A Bayesian Approach for Integrated Raindrop Size Distribution (DSD) Retrieval on an X-Band Dual-Polarization Radar Network

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Abstract

A raindrop size distribution (DSD) retrieval method for a weather radar network consisting of several X-band dual-polarization radars is proposed. An iterative maximum likelihood (ML) estimator for DSD retrieval in a single radar was developed in the authors’ previous work, and the proposed algorithm in this paper extends the single-radar retrieval to radar-networked retrieval, where ML solutions in each single-radar node are integrated based on a Bayesian scheme in order to reduce estimation errors and to enhance accuracy. Statistical evaluations of the proposed algorithm were carried out using numerical simulations. The results with eight radar nodes showed that the bias and standard errors are 0.05 and 0.09 in \( \log(N_w) \); and \( N_w \) (mm \(^{-1}\) m\(^3\)) and 0.04 and 0.09 in \( D_0 \) (mm) in an environment with fluctuations in dual-polarization radar measurements (normal distributions with standard deviations of 0.8 dB, 0.2 dB, and 1.5° in \( Z_{HH} \), \( Z_{DR} \), and \( \Phi_{DP} \), respectively). Further error analyses indicated that the estimation accuracy depended on the number of radar nodes, the ranges of varying \( m \), the raindrop axis ratio model, and the system bias errors in dual-polarization radar measurements.

1. Introduction

Raindrop size distribution (DSD), which is defined as the number density of raindrops in a unit interval of diameter in a unit volume (mm \(^{-1}\) m\(^3\)), is essential for radar meteorology. Precipitation radar is a requisite for current scientific weather observation campaigns and practical weather disaster prevention because radar covers a wider area in a shorter time than weather observation devices, such as rain gauges, disdrometers, and radiosondes. Because precipitation radar is a remote sensing device that measures reflections of electromagnetic waves from precipitation media, it is necessary to convert a radar measurement, such as equivalent reflectivity factor, to physical parameters of precipitation, such as rainfall rate. Traditionally, the physical parameters of precipitation have been estimated from radar measurements using empirical power laws directly connecting each other. These power laws, however, depend heavily on DSD and produce large estimation errors in many cases (Bringi and Chandrasekar 2001).

Dual-polarization radar observation allows us to obtain deeper insight into precipitation microphysics than single polarization. Three dual-polarization parameters—equivalent reflectivity factor at horizontal polarization, differential reflectivity, and differential propagation phase (Bringi and Chandrasekar 2001)—can determine parameters of the normalized gamma DSD (Ulbrich 1983; Testud et al. 2001), which defines the DSD by three parameters to sufficiently express natural variation in precipitation. Bringi et al. (2002) and Gorgucci et al. (2002) proposed algorithms that successfully retrieve DSD with an S-band dual-polarization radar. In contrast, DSD retrieval is adversely
affected by precipitation attenuation, which appears more heavily in modern higher-frequency radars, such as at C, X, Ku, and Ka bands. As proposed in Anagnostou et al. (2004), Gorgucci et al. (2006, 2008), and Kalogiros et al. (2013), it is necessary in high-frequency radars to retrieve DSD after precipitation attenuation is corrected.

However, since the amount of precipitation attenuation depends on DSD, there is normally a conflict between the steps of precipitation attenuation correction and DSD retrieval, resulting in errors in retrieved DSD. Yoshikawa et al. (2014) proposed a DSD retrieval algorithm to avoid this two-step problem. In this algorithm, a range profile of the normalized gamma DSD is iteratively optimized to match the dual-polarization measurements by simultaneously formulating forward scattering and backscattering. Even if the two-step problem is avoided, fluctuations in dual-polarization measurements yield ambiguity of the retrieved DSD. As this algorithm applies a minimization of mean square error, larger fluctuations in the dual-polarization measurements generally lead to larger ambiguity in retrieved DSD. Yoshikawa et al. (2014) pointed out that the ambiguity in retrieved DSD becomes higher in weaker precipitation, especially with weak reflectivity less than 30 dBZ, throughout a radar beam.

A concept of a radar network in which an area of interest is redundantly covered by several short-range radars is a new approach for observing precipitation with high measurement accuracy and high spatial and temporal resolution (Junyent and Chandrasekar 2009). The Center for Collaborative Adaptive Sensing of the Atmosphere (CASA) project is well known as a major radar network system (Junyent et al. 2010). In the HydroRad project in Moldova, a radar network consisting of small-sized X-band radars is being tested (Picciotti et al. 2013). Also, a Ku-band broadband radar (BBR) network has been proposed that observes a narrower region with finer resolution than the CASA radar network (Mega et al. 2007; Yoshikawa et al. 2010, 2012). In these projects, data processing methods to enhance estimation accuracy based on the radar network environment are currently attracting significant attention and some methods have already been proposed (Bharadwaj and Chandrasekar 2011; Chandrasekar and Lim 2008; Lim et al. 2011). A main advantage of the radar network environment is multi-directional observation, which compensates for problems caused by single-directional observations such as the problem of weak reflectivity throughout a radar beam, indicated by Yoshikawa et al. (2014).

