A Dual-Wavelength Radar Method for Ice–Water Characterization in Mixed-Phase Clouds

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ABSTRACT

A dual-wavelength method to differentiate supercooled water from ice and to measure mass content in each phase in cold stratiform clouds is proposed and discussed. The differential attenuation $A_d$, whose direct measurement is available with dual-wavelength radar, is a linear function of the liquid water content $M_L$ (the contribution from ice hydrometeors is negligible in comparison). Measuring both $A_d$ and the radar reflectivity factor $Z$, leads to a system of two equations expressed as functions of $M_L$ and $M_t$ (ice water content); its solution provides the values of both $M_L$ and $M_t$ between any two ranges along the radar beam, and as a consequence the distribution pattern of these two parameters within the cloud. Simulations of the method on two idealized cloud structures with various spatial distributions of $M_L$ and $M_t$ are shown. From a comparative study, the wavelength couple of 3.2 cm and 0.86 cm has been selected as the most suitable one, either for ground-based midrange cloud observations or for an airborne radar.

1. Introduction

The discrimination between liquid and solid hydrometeors in supercooled clouds is extremely important for a better understanding of the processes involved in the microwave radiometric measurements and in the precipitation formation, as well as for analyzing and modeling radiative cloud properties. Other fields of application are also concerned, such as aircraft-icing prevention.

Using polarimetric radar techniques, some authors suggest that the shape of the hydrometeors could be used as a discriminating factor between solid and liquid phases (Hall et al. 1984; McCormick and Hendry 1975). In rainy conditions the polarimetric measurements are efficient and permit a quantification of the process, as the medium is homogeneous, characterized by an almost constant dielectric factor, a simple relation between the size and axial ratio of the drops, and a common horizontal orientation of their main axes. But, in the presence of ice and mixed-phase clouds, only qualitative information about the nature of the main contribution (hail, rain, snow) could be deduced from polarimetric observations. The reasons are the following: 1) solid hydrometeor shape is complex and independent from their size, 2) their orientation is random except in a few cases such as columnar or platelike ice crystals (Sauvageot et al. 1986), and 3) their density, mainly variable, affects their apparent axial ratio. As a consequence, the hydrometeor shape is not a good parameter.

In addition to their shape, the hydrometeors are characterized by their dielectric factor, which is a primary factor in the computation of scattering and attenuation cross section. The dielectric properties, and particularly their frequency dependence, differ notably for water and ice. This suggests basing the water–ice discrimination on the measurement of these properties, through their effects on the attenuation and radar reflectivity terms (Westwater and Kropfli 1989; Gosset and Sauvageot 1991). In this paper, the conditions in which a dual-wavelength radar may enable the quantification of the water and ice components in mixed cold clouds are discussed.

The dual-wavelength radar technique has been used by several authors. Eccles and Mueller (1971) attempted to determine the mass content of liquid precipitation by measuring the differential attenuation with a dual-wavelength radar. Their method addressed the difficulty of establishing a simple and reliable inversion formula between the water content and the measured attenuation for any drop-size distribution (DSD). Dual-wavelength radars were also proposed by Eccles and Atlas (1973) to locate hail in storms: non-Rayleigh scattering by hailstones implies that hail shafts may be characterized by measurably different equivalent reflectivity factors for two selected wavelengths. Various measurement difficulties, however, make the interpretation difficult and limit the implementation of this method to an essentially qualitative aspect (Rinehart and Tuttle 1982).

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The originality of the present work comes from the fact that the remote sensing of mixed clouds is proposed. For nonprecipitating clouds characterized by small-size particles and simple relations between radar observables and physical parameters, this new approach is well adapted to an unambiguous discrimination between the solid and liquid water phases and an estimation of water and ice contents.

2. Principles

a. Measurement of the differential attenuation

In case of a dual-wavelength radar, assuming beams and sample volumes perfectly matched, the average power received, coming from a volume $V$, located at a distance $r$, is, for each wavelength,

$$ P_r = \frac{C_\alpha}{r^2} Z_{r\alpha}, $$

where subscript $\alpha = l$ corresponds to the longest wavelength $\lambda_l$ and $\alpha = s$ to the shortest one $\lambda_s$; $C_\alpha$ is the radar calibration factor; $Z_{r\alpha}$ is the measured reflectivity factor. This last term is related to the equivalent reflectivity factor $Z_{eq}$ and the attenuation factor for the two-way path $A_r$ (dB km$^{-1}$), for the given wavelength, by the expression

$$ Z_{r\alpha} = Z_{eq} 10^{-0.1 \int_0^r A_r(u) du}. $$

(2)

Here $A_r$ is the summation of two terms: the attenuation by the atmospheric gases, and, when going through clouds, the attenuation by hydrometeors.

