THE VARIATION WITH HEIGHT OF THE VERTICAL FLUX OF HEAT
AND MOMENTUM

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ABSTRACT

Direct measurements have been made of the vertical flux of heat and momentum in the layer from 2 to 12 meters. Eddy velocities were obtained from hot-wire anemometers and light bivanes, mounted at four levels; temperature fluctuations were measured with fast-response thermocouples, mounted at three levels. Data were recorded by taking photographs of indicating dials, at the rate of one exposure per second. Six sets of data have been analyzed, each set corresponding to one period of observation approximately 10 minutes in length. In four sets of data, the flow was over a rough land surface; in one set, the flow came directly from a water (ocean) surface; in the remaining set, the flow was principally over water except for a short land trajectory immediately upwind from the point of observation.

The flux data show a maximum variation from two- to four-fold within the layer. Over land, the shearing stress tends to decrease with height during the day and to increase with height at night; over water, both the heat flux and the momentum flux tend to decrease with height and also are significantly smaller at all levels than over a land surface. Values are presented for the coefficients of eddy viscosity $K_n$, eddy conductivity $K_a$, surface drag $C_D$, and for von Kármán’s constant $k$. For a land surface, the eddy coefficients are approximately equal near the ground; $K_n$ increases with height at a slower rate than $K_a$ during the day, while at night the reverse is true. Over a water surface, $K_n$ is considerably larger than $K_a$ at all levels. The values for $C_D$ are in good agreement with previous estimates, based on less direct measurements. The rather complicated variation of $k$ with stability and height is discussed; the average of all determinations is approximately 0.4.

1. Introduction

Many theoretical developments in atmospheric turbulence are based on the assumption that, near the ground, the Reynolds stress $\tau$ is invariant with height. A similar assumption is usually made in regard to the vertical flux of heat and other transferable properties. Recent surveys of turbulence and diffusion [12; 18] have emphasized the fact that field studies necessary to test the validity of these assumptions are almost entirely lacking. The purpose of this paper is to present the results of a series of direct measurements of the shearing stress and the heat flux, in the layer from 2–12 m. The coefficients of eddy viscosity and eddy conductivity, and von Kármán’s constant, have also been determined for the same height interval. The observations which form the basis of this investigation were obtained principally over a rough land surface, under varying conditions of thermal stratification; one set of observations was obtained for flow coming directly over a water (ocean) surface. These data were collected as part of a program for studying the basic features of fluctuations in wind speed, wind direction, and air temperature; other phases of this investigation have not yet been completed.

2. Instrumentation

Direct measurements of the shearing stress and the turbulent heat flux require a knowledge of the quantities $\bar{u}'w'$ and $\bar{u}'T'$ (see section 4, below). The eddy velocities $u'$ and $w'$ were obtained from the records of hot-wire anemometers and sensitive bivanes, mounted at four levels on a mobile tower: 11.9, 6.4, 3.7, and 2.3 m. Temperature fluctuations were measured by fast-response thermocouples, installed at three levels: 11.9, 6.4, and 2.3 m. A photograph of this instrumentation and the mobile tower appears in fig. 1. Cup anemometers of conventional design, and slow-response ventilated thermocouples, may be seen in the figure to the left of the fast-response instruments. The box at the base of the tower contains pen recorders and auxiliary equipment, required to record the wind-speed and temperature data of the slower instruments. These data are used in calculating the profiles of mean wind speed and air temperature.

A typical installation of the fast-response instrumentation is shown in fig. 2; the bivane, and the hot-wire probe which is attached to it, are at the right, while the thermocouple is located inside the small bakelite tube appearing in the center. The airfoil sections of the bivane are made of balsa wood, each section being about 10 cm long and 4 cm wide. Variations in azimuth and elevation angle are transmitted through miniature selsyn systems, to two pointers.