In this paper, an enhanced DSD retrieval algorithm based on an X-band dual-polarization radar network is proposed. The algorithm is derived by extending the work proposed in Yoshikawa et al. (2014). To avoid confusion, the algorithms proposed in our previous work and in this paper are, hereafter, called single-radar retrieval (SRR) and networked retrieval (NTR). SRR is understood as a maximum likelihood (ML) estimator. NTR integrates the SRR solutions in each radar node based on a Bayesian scheme under a Gaussian model of probability density function (PDF) (Bishop 2006) using the following procedure: 1) transformation of ML solutions of the SRR to posterior PDF, 2) transition of posterior PDF of the SRR solutions from a polar gridded one of each radar node to a common Cartesian gridded one, and 3) optimum integration of the posterior PDFs of the SRR solutions onto the common Cartesian grid. Here, the radar node is defined as radar composed in a radar network. Thus, estimation accuracy of the SRR is improved.

This paper is organized as follows. In section 2, the methodology of the NTR is elaborated with descriptions of governing formulations between dual-polarization measurements and DSD parameters and the SRR methodology. Section 3 shows evaluations of the NTR by numerical simulations. The improvement from the SRR is indicated with an example of simulation, and errors due to possible properties of precipitation are statistically evaluated. Section 4 concludes this paper.

2. Methodology

a. Background

The three dual-polarization measurements of measured reflectivity—horizontal polarization, \(Z_{Hm}\); measured differential reflectivity, \(Z_{DRm}\); and measured differential propagation phase, \(\Phi_{DPm}\)—are the inputs of the NTR. Strictly speaking, these three measurements are input to the SRR, and then the results of the SRR are input to the NTR. In this subsection, the relations between the three dual-polarization measurements and the DSD are described as follows:

\[
Z_{Hm}(r) = Z_H(r) - 2 \times 10^{-3} \int_r A_H(r) dr + n_{Z_H}(r),
\]

\[
Z_{DRm}(r) = Z_{DR}(r) - 2 \times 10^{-3} \int_r A_{DP}(r) dr + n_{Z_{DR}}(r),
\]

\[
\Phi_{DPm}(r) = 2 \times 10^{-3} \int_r K_{DP}(r) dr + n_{\Phi_{DP}}(r),
\]

where \(Z_H, Z_{DR}, \) and \(K_{DP}\) are equivalent reflectivity factor at horizontal polarization, differential reflectivity, and differential propagation phase, respectively. Terms \(A_H\) and \(A_{DP}\) are precipitation attenuations for \(Z_H\) and \(Z_{DR}\), respectively. And \(n_{Z_{Hm}}, n_{Z_{DRm}}, \) and \(n_{\Phi_{DPm}}\) are random variables corresponding to the statistical properties of \(Z_{Hm}, Z_{DRm},\) and \(\Phi_{DPm}\), respectively. Thus, \(Z_{Hm}, Z_{DRm},\) and \(\Phi_{DPm}\) are also random variables in the NTR. Terms \(Z_H, Z_{DR}, K_{DP}, A_H,\) and \(A_{DP}\) are expressed by DSD, \(N(D),\) as
\[
\xi_h(r) = \frac{\lambda^4}{\pi^3 |Kw|^2} \int_D 4\pi |s_{hh}(D)|^2 N(D, r) dD; \quad (\text{mm}^2 \text{m}^{-3}),
\]

\[
\xi_{hh}(r) = \int_D |s_{hh}(D)|^2 N(D, r) dD
\]

\[
K_{DP}(r) = \frac{180\lambda}{\pi} \int_D \text{Re}[f_{hh}(D) - f_{vv}(D)] N(D, r) dD; \quad (\text{deg km}^{-1}),
\]

\[
A_{HH}(r) = 8.68\lambda \int_D \text{Im}[f_{hh}(D)] N(D, r) dD; \quad (\text{dB km}^{-1}),
\]

\[
A_{DP}(r) = 8.68\lambda \int_D \text{Im}[f_{hh}(D) - f_{vv}(D)] N(D, r) dD; \quad (\text{dB km}^{-1}),
\]

where \(\xi_h(r) = 10^{1.12_{HH}}\) and \(\xi_{hh}(r) = 10^{1.12_{HH}}\). Terms \(s_{hh, vv}(D)\) are backscattering amplitudes at horizontal and vertical polarizations, and \(f_{hh, vv}(D)\) are forward-scattering amplitudes at horizontal and vertical polarizations, respectively. Terms \(\text{Re}[\cdot]\) and \(\text{Im}[\cdot]\) are a real part and an imaginary part, respectively. Term \(\Lambda\) is the wavelength of transmitting electromagnetic wave (mm). Term \(K_w\) is defined by a complex index of refraction of water \(\varepsilon\) as

\[
K_w = \frac{\varepsilon^2 - 1}{\varepsilon^2 + 2}.
\]