The dual-wavelength ratio $y$ was defined by Eccles and Mueller (1971) (see also Atlas 1954) as the logarithm of the ratio of the mean powers received at each wavelength, from a volume at range $r$; that is,

$$ y(r) = 10 \log \left( \frac{Z_{ml}}{Z_{ms}} \right) = 10 \log \left( \frac{Z_{cl}}{Z_{es}} \right) + \int_0^r [A_s(u) - A_l(u)] du. $$

(3)

The basis of the method is the measurement of the difference $\Delta y$ of the dual-wavelength ratios of two volumes, $V$ and $V'$, of the radar beam, separated by a range increment $\Delta r$ (km):

$$ \Delta y = y(r + \Delta r) - y(r) $$

$$ = 10 \log \left[ \frac{Z_{cl}(r+\Delta r)}{Z_{es}(r+\Delta r)} \right] - 10 \log \left[ \frac{Z_{cl}(r)}{Z_{es}(r)} \right] + \int_r^{r+\Delta r} (A_s - A_l) du. $$

(4)

Thus, $\Delta y$ is the sum of two terms: a term $\Delta y_0$ plus the total differential attenuation undergone along the path $\Delta r$.

b. $\Delta y_0$ term

The equivalent reflectivity factor for a mixed cloud is the sum of the contribution of the liquid water hydrometeors ($Z_{eq}$) and of the ice hydrometeors ($Z_{ei}$):

$$ Z_{eq} = Z_{ew} + Z_{ei}. $$

(5)

In nonprecipitating clouds, these hydrometeors (approximated by water or ice spheres) belong to the Rayleigh-scattering region for the considered radar frequencies (see section 5). The equivalent reflectivity factor of the droplets is given by

$$ Z_{ew} = \frac{|K_{w}|^2}{|K_0|^2} Z_w, $$

(6)

with $K = (\epsilon - 1)(\epsilon + 2)^{-1}$ where $\epsilon$ is the complex permittivity of the considered medium, and $|K_0|^2 = 0.93$ (reference value for water); $|K_{w}|^2$ is a function of the temperature and the wavelength, as shown in the Appendix where the variation models of dielectric properties of water and ice used in this paper are presented, as well as the calculated values of factors $|K|^2$ and $\text{Im}(-K)$. Here $Z_w$ is the reflectivity factor of water:

$$ Z_w = \int N_w(D) D^6 dD, $$

where $N_w(D)$ is the DSD.

In the same way, for ice crystals

$$ Z_{ei} = \frac{|K_{i}|^2}{|K_0|^2} Z_i, $$

(7)

with the same definitions as for water, but for ice the term $|K_{i}|^2$ is independent of the wavelength (cf. the Appendix).

Finally $\Delta y_0$ has the following expression:

$$ \Delta y_0 = 10 \log \left( \frac{|K_i|^2}{|K_0|^2} \frac{Z_i}{Z_w} \right) \left\{ \frac{|K_i|^2}{|K_0|^2} \frac{Z_i(\Delta r)}{Z_w(\Delta r)} \right\} - 10 \log \left( \frac{|K_l|^2}{|K_0|^2} \frac{Z_l}{Z_w} \right) \left\{ \frac{|K_l|^2}{|K_0|^2} \frac{Z_l(\Delta r)}{Z_w(\Delta r)} \right\}. $$

(8)

If one of the populations is preponderant ($Z_l/Z_w \gg 1$ or $Z_l/Z_w \ll 1$) or if the proportions are homogeneous for the two volumes ($Z_l(r)/Z_w(r) = Z_l(r+\Delta r)/Z_w(r+\Delta r)$) then $\Delta y_0 \approx 0$. The calculations (cf. simulation in section 3) show that for all cases $\Delta y_0$ is very close to zero and we can write

$$ \Delta y = \int_r^{r+\Delta r} (A_s - A_l) du = A_d \Delta r. $$

(9)

The mean differential attenuation $A_d$ along the path from $r$ to $r + \Delta r$ is thus directly obtained from $\Delta y$. 

