The Diurnal Precipitation Change over the Sea

E. B. Kraus

Woods Hole Oceanographic Institution

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ABSTRACT

Analysis of nine weathership records indicates that maritime precipitation is significantly more frequent at night. The effect varies with season and latitude. The diurnal variation is related to the absorption of solar radiation. Non-adiabatic heating may cause a reduction of liquid water production within rising clouds; this factor becomes important when vertical velocities are not too high. At the top of layer clouds, the diurnal rhythm of irradiation can also cause a time-lagged diurnal change of inversion levels and cloud thickness. The last inference is supported by observational data.

1. Introduction

The existence of a maritime rainfall maximum during night is part of meteorological lore. The evidence seems to have come mainly from the impression of seafarers and from coastal observations which may be influenced by a reversing land/sea breeze. The daily variation in cloudiness that may be caused even by very small islands in the trades has been discussed by Charnock et al. (1956) and many others. A demonstration that rainfall does vary diurnally over the open sea could have some bearing on our general understanding of rain formation and of the role that radiation plays in the process.

The present analysis is based on data from the weatherships listed in Table 1. These operate at fixed stations remote from meso-scale coastal effects. The National Weather Records Center of the U.S. Weather Bureau supplied, for each calendar month, separate sums of convective and non-convective, liquid and frozen precipitation incidence—as well as the number of occasions without precipitation (weather code 00-49)

Table 2. Significance of daily precipitation change in July. [D day, (N) night].

---for each of the eight daily synoptic hours. Some 2000–2300 observations per ship record month were therefore available.

2. Demonstration of the nighttime maximum

Table 2 lists separately the observed number of occasions with and without precipitation at night (N) and day (D) during July. Observations at the three international synoptic hours that fall between 21 and 06 local time are defined as night observations; the three observations between 09 and 18 local time provide the

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<th>Ship</th>
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1 Contribution No. 1381 from the Woods Hole Oceanographic Institution, Woods Hole, Mass.
Table 3. Mean number of precipitation reports in 1000 observations.

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<tr>
<th>Ship</th>
<th>J</th>
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<td>197</td>
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Paytime data. The two remaining synoptic observations in the transition hours have not been used for Table 2.

The table shows a preponderance of nocturnal precipitation at each station. It might be noted that the effect is at least as large for non-convective as for convective rain. In fact, the mid-latitude regions where rain is mostly non-convective have the largest diurnal amplitude. The smallest is found near the center of the Bermuda high at Station E, with predominantly convective rain. Station N also reports much convective rain, but this cannot be ordinary cumulus convection. The station has a complete overcast about 70–80 percent of time in July. Rain appears to originate intermittently from a strato-cumulus deck; the cause for its variation will be discussed specifically in Section 5.

It is sometimes difficult to distinguish convective and nonconvective rain. The significance of the night-to-day change has therefore been tested without regard to rainfall type. The difference between day and night has the same sign for all nine ships. The probability of this occurring by chance is only 2^{-8} or 1/256. In addition, chi-squares computed in the usual way are listed in Table 2. With one degree of freedom a value of $x^2 = 10.83$ denotes a significance on the 1:1000 level. This level is surpassed for six ships. The difference between night and day is insignificant only at Station E.

The sum of all the nine individual values $\sum x^2$ is 154. With eight degrees of freedom the 1:1000 significance level would be surpassed by any value above 26. This seems to prove the existence of the night-to-day difference of rainfall incidence in July beyond reasonable doubt.

3. The change of the daily amplitude with season and latitude

The mean number of precipitation reports ($P_{mt}$) in 1000 observations at each of the eight synoptic hours ($t$) may be represented separately for each month ($m$) by an expression:

$$P_{mt} = P_m + A_m \cos 2\pi (t - a_m)/24 + B_{mt}.$$  

The first right hand term $P_m$ represents the mean monthly number of precipitation reports per thousand observations; $A_m$ is the amplitude of the daily harmonic for month $m$; $a_m$ the phase angle and $B_{mt}$ the variations that are not associated with the daily wave.