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that rotate about a common dial. The probe of the hot-wire anemometer is fastened to the shaft that transmits the azimuth position of the bivane; thus, the probe is headed directly into the wind by the action of the bivane. The fine wires of the probe are of gold-palladium and platinum-rhodium composition, and have a diameter of approximately 0.0075 cm. The wires are so arranged that the probe indicates total wind speed as long as the angle between the probe axis and the azimuth wind vector is less than 10 deg; for larger deviations, the indicated speed is approximately equal to the total speed multiplied by the cosine of the angle. The response of the instrument is fully compensated for changes in ambient air temperature. The thermocouple, used to measure temperature fluctuations, is constructed of copper-constantan wire about 0.0025 cm in diameter. The outer surface of the small bakelite tube in which the couple is housed is coated with gold leaf, for shielding purposes. The unit is ventilated, the rate of air passage being approximately 7 m/sec. The output from the thermocouple is led to a galvanometer that has a time constant of about 0.3 sec; this is the limiting factor in the response time of the thermocouple assembly. The lateral separation of the hot-wire probe and the thermocouple, which are mounted at the same height above ground, is about 25 cm. The airfoil sections of the bivane are located about 10 cm above and 20 cm downwind from the

Fig. 1. View of instrumentation mounted on mobile tower.
Fig. 2. Typical installation of fast-response instruments.

Table 1. Comparison of response characteristics of the bivane, hot-wire anemometer, and ventilated thermocouple for two mean wind speeds. Values are the ratio of the indicated amplitude to the true amplitude of simple sinusoidal fluctuations.

<table>
<thead>
<tr>
<th>Period of fluctuation (sec)</th>
<th>Bivane</th>
<th>Anemometer Mean wind speed = 2.5 m/sec</th>
<th>Thermocouple</th>
<th>Bivane</th>
<th>Anemometer Mean wind speed = 5.5 m/sec</th>
<th>Thermocouple</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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<td>1.00</td>
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</tr>
<tr>
<td>5</td>
<td>1.02</td>
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<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>4</td>
<td>1.05</td>
<td>0.96</td>
<td>0.97</td>
<td>1.01</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>3</td>
<td>1.09</td>
<td>0.91</td>
<td>0.90</td>
<td>1.04</td>
<td>0.96</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>1.04</td>
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<td>1.09</td>
<td>0.92</td>
<td>0.80</td>
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<tr>
<td>1.5</td>
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<tr>
<td>1.0</td>
<td></td>
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</tr>
</tbody>
</table>
hot-wire probe. This arrangement was adopted to minimize the interference between the various components, while at the same time maintaining a reasonable degree of proximity.

The response characteristics of the instruments used to measure the flux of heat and momentum are given in Table 1. Ratios of the indicated amplitude to the true amplitude for simple sinusoidal fluctuations are presented for each instrument, as a function of the period of the fluctuations at two values of the mean wind speed. These data show that the response times of the bivane, hot-wire anemometer, and ventilated thermocouple are remarkably well matched; also, the amplitude ratios are within a few percent of the optimum value for fluctuations having a period of 4 sec, and are within 10 percent of unity when the period of the fluctuations is 2 sec. It is concluded that the fast-response instrumentation is sufficiently sensitive to resolve satisfactorily fluctuations of period 2 sec or longer, at mean wind speeds above 2.5 m/sec. One feature of the operation of the bivane deserves comment. When the fin assembly of the bivane moves in a circular path in response to a sudden change in the azimuth direction of the wind, a centrifugal force exists. If the elevation angle of the bivane is different from zero, there is a component of this force which tends to bring the vane to a horizontal position. Wind tunnel tests have shown that, under conditions likely to be encountered in the field, the maximum deviation of the elevation angle from its true value due to this factor is less than 1 deg. This is within the limits of accuracy of the recording mechanism. A more detailed account of this instrumentation, which was developed by G. C. Gill, is given elsewhere [2, 3, 4].

Data are recorded by taking lapse-time photographs with two 35-mm movie cameras. Wind speed and wind direction record on meters placed in one panel, while fluctuations in temperature register on the scales of galvanometers mounted in a second panel. Simultaneous recordings are ensured, since the same electrical timing device operates the shutters of both cameras. The sampling time is limited by the capacity of the spring-driven film advance mechanism of the movie cameras. For the usual exposure rate of one frame per second, the maximum continuous sampling time is 10–12 min.

