop oscillations are included into the methodology. Rayleigh scattering is used to show that the estimator for $\beta$ (in $r = 1 - \beta D$) proposed by Gorgucci et al. (2000) as a power law involving $Z_{dr}$, $K_{dp}$, and $Z_{hh}$ is of the correct form. Next we analyze changes in $\beta$ using National Center for Atmospheric Research–S-band Polarimetric (NCAR–SPOL) radar data from a convective rain event previously analyzed in detail by Atlas and Williams (2003a,b). This particular event was characterized by detailed measurements of an evolving cell from rapid “growth” phase to the mature downdraft phase; further, the shift in $\beta$ from the equilibrium value might be reflective of collisional forcing of drop oscillations during the mature phase. Next we evaluate the pol-based Z–R algorithm using SPOL radar data and gauge network data from three events during the Tropical Rainfall Measuring Mission–Large-Scale Biosphere–Atmosphere (TRMM–LBA) project in Brazil and three events during the Texas and Florida Underflights Experiment-B (TEFLUN-B) project in central Florida. It is shown that the pol-based Z–R method implemented here reduces the normalized bias in rainfall accumulation to 6% from the −18% bias if a combined stratiform/convective Z–R is used.

2. Background

a. General

The methodology of deriving a pol-based $Z$–$R$ relation is based on the assumption of a normalized gamma DSD of the form

$$N(D) = N_{w}f(\mu)\left(\frac{D}{D_{0}}\right)^{\alpha} \exp\left[-(3.67 + \mu)\frac{D}{D_{0}}\right]$$

(1)

where

$$f(\mu) = \frac{6}{(3.67)^{4}} \frac{(3.67 + \mu)^{\mu + 4}}{\Gamma(\mu + 4)}$$

and

$$N_{w} = \frac{(3.67)^{4}}{\pi\rho_{w}} \left(\frac{W}{D_{0}}\right).$$

(2)

The above form is the so-called normalized gamma form (Willis 1984; Testud et al. 2001); note that $W$ in (3) is the water content and $\rho_{w}$ is the water density. Assuming Rayleigh scattering and a power-law form for the terminal fall speed of drops, $V(D) = 3.78D^{0.67}$ (Atlas and Ulbrich 1977), it is easy to show that the reflectivity factor $Z$ is related to rain rate $R$ as

$$Z = \frac{a(\mu)}{\sqrt{N_{w}}} R^{b},$$

(4)

where $a(\mu)$ depends on $\mu$ only and other constants. Testud et al. (2001) further made the important generalization that the form of (4) is valid for any general DSD, not necessarily of the gamma form. They also showed that the sensitivity of rain rate to $\mu$ is only around 20%, very much less than the sensitivity to the range of $N_{w}$ (factor of 100) in different rain types (Bringi et al. 2003). Note that if $N_{w} = \text{constant}$ then the exponent is fixed at 1.5 in (4), whereas if $N_{w}$ varies linearly with $R$ (as in the case of an equilibrium type DSD), then the exponent in (4) becomes unity.

In several recent articles it has been shown that $D_{0}$, $N_{w}$, and (to some extent) $\mu$ can be retrieved from polarimetric radar measureables of $Z_{dr}$, $Z_{hh}$, and $K_{dp}$ (Gorgucci et al. 2002; Bringi et al. 2002). A major development has been in methods to account for drop oscillations in an “effective” manner, and to extend the retrieval methodology to cases where $Z_{dr}$ and $K_{dp}$ tend to be “noisy” because of measurement fluctuations, especially in light rainfall events. Details of the algorithm are given in the appendix. The basic argument is that the nonlinear axis ratio ($r$) versus drop diameter ($D$) relation (e.g., Beard and Chuang 1987) can be replaced by an “effective” linear model of the form $r = 1 - \beta D$, which gives the same $K_{dp}$ as the nonlinear model for fixed $W$ and $D_{0}$. Further, Gorgucci et al. (2002) used scattering simulations based on normalized gamma DSD and $\beta$ parameter variations to find the “best fit” estimator for $\beta$ in terms of $Z_{dr}$, $Z_{hh}$, and $K_{dp}$. However, the form chosen was ad hoc and was validated by comparing the estimated $\beta$ with the “true” or assumed $\beta$. Note
that $\beta$ for the equilibrium drop axis ratios of Pruppacher and Beard (1970) is 0.062 mm$^{-1}$; however, in the presence of drop oscillations it can reduce to around 0.045 mm$^{-1}$.