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c. Expression of the differential attenuation as a function of the water content.

The attenuation factor of a volume full of hydrometeors can be written

\[ A = (2)(0.434) \int Q_T(D)N(D)dD, \]

where \( A \) is in decibels per kilometer (dB km\(^{-1}\)), \( Q_T(D) \), the extinction cross section of a hydrometeor with a diameter \( D \), is in square centimeters (cm\(^2\)) and \( N(D) \) is per cubic meter (m\(^{-3}\)). The contributions of each phase can be added as discussed previously. For each wavelength we have

\[ A_\alpha = A_{aw} + A_{al}. \]  

(10)

For cloud droplets, \( Q_T \) is reduced to an absorption term that, in the Rayleigh approximation, is written as (Gunn and East 1954)

\[ Q_T = \left( \frac{\pi^2}{\lambda} \right) D^3 \text{Im}(−K), \]

from which the following expression is deduced:

\[ A_{aw} = 2C_{aw}M_w, \]

(11)

where \( M_w \) (g m\(^{-3}\)) is the liquid water content [\( = \rho \pi/6 \int D^3N(D)dD \) where \( \rho \) is the density of water]. The water attenuation coefficient

\[ C_{aw} = \frac{0.434}{\rho \lambda} \frac{6\pi}{\rho \lambda} \text{Im}(−K) \]

thus defined is a function of the wavelength and the temperature only (Fig. 1); the attenuation by the droplets is independent of their distribution and is a linear function of the liquid water content.

The attenuation by the ice crystals is the sum of an absorption term and a nonnegligible scattering term (Gunn and East 1954):

\[ Q_T(D) = \frac{\pi^2}{\lambda} D^3 \text{Im}(−K) + \frac{2\pi^5}{3\lambda^4} |K|^2 D^6, \]

which leads to

\[ A_{al} = 2(C_1 M_i + C_2 Z_i), \]

where \( M_i \) (g m\(^{-3}\)) is the ice water content. By expressing \( Z_i \) as a function of \( M_i \), we have:

\[ Z_i = M_iD_0^3 \left( \frac{g}{\pi \rho/6} \right), \]

where \( D_0 \) is the median volume diameter and

\[ g = \frac{\int D^6N_i(D)dD}{D_0^3 \int D^3N(D)dD} \]

is a function of \( N_i(D) \), the ice-crystal-size distribution (ICSD) (Atlas 1954). Thus, a coefficient of attenuation by the ice \( C_{ai} \) is defined as a function of temperature, wavelength, and ICSD, such as

\[ A_{ai} = 2C_{ai}M_i, \]

(12)

where

\[ C_{ai} = 0.434 \left[ \frac{\pi^2}{\rho \lambda} \text{Im}(−K) + \frac{4\pi^4}{\rho \lambda^4} |K|^2 gD_0^3 \right], \]

with \( D_0 \) in centimeters.

Figure 2 shows the influence of the scattering term on the coefficient of attenuation by ice, for monodispersed populations (\( g = 1 \)) of ice spheres with growing diameters.

The coefficients of attenuation for water and ice, according to the temperature and for various wavelengths from centimeter to the millimeter-wavelength regions as represented in Fig. 1, show that the attenuation increases with frequency and that the attenuation by ice is negligible compared to the attenuation by water. So, (9) can be rewritten

\[ \frac{\Delta y}{\Delta r} \approx A_{dw} = 2(C_{sw} - C_{lw})M_w, \]

(13)

or

\[ M_w = \frac{\Delta y/\Delta r}{2(C_{sw} - C_{lw})}, \]

\[ \text{Temperature (°C)} \]

Fig. 1. Attenuation coefficient of clouds versus temperature for Rayleigh approximation. Solid curves represent attenuation by water clouds for several wavelengths. Dashed lines are attenuation by a cloud composed of a monodisperse population of ice spheres (diameter = 200 µm).
whose coefficients are calculated from the measured parameters \(Z_e, A_d, \) and \(T,\) and whose solution gives the values of the liquid \((M_w)\) and ice \((M_i)\) water contents in the considered volume.