In the NTR, \(N(D)\) is defined by the normalized gamma DSD (Ulbrich 1983; Testud et al. 2001) to describe the natural variability of the DSD, as below:

\[
N(D) = N_w f(\mu) \left( \frac{D}{D_0} \right)^\mu \exp(-\Lambda D); \quad (\text{mm}^{-1} \text{m}^{-3}),
\]

where

\[
\Lambda = \frac{3.67 + \mu}{D_0},
\]

\[
f(\mu) = \frac{6}{3.67^4 \Gamma(\mu + 4)} (3.67 + \mu)^{\mu+4},
\]

and \(D\) is a diameter of a raindrop (mm), \(N_w\) is an intercept parameter (mm\(^{-1}\) m\(^{-3}\)), \(D_0\) is a median volume diameter (mm), \(\mu\) is a shape factor that is dimensionless, and \(\Gamma(\cdot)\) is a gamma function.

b. SRR

This subsection describes the SRR, whose details are elaborated in Yoshikawa et al. (2014). The SRR is implemented as the preprocessing of the NTR, as shown in Fig. 1. The SRR is an iterative ML approach to fit a range...
profile of the DSD parameters to dual-polarization measurements. A likelihood function of a range profile in the $n$th beam of the $l$th radar node is expressed as

$$p(y^{(m,l)} | x^{(m,l)}) = N(y^{(m,l)} | F(x^{(m,l)}), \Sigma^{(m,l)})$$

$(m = 1, \ldots, M, \quad l = 1, \ldots, L), \quad (13)$

where $N(x | \mu, \Sigma)$ is a Gaussian PDF whose mean vector and covariance matrix are represented as $\mu$ and $\Sigma$, respectively. Terms $x^{(m,l)}$ and $y^{(m,l)}$ are the range profiles of the DSD parameters and the dual-polarization measurements, respectively. Term $F(x^{(m,l)})$ is the relation between $x^{(m,l)}$ and $y^{(m,l)}$, corresponding to Eqs. (1)–(12). Term $\Sigma^{(m,l)}$ is a covariance matrix of $y^{(m,l)}$, which is assumed to be known. Term $M$ is the number of beams, and $L$ is the number of radar nodes in the radar network. Terms $x^{(m,l)}$ and $y^{(m,l)}$ are represented as follows:

$$x^{(m,l)} = (N_w^{(1)}, \ldots, N_w^{(n)}, \ldots, N_w^{(N)}, D_0^{(1)}, \ldots, D_0^{(n)}, \ldots, D_0^{(N)}, \mu')^{(m,l)}$$

$$y^{(m,l)} = (Z_H^{(1)}, \ldots, Z_H^{(n)}, \ldots, Z_H^{(N)}, Z_{DR}^{(1)}, \ldots, Z_{DR}^{(n)}, \ldots, Z_{DR}^{(N)}, \Phi_{DPm}^{(1)}, \ldots, \Phi_{DPm}^{(n)}, \ldots, \Phi_{DPm}^{(N)})^{(m,l)},$$

where the superscript $n$ indicates a parameter in the $n$th range bin $r_n$ of $N$ range bins. Terms $N_w$, $D_0$, and $\mu'$ are applied as state variables, which are defined to avoid physical impossibilities through the iterative process.

$$N_w'(r) = \ln N_w(r),$$

$$D_0'(r) = \ln D_0(r),$$

$$\mu' = \frac{6}{1 + \exp(-\mu_C)} - 1,$$

where $\mu$ is assumed to be a constant value, $\mu_C$, in whole range of a radar beam as

$$\mu(r) = \mu_C.$$ (19)

Thus, $N_w$ and $D_0$ are kept positive and $\mu$ is squashed from $-1$ to 5 without any constraints to $N_w'$, $D_0'$, and $\mu'$. In summary, the SRR solution, $\tilde{x}^{(m,l)}$, is calculated by minimizing a cost function, which is derived from the likelihood function of Eq. (13), as shown below:

$$J(x^{(m,l)}) = -\ln p(y^{(m,l)} | x^{(m,l)})$$

$$= \frac{1}{2} \sum_{n=1}^{N} \frac{(Z_H^{(n)} - \hat{Z}_H^{(n)})^2}{\sigma_{ZH}^2} + \frac{1}{2} \sum_{n=1}^{N} \frac{(Z_{DR}^{(n)} - \hat{Z}_{DR}^{(n)})^2}{\sigma_{ZDR}^2}$$

$$+ \frac{1}{2} \sum_{n=1}^{N} \frac{(\Phi_{DPm}^{(n)} - \hat{\Phi}_{DPm}^{(n)})^2}{\sigma_{\Phi DP}^2} \rightarrow \min,$$ (20)

where $\sigma_{ZH}$, $\sigma_{ZDR}$, and $\sigma_{\Phi DP}$ are standard deviations for each dual-polarization measurement, respectively, which are diagonal elements of $\Sigma^{(m,l)}$. The iterative minimization is reasonably converged by the Gauss–Newton (GN) method (Nocedal and Wright 1999).

c. NTR

A flowchart of the NTR is shown in Fig. 2, which indicates that the NTR integrates the SRR solutions through three steps. In step 1, posterior PDFs in each radar node are calculated from the SRR solutions. Step 2 transforms the posterior PDFs on the radar polar coordinate of each radar node to those on the common Cartesian coordinate defined in the radar network. In step 3, the posterior PDFs of each radar node on the common Cartesian grid are integrated optimally in a Gaussian stochastic sense.