There are two fundamental modes of oscillation termed as transverse and axisymmetric by Beard and Kubesh (1991). In natural rain the preferred mode or oscillation amplitudes are not known but it is hypothesized that drops from about 1.5 to 4 mm may preferentially oscillate in the transverse mode because of "off-center" collisions (Andsager et al. 1999). There is some indirect evidence that drops larger than 4 mm may oscillate in the axisymmetric mode (Bringi et al. 2003). Mixed mode oscillations are also possible (Beard 1984). If drop oscillation mode and amplitudes were steady and caused by "intrinsic" factors (Tokay and Beard 1996), then it is expected that $\beta$ would be nearly uniform. For example, in light rainfall or in stratiform rain events, the axis ratio fit recommended by Andsager et al. (1999) might be most suitable for $1 < D < 4$ mm. In heavier rainfall, it is possible that collisional forcing of drop oscillations can occur (Beard and Johnson 1984) causing $\beta$ to decrease from its equilibrium value. The collisional probability model predicts $\beta$ to decrease from 0.058 mm$^{-1}$ at 10 mm h$^{-1}$ to 0.0475 mm$^{-1}$ at 100 mm h$^{-1}$. The effect would be most noticeable in the measurement of $K_{dp}$ since, to first order, $K_{dp}$ is nearly linear with $\beta$ (Bringi and Chandrasekar 2001). There is indirect experimental evidence that the $R = aK_{dp}$ relation needs to be modified in tropical rain due to drop oscillations (May et al. 1999; Bringi et al. 2001a; Gorgucci et al. 2001). We note that Matrosov et al. (2002) have adapted the Gorgucci et al. (2001) methodology for X-band radar rainfall retrievals. The algorithm for estimating $\beta$ and subsequently estimating $N_u$, $D_u$, and $\mu$ is fully explained in Gorgucci et al. (2002); see also the appendix. Simulations of the algorithm and errors in the estimation of the DSD parameters using distrometer data can be found in Gorgucci et al. (2002). Since their form for the $\beta$ estimator is ad hoc, in this background section we use Rayleigh scattering to show that the form is, in essence, correct from first principles.

b. The form for $\beta$ using Rayleigh scattering

Following Bringi and Chandrasekar (2001), the elements of the backscattering matrix $\mathbf{S}$ for Rayleigh–Gans scattering can be written in terms of the dipole moments $\alpha$, $\alpha_z$, and the canting angle $\phi$ as

$$S_{hh} = \frac{k_0^2}{4\pi\epsilon_0}(\alpha_x \sin^2 \phi + \alpha \cos^2 \phi),$$

$$S_{hv} = \frac{k_0^2}{4\pi\epsilon_0}(\alpha_x \cos^2 \phi + \alpha \sin^2 \phi),$$

$$S_{vv} = S_{hv} = \frac{k_0^2}{4\pi\epsilon_0}\left[\frac{\alpha_x - \alpha}{2} \sin 2\phi\right],$$

where

$$\alpha = \frac{1}{6} \pi D^3 \varepsilon_0 \Lambda_h, \quad \alpha_z = \frac{1}{6} \pi D^3 \varepsilon_0 \Lambda_v,$$

and $\Lambda_h$ and $\Lambda_v$ are defined as

$$\Lambda_h = \frac{\varepsilon - 1}{1 + \frac{1}{2}(1 - \Lambda_v)(\varepsilon - 1)},$$

$$\Lambda_v = \frac{\varepsilon - 1}{1 + \Lambda_v(\varepsilon - 1)},$$

where

$$\lambda_z = \frac{1 + f^2}{\sqrt{\lambda - 1}}, \quad f = \frac{a}{b} \equiv 1.$$

Note that $a$ and $b$ in (12) are the semimajor and semiminor axes, respectively (the axis ratio is defined as $r = b/a$). Assuming the DSD and canting angle distributions are independent, radar "measurables" are calculated from the backscattering matrix as

$$Z_h = \frac{1}{9|K_{\alpha}|^2}[(\cos^4 \phi)(D^4|\Lambda_h|^2) + (\sin^4 \phi)(D^4|\Lambda_v|^2)]$$

$$+ (2 \sin^2 \phi \cos^2 \phi)(D^2 Re\{\Lambda_h \Lambda_v^*\}),$$

$$Z_v = \frac{1}{9|K_{\alpha}|^2}[(\sin^4 \phi)(D^4|\Lambda_h|^2) + (\cos^4 \phi)(D^4|\Lambda_v|^2)]$$

$$+ (2 \sin^2 \phi \cos^2 \phi)(D^2 Re\{\Lambda_h \Lambda_v^*\}),$$

$$K_{dp} = C_D (\cos^2 \phi - \sin^2 \phi) \cdot D \cdot Re\{\Lambda_h - \Lambda_v\},$$

$$|\Lambda_h|^2 \approx a_0 + a_1 BD + a_2(BD)^2,$$

$$|\Lambda_v|^2 = b_0 + b_1 BD + b_2(BD)^2,$$

$$Re\{\Lambda_h \Lambda_v^*\} \approx c_0 + c_1 BD + c_2(BD)^2,$$

$$Re\{\Lambda_h - \Lambda_v\} = d_0 + d_1 BD + d_2(BD)^2.$$
$d_n$ will be very small in comparison to the other terms in the expansion. Assuming the normalized gamma DSD model,
\[
\left\langle D^4 | \Lambda_n |^2 \right\rangle = \left\langle D^4 \right\rangle (a_0 + a_1 D_T + a_2 D_T^2),
\]
\[
\left\langle D^4 \text{Re} \{ \Lambda_n - \Lambda_n \} \right\rangle = \left\langle D^4 \right\rangle (d_0 + d_1 D_T + d_2 D_T^2),
\]
where
\[
D_T = \frac{\left\langle D^4 \right\rangle}{\left\langle D^4 \right\rangle} = \frac{\mu + 7}{\mu + 3.67} \beta D_0,
\]
\[
D_k = \frac{\left\langle D^4 \right\rangle}{\left\langle D^4 \right\rangle} = \frac{\mu + 4}{\mu + 3.67} \beta D_0.
\]

Similar approximations hold for $\left\langle D^4 | \Lambda_n |^2 \right\rangle$ and $\left\langle D^4 \text{Re} \{ \Lambda_n \Lambda_n^* \} \right\rangle$. With these approximate formulas, the radar measureables $Z_n$, $Z_v$, and $K_m$ can be well approximated by polynomials of $D_T$, $D_k$, and the coefficients of the polynomials are only determined by the canting angle distribution (assumed to have mean of 0 and standard deviation $\sigma_c$).