The monthly variance within days is then:

$$\frac{1}{8} \sum P_{mt}^2 - P_m = \frac{1}{2} A_m^2 + V_m(B),$$

where $V_m(B)$ is the variance associated with $B_{mt}$. If rainfall were equally probable at any time of day, the expected value of the ratio $A_m^2/2V_m$ is $\frac{1}{8}$. If this ratio has a value of 0.5 or 0.75, it would explain half or three quarters of the variance within days as a daily harmonic.

Table 3 shows that maritime precipitation incidence everywhere tends to have a maximum in winter and a minimum in summer.

The quantity $A_m$ is listed in Table 4 for all the sub-sets for which $A_m^2/2V_m \geq 0.5$. Entries where the latter

Table 4. The daily variation of rainfall incidence—amplitude ($A_m$) of first harmonic in per mile. (Only values which explain at least half the variance within days are listed. Values in heavy type explain at least 0.75 of variance.)

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<tr>
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Table 5. Phase angle ($a_m$) of daily harmonic maximum (to nearest three hour local time). (Listing restricted as in Table 4.)

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quantity is larger than 0.75 are in heavy type. The table gives therefore some indication of significance as well as absolute values of the daily amplitude. It may be noted that the daily harmonic tends to be largest in mid-latitudes (ships K, P, C) during summer. There is no evidence of a significant diurnal rainfall variation anywhere in December. On the other hand it does exist everywhere in June and July except for position E.

Table 5 suggests that precipitation falls most frequently in the second half of the night. The value of the phase angle is remarkably uniform, particularly for the most significant summer amplitudes.

Tables 4 and 5 seem to indicate a secondary maximum of the daily variation in February with peak values at about 21 local time. No attempt is made in the present paper to explain this interesting and somewhat puzzling feature. The summer data are undoubtedly more consistent over a longer period of months.

4. Discussion

The available weathership records come from northern temperate latitudes. There appears to be, however, no prima facie statistical evidence for a different daily pattern over the open tropical and southern oceans.

A direct explanation may be based on considerations of liquid cloud water content. Solar radiation is partly absorbed by clouds. From figures and data published by Fritz (1958) it can be inferred that in a deep cloud of maritime character more than half of this absorption is likely to occur within a depth from the cloud boundary of about 50–100 meters. The penetration depth seems to vary little with radiance incidence angles between 0 and 60 degrees. A substantial part of the absorbed solar energy will be used to evaporate cloud drops. This reduces the amount of water available for precipitation.

The observed seasonal variation in the daily precipitation amplitude may be related on this basis to the seasonal change in solar radiation. It may also be affected by differences in the energy transformation at different temperatures. For example, it can be readily computed that 60 per cent of the absorbed energy will be used to evaporate water and 40 per cent will be used to increase the temperature in a cloud with a temperature of 10°C. In a cloud with a temperature of −5°C, only 35 per cent of the absorbed energy will be used to evaporate water and 65 per cent will be used to increase the temperature. The relative humidity has been assumed to remain constant at 100 per cent in both cases.

Before presenting an analytical discussion of these effects, it may be of interest to discuss briefly some other mechanisms. The harmonic analysis indicated that the amplitudes of higher harmonics have no systematic statistical significance. The variance in the present set of rainfall observation cannot be related therefore to the

<table>
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Table 6.

a) Difference between day and night time temperatures (°C) at fixed pressure levels in July (1958–62).

b) Phoenix, Arizona—Mean daily heating rate, July 1957 [after McDonald (1960b)].

\[
\begin{array}{cccccc}
\text{Pressure level} & 1000 & 850 & 700 & 500 & 300 \\
\hline
N & 0.57 & 0.56 & 0.68 & 0.65 & 0.40 \\
\end{array}
\]

S₂ solar tide, which should exhibit a semi-diurnal variation.

Variations in lapse rate and therefore convective activity might be related to the absorption of solar radiation by water vapor. During several hours of the day, solar heating in the free atmosphere can be larger than the cooling caused by the divergence of the infrared radiation flux. Table 6 illustrates the mean temperature changes over stations N and P which may result from this absorption.

To show that this temperature change is genuine and not due to an instrumental radiation error, corresponding data from station E have also been listed. Soundings at N and E are made at about the same solar height but with a phase difference of six hours in local time. There is no systematic temperature difference apparent between the two ascents at E. Both stations are American and use presumably the same equipment.