3. Description of conditions under which observations were made

This study is based upon six sets of measurements; for each set, the indicating dials were photographed at the rate of one frame per second, and the average sampling time was about 11 min. Five sets of observations were made in a large field, during the period 10–11 June 1952. A single set of data was obtained on 20 August 1952, at a seaplane ramp. For the June studies, the mobile tower was located on a level plot of hard sandy soil, about 65 m in diameter; this plot is encircled by a large field, containing scrub cedar trees, low brush, and tall grass. The average height of these roughness elements was estimated to be 50 cm; roughness elements in the immediate vicinity of the mobile tower, consisting of irregularities in the sandy soil, isolated tufts of grass, and occasional stones, were estimated to be about 5 cm high on the average. Portions of the site are visible in Fig. 1. The shoreline of Buzzards Bay, an inlet of the Atlantic Ocean, is found approximately 150 m directly south of the point of observation. The terrain is generally flat, except for a low ridge about 20 m in elevation, located 1 km to the west. The only major obstacles in the vicinity of the site are a grove of trees about 350 m to the west and several two-story buildings situated at approximately the same distance to the north. A sketch, showing the principal topographic features and the configuration of the shoreline in the area where the measurements were made, is presented in Fig. 3.

The weather for the period 10–11 June 1952 was generally fair, and was characterized by the presence of a fresh polar continental air-mass. During the afternoon of 10 June, when one set of observations was collected, the sky was about 6/10 covered by cumulus and altocumulus clouds; the surface wind was from a southwesterly direction. It is evident, from Fig. 3, that air from this direction has a trajectory principally.

![Fig. 3. Map of general area in which measurements were made; stippled area shows boundary of Round Hill Field Station, and rectangle denotes approximate site of observations. Contours are labeled in feet.](image-url)
over water until the shoreline is reached. On 11 June, the surface wind was from the northwest; cumulus clouds covered about 6/10 of the sky during the day but, at night, the skies were clear. The air trajectory for these observations was entirely over land (see fig. 3). Profiles of mean wind speed and air temperature for each period of measurement are shown in fig. 4; the basic data for the profiles are summarized in table 2.

On 20 August 1952 the mobile tower was towed to the shore of Buzzards Bay, to measure the eddy fluxes of heat and momentum in air coming directly over a water (ocean) surface. The seaplane ramp on which the tower was located has a gentle slope of about 3 deg, and is constructed of smooth wood planking. During the period of observation, the wind was directly from the south, and the sky was clear except for scattered high clouds. The distance from the edge of the water to a point directly beneath the tower instruments was about 8 m; with the exception of this factor, the trajectory was entirely over water. The water surface was characterized by the presence of wind-generated waves, about 0.5 m in height.

4. Measurements of eddy flux of heat and momentum

The shearing stress \( \tau \), which represents the vertical flux of momentum, is defined by the equation

\[
\tau = - \rho \bar{u}' \bar{w}',
\]

where \( \rho \) is the density, \( u' \) and \( w' \) are the eddy velocities in the direction of flow and along the vertical coordinate, respectively, and the bar denotes a time-mean value of the product. The flux of heat may be written, in a similar manner, as

\[
q = c_p \bar{u}' \bar{T}',
\]

where \( c_p \) is the specific heat, and \( T' \) is the temperature anomaly. The eddy velocities and the temperature anomaly are here defined with reference to average values taken over the period of sampling. This definition of the eddy flux excludes the contribution due to fluctuations whose period is of the same order or greater than the duration of the sampling interval. A similar concept of the eddy transfer has recently been proposed by Priestley and Sheppard [12]. In symbolic form,

\[
\bar{u} = \bar{u} + u', \quad \bar{w} = \bar{w} + w', \quad T = \bar{T} + T',
\]

where the bar refers to the arithmetic mean for the period of observation.

Eddy velocities were obtained from the bivane and hot-wire data, by the following procedure. The azimuth and elevation angles at each level were summed and the arithmetic means determined for each set of observations. Working values for the elevation angle \( E \) and the azimuth angle \( A \) were secured, by taking the differences between these means and the individual data. Since the hot-wire anemometer measures total wind speed \( V \), we may write

\[
w = V \sin E, \quad \bar{V}_h = V \cos E,
\]

\[
\bar{v} = \bar{V}_h \sin A, \quad \bar{v} = \bar{V}_h \cos A
\]

where \( \bar{V}_h \) is the total horizontal wind speed, and \( v \) is the component of wind speed in the azimuth plane directed normal to the mean direction of flow.

Values for the temperature \( T \) were read directly from the galvanometer scales. In computation of the stress and the heat flux, the density \( \rho \) has been assumed constant; while it is admitted that this is not strictly true, it appears that this simplification leads to errors which are negligible in view of the precision with which \( u', w' \) and \( T' \) can be determined. It is estimated that individual values of \( w' \) are accurate to \( \pm 5 \text{ cm/sec} \), \( u' \) and \( v' \) to \( \pm 10 \text{ cm/sec} \), and that \( T' \) is accurate to \( \pm 0.05 \text{ C} \).