1) STEP 1: POSTERIORS OF RADAR NODE

A posterior PDF in the $m$th beam of the $l$th radar node is theoretically derived by the product of a likelihood function and a prior PDF as

$$p(x^{(m,l)} | y^{(m,l)}) \propto p(y^{(m,l)} | x^{(m,l)}) p(x^{(m,l)}).$$ (21)

With a linear approximation of $F(x^{(m,l)})$ around $\hat{x}^{(m,l)}$, the likelihood function of Eq. (13) is transformed as

$$p(y^{(m,l)} | x^{(m,l)}) \approx N(F(m,l) | x^{(m,l)}, \Sigma_y^{(m,l)});$$

$(m = 1, \ldots, M, \quad l = 1, \ldots, L), \quad (22)$

where

$$F^{(m,l)} = \frac{\partial F(x^{(m,l)})}{\partial x^{(m,l)}} \bigg|_{x^{(m,l)} = \hat{x}^{(m,l)}}.$$ (23)

Supposing a situation without any other prior information, a noninformative PDF, $p_{\text{no-info}}(x) = N(x | 0, \alpha) (\alpha \to \infty)$, is applied to the prior PDF. Thus, Eq. (21) is transformed as
\[
p(x^{(m,j)} | y^{(m,j)}) \approx N(x^{(m,j)} | \hat{x}^{(m,j)}, \Sigma_x^{(m,j)}),
\]
where
\[
\Sigma_x^{(m,j)} = (P^{(m,j)T} \Sigma_y^{(m,j)} - 1 P^{(m,j)})^{-1}.
\]
Assuming that range profiles of DSD parameters are independent of each other, the posterior PDF, including all the beams of the \( l \)th radar node, is expressed as
\[
p(x^{(l)} | y^{(l)}) \approx N(x^{(l)} | \hat{x}^{(l)}, \Sigma_x^{(l)}),
\]
where
\[
\hat{x}^{(l)} = [\hat{x}^{(1,lT)} \ldots \hat{x}^{(M,lT)}]^T
\]
\[
\Sigma_x^{(l)} = \text{diag}(\Sigma_x^{(1,l)} \ldots \Sigma_x^{(M,l)}).
\]

2) STEP 2: TRANSITIONS TO A COMMON CARTESIAN GRID

A posterior PDF in the \( l \)th radar node on the \( l \)th radar polar coordinate is transited to that on a common Cartesian coordinate, as shown below:
\[
p(x_C^{(l)} | y^{(l)}) = N(x_C^{(l)} | \hat{x}_C^{(l)}, \Sigma_C^{(l)})
\]
\[
= N(x_C^{(l)} | T^{(l)} \hat{x}^{(l)}, T^{(l)} \Sigma_x^{(l)} T^{(l)T}),
\]
where \( x_C^{(l)} \) is a vector including the two DSD parameters in the \( l \)th grid of \( I \) common Cartesian grids, defined as
\[
x_C^{(l)} = [N_w', D_0']^T; \quad (i = 1, \ldots, I).
\]
methods from polar to Cartesian coordinates have been described (Trapp and Doswell 2000). The closest-point method, which is primitive in radar meteorology, is applied in this paper. Equation (29) indicates that other linear transition methods can be alternated.

3) STEP 3: INTEGRATION

Posterior PDFs of all the radar nodes on the common Cartesian coordinate have now been prepared. Assuming that posterior PDFs calculated by each radar node are independent and identically distributed (i.i.d), their joint distribution is theoretically given as

\[ p(x_C^{(i)} | y^{(1)}, \ldots, y^{(L)}) = \mathcal{N}(x_C^{(i)} | \bar{x}_C^{(i)}, \Sigma_C^{(i)}) \]

\[ = \prod_{i=1}^{L} p(x_C^{(i)} | y^{(i)}), \quad (31) \]

where

\[ \bar{x}_C^{(i)} = \left( \sum_{l=1}^{L} \Sigma_C^{(i,l)} \right)^{-1} \left( \sum_{l=1}^{L} \Sigma_C^{(i,l)-1} x_C^{(i,l)} \right) \]

\[ \Sigma_C^{(i)} = \left( \sum_{l=1}^{L} \Sigma_C^{(i,l)} \right)^{-1} \quad (32) \]

Thus, the integrated solution \( \bar{x}_C^{(i)} \) is the best solution of the radar network in a Gaussian stochastic sense.

3. Evaluation by numerical simulation

a. Specifications of numerical simulation

Numerical simulations of the NTR are carried out to confirm their performance. Two-dimensional distributions of reference values of the DSD parameters are shown in Fig. 3, where \( N_w \) and \( D_0 \) are in Figs. 3a and 3b, respectively. The reference values of \( N_w \) and \( D_0 \) were generated from observed data on the Colorado State University–University of Chicago–Illinois State Weather Survey (CSU–CHILL) radar. The observation was carried out on 20 June 2000. The area shown in Fig. 3 is placed inside the area of the observation. The observed data were converted to the reference values of DSD parameters with the use of the algorithm proposed by Gorgucci et al. (2002). The algorithm by Bringi et al. (2002) was applied in cases where the obtained \( K_{DP} \) was noisy. Since it is well known that \( m \) varies depending on \( D_0 \), a varying \( m \) model is applied in order to simulate a natural variation of \( m \). The \( m \) of each range bin is determined by a linear \( m-D_0 \) relation as