It is of some interest that the observed mean daily temperature change at stations N and P is of the same order as that calculated theoretically by McDonald (1960).

At 1000 mb the daily temperature amplitude over N and P is of the same order as the mean air-sea temperature difference. This could lead conceivably to a daily variation in the heat and moisture flux from the surface. A direct connection between these flux variations and the daily precipitation pattern is not impossible, but it is unlikely. Rainfall over P, in particular, was shown in Table 2 to be predominantly nonconvective. Its time scale cannot be expected to be influenced significantly by local time variations and surface flux.

There is some indication of a daily variation in mean vertical stability over N and P, but this reverses in sign with height and is statistically insignificant on the basis of the analyzed data. The relative uniformity of solar tropospheric heating with height has been associated by several authors with the opposing variations of mixing ratio and of solar beam intensity with attitude. Conditions are different when clouds are present. As mentioned above, though, warm cloud will not change.
greatly in temperature through insolation, because the energy is being used mainly to evaporate water. On the other hand, the temperature of cold and high cloud may be raised to a greater extent by insolation during the day with a corresponding fall at night time. This might contribute to increased nocturnal convective activity in high tropical cumulus for example. The process does not seem to be very relevant, however, to the data presented above, which show the greatest variations between day and night at places and during times—that is during the maritime summer—when convective activity is at a minimum.

There remains then the relation between solar absorption in clouds and liquid water content. This relation is direct and unequivocal. It may or may not account completely for the observed daily rainfall variation, but it will always act in the right direction. In particular, absorption within clouds should cause a reduction of the condensation rate in rising air and therefore a reduction of the liquid water content per unit volume. Evaporation resulting from absorption near the upper surface of a strato-cumulus deck would tend to reduce cloud height and thickness; this again reduces rainfall likelihood. Both processes will be discussed in the two remaining sections.

5. The condensation rate

Condensation in adiabatically rising air increases with vertical velocity. On the other hand, radiational effects do not depend on vertical velocity. The relative importance of radiational heating is therefore likely to increase if vertical velocities are small. Non-convective rain is associated with slow up-drafts and this may have a bearing on its pronounced diurnal variability in maritime conditions.

Following a procedure similar to that developed by Fulks (1935) we compute the steady state condensation rate $C$ above a horizontally fixed location, in air which rises with a vertical velocity component $w$, through a layer of thickness $\Delta p$, centered at a pressure level $p$. From the definition of the saturation mixing ratio $q_s$ and the Clausius-Clapeyron equation, we obtain with sufficient approximation for the present purpose:

$$C \approx \frac{\partial q_s}{\partial p} \Delta p = \rho \alpha \Delta p \left( \frac{\partial q_s}{\partial T} \frac{dT}{dp} + \frac{\partial q_s}{\partial p} \right)$$

$$= \rho \alpha \Delta p \left( \frac{L q_s}{R_w T^2 \frac{dT}{dp}} \frac{dT}{dp} + 1 \right)$$

(1)

\((dT/dp)\text{physical change in temperature } T; L=\text{latent heat}; R_w=\text{gas constant for water vapor.})

On a similar level of approximation, the rate of radiational heating $Q^*$ for the layer $\Delta p$, is related through the first law, to the temperature change by:

$$Q^* = \rho \alpha \Delta p \left( -\rho c_p \frac{dT}{dp} - \frac{\partial q_s}{\partial p} \right)$$

$$= -\rho \alpha \Delta p \left( c_p + \frac{L q_s}{R_w T} \right) \frac{dT}{dp} + w \Delta p \frac{dT}{dp} \frac{L q_s}{R_w T}$$

(2)

\((R_w \text{is the gas constant for air.})

We set $C$ equal to $C_0$ plus a correction factor, where $C_0$ is the condensation rate without radiation effects ($Q^*=0$) and can be obtained from the Smithsonian Meteorological tables.

$$C = C_0 (1+r).$$

(3)

For the radiational correction term $r$, we obtain, after elimination of $dT/dp$ between (1) and (2) and rearrangement,

$$r = -\left(1 + \frac{c_p T}{L} \frac{R_w}{R_w} \right) \frac{Q^*}{w \Delta p} \equiv -\frac{\alpha Q^*}{w \Delta p}.$$  

(4)

The dimensionless quantity $\alpha$ increases slowly with $T$. A constant value $\alpha = 1.22$ is appropriate for $260 \text{K} \leq T \leq 280 \text{K}$.