Figure 3 shows a graphic example of the solution of the system (17), in the \((M_w, M_i)\) plane. The \(Z-M\) relations used in Fig. 3 are

- for liquid water (Sauvageot and Omar 1987):
  \[ Z_w = 0.068M_i^{1.94} \]  
  \[ \text{(18)} \]

- for ice (Sassen 1987):
  \[ Z_i = 96.7M_i^{1.44} \]  
  \[ Z_i = 773M_i^{1.98} \]  
  \[ Z_i = 1876M_i^{1.89} \]  
  \[ \text{(19a)} \]
  \[ \text{(19b)} \]
  \[ \text{(19c)} \]

where \(Z\) is in standard units \((\text{mm}^6 \text{m}^{-3})\) and \(M\) in grams per cubic meter \((\text{g m}^{-3})\). The coefficients of relation (18) are similar to those obtained by Atlas (1954) and correspond to stratiform and shallow convective clouds. No other values have been proposed for liquid water clouds in the literature. For ice, (19a) corresponds to stratus clouds with small ice crystals, (19b) is associated with deep ice clouds of large-scale weather systems, and (19c) is given for the thunderstorm anvils.

In Fig. 3 solid lines correspond to (17a) for the indicated values of \(A_d\) and temperature; it can be seen that ice contribution to \(A_d\) is so small that the curves are almost vertical. Dotted lines correspond to the reflectivity factor equation (17b) for various values of \(Z_e\) and for the \(Z_e-M_i\) relations indicated in the brackets; because the contribution of ice to \(Z_e\) is largely dominant, the curves are almost horizontal. Figure 3 shows that the choice of the \(Z_e-M_i\) coefficients do not change the results for \(M_w\) but affects the values of \(Z_i\) and \(M_i\).

\[ \text{FIG. 3. Example of graphic solution of system (17). Solid lines correspond to (17a) for the indicated values of } A_d \text{ and temperature. Dotted lines correspond to (17b) for various values of } Z_e \text{ (dBZ)} \text{ and the } Z_i-M_i \text{ relations indicated in the brackets. The intersection points give } M_w \text{ and } M_i \text{ for the corresponding case (see text).} \]

Environmental temperature is used as an input parameter to compute precisely attenuation coefficient, and the liquid water content value can be determined from \(\Delta y/\Delta r\) measurement.

d. Determination of ice water content and generalization

The volume where the attenuation factor \(A_d\) has been determined is also characterized by its equivalent reflectivity factor \(Z_e\) measured at the less-attenuated wavelength (i.e., \(\lambda_i\)). From (5), (6), and (7), we deduce the expression of \(Z_e:\)

\[ Z_e = \frac{1}{|K_0|^2} \left( |K|^2_0 Z_w + |K|^2_i Z_i \right) . \]  
  \[ \text{(14)} \]

From the literature, we can choose relations between reflectivity factor and water content for each phase:

\[ Z_w = \delta_w M_w^{\beta_w} , \]  
  \[ \text{(15)} \]

\[ Z_i = \delta_i M_i^{\beta_i} , \]  
  \[ \text{(16)} \]

where \(\delta\) and \(\beta\) are coefficients adapted to the type of cloud observed.

We obtain a system of two equations with two unknowns \((M_w, M_i)\):

\[
\begin{align*}
\frac{A_d}{2} &= (C_{sw} - C_{iw})M_w + (C_{si} - C_{ij})M_i, \\
|K_0|^2Z_e &= |K|^2_0 \delta_w M_w^{\beta_w} + |K|^2_i \delta_i M_i^{\beta_i},
\end{align*}
\]  
  \[ \text{(17a)} \]
  \[ \text{(17b)} \]

\[ \text{FIG. 2. Attenuation coefficient versus diameter, for a monodispersed distribution of ice spheres. For each wavelength, absorption term only (dashed lines) and total attenuation (solid lines) are represented.} \]

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From the system (17), by an iterative process, the distribution of the water and ice contents is determined, volume by volume, in the entire cloud.

Depending on whether the precision of $M_w$, $M_i$, measurement or the spatial resolution is favored, and depending on the technical capacities of the radar, a gate-to-gate resolution is made or values of reflectivity and attenuation are integrated on a larger volume.

3. Simulations of differential-attenuation signal

The simulations of horizontal, monodirectional, dual-wavelength soundings of various mixed-phase clouds were performed. Two of them are presented as examples. Figure 4a corresponds to the case of supercooled water cells maintained by shallow convection in a stratiform ice cloud; Fig. 4b represents ice particles (fall streaks) precipitating through a liquid stratiform cloud. Both structures are idealized as follows: water content for one of the phases is kept constant all over the path, while the water content of the other phase is a triangular function of the distance.