\[ m = -\frac{\mu_x - \mu_y}{D_{0_{max}} - D_{0_{min}}} (D_0 - D_{0_{min}}) + \mu_x, \quad (34) \]

where \( D_{0_{max}} \) and \( D_{0_{min}} \) are the maximum and minimum \( D_0 \) in the dataset, respectively. Terms \( \mu_x \) and \( \mu_y \) are the largest and smallest values in a horizontal profile of \( \mu \). In this section, 2.0 and \(-0.5\) were selected for \( \mu_x \) and \( \mu_y \), respectively. Observed parameters \( Z_{Hm}, Z_{DRm} \), and \( \Phi_{DPm} \) at a frequency of 9.5 GHz (X band) were calculated from these generated reference values of DSD parameters on the assumption of spheroidal raindrops with the axis ratio of the Beard and Chuang (BC) model (Beard and Chuang 1987). Then, normally distributed fluctuations were added to the three observed parameters, whose standard deviations are 0.8 dBZ, 0.2 dB, and 1.5 dB, respectively. These values of the standard deviations were chosen by referring to Bringi and
Chandrasekar (2001, chapter 6), wherein the theoretical fluctuations in the dual-polarization measurements are derived. The “×” marks in both panels of Fig. 3 indicate the positions of each radar node, which are assumed to be deployed with the same interval on a circle whose diameter is 30 km as connected by dashed lines. Numbers labeled at each radar node are noted beside each mark. A square area bounded by the dotted line (its side length is about 21 km) is an area of interest in which DSD parameters are retrieved by the NTR. It is assumed that eight X-band radars are deployed on the field with a maximum observation range of 30 km and 1024 range bins; that is, each range bin is about 30 m long. A disagreement between polar coordinates of the CSU–CHILL radar and the assumed X-band radar was solved by linear interpolation.

In the proposed algorithm, the same assumptions as in the calculation of the dual-polarization measurements from the reference values of DSD parameters—spheroidal raindrops and the BC model—are applied (see Bringi and Chandrasekar 2001, chapter 4). In the zeroth step (the SRR), it is assumed in the algorithm that 16 adjacent range bins have the same DSD in order to reduce the effect of fluctuations in measurements and calculation costs. Thus, the algorithm has 64 state variables for both \( N_w \) and \( D_0 \) in a radar beam. On the basis of this assumption, three measurements in a chunk of 16 range bins are calculated by one DSD profile in the GN method. Term \( M \) is the number of chunks of range bins (≈64 in this paper).

The integration of the NTR is performed in each common Cartesian grid as shown in Eq. (31); that is, 16,384 retrievals of the NTR are evaluated in the single 2D distribution of precipitation. In addition, the dataset of DSD parameters is the same as the one used in our previous work (Yoshikawa et al. 2014). An area of interest for the NTR is separated by 128 \( 3 \times 128 \) square grids. The closest-point method is applied for transition to the common grid. All these settings of the simulation are summarized in Table 1.

### Table 1. Simulation characteristics.

<table>
<thead>
<tr>
<th>Radar</th>
<th>Coverage range</th>
<th>No. of range bins</th>
<th>Fluctuations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0–30 km</td>
<td>1024</td>
<td>Normal distributions with std dev of 0.8 dB, 0.2 dB, and 1.5° in ( Z_{\text{HN}} ), ( Z_{\text{DRm}} ), and ( \Phi_{\text{DPm}} ), respectively</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulated precipitation</th>
<th>Dataset</th>
<th>DSD model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>269 range profiles of ( N_w ) and ( D_0 ) generated from data observed in the CSU–CHILL radar on 20 Jun 2000</td>
<td>Normalized gamma DSD (Testud et al. 2001) with every 0.1-mm equivolume spherical diameter from 0.5 to 8.0 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Raindrop shape</th>
<th>Axis ratio model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spheroid</td>
<td>PB, BC, or TB model</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>SRR</th>
<th>No. of chunks of range bins</th>
<th>Raindrop shape</th>
<th>Axis ratio model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>16</td>
<td>Spheroid</td>
<td>BC model</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stabilization factors (( \sigma_{N_w}, \sigma_{D_0} ))</th>
<th>(2.00, 3.00)</th>
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<table>
<thead>
<tr>
<th>Convergence criteria</th>
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where \( \Delta x \) is a vector of a DSD profile in the \( i \)th iteration and \( \Delta x \) is an update vector of a DSD profile in the GN method. Term \( M \) is the number of chunks of range bins (=64 in this paper).

<table>
<thead>
<tr>
<th>NTR</th>
<th>Integrated 2D grid field</th>
<th>Number of radar nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>128 \times 128 grids in 21 km ( \times 21 ) km</td>
<td>2, 4, and 8</td>
</tr>
</tbody>
</table>

Chandrasekar (2001, chapter 6), wherein the theoretical fluctuations in the dual-polarization measurements are derived. The “×” marks in both panels of Fig. 3 indicate the positions of each radar node, which are assumed to be deployed with the same interval on a circle whose diameter is 30 km as connected by dashed lines. Numbers labeled at each radar node are noted beside each mark. A square area bounded by the dotted line (its side length is about 21 km) is an area of interest in which DSD parameters are retrieved by the NTR. It is assumed that eight X-band radars are deployed on the field with a maximum observation range of 30 km and 1024 range bins; that is, each range bin is about 30 m long. A disagreement between polar coordinates of the CSU–CHILL radar and the assumed X-band radar was solved by linear interpolation.