When the standard deviation of the canting angle is small, a new $\beta$ estimator can be derived from the above approximate forms for $Z_n$, $\xi_d$, and $K_{d\phi}$ as
\[
\hat{\beta} = 3.0 \frac{\left( \frac{K_{d\phi}}{Z_n} \right)^{1/3} (\xi_d - 1)^{1/3}}{e^{-4.18 e_{d/3}}},
\]
where $\xi_d$ is the ratio $Z_d/Z_n$ (ratio form of conventional $Z_d$ in dB). This formulation follows from the fact that $K_{d\phi}/Z_n(D^4)\right\rangle$ can be approximately treated as a linear function of $D_T$, whereas the second-order term of $D_k$ in the polynomial formulation of $\xi_d$ contributes most to $\xi_d - 1$. The small uncertainty caused by $\mu$ variability is not essential for this estimator. Since the form for $\hat{\beta}$ is now known, it is possible to drop the Rayleigh approximation and estimate the multiplicative coefficient and the exponents of $(K_{d\phi}/Z_n)\right\rangle$ and $(\xi_d - 1)$ in (24) using simulations similar to Gorgucci et al. (2001) but using a different set of gamma DSD fitted to measured 2D-video disdrometer from different climatic regimes (see Bringi et al. 2003 for details on the 2D-video dataset and fitting method). The input axis ratio relation is assumed to be $1.03 - \beta D$ with $\beta$ varying from $0.032$ to $0.092$ mm$^{-1}$ in steps of $0.01$. For each DSD, the $Z_n$, $Z_d$, and $K_{d\phi}$ are computed and nonlinear fitting is used to estimate the coefficient and exponents in (24) leading to
\[
\hat{\beta} = 3.25 \left( \frac{K_{d\phi}}{Z_n} \right)^{0.34} (\xi_d - 1)^{0.36}.
\]

Note the above fitted coefficient and exponents are very close to those obtained theoretically in (24).

The Gorgucci et al. (2001) estimator based on simulations using random variations in the gamma DSD parameters ($N_w$, $D_o$, and $\mu$) is

\[
\tilde{\beta} = 2.08 K_{d\phi}^{0.38} Z_n^{-0.365} \xi_d^{0.965}.
\]

Figure 1 compares the mean value of $\tilde{\beta}$ from (25) and $\hat{\beta}$ from (26) versus the true $\beta$. Note that, in the expected range of mean $\beta$ (<0.07 mm$^{-1}$), the two estimators are in very good agreement. In essence, the estimator derived by Gorgucci et al. (2001) is close to that obtained from first principles using Rayleigh scattering.

3. Changes in $\beta$ in an evolving rain cell

It is difficult to estimate changes in $\beta$ and attribute it to drop oscillations using radar measurements alone, when the DSD itself is changing. One possibility is to document the changes in $\beta$ as a storm cell evolves in time from the growth phase to the mature phase, especially if the rain rate in the growth phase is low and the DSD has a high value for $D_o$ (with low $N_w$) while the rain rate in the mature phase is high and the DSD tends to the equilibrium-type where the $D_o$ tends to a constant in the rain cell and the rain rate is then proportional to $N_w$. The collisional probability model of Beard and Johnson (1984) predicts that, in intense rain rates of 100 mm h$^{-1}$ and if the DSD follows the Sekhon and Srivastava (1971) model with $(D_o$ near 1.84 mm and $N_w$ near 38,000 mm$^{-1}$ m$^{-3}$), the $\beta$ should be close to 0.0475 mm$^{-1}$.