The DSD of the hydrometeors are of the modified gamma type with $\mu = 2$, that is,

$$N(D) = N_0 D^2 e^{-\Delta D},$$

where the parameters $\Delta$ and $N_0$ are determined by subjecting each population to a reflectivity–water-content relation. For the simulation presented in Fig. 4, relations (18) and (19b) have been used.

Figure 4 confirms that

- reflectivity factor is dominated by the ice component [Figs. 4a(II) and 4b(II)],
- differential attenuation is a measurement of the liquid water component [Figs. 4a(III) and 4b(III)],
- attenuation by ice can be neglected, even if ice content is higher than liquid water content [Figs. 4a(IV) and 4b(IV)].

The influence of the $\Delta y_0$ term of (4) can induce [Fig. 4b(V)] a slight overestimation or underestimation of the differential attenuation for gradients of the water–ice ratio, positive or negative, respectively. For all the simulated examples, the deviation is never over 10%.

The relation between the differential attenuation and the liquid water content enables the location and the estimation of the quantity of supercooled water, both precisely and easily.

4. Detectability threshold and spatial resolution

Eccles and Mueller (1971) deduced the statistical properties of the dual-wavelength ratio from the characteristics of the received signal $P_r$. They showed that the minimum detectable mean attenuation has the following expression:

$$A_{d,\text{min}} = \frac{B}{\Delta \rho k^{0.5}},$$  \hspace{1cm} (20)

where $B$ is a function of the maximum value of the ratio $\sigma/A_{\text{min}}$ ($\sigma_A$ is the standard deviation of $A$) tolerable to validate the estimation of $A$ (with $B = 19.4$ for $\sigma_A/A = 5^{-0.5}$) and $k$ is the total number of samples used in the measurement, namely:

$$k = \frac{U_T (U_r - 1)(U_\theta - 1)(U_\phi - 1)}{U_T - 1}. \hspace{1cm} (21)$$

In (21) $U_T$ is the number of independent samples in time; $U_r$, $U_\theta$, and $U_\phi$ are the number of measurements when a spatial integration is made in range, elevation, and azimuth, respectively. The three-dimensional integration requires a postprocessing (after recording); then, the sensitivity can be improved to the detriment of the spatial resolution.

For a bidimensional scanning, with $8 \times 8$ measurements, 32 independent samples, and an intervolume distance $\Delta r = 2$ km, we find $A_{d,\text{min}} \approx 0.25$ dB km$^{-1}$ (two-way path). The minimum detectable water content is calculated with (13):

$$M_{w,\text{min}} = \frac{A_{d,\text{min}}}{2(C_{sw} - C_{hw})}.$$  \hspace{1cm} (22)

Its value depends on the used wavelengths; it is given for various frequency pairs in section 5.

An upper limit of detectability is imposed by the oscillations in the $Z$ and $Q_r(D)$ functions of hydrometeors belonging to the Mie region. This limit depends on the wavelengths used, as discussed in section 6.

5. Choice of the radar frequency pairs

In the spectral window for atmospheric gas attenuation (Fig. 5), several pairs can be considered a priori. Various criteria were analyzed to determine the frequencies best adapted to our application. The following discussion is summarized in Table 1.

a. Minimum detectable water content

Equation (22) and the calculated values of the attenuation coefficients (Fig. 1) clearly show that the choice of a millimetric wavelength (0.86 cm) for $\lambda_1$, and of a centimetric wavelength for $\lambda_3$, with a negligible attenuation (3.2 cm is sufficient), is an advantage.

For the example of section 4, $(A_{d,\text{min}} = 0.25$ dB km$^{-1}$), we find $M_{w,\text{min}} = 0.028, 0.035, and 0.14 g m^{-3}$ for the pairs: 3.21 cm with 0.32 cm; 0.86 cm with 0.32 cm, and 3.2 cm with 0.86 cm, respectively.