In the proposed algorithm, the same assumptions as in the calculation of the dual-polarization measurements from the reference values of DSD parameters—spheroidal raindrops and the BC model—are applied (see Bringi and Chandrasekar 2001, chapter 4). In the zeroth step (the SRR), it is assumed in the algorithm that 16 adjacent range bins have the same DSD in order to reduce the effect of fluctuations in measurements and calculation costs. Thus, the algorithm has 64 state variables for both \( N_w \) and \( D_0 \) in a radar beam. On the basis of this assumption, three measurements in a chunk of 16 range bins are calculated by one DSD profile in the GN method. Term \( M \) is the number of chunks of range bins (≈64 in this paper).

The integration of the NTR is performed in each common Cartesian grid as shown in Eq. (31); that is, 16,384 retrievals of the NTR are evaluated in the single 2D distribution of precipitation. In addition, the dataset of DSD parameters is the same as the one used in our previous work (Yoshikawa et al. 2014). An area of interest for the NTR is separated by 128 \( 3 \times 128 \) square grids. The closest-point method is applied for transition to the common grid. All these settings of the simulation are summarized in Table 1.

The integration of the NTR is performed in each common Cartesian grid as shown in Eq. (31); that is, 16,384 retrievals of the NTR are evaluated in the single 2D distribution of precipitation. In addition, the dataset of DSD parameters is the same as the one used in our previous work (Yoshikawa et al. 2014), where it is indicated that the statistical properties of the reference values almost agree with a long-term observational result by a Joss–Waldvogel disdrometer [see Fig. 2 in Yoshikawa et al. (2014) and Fig. 7.17 in Bringi and Chandrasekar (2001)]. Therefore, this numerical simulation covers a wide range of precipitation and includes a sufficient number of attempts to confirm the theoretical performance of the NTR.
b. Improvement from the SRR

Figure 4 shows the retrieved results. Figures 4a and 4b are horizontal distributions of $N_w$ retrieved by the SRR on nodes 0 and 4, respectively, and Figs. 4d and 4e are those of $D_0$. The NTR results of $N_w$ and $D_0$ are shown in Figs. 4c and 4f respectively, where they are retrieved by the NTR integration with nodes 0 and 4 (2-NTR), respectively. On the upper sides of Figs. 4a and 4d which indicate a weak precipitation region,
discontinuities along the azimuth direction significantly appear. The $Z_H$ are roughly 25–31 dBZ in this area, which is reasonable to make the SRR unstable. Compared with the reference shown in Fig. 3, the retrievals in this area have larger errors than ones in the other areas. As seen in Figs. 4c and 4f, the NTR properly corrects the discontinuity in the upper area of node 0. Figure 5, showing horizontal distributions of standard deviations of the SRR solutions of nodes 0 and 4 (corresponding to the square roots of the diagonal elements of $\Sigma_0^{0}$ and $\Sigma_0^{4}$, respectively), indicates how integration of the NTR works. Figures 5a–5d show those of $N_w$ in node 0, $N_w$ in node 4, $D_0$ in node 0, and $D_0$ in node 4, respectively. As pointed out in Yoshikawa et al. (2014), the SRR is unstable in the case of weak precipitation throughout a beam because the DSD parameters have less sensitivity to the dual-polarization measurements in such a case due to a characteristic of the nonlinear function expressed in Eqs. (1)–(12). However, the covariance matrix, $\Sigma_0^{p}$, includes large variances in the diagonal elements corresponding to those unstable solutions. This is seen in the upper side of Figs. 5a and 5c. In the NTR, the DSD parameters are integrated considering each covariance based on a Bayesian scheme—that is, an accurate solution is obtained as long as one node in a radar network has a stable solution by the NTR utilizing a multidirectional observation environment. On the other hand, in the case of all nodes having a stable solution in a desired grid, the SRR retrievals in each node are integrated with almost equivalent averaging to derive retrieval with lower fluctuations.

c. Error analyses

To confirm the performance of the NTR with possible radar network situations, statistical evaluations are carried out by changing setting parameters of the numerical simulation with respect to the number of radar nodes, ranges of varying $\mu$, axis ratio models, and system bias errors. Hereafter, 8-NTR is implemented on the
Table 2. Bias and standard errors of log($N_w$) [$N_w$ (mm$^{-1}$ m$^{-3}$)] and $D_0$ (mm) resulting from the numerical simulations; bias error (standard error). Standard setting; 8-NTR, $(\mu_u, \mu_l) = (2.0, -0.5)$, BC model, and no-biased measurements. In the others, the standard setting is applied if not specified.