Atlas and Williams (2003a,b) have conducted a detailed analysis of a storm cell on 17 February 1999 during the TRMM/Brazil field campaign using Doppler velocity spectral data from the vertically pointing profiler over which the storm cell evolved from the early growth phase to mature phase with microburst and intense rain rates. It is not the intent here to describe their results in any detail other than to support our interpretation of the $Z_n$, $Z_d$, and $K_{d\phi}$ data, and, in particular, to support our hypothesis that the reduction in $\beta$ in the rain cell, on average, as the cell evolves to maturity is related to increased drop oscillations, perhaps forced by...
collisions in intense rainfall as predicted by Beard and Johnson (1984). Figure 2 shows a six-panel set of range height indicator data from the NCAR–SPOL radar over the location of the profiler at 1711 (top panels) 1718 (middle panels) and 1728 UTC (lowest panels). The profiler is located within the gauge cluster at 40 km range in Fig. 8b. The storm cell analyzed by Atlas and Williams (2003a,b) is located along the range interval 42–47 km. They reported that between 1708 and 1711 UTC the cell was in its early growth phase, that it was in its vigorous growth phase at 1718 UTC, and by 1728 UTC...
had produced a microburst with peak rain rates of 124 mm h\(^{-1}\). While there are notable features in the \(Z_a\) and \(K_a\) data within this rain cell at these times that can be correlated with the Atlas and Williams (2003a,b) analysis, our goal is more specific as alluded to earlier. Using the methodology developed in Bringi et al. (2002) (described in more detail in the appendix) the average values of \(\beta\), \(D_0\), \(N_w\), and \(R\) over a 2-km-range interval (approximately centered on the cell for each of the 3 RHIs) are calculated, and the vertical profile of this average is plotted in Figs. 3–6, respectively. In Fig. 3 note the systematic shift in average \(\beta\) from 0.055 to 0.047 mm\(^2\) in the height interval 1–2 km as one progresses in time from 1711 (early growth phase) to 1728 UTC (mature phase with microburst and intense rain rates). The corresponding change in \(D_0\) is from 2.8 mm to 1.8 mm, \(\log_{10}(N_w)\) from 3.2 to 4.4 and \(R\) from 40 to 110 mm h\(^{-1}\). Note that \(N_w\) is in units of mm\(^{-1}\) m\(^{-3}\) while \(R\) is calculated from the pol-based \(Z-R\) relation to be described later. At 1728 UTC, the radar-inferred \(D_0\), \(\log_{10}(N_w)\), and \(R\) values in the height interval 1–2 km are fairly close to the values used by Beard and Johnson (1984) to predict \(\beta\) of 0.0475 mm\(^{-1}\) at rain rate of 100 mm h\(^{-1}\) [recall their \(D_0\) was near 1.8 mm and \(\log_{10}(N_w)\) was near 4.5]. The radar-inferred \(\beta\) is remarkably close to their model result supporting our hypothesis that collisional forcing of drop oscillations may have been an important factor at 1728 UTC.

The NCAR–SPOL radar, apart from collecting 3 RHI datasets over the profiler location at 1711, 1718, and 1728 UTC, was also scanning in the PPI mode during the intervening times. It is illustrative to plot the average \(\beta\) over a small 2 km \(\times\) 2 km area centered near the profiler versus the average radar-inferred \(R\), especially for the higher rain rates (say, \(R > 40\) mm h\(^{-1}\)). The 0.5° elevation angle sweep is used to construct Fig. 7, which shows \(\beta\) versus \(R\) without regard to the stage of evolution of the cell but from scans taken between 1720 and 1733 UTC. There is systematic decrease in \(\beta\) with increasing \(R\) in
general agreement with the model of Beard and Johnson (1984) and, in particular, the excellent agreement with their prediction of $\beta = 0.0475 \text{ mm}^{-1}$ at $R = 100 \text{ mm}^{-1}$ noted in Fig. 7. It was not possible to compare their predictions at lower $R$ of 30 and 10 mm h$^{-1}$ since their DSD parameter values were not comparable to the radar-inferred ones at the same $R$, that is, the radar-inferred DSD was substantially different from the Sekhon–Srivastava model at the same $R$.

Note that the radar-based inference of $\beta$ will be accurate only under the condition of moderate-to-intense rain rates when the corresponding $K_{dp}$ measurement will also be accurate (at S-band this implies a reflectivity $>35 - 40 \text{ dBZ}$), see appendix [Eqs. (A.1)–(A.14)]. But this is precisely the condition when collisional forcing of drop oscillations may become important enough to influence the retrieval of rain rate or DSD parameters. At lower rain rates we use the nonlinear axis ratio versus $D$ fit recommended by Andsager et al. (1999) for $1 < D < 4 \text{ mm}$ (which accounts for transverse oscillations) composited with the Beard and Chuang (1987) fit for $D < 1$ and $D > 4 \text{ mm}$. Bringi et al. (2002, 2003) used this composite but added in a Gaussian canting angle model with mean 0 and standard deviation of $10^\circ$ (partly to account for turbulence and partly to account for oscillations for $D > 4 \text{ mm}$).

The field observations of Tokay and Beard (1996) support the idea that for low-to-moderate $R$, the drop oscillations are due to intrinsic reasons, for example, resonance with the natural oscillation frequency of the drop, which is maintained by vortex shedding, rather than due to extrinsic reasons such as, for example, collisional forcing. Thus, we use the “fixed” nonlinear shape model when the $K_{dp}$ data are “noisy” to deduce the $D_0$, $N_w$, and $R$ using just $Z_h$ and $Z_{dp}$ data [see appendix Eqs. (A.15)–(A.17)]. When $Z_{dp}$ is also noisy we have altered the methodology suggested by Bringi et al. (2002) as described in the appendix [Eqs. (A.18)–(A.21)]. The estimate of DSD shape parameter $\mu$ is also possible using the algorithm proposed by Gorgucci et al. (2002) using $Z_h$, $K_{dp}$, and $Z_{dp}$ data. Whereas not very accurate it is useful in distinguishing certain ranges of $\mu$, for example, near 0 to greater than 5. Similar to the estimator for $\beta$, the $\mu$ estimator is accurate only for moderate-to-high rain rates. When $K_{dp}$ or $Z_{dp}$ is noisy, that is, at the lower rain rates a fixed $\mu = 3$ is used.