Moreover, the decorrelation time of the meteorological signal is a decreasing function of the frequency; the use of short wavelengths enables the increase of the number of independent samples $(U_T)$ for a same observation time, and the decrease [see (20) and (21)] of the detectability threshold of the differential atten-
FIG. 4. Simulations of radial sounding with the proposed dual-wavelength method. (a) Simulation of supercooled-water cells embedded in a layer of ice crystals. (b) Simulation of ice-crystal trails precipitating in a layer of supercooled droplets. (1) Water content versus range-gate number (in the example the interval between gates is 200 m; i.e., maximum range is 10 km). (II) Calculated values of $Z_w$ for the two wavelengths. (III) Differential-attenuation signal (for two ways): — solid line represent differential attenuation only, — dotted line (if the two curves do not match) is for the measured signal ($\Delta Z_w$ term included). (IV) Percentage of total attenuation by ice. Panel [b(V)] is the same as panel [b(III)] with vertical scale multiplied by 10. In both examples temperature is 0°C.
The importance of the attenuation of the millimetric waves by the hydrometeors and the existence of a residual attenuation by the atmospheric water vapor, highly perceptible at 0.32 cm, are very penalizing.

The radar equation can be restated as

$$\bar{P}_r = \left( \frac{P_t \beta G^2 \lambda^2}{2^6 \pi^3 r^4} \right) \frac{h}{2} \frac{\pi^5}{\lambda^4} \left( K^2 Z_e 10^{-0.1 \mu A d r} \right),$$

(23)

where $\bar{P}_r$ is the received power, $P_t$ the transmitted peak power, $G$ the gain, $h$ the pulse length, $\theta$ the half-power beamwidth, and $\beta$ a loss factor (see, for example, Sauvageot 1991).

Assuming radars with different frequencies but with the same technical characteristics (i.e., identical values for $P_t$, $G$, $h$, $\theta$, and the minimum detectable power $P_r_{\text{min}}$), the minimum detectable reflectivity factor at the distance $r$, for each of them, depends only on the wavelength:

$$\left( \frac{Z_e}{r} \right)_{\text{min}} (\text{dB}) = C + 20 \log \lambda + \int_0^\infty A(\lambda) d\lambda, \quad (24)$$

where $C$ is a coefficient depending on the radar technical characteristics.

A very short wavelength ($\lambda = 0.32$ cm) increases the sensitivity of the system ($\lambda^2$ term) at short distances only. For example, the crossing of 1 km of cloud with 1 g m$^{-3}$ of water or 2 km of warm saturated air at 25°C is enough to cancel the improvement of 0.32 cm, on 0.86 cm (cf. also Table 1).

To conclude, the 0.86- with 0.32-cm pair could be interesting for a short-range airborne system, but for medium-range observations a 3.2-cm with 0.86-cm radar seems to be the best adapted, both for an airborne and for ground-based system.

### 6. Extension to the Mie hydrometeors and validity limit of the method

For hydrometeors not meeting the Rayleigh scattering conditions, reflectivity and attenuation factors must

<table>
<thead>
<tr>
<th>Wavelength $\lambda$ (cm)</th>
<th>Atmospheric gases attenuation worst case</th>
<th>Attenuation by a water cloud ($M = 1$ g m$^{-3}$, $T = 0°C$)</th>
<th>20 log($\lambda/3.21$) (dB)</th>
<th>Needed antenna diameter for a half-power beamwidth of 1°</th>
<th>Detectable water content for minimum differential attenuation $A_d_{\text{min}} = 1$ dB km$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.32</td>
<td>&lt;4.5</td>
<td>4.6</td>
<td>-20</td>
<td>(22 cm) ($D_h$)</td>
<td>0.32; 3.2; 0.11</td>
</tr>
<tr>
<td>0.86</td>
<td>&lt;0.8</td>
<td>1</td>
<td>-11.4</td>
<td>(60 cm) ($2.7 D_h$)</td>
<td>0.32; 0.86; 0.14</td>
</tr>
<tr>
<td>3.21</td>
<td>&lt;0.1</td>
<td>0.08</td>
<td>0</td>
<td>(2.2 m) ($10 D_h$)</td>
<td>0.86; 3.2; 0.54</td>
</tr>
</tbody>
</table>

* From Liebe (1985); attenuation is given for a relative humidity of 100%, and a temperature of 37°C.
be calculated from the Mie formulas. This will be the case for the proposed wavelengths in precipitation.

a. Attenuation term

Figure 6a represents the differential attenuation $A_d$ (dB km$^{-1}$ g$^{-1}$ m$^3$) calculated for the distributions of Marshall and Palmer (1948) for wavelength pair 3.21 and 0.86 cm. For rain, with a median volume diameter $D_0 > 800$ $\mu$m, the attenuation is no longer a linear function of the water content. Atlas and Ulbrich (1974) showed that it is, however, possible to find a relation in the form $A = kM_w^\alpha$ (the coefficients $k$ and $\alpha$ are adapted to the type of rain considered) and to estimate the water content by measuring the attenuation.

b. $\Delta y_0$ term

The ratio of the reflectivity factors is no longer a constant for each phase but depends on the droplet or ice-crystal distribution. Figure 6b represents the ratio $(Z_{3.21\, cm}/Z_{0.86\, cm})$ for Marshall–Palmer distributions of water or ice spheres as a function of $D_0$.