<table>
<thead>
<tr>
<th></th>
<th>log($N_w$)</th>
<th>$D_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard setting</td>
<td>-0.05 (0.09)</td>
<td>0.04 (0.09)</td>
</tr>
<tr>
<td>2-NTR</td>
<td>-0.05 (0.12)</td>
<td>0.03 (0.12)</td>
</tr>
<tr>
<td>4-NTR</td>
<td>-0.05 (0.10)</td>
<td>0.04 (0.09)</td>
</tr>
<tr>
<td>$(\mu_u, \mu_l) = (0.0, 0.0)$</td>
<td>-0.00 (0.05)</td>
<td>0.00 (0.06)</td>
</tr>
<tr>
<td>$(\mu_u, \mu_l) = (5.0, -1.0)$</td>
<td>-0.10 (0.14)</td>
<td>0.07 (0.13)</td>
</tr>
<tr>
<td>PB model</td>
<td>-0.11 (0.15)</td>
<td>0.12 (0.12)</td>
</tr>
<tr>
<td>TB model</td>
<td>-0.22 (0.21)</td>
<td>0.16 (0.24)</td>
</tr>
<tr>
<td>Randomly biased</td>
<td>-0.09 (0.15)</td>
<td>0.07 (0.14)</td>
</tr>
</tbody>
</table>

2) RANGES OF VARYING $\mu$

Varying $\mu$ of the actual precipitation could give significant errors to the NTR since constant $\mu$ is assumed in each radar beam in the zeroth step (SRR). To evaluate those errors, dual-polarization measurements were calculated with three different ranges of varying $\mu$, where $\mu_u$ and $\mu_l$ are 1) 0.0 and 0.0; 2) 2.0 and $-0.5$; and 3) 5.0 and $-1.0$, respectively. The second $\mu$ range is the same one as in section 3a. These values of $\mu_u$ and $\mu_l$ were prepared by referring to Ulbrich and Atlas (1998) and Moisseev and Chandrasekar (2007). The expression $(\mu_u, \mu_l) = (0.0, 0.0)$ results in a constant $\mu$ model and is equivalent to the Marshall–Palmer DSD (Marshall and Palmer 1948). In Fig. 7, bias and standard errors of $N_w$ and $D_0$ are shown. Dotted, dashed, and solid lines correspond to $\mu$ ranges 1–3, respectively. With the $\mu$ range 1, the NTR outputs retrievals with bias and standard errors of $-0.00$ and $0.05$ in log($N_w$), and $0.00$ and $0.06$ in $D_0$, respectively, which are the best accuracy throughout the error analysis, as shown in Table 2. The bias errors occur only because of spatial disagreements between the reference and the NTR. While the reference values of DSD parameters were calculated on the common Cartesian coordinate, dual-polarization measurements in each radar node were calculated from DSD parameters on each radar polar coordinate. Then the standard deviations are due to a combination of the coordinate disagreement and the fluctuations in the dual-polarization measurements. Errors due to the constant $\mu$ assumption of the SRR appear in the $\mu$ ranges 2 and 3, as shown in Fig. 7, where $\mu$ monotonically decreases along $D_0$. This is because of the disagreement of the SRR’s assumption of constant $\mu$ with the spatial variation of $\mu$. Both bias and standard errors increase with a higher variation of $\mu$. Table 2 indicates that evaluation with the $\mu$ range 3 outputs the largest bias and standard errors in the three $\mu$ ranges, which are $-0.10$ and $0.14$ in log($N_w$), and $0.07$ and $0.13$ in $D_0$, respectively. These evaluated values are possibly the worst errors caused by the variation of $\mu$ because varying $\mu$ from $-1.0$ to $5.0$ is naturally too large to exist in the entire target area.

3) AXIS RATIO MODELS

Errors caused by incompatibility of the axis ratio between the NTR assumption (the BC model used in this paper) and actual precipitation were confirmed in this subsection. Two numerical simulations where dual-polarization measurements were calculated using the Pruppacher and Beard (PB) model (Pruppacher and Beard 1970) and the Thurai and Bringi (TB) model (Thurai and Bringi 2005) are compared with the BC model. In Fig. 8, the bias and standard errors of $N_w$ and $D_0$ are shown. Dotted, dashed, and solid lines correspond to

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the PB, TB, and BC models, respectively. As in the error analysis of the $m$ range, it is seen in Fig. 8b that the bias error of the PB and TB models monotonically depend on $D_0$. This is because an axis ratio is a function of $D_0$. In the simulation with the PB model, the bias and standard errors are $-0.11$ and $0.15$ in $\log(N_w)$, and $0.12$ and $0.12$ in $D_0$, respectively. In the TB model, those errors are $-0.22$ and $0.21$ in $\log(N_w)$, and $0.16$ and $0.24$ in $D_0$, respectively. According to Table 2, the bias and standard errors could be affected up to 2 times greater than those of the varying $m$ from $-1.0$ to $5.0$. Note that the area of interest is too large to have precipitation with a uniform axis ratio model. Therefore, the evaluated bias and standard errors are the possible worst errors caused by the axis ratio model.