4. Radar data processing

Polarimetric radar data used in this study are from the NCAR–SPOL radar during the TRMM–LBA and TEFLUN-B programs (locations are Brazil and Florida, respectively). Evaluation of the pol-based $Z$–$R$ algorithm is based on individual gauge comparisons; the gauge locations for TRMM–LBA and TEFLUN-B are shown in Figs. 8a,b. The radar data stream consists of $Z_h$, $Z_{dp}$, and $\phi_{dp}$ (differential propagation phase) available every 150 m in range. The $\phi_{dp}$ data are filtered in
range using the iterative method of Hubbert and Bringi (1995). The filtered $\phi_{\text{m}}$ range profiles are used to estimate $K_{\text{dp}}$. The $Z_a$ data are corrected for rain attenuation using Testud et al. (2000) adapted for S-band, while $Z_r$ is corrected for differential attenuation using a self-consistent, constraint-based algorithm described by Bringi et al. (2001b). These attenuation corrections are significant only when the total $\phi_{\text{m}}$ exceeds around 50$^\text{o}$. In addition, gaseous attenuation is estimated from Doviak and Zrnić (1993). The $Z_a$ calibration is based on examining the values in the ice region of storms where $Z_a$ values should be 0 dB because of low density and isotropic orientation (this method was used to fine tune the $Z_a$ calibration provided by NCAR, which was based on vertical pointing data in light rain, which was only available on certain days during the experiment). Such fine-tuning showed that the nominal NCAR–SPOL calibrated $Z_a$ values had to be adjusted lower by 0.15 dB for both TRMM–LBA and TEFLUN–B experiments. The system gain (or, radar constant) was fine tuned using reconstruction of $\phi_{\text{m}}$ range profiles [derived by integrating $K_{\text{dp}} = f(Z_a, Z_r)$ with range] and comparing with the measured $\phi_{\text{m}}$ range profiles. An upper/lower bound method was used resulting in a reflectivity adjustment 1.25 dB for TRMM–LBA (nominal NCAR–SPOL reflectivity values were increased by 1.25 dB) and 0 adjustment for TEFLUN–B. Our TRMM–LBA adjustment for reflectivity is in excellent agreement with other independent measures (Anagnostou et al. 2001). Such accurate calibration and inclusion of rain and gas attenuation corrections are necessary for proper evaluation of polarimetric rainfall algorithms.

For comparison against gauges, a small polar “box” is constructed around each gauge location (nominally 1 km in range and 1.5$^\text{o}$ in azimuth angle) and it is assumed that each “point” gauge rain-rate estimate represents this polar area. Data from radar beams that intersect this polar area are used to estimate an average $\mu$ and $N_w$ for the area (equations are given in the appendix); then (4) is applied to find the average rain rate over the area. We refer to the pol-based $Z$–$R$ relation as $Z = \bar{R}^{1.5}$, where $\bar{R}$ is the multiplicative coefficient obtained as $\bar{R} = a^\prime (\langle \mu \rangle / V (N_w))$ (angle brackets represent areal averages). The hypothesis here is that over short time intervals the DSD shape parameter $\mu$ and the generalized intercept parameter $N_w$ are stationary over the small polar area. However, because of the nonlinearities involved, such a procedure can give a small bias in the retrieval of the average rain rate but we have estimated this bias (based on simulations) to be less than about 10%. We have also determined (from the data itself) that the assumption of a stationary $N_w$ over small areas is quite reasonable for the events analyzed herein [in general agreement with Testud et al. (2001)].

5. Results

Radar/gauge datasets were analyzed for three storm days during TRMM–LBA (15, 18, and 27 February 1999) and three storm days during TEFLUN–B (21 August and 7 and 17 September 1998), see Table 1. The gauge data consisted of time series of 1-min rain rates. The radar scanning strategy was variable, but generally the sampling interval was around 1–2 min except for events of 18 and 27 February 1999 when it was between 5 and 10 min. In all cases, only data from low elevation angles were used (0.5 to about 1$^\text{o}$). Nonmeteorological echoes were removed by using the radial standard deviation of $\phi_{\text{m}}$ over 10 consecutive gates, the mean $\rho_m$, and the signal-to-noise ratio (SNR) as the data “quality” indicators. The use of polarimetric data to identify nonmeteorological echoes and to generally improve data quality as compared with a conventional Doppler radar cannot be overemphasized.

Figure 9 shows the time series of 5-min-averaged rain rates over a single gauge location for the event of 17 September 1998. The standard error bars on the pol-based $Z$–$R$ estimate reflect both measurement fluctuation errors as well as algorithm errors [these are based on disdrometer simulations and shown in Fig. 10 based on section 8.3 of Bringi and Chandrasekar (2001)]. Figure 9 also shows the 5-min-averaged gauge rain rates as well as rain rate from using the conventional $Z$–$R$ method after

### Table 1. Dates, times, and gauge accumulation (for the gauge with maximum $R$ in the network) during TRMM-LBA and TEFLUN-B.