From (3) and (4) it can be readily seen that the ratio between the day and night time condensation rates will be the larger, the smaller the vertical velocity. For example in a cloud with a thickness of 50 mb, which is heated at the rate of 1 cal cm$^{-2}$ hr$^{-1}$ during the middle of the day, and which loses heat at a rate of 3 cal cm$^{-2}$ hr$^{-1}$ at night, the ratio of the day and night condensation rates $[1+r(D)]/[1+r(N)]$ would be 0.4 for $w=1$ cm sec$^{-1}$; 0.77 for $w=4$ cm sec$^{-1}$ and 0.89 for $w=10$ cm sec$^{-1}$. At larger vertical velocity values, and for thicker clouds, radiative contributions to influence the mean condensation rate significantly.

6. The daily variation in inversion level and cloud thickness

Variations in cloud top level can be related to existing data, which make this somewhat more interesting than the hypothetical changes in the condensation rate. As the cloud top height and therefore the cloud depth increases, precipitation becomes more probable—an effect discussed by Battan and Braham (1956), Fletcher (1962) and others. The data presented at the end of this section deal with the diurnal rise and fall of a quasi-permanent maritime inversion. The cloud deck below such an inversion tends to lose heat by radiation. This loss varies diurnally. It may even be reversed some of the time. The resulting change in cloud height can be inferred as follows.

A layer cloud with an upper surface at $z=h$ may
vary by $\Delta h$ during the short time interval $\Delta t$. Let $D$
indicate the penetration half-depth of the absorbed com-
ponent of solar radiation. We allow for a mean vertical
mass flux of air $\rho w$ through the layer $D+\Delta h$. Typical
mixing ratios and temperatures just above and below

$$\rho(q_a-q_e-q_L)\Delta h=\rho[w(q_a-q_e-q_L)-\overline{w'q_a'}-\overline{w'q_e'}+\overline{w'q_L'}]\Delta t+P\Delta t.$$  (5)

The last three terms in the square bracket represent
the turbulent moisture flux through the levels $h-D$ and
$h+\Delta h$.

In a similar way we obtain for the heat balance:

$$\rho[c_p(T_a-T_e)+Lq_L]\Delta h=\rho c_p[w(T_a-T_e)+\overline{w'T_a'}-\overline{w'T_e'}]\Delta t+\rho L(wq_L+\overline{w'q_L'}) \Delta t-(LP+Q)\Delta t$$  (6)

where $Q$ now indicates the convergence of the radiational
heat flux in the layer $\Delta h+D$, and it has been assumed
that variations of $q_L$ within the layer $D$ are negligible. They are not negligible in $\Delta h$ where the cloud appears or
vanishes.

Multiplication of (5) with $L$ and addition to (6) eliminates all the liquid water terms.

$$\rho[c_p(T_a-T_e)+L(q_a-q_e)]\Delta h=\rho w[c_p(T_a-T_e)+Lq_L]\Delta t+[c_p(\overline{w'T_a'}-\overline{w'T_e'})+L(\overline{w'q_a'}-\overline{w'q_e'})]P\Delta t-Q\Delta t.$$  (7)

Introduction of the (isobaric) equivalent temperature, $T^*=T+(L/c_p)q$, as defined by Van Miegheem (1943),
transforms (7) into:

$$\rho c_p(T_a*-T_e*)\Delta h=\rho c_p[w(T_a*-T_e*+\overline{w'T_a*'}-\overline{w'T_e*'})]\Delta t-Q\Delta t.$$  (8)

This is simply a balance equation for equivalent—or latent plus sensible—heat. After division by $\rho c_p(T_a*-T_e*)\Delta t$
this becomes in the limit, for $\Delta t \to 0$:

$$\frac{dh}{dt}=\frac{\overline{w'T_a*'}-\overline{w'T_e*'}}{T_a*-T_e*}-\frac{Q}{\rho c_p(T_a*-T_e*)}.$$  (9)

The analysis could have been somewhat sharpened
by the use of potential temperatures, but the resulting
difference would be relatively small.