The results of the direct measurements of the shearing stress and the eddy heat flux are summarized in table 3. Although it is not possible to state with absolute certainty the limits of error in these measure-

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**Table 2. Mean wind speeds and air temperatures. Time is Eastern Standard and the temperature scale has an arbitrary base.**

<table>
<thead>
<tr>
<th></th>
<th>10 June 1952</th>
<th>11 June 1952</th>
<th>20 August 1952</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z ) (m)</td>
<td>1500–1512</td>
<td>0955–1006</td>
<td>1340–1357</td>
</tr>
<tr>
<td>11.4</td>
<td>6.33</td>
<td>5.60</td>
<td>5.55</td>
</tr>
<tr>
<td>5.9</td>
<td>5.24</td>
<td>5.01</td>
<td>4.92</td>
</tr>
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<td>4.43</td>
<td>4.52</td>
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</tr>
<tr>
<td>1.8</td>
<td>3.60</td>
<td>4.16</td>
<td>4.06</td>
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</table>

\( T \) (deg C)

<table>
<thead>
<tr>
<th></th>
<th>11.4</th>
<th>2.35</th>
<th>3.00</th>
<th>3.18</th>
<th>1.24</th>
<th>1.12</th>
<th>1.64</th>
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<tr>
<td>5.9</td>
<td>2.66</td>
<td>3.13</td>
<td>3.39</td>
<td>1.34</td>
<td>1.02</td>
<td>1.45</td>
<td></td>
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<tr>
<td>3.2</td>
<td>2.95</td>
<td>3.30</td>
<td>3.55</td>
<td>1.46</td>
<td>0.98</td>
<td>1.65</td>
<td></td>
</tr>
<tr>
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<td>3.47</td>
<td>3.76</td>
<td>1.58</td>
<td>0.94</td>
<td>1.90</td>
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</tbody>
</table>
ments, the tabulated values are considered correct to the first significant figure, and the second figure, when given, is considered correct to \( \pm 2 \) units. This estimate makes no allowance for the response characteristics of the instruments. In this regard, considering both the time constants of the instrumentation and the method of sampling, the computed fluxes represent the contributions of fluctuations having periods from about 4 sec to several minutes. The upper limit is set by the length of the sampling interval.

Previous measurements of \( \tau \), with the use of bivanes, have been made by Scrase [13], Panofsky [7] and Frankenberger [1], while determinations of the stress at the earth’s surface have been carried out by Shepard [14], Poppendieck and Vehrencamp [10], and others. The heat-flux data agree well with measured values reported by Swinbank [19] and by Pasquill [8], and with estimates prepared by Sutton [18].

The marked change in \( \tau \) and \( q \) with height in the data for 10 June is probably due to the trajectory. As pointed out previously, the wind direction for this set of observations was such that the flow was principally over a water surface, except for a distance of about 150 m directly upwind from the point of measurement. Below a height of 6 m, the flux values are similar to those obtained during the day on 11 June, when the trajectory was entirely over land; above 6 m, the data resemble those obtained on 20 August for a water trajectory. With this exception, the maximum variation of both \( \tau \) and \( q \) in the layer from 2-12 m is from two- to four-fold. There is some evidence that, over land, \( \tau \) tends to decrease with height during the day and tends to increase with height at other times. It is evident, from the tabulated stress values, that \( \tau \) frequently has a minimum at the 6.4-m level; inspection of the instrumentation both before and after the measurements were made did not reveal any instrumental deficiencies to which this behavior might be ascribed. Although the possibility still exists that this feature of the data is due to undetected experimental error, it seems more reasonable to conclude that the indicated variation is genuine.

5. Determination of the eddy viscosity and the eddy conductivity

It is customary to express the eddy flux in terms of a transfer coefficient \( K \) and a mean gradient. Although expressions of this type are usually identified with the mixing-length theory, they may be derived from purely dimensional considerations [12]. There is, of course, little physical justification for assuming that the flux is dependent only upon the mean gradient. This convention is adopted here, to arrive at results which may be compared with those obtained previously. The relation between the transfer coefficients and the eddy fluxes may be written

\[
\tau = \rho K_n \frac{\partial \bar{u}}{\partial z},
\]

and

\[
q = -c_p K_n (\partial \bar{T}/\partial z + \Gamma),
\]

in the usual notation. It has been suggested [11] that (4) should contain an additional term on the right, to compensate for temperature anomalies existing at the level of origin of the eddies. However, there does not appear to be any satisfactory method by which this term may be computed.