<table>
<thead>
<tr>
<th>Gauge</th>
<th>Date</th>
<th>Duration (UTC)</th>
<th>Accumulation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFB 109</td>
<td>21 Aug 1998</td>
<td>1431–1556</td>
<td>21.4</td>
</tr>
<tr>
<td>TFB 116</td>
<td>7 Sep 1998</td>
<td>2103–2208</td>
<td>8.8</td>
</tr>
<tr>
<td>TFB 101</td>
<td>17 Sep 1998</td>
<td>1915–2245</td>
<td>63.4</td>
</tr>
<tr>
<td>LBA 19</td>
<td>15 Feb 1999</td>
<td>0348–0708</td>
<td>56.3</td>
</tr>
<tr>
<td>LBA 38</td>
<td>18 Feb 1999</td>
<td>0533–0838</td>
<td>25.4</td>
</tr>
<tr>
<td>LBA 1</td>
<td>27 Feb 1999</td>
<td>1009–1209</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Fig. 9. Time series of 5-min-averaged rain rates for one event over a particular gauge location.
separating stratiform and convective rain types. The separation method is simple and based on the mean and standard deviation of five consecutive 1-min gauge rain-rate samples with stratiform rain classified when the mean is \( \leq 5 \text{ mm h}^{-1} \) and the 5-point \( \sigma \) is \( < 1.5 \text{ mm h}^{-1} \). The stratiform \( Z-R \) chosen herein is \( Z = 285R^{1.5} \) whereas the convective \( Z-R \) is \( Z = 113R^{1.5} \). The constant \( N_w \) values chosen for the convective and stratiform rain \( Z-R \) relations are based on an average for tropical regions (from Figs. 10 and 11 in Bringi et al. 2003). Figure 11 shows the time series plot of the pol-based coefficient, \( a \), which changes continuously (this plot may be compared with Fig. 9). Examination of Figs. 9 and 11 shows that the fixed \( Z-R \) underestimates the peak gauge rain rates whereas the pol-based \( Z-R \) is able to match better the peak gauge values (the pol-based coefficient \( a \) is lower than the fixed convective coefficient within the peak rain cell giving higher rain rates for the pol-based estimator). This suggests that to avoid rainfall accumulation bias with a nonpolarimetric radar, it is necessary to have the correct \( Z-R \) relation even after convective/stratiform separation. Data similar to Fig. 9 were generated for each event and for small polar areas over each gauge. The mean rain rate for each event (over a gauge location) is defined as the average of the 5-min estimates for the whole event over that particular gauge location. Figure 12a shows the plot of the mean rain rate (with estimated error bars) for each gauge location coded for each of the six storm events. This plot shows the accuracy of the pol-based \( Z-R \) estimator when compared against gauges. The normalized bias is around 6\% and the normalized standard error is around 20\% (we use the absolute deviation between radar and gauge data as the measure rather than the deviation squared). Defining \( \Delta = R - R_g \) where \( R \) is the radar estimate and \( R_g \) is the gauge measurement, the normalized bias is
defined as $\langle \Delta \rangle / \langle R \rangle$ and normalized standard error as $\langle |\Delta| \rangle / \langle R \rangle$; angle brackets denote average values. The small normalized bias is less than that due to uncertainty of gauge measurements [estimated at 10% by Matrosov et al. (2002)]. Figure 12b shows a similar comparison except for the conventional $Z-R$ estimator (with estimated error bars) after convective/stratiform separation. The normalized bias is now about $-18\%$ (underestimate relative to gauge) and the normalized standard error is $24\%$. This normalized bias value of $-18\%$ is consistent with Matrosov et al. (2002), who also obtained a bias of $-18\%$ when using a fixed (mean) $Z-R$ relation at X-band (after correcting for attenuation). They were able to reduce the bias to $-8\%$ when using a polarimetric method based on Gorgucci et al. (2000) but adapted for X-band. Thus we conclude that the pol-based $Z-R$ method offers higher accuracy and significantly lower bias as compared with conventional $Z-R$ for estimating storm-total rainfall. The continuous pol-tuning of the coefficient $\hat{a}$ in $Z = aR^{1.5}$ automatically accounts for evolution of the DSD, and indeed demonstrates that tracking the evolution is necessary for accurate rain-rate estimation.

6. Discussion and conclusions

This study continues a series of several recent articles involving comparison between different pol-based rain-rate algorithms against a network of gauges (e.g., Bringi et al. 2001a; May et al. 1999; Ryzhkov et al. 2000; Brandes et al. 2003; Le Bouar et al. 2001). It should be mentioned that the TRMM field campaigns in Brazil and Florida have provided an important database for such comparisons although encompassing only one season. Few organizations are equipped with both a high-quality dual-polarized radar and access to a dense network of gauges over many seasons (notable exceptions are the National Severe Storms Laboratory and the Bureau of Meteorology Research Center in Darwin, Australia). Such large databases are needed to develop an optimal pol-based rain-rate algorithm not only for research but also for future operational needs.