4) SYSTEM BIAS ERRORS

Radars may have system bias errors in their measurements from several causes such as a water-coated radome, temperature-dependent amplifier gain, and so on. Although these kinds of errors are typically calibrated in the quality control process, it is difficult to correct them completely. To make an error analysis of this problem, the $Z_{Hm}$ and $Z_{DRm}$ of all eight nodes were independently and randomly biased. Bias values of $Z_{Hm}$ were generated by a Gaussian random number generator with a mean value of $0$ dB and a standard deviation of $1$ dBZ. Random biases with a mean value of $0$ dB and a standard deviation of $0.2$ dB were given for $Z_{DRm}$. Note that the measurements are biased by the same value in an arbitrary radar node. Twenty independent simulations of 8-NTR showed that the bias and standard errors of $\log(N_w)$ are $-0.09$ and $0.15$, and that those of $D_0$ are $0.07$ and $0.14$, respectively. As shown in Table 2, these system biases could give almost 2 times worse bias errors and about 1.5 times worse standard errors than the standard setting in both $\log(N_w)$ and $D_0$ retrievals.
4. Conclusions

An integrated DSD retrieval algorithm for an X-band dual-polarization radar network is proposed. The proposed algorithm, the NTR, is based on our previous work, which is a DSD retrieval method on a single radar, the SRR (Yoshikawa et al. 2014). The NTR is configured with the three steps, as below. In the preface, the SRR is implemented in all the radar beams of all the radar nodes included in a radar network. In the first step, the SRR solutions in each radar node are transformed to a posterior PDF; that is, a covariance matrix of the SRR solutions is calculated from known error covariance by linearly approximating the governing equation around the SRR solution. In the second step, the SRR solutions and their covariance matrices are transited from polar gridded ones to a common Cartesian gridded one using a transition matrix. In the final step, the SRR solutions and their covariance matrices on the common Cartesian grid are integrated based on a Bayesian scheme. It is concluded that the integrated solution of the NTR is theoretically the best estimation in the Gaussian stochastic sense.

Numerical simulations were carried out to verify the estimation accuracy, supposing a radar network in which two, four, or eight X-band radars are deployed on a circle whose diameter is 30 km. The reference value of DSD parameters was created from the CSU–CHILL radar data. Dual-polarization radar measurements of each radar node were calculated from the reference values of DSD parameters on the assumption of a spheroid raindrop shape and a fixed axis ratio model (see Table 1 for details of setting parameters.) As described in Fig. 4, the NTR properly corrects the wrong spatial discontinuities of the SRR, which is because the SRR retrieves DSD parameters in respect to each radar beam independently. Error analyses to evaluate the performance of the NTR in possible situations of a radar network were carried out. Specifically, statistical evaluations were implemented with the numerical simulation by changing the setting parameters of the numerical simulation with respect to the number of radar nodes, ranges of varying $m$, axis ratio models, and system bias errors. Because the NTR is based on a Bayesian scheme, the number of radar nodes works to reduce standard errors in the retrievals. The constant $m$ assumption of the SRR yields bias and standard errors of $-0.10$ and $0.14$ in $\log(N_w)$, and $0.07$ and $0.13$ in $D_0$ with varying $m$ from $1.0$ to $5.0$, respectively. Incompatibility of axis ratio models between the NTR assumption and actual precipitation gives retrieval errors up to 2 times greater than varying $m$ from $1.0$ to $5.0$. Further, system bias errors of about 1 dBZ and 0.2 dB (both of which are root-mean-square values) in $Z_{Hm}$ and $Z_{DRm}$, respectively, give bias and standard errors of $-0.09$ and $0.15$ in $\log(N_w)$, and $0.07$ and $0.14$ in $D_0$, respectively. In actual observation, a natural combination of these error sources should be included, and each of the error sources could be heavier than the settings of the error analyses. Our future work will entail performance evaluations in the CASA experiment or the Midlatitude Continental Convective Clouds Experiment (MC3E) by comparing ground-based devices, such as disdrometers, with DSD parameters and rainfall rate.

The concept of the NTR can be expanded from a radar network to a multisensor network, and the expansion will be a part of our future work. In the case of a weather observation campaign, many kinds of sensors such as radars at several frequencies, rain gauges, and disdrometers are deployed. In general, ground-based devices are utilized as hard constraints, where radar-retrieved physical parameters of precipitation are corrected so as to exactly match a measurement of ground-based devices.

![Fig. 8](image-url) Result of error analysis with regard to the axis ratio models. Bias and standard errors of (a) $\log(N_w)$ and (b) $D_0$. The errors are represented with six different quantiles (i.e., $<2, 2–2.5, 2.5–3, 3–3.5, 3.5–4, >4$ for $\log(N_w)$, and $0–1.5, 1.5–2, 2–2.5, 2.5–3, 3–3.5, >3.5$ for $D_0$.) In each panel, the dotted, dashed, and solid lines correspond to the PB, TB, and BC models, respectively (lines are drawn with slight shifts for clarity).
However, a ground-based device would have uncertainty due to not only its accuracy but also gaps of positions and resolution volumes. Therefore, a hard constraint generally gives a bias error by neglecting those uncertainties. On the other hand, the concept of the NTR offers soft constraints based on a Bayesian scheme, which appropriately improves radar-retrieved physical parameters of precipitation by considering both uncertainties of a radar retrieval and measurement of a ground-based device.

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