The quantities \( \partial \bar{u}/\partial z \) and \( \partial \bar{T}/\partial z \) were obtained from the data given in table 2. As a matter of convenience, and in the interest of uniformity, the slopes of the temperature and velocity profiles were calculated from the power-law expressions used to compute von Kármán’s constant. The derivation of these equations is described in the following section. The values for \( \partial \bar{u}/\partial z \) and \( \partial \bar{T}/\partial z \) so obtained were compared with those resulting from direct measurement by the method of tangents, applied to the profiles of mean temperature and wind speed drawn on rectangular coordinate paper. The agreement was very good and, in practice, either procedure resulted in approximately the same values for the transfer coefficients.

The results of the calculations are presented in table 4. In general, the tabulated data confirm conclusions reached by previous investigators who, in the absence of direct measurements, were forced to pro-

<table>
<thead>
<tr>
<th>10 June 1952</th>
<th>11 June 1952</th>
<th>20 August 1952</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z (m) )</td>
<td>( 1500-1512 )</td>
<td>( 1512-1522 )</td>
</tr>
<tr>
<td>( \tau ) (dyne/cm(^2))</td>
<td>11.9</td>
<td>0.2</td>
</tr>
<tr>
<td>( q ) (cal cm(^{-3}) sec(^{-1})</td>
<td>11.9</td>
<td>0.1</td>
</tr>
</tbody>
</table>

| \( K_n \) (cm\(^3\) sec\(^{-1}\) \times 10\(^9\)) | 11.9 | 1.4 | 16.8 | 25.9 | 53.0 | 15.0 | 17.8 |
| \( K_s \) (cm\(^3\) sec\(^{-1}\) \times 10\(^9\)) | 11.9 | 1.1 | 42.5 | 39.6 | 81.1 | 5.5 | -1.4* |

* The minus sign results from an upward heat flux and a temperature inversion.

Table 3. Measurements of shearing stress \( \tau \) and heat flux \( q \).

Table 4. Computed values for \( K_n \) and \( K_s \).
ceed by indirect methods. These conclusions, which are described in detail elsewhere [17], may be briefly summarized as follows: (1) a rapid increase with height in the surface layer for all transfer coefficients; (2) a wide range in the absolute values of the coefficients, dependent upon the thermal stratification; (3) $K_h > K_m$ for steep lapse rates of temperature, and $K_h < K_m$ for temperature inversions. Certain features of the data in table 4, namely the variations of $K_m$ and $K_h$ with height on 10 June and the tendency for low values of the coefficients to occur at the 6.4-m level, are presumably explained by factors discussed previously in connection with the flux measurements. The range in absolute value for $K_m$ is $1.4 - 53.0 \times 10^6$ cm$^2$/sec, and the range for $K_h$ in the same units is $0.8 - 81.1 \times 10^4$. For flow over a land surface, the two coefficients are approximately the same near the ground; the 20 August data, which refer to a water trajectory, show $K_m$ considerably larger than $K_h$ near the ground. With the exception of the 10 June data, $K_m$ has a pronounced maximum at 11.9 m, for both land and water trajectories and at night as well as during the day. The eddy conductivity for 11 June has a pronounced maximum at 11.9 m during the day over land, and this maximum is about 1.5 to 2 times as large as that for the eddy viscosity; at night, the maximum for $K_h$ is at the 6.4-m level. For a water trajectory, $K_h$ is considerably smaller than $K_m$ at all levels, although the comparison is somewhat uncertain because of the discounting negative sign for $K_h$ at 11.9 m. The change in sign results from the combination of a positive heat flux (see table 3) and an inversion in the profile of mean temperature (see table 2). At the top level, $\partial T/\partial z$ could not be determined from the power law and was assumed equal to the average value between 5.9 and 11.4 m. Apparently the simple hypothesis used to define $K$ is not applicable in this instance. Pansky [6] has recently found $K_m > K_h$ at about 100 m in air that moved inland after a water trajectory. Eddy-diffusivity profiles that show an approximately linear increase with height have been described by Poppendieck [9].