The approach here differs from other recent studies in two important respects, namely, the method of formulating the rain rate in moderate-to-intense events to account for the effects of drop oscillations beyond using the axis ratio versus $D$ fits recommended by Andsager et al. (1999) or other empirical fits such as used by May et al. (1999) or Brandes et al. (2003). Further, the pol-based data is recast in the familiar form of a $Z-R$ relation whose multiplicative coefficient is adjusted continuously in space and time. Implicit in this formulation is that the $N_\omega$ and $\mu$ parameters are nearly uniform over small areas around $2 \text{ km} \times 2 \text{ km}$ in size. The method also accounts for the noisy $K_{sp}$ and $Ze$ measurements in light rain-rate conditions in a systematic manner. Implicit is the requirement for high accuracy in the $Z_i$, $Ze$, and $K_{sp}$ data, which is likely to be difficult to achieve in an operational setting or at more attenuating, higher frequencies. We note, however, that Matrosov et al. (2002) have been able to essentially apply the Gorgucci et al. (2002) methodology at X-band (after correcting the $Z_i$ and $Ze$ data for attenuation) and were able to obtain similar normalized bias ($-8\%$) and normalized standard error ($22\%$) as obtained herein. We also note the ZPHI method applied at S-band by Le Bouar et al. (2001), which belongs to a different class of profiling algorithms than used herein. However, we have been able to obtain reasonably good comparisons between the log$_10$($N_\omega$) predicted by our algorithm and that predicted by the ZPHI technique at S-band when the differential phase shift across the rain cell is large ($>50^\circ$).

This work also shows, using simple Rayleigh scattering and analytical approximations, that the estimator for $\beta$ proposed by Gorgucci et al. (2000) is of the correct form. In one event, well documented by Atlas and Williams (2003a,b), it is shown that the radar-inferred reduction of $\beta$ at high rain rates (100–120 mm h$^{-1}$) is consistent with the collisional probability model of Beard and Johnson (1984). This could be demonstrated only after averaging $\beta$ over the width of the rain cell (around $2 \text{ km}$) since the estimate of $\beta$ is known to be quite noisy at the basic resolution volume of the radar system (e.g., every 150 m in range). The $Z_i$ and $Ze$ data are available at this basic range resolution as compared with the $K_{sp}$ data (since the differential phase range profile needs to be smoothed before the range derivative can be computed). In our filtering methodology the $K_{sp}$ resolution is around 2–3 km. Hence, we further smooth the $Z_i$ and $Ze$ data in range to be compatible with the $K_{sp}$ data, which increases the stability of the $\beta$ retrieval. Needless to say, the estimate of $\beta$ is unbiased only in homogeneous sections of rainfall. Further averaging over a basic rainfall measurement area of $2 \text{ km} \times 2 \text{ km}$ improves the accuracy of $\beta$ especially for moderate-to-intense rain rates ($R > 40 \text{ mm h}^{-1}$) and this may be sufficient to account for possible collisionally forced drop oscillations. The empirical adjustments to account for biases when rain is dominated by large or small drops in the algorithm proposed by Ryzhkov et al. (2003) could be interpreted in terms of small systematic shifts in the effective $\beta$ parameter though these authors do not use such an interpretation. It is possible that for rain with large drops the dominant mode of oscillation could be different from that in rain with small drops, for example, the transition from axisymmetric oscillations ($\beta$ slightly larger than that assumed in the retrieval shape model) to transverse oscillations ($\beta$ slightly lower than assumed). The example shown in Fig. 3 is suggestive of this behavior as the storm cell evolves from growth phase (large drops reaching the surface after falling in an updraft) to mature phase (more equilibrium-type of drop size distribution with large numbers of small and moderate-sized drops in strong downdraft).

The comparison of rain rates using the pol-based $Z-R$ method with a network of rain gauges from two different climatic regions (though it could be argued that
the rain type is predominately tropical) covering three storm events from each region is very encouraging, the normalized bias being near 6\% and normalized standard error near 20\%. This is a substantial improvement over using two different \(Z-R\) relations for stratiform and convective rainfall types, the corresponding values being \(-18\%\) and \(24\%\). Even though our stratiform/convective classification is primitive it is unlikely that a more elaborate classification method is justified for our application.

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APPENDIX

Radar Retrieval of DSD Parameters

The estimators for \(\beta\), \(D_0\), \(N_w\), and \(\mu\) are taken from Gorgucci et al. (2002) and are valid for a radar operating frequency near 3 GHz. The following conditions for the radar measurements must be satisfied (\(Z_{\text{h}} \geq 35\text{ dBZ}\), \(K_{\text{dp}} \geq 0.3^9\text{ km}^{-1}\), and \(Z_{\text{dr}} \geq 0.2\text{ dB}\)). First, the \(\beta\) is estimated as

\[
\beta = 2.08Z_{\text{h}}^{-0.365}K_{\text{dp}}^{0.38}\frac{2^0.65}{\text{mm}^{-1}}, \quad (A.1)
\]

where \(Z_{\text{h}}\) is in \(\text{mm}^6\text{ m}^{-3}\), \(K_{\text{dp}}\) in \(\text{km}^{-1}\) and \(\xi_{\text{dr}}\) is the differential reflectivity ratio (\(Z_{\text{dr}} = 10\log_{10}\xi_{\text{dr}}\)). The \(D_0\) estimator is

\[
D_0 = aZ_{\text{h}}\xi_{\text{dr}}\text{ (mm)}, \quad (A.2)
\]

where

\[
a = 0.56, \quad (A.3)
\]

\[
b = 0.064, \quad (A.4)
\]

\[
c = 0.024\beta^{-1.42}, \quad (A.5)
\]