In an inversion $T_a$ is greater than $T_e$. If the situation
is conditionally stable, the relation $T_a*-T_e*$ has to be
satisfied also. The difference $T_a*-T_e*$ is always smaller
than $T_a-T_e$ because the clear air is drier. The mean
velocity $w$ is generally negative for a persistent inversion.
Its value depends on the general synoptic situation
and cannot be expected to be correlated significantly with
local time. The quantity $\overline{w'T_a*'}-\overline{w'T_e*'}$ represents the
difference of the turbulent flux of $T^*$ above and within
the cloud. It may have a systematic daily variation,
but this is likely to be small compared to the variation
in the last term, which is related to the radiational
energy change in the layer where more than half the
absorption and almost all the emission occurs.

On the average (over 24 hours) $dh/dt$ will tend to be
zero. The mean value of $Q$ must be negative because the
cloud surface is cooled on the average. Subtraction of
suitable means from (9) and multiplication by $-\rho g$
gives therefore as a first approximation:

$$\frac{dh}{dt}=-\frac{d\rho h}{dt}=-\frac{Q-Q}{\rho c_p}.$$  (10)

It can be inferred from (10) that the cloud surface
will rise during the night when the cooling is more than
average, $(Q-Q)<0$. It will sink during the day when
the cooling is less than average. The actual value of
$h$ (or $-\rho_h$) should lag up to 90° or six hours behind $Q$.

Radioaonde observations from weathership N at 0300
and 1500 local time have been used to verify this inference.
July conditions there are characterized by a quasi-
permanent inversion. Values of $(T_a-T_e)$ between 2C
and 7C, $(T_a*-T_e*)$ between 0C and 1C, and $\rho_h$ between
900 and 800 mb are common and appear to be represen-
tative. The clouds are shown by Riehl, et al. (1951) to
reach from about 930 mb up to that level, with air
sinking through the inversion from above. The sky is
fully overcast most of the time. With an assumed value of
$T_a*-T_e*=0.8$ C, a daytime value of $Q(D)=+1$ cal
$\text{cm}^{-2}\text{hr}^{-1}$ and a nighttime value $Q(N)=-3$ cal
$\text{cm}^{-2}\text{hr}^{-1}$, we obtain from (10) vertical changes of the upper
cloud surface which are of order 100 m $\text{hr}^{-1}$.

Fig. 1 illustrates the actual changes of the pressure
level $\rho_h$—that is the level at which the temperature
has a minimum—which occurred during two weeks in
July 1952. The height of the inversion can vary con-
siderably on the larger "synoptic" time scale. The result-
ing variance makes it difficult to establish the
statistical significance of a daily variation directly,
There is a prevalence of $F(-)$—that is, of lower pressures and of higher inversion and cloud top levels during night—in each of the five years. The total number of observations ($N$) is 136. If there were no daily variation, that is, if the distribution of $F$ were random, the expected values would be $68F(+)$ and $68F(-)$ with a standard deviation equal to $\sqrt{N/2}=5.8$. The observed values deviate by five standard deviations, which is extremely significant.²

It is concluded that the daily variations of the inversion are a characteristic phenomenon. They may change cloud thickness by a considerable fraction, with a corresponding change in rainfall probability. The phase lag between $\theta^*$ and $p_h$ which follows from (10) may explain why it rains most frequently in the later hours of the night. It can further be predicted from (10) that the amplitude of the daily change should be large when $T_o^* - T_e^*$ is small. This should be the case when the upper clear air is relatively moist. No attempt has been made to verify this prediction.

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² Since this paper was first submitted, it was pointed out by Dr. Murray Mitchell, Jr., that the expected value of the standard deviation may be affected by the use of the same $p_h$ (day) figures in two successive values of $F$, and also by Markov type persistence. The two effects oppose each other. Dr. Mitchell kindly computed both and found that this increases the expected standard deviation from 5.8 to about 7.1. He pointed out that this is still only a quarter of the observed deviation. The observations listed in Table 2 remain therefore highly significant, even if one allows for persistence.

REFERENCES