If the sampled environment is homogeneous, $\Delta y_0$ [(4)] remains equal to 0 and the gradient of the measured dual-wavelength ratio ($\Delta y$) is equal, as previously seen (section 2b), to the differential attenuation undergone along the path. The proposed method remains applicable as long as the signal remains detectable. This may be the case for stratiform, light precipitation.

When the size distributions of the precipitating hydrometeors vary in space, or in the presence of high gradients of the ice–water ratio, $\Delta y$ is no longer a measurement of the differential attenuation, and the ratio of the reflectivity factors at the two wavelengths $(\Delta y_0)$ becomes the preponderant term. This is likely to be the case for convective precipitation. In fact it is this property of $\Delta y$ which is exploited for hail detection (Eccles and Atlas 1973).

7. Summary

An original application of the dual-wavelength radar technique is proposed for the discrimination and characterization of ice and supercooled water in clouds.

As attenuation by ice is negligible and cloud particles are Rayleigh scatterers; the mean differential attenuation $A_d$, measured between two points of the radar beam, is a linear function of the liquid water content $M_w$ of the volume crossed; the radar reflectivity factor $Z_r$, averaged along the same path, is the sum of both ice and water contribution, and with appropriate $Z$–$M_w$ and $Z$–$M_i$ relations for each phase of water, $Z_r$ is shown to have a power-law dependence on $M_w$ and $M_i$.

From the measured parameters ($A_d$, $Z_r$), with an additional knowledge of the temperature, a systematic inversion is processed and the values of $M_w$ and $M_i$ are computed for each resolution volume. By iterative process, the method allows step-by-step determination of the distribution pattern of $M_w$ and $M_i$ within a cloud.

The comparative study of various wavelength pairs led to the selection of the 3.21- and 0.86-cm wavelength pair, which, for all its advantages, is the best adapted to the sounding of mixed clouds or light precipitation, whether the radar is ground based or airborne. Our conclusion is that a radar with X-band and $K_a$-band wavelengths would be a good means for study of the inner structure of cold clouds, and in particular a tool to help the prevention of aircraft icing by the localization of hazardous areas of supercooled liquid water.

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APPENDIX

Dielectric Properties of Water and Ice

For the calculation of the dielectric properties of water and ice we have used the variation model of the complex permittivities $\epsilon$ proposed by Ray (1972) and updated by Liebe et al. (1989). The water permittivity is given by the Debye (1929) formula:

$$\epsilon = \epsilon_0 + \frac{\epsilon_0 - \epsilon_p}{1 - j(\lambda_p/\lambda)},$$
where $\varepsilon_0$ is the static dielectric constant, $\varepsilon_p$ is the optical dielectric constant, $\lambda_p$ is the relaxation wavelength, and $j = (-1)^{0.5}$. Parameters $\varepsilon_0$, $\varepsilon_p$, and $\lambda_p$ are expressed in function of the wavelength and the temperature (a more complete model, the “double Debye formulation,” valid above 100 GHz, is also proposed by Liebe et al. 1989).

For the ice, this model gives the following expression:

$$\epsilon = 3.15 - j\left(\frac{A_i}{f} + B_i f\right),$$

where $A_i$ and $B_i$ are functions of $T$ and $\lambda$ and $f$ is the frequency.
From these expressions, \( K = (\epsilon - 1)(\epsilon + 2)^{-1} \), \(|K|^2\), \(\text{Im}(-K)\) and the extinction cross sections are calculated for each phase. Figures A1a–d show the variations of the terms \(|K|^2\) and \(\text{Im}(-K)\) according to the temperature, for water and ice, at wavelengths 10, 3, 1.24, 0.86, 0.62, 0.32 cm. The Ray model and the double Debye model (valid above 100 GHz) are represented for information only. It is noteworthy that the two models sensibly differ for ice (Figs. A1c and A1d).

REFERENCES