The estimator for \(\log_{10}N_w\) is

\[
\log_{10}N_w = aZ_{\text{h}}\xi_{\text{dr}}, \quad (A.6)
\]

where now

\[
a = 3.29, \quad (A.7)
\]

\[
b = 0.058, \quad (A.8)
\]

\[
c = -0.023\beta^{-1.389}, \quad (A.9)
\]

Note that the unit for \(N_w\) is \(\text{mm}^{-1}\text{ m}^{-3}\). The estimator for \(\mu\) (which is not very accurate) is

\[
\mu = a\frac{D_0}{\xi_{\text{dr}}} - c\xi_{\text{dr}}, \quad (A.10)
\]

where

\[
a = 203\beta^{0.89}, \quad (A.11)
\]

\[
b = 2.23\beta^{0.0388}, \quad (A.12)
\]

\[
c = 3.16\beta^{-0.0463}, \quad (A.13)
\]

\[
d = 0.374\beta^{-0.355}. \quad (A.14)
\]

In moderate rain rates, when the \(K_{\text{dp}}\) data may be noisy (i.e., \(K_{\text{dp}} < 0.3^9\text{ km}^{-1}\)), but the \(Z_{\text{dr}}\) satisfies \(Z_{\text{dr}} \geq 0.5\text{ dB}\), then the following estimator from Bringi et al. (2002) is used:

\[
\mu = 3, \quad (A.15)
\]

\[
D_0 = 1.81Z_{\text{dr}}^{0.486} \quad (\text{mm}), \quad (A.16)
\]

\[
N_w = \frac{21Z_{\text{dr}}}{D_0^{3.3529}} \quad (\text{mm}^{-1}\text{ m}^{-3}). \quad (A.17)
\]

Note that \(Z_{\text{dr}}\) is in \(\text{dB}\) in (A.16). In light rain rates and when \(Z_{\text{dr}}\) is in the range \(0.2 \leq Z_{\text{dr}} < 0.5\text{ dB}\), the algorithm for estimating \(D_0\) is slightly altered from (A.16) to provide a smooth transition to the case (described below) when \(Z_{\text{dr}}\) can be negative due to statistical fluctuations in light rainfall,

\[
D_0 = \frac{Z_{\text{dr}} - 0.2}{0.3}(1.81)Z_{\text{dr}}^{0.486} + \frac{0.5 - Z_{\text{dr}}}{0.3}(0.6096)Z_{\text{h}}^{0.3516}\xi_{\text{dr}}^{1.111} \quad (\text{mm}), \quad (A.18)
\]

\[
N_w = \frac{21Z_{\text{dr}}}{D_0^{3.3529}} \quad (\text{mm}^{-1}\text{ m}^{-3}). \quad (A.19)
\]

Furthermore, in light rain-rate conditions when statistical fluctuations in \(Z_{\text{dr}}\) cause it to be negative (\(-0.5 \leq Z_{\text{dr}} < 0.2\text{ dB}\)) the \(D_0\) estimator is approximated as follows:

\[
D_0 = 0.6096Z_{\text{h}}^{0.3516}\xi_{\text{dr}}^{1.111} \quad (\text{mm}), \quad (A.20)
\]

\[
N_w = \frac{21Z_{\text{dr}}}{D_0^{3.3529}} \quad (\text{mm}^{-1}\text{ m}^{-3}). \quad (A.21)
\]

Otherwise, the DSD parameters are not retrievable. Note that the method used herein for estimating \(D_0\) under light rain condition [(A.18)–(A.21)] is different from that described in Bringi et al. (2002). The nonlinear fit relations are based on adding Gaussian noise to the \(Z_{\text{dr}}\) simulation in light-rain-rate conditions.

Once \(\mu\) and \(N_w\) are estimated for the different rain-rate conditions, the coefficient of the \(Z = a\beta R^{1.5}\) relation is calculated as \(\tilde{a} = a'(\mu)\sqrt{N_w}\) (see 4) where \(a'(\mu)\) is obtained as [see, also, Bringi and Chandrasekar 2001, their Eq. (7.71)],

\[
a'(\mu) = \frac{F_z(\mu)}{[F_z(\mu)]^{1/3}}, \quad (A.22)
\]

where

\[
F_z(\mu) = \frac{f(\mu)\Gamma(7 + \mu)}{(3.67 + \mu)^{7+\mu}} \quad (A.23)
\]

\[
F_z(\mu) = (0.6 \times 10^{-3}\pi)(3.78)f(\mu) \times \frac{\Gamma(4.67 + \mu)}{(3.67 + \mu)^{4.67+\mu}} \quad (A.24)
\]
Note that the assumed terminal fall speed in $v(D) = 3.78D^{0.67}$ m s$^{-1}$ ($D$ in mm).

### REFERENCES