6. von Kármán’s constant

According to the mixing-length theory, the shearing stress is related to the profile of mean velocity by the expression

$$\tau = -\rho \bar{u} w' = \rho \bar{u}^2 \frac{\partial \bar{u}}{\partial z},$$

where $l$ is the mixing length, and the other symbols have their usual significance. A simple hypothesis for predicting $l$ is due to von Kármán [5], who deduced the relation

$$l = k \frac{\partial \bar{u}}{\partial z},$$

where $k$ is a pure number, known as von Kármán’s constant. Measurements of turbulent flow on the laboratory scale have indicated that $k = 0.40$ is a universal constant for all fluids in isothermal flow. The only determination of $k$ for the lower atmosphere is due to Sheppard [14], who measured the aerodynamic drag of a smooth (cement) surface. His results show that, in near-neutral equilibrium, $k$ is about the same in the lower atmosphere as in the small-scale flow of the laboratory. In addition, Sheppard inferred that there was a significant variation in $k$ for non-adiabatic temperature gradients, i.e., $k > 0.4$ for temperature lapse and $k < 0.4$ for inversions. These conclusions are qualified by three assumptions: (1) that the deflection of the test surface used to measure the drag was a reliable measure of the aerodynamic drag, (2) that logarithmic velocity profiles are characteristic of both neutral equilibrium and steep lapse rates, and (3) that the shearing stress is invariant with height. In the present study, it is not necessary to make any of these assumptions.

The principal difficulty in determining von Kármán’s constant, as Sheppard pointed out [15], is the evaluation of the second derivative of the mean velocity profile. The limited data available for this purpose (see table 2) are clearly insufficient for a precise and reliable determination. However, it is felt that these data are sufficient for a gross measurement of $k$, and it also appears that they compare favorably with Sheppard’s data in this respect. The velocity profiles for three sets of observations agree closely with a logarithmic law, while a power law is more suitable for the remaining three sets (11 June 1952: 0955–1006, 1310–1322, 2119–2130 EST). The mean wind speeds for each period were fitted to a power law of the form

$$\bar{u} = a + bx^n,$$

Table 5. von Kármán’s constant

<table>
<thead>
<tr>
<th>Date</th>
<th>10 June 1952</th>
<th>20 August 1952</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1500–1512</td>
<td>1557–1609</td>
</tr>
<tr>
<td>$z$ (m)</td>
<td>0955–1006</td>
<td>$\bar{u}_{30m}$ (m/sec)</td>
</tr>
<tr>
<td>11.9</td>
<td>0.9</td>
<td>0.3</td>
</tr>
<tr>
<td>6.4</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>3.7</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>2.3</td>
<td>0.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>
where $a$, $b$ and $n$ are constants. For the three cases in which the velocity profile was approximately logarithmic, there is no significant difference in either $\partial^2 \bar{u} / \partial z^2$ or $k$, no matter which law is used.

The results of the computations are shown in table 5; values for the mixing length $l$, required to evaluate $k$, were obtained from the relation shown in (5). The range in the absolute value of $k$ is very similar to that predicted by Sheppard; however, the observed variation in $k$ with lapse rate appears to be more complex than he suggested. These results confirm the previous conclusion that $k$ is of the same order of magnitude in the lower atmosphere as in the laboratory-scale flow; the average of all the tabulated data is 0.43.

7. Surface drag coefficient

The drag, or skin friction, coefficient $C_d$ is defined in terms of the surface stress $\tau_0$ and the mean wind speed at a fixed height,

$$\tau_0 = C_d \bar{u}^2.\hspace{1cm}(7)$$

Since few direct measurements of the surface stress have been made, the variation in the drag coefficient has largely been studied by indirect methods. Tabulated values of $C_d$ obtained by this procedure have been given by Sutton [17] and Sheppard [14]. The drag coefficient has been computed from the observations described in this paper, by substitution of the measured stress at 2.3 m for the surface stress and by evaluation of the mean wind speed at 2 m and 10 m, since both levels have been previously used for reference. The results are shown in table 6, and are in good agreement with previous estimates. The surface drag of the ocean in the trade-wind belt has recently been determined by Sheppard and Omar [16], using the method of geostrophic departure. Their results indicate significantly smaller values for $C_d$ than the 20 August data above.

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