Diagnostic Determination of Cumulus Cloud Populations from Observed Large-Scale Variables

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ABSTRACT

A method is proposed whereby some properties of cumulus cloud populations are determined from observed large-scale meteorological variables. This method combines large-scale heat and moisture balance considerations with a simple steady-state one-dimensional model for an individual cumulus cloud. When applied to the mean summer-time conditions over the Marshall Islands, the method gives a bimodal distribution of the vertical mass flux at the cloud base in terms of the cloud top height: one group of clouds penetrates up to the 200-300 mb level and another group stays below the 600 mb level, while relatively few cloud tops lie between 400 and 600 mb. The total percentage area covered by all clouds is found to be a few percent and the vertical velocity inside the cloud at the cloud base is estimated to be somewhere near 1 m sec\(^{-1}\) for most clouds. Because the observed large-scale averaged upward velocity at that level is of the order of 0.1 cm sec\(^{-1}\), this result indicates that the total mass entering into ascending clouds through the cloud base is one order of magnitude larger than the mass brought into the subcloud layer by large-scale convergence. This excess mass is compensated by the downward motion with a velocity of the order of 1 cm sec\(^{-1}\) in the region between clouds at the cloud base.

1. Introduction

It has been well recognized that: 1) many large-scale circulations in the tropical atmosphere are driven by the release of latent heat of condensation; 2) the release of latent heat in these large-scale circulations takes place mostly in cumulus and cumulonimbus clouds; 3) the typical horizontal scale of cumulus clouds is one to three orders of magnitude smaller than meso- or wave-like large-scale disturbances observed in the tropics; and that consequently, 4) the collective effect of cumulus ensembles must be incorporated into a large-scale model in a parameterized fashion.

Cumulus parameterization schemes previously proposed for large-scale prediction models have been reviewed by Yanai (1971) and Ogura (1972) and also in the GARP Publication Series No. 8 (Parameterization of Sub-Grid Scale Processes, 1972).

In the last couple of years, a new approach in cumulus parameterization has been developed by Ooyama (1971) and Arakawa (1971a) in which a one-dimensional cumulus cloud model is incorporated. Their formulations present clear mathematical expressions for physical processes through which the vertical transport of heat and moisture inside cumulus clouds affects the evolution of large-scale circulation. Another important aspect included both in Arakawa’s and Ooyama’s work is the simultaneous existence of cumulus clouds of different sizes at any instance. This is in contrast to previous works in the area of dynamics of cloud groups where attention was focused on finding a preferred horizontal scale of cumulus clouds in a given environment (e.g., Assi and Kasahara, 1967). Ooyama applied an empirical formula for the spectral distribution of vertical mass flux at the cloud base and evaluated the contribution of cumulus clouds to the evolution of large-scale circulations. On the other hand, Arakawa proposed a dynamical constraint for a cloud ensemble which gives an equation for this spectral distribution, thereby providing a closed parameterization of cumulus cloud.

From the diagnostic viewpoint, attempts have been made to estimate the vertical transport of water vapor and sensible heat associated with convective clouds in large-scale circulations. There is the pioneering work by Riehl and Malkus (1958), which pointed out the importance of cumulus clouds in the heat balance of the tropical atmosphere, and works by Riehl and Malkus (1961), Yanai (1961), Matsumoto (1967) and Ninomiya (1968). Riehl and Malkus, and Yanai discussed the role of penetrative cumulus towers in the development of tropical cyclones, while Matsumoto and Ninomiya evaluated the vertical transport of heat and moisture by cumulus clouds observed in the northwesterly winter monsoon over the Sea of Japan.

More recently, Reed and Recker (1971) obtained the vertical profiles of diabatic heating in different segments of propagating tropical waves from heat and moisture budget computations. In doing this, they considered
the cumulus updraft, the saturation downdraft, and the slow descent in the environment. Nitta (1972) also estimated the contribution of convective clouds in the large-scale heat and water budgets using observed 1956 and 1958 Marshall Islands data.

Yanai et al. (1973) demonstrated that the average properties of tropical cloud clusters, such as the vertical mass flux in updrafts, and the excess temperature, moisture and liquid water content of the clouds, can be determined from a combination of a one-dimensional cloud model and the large-scale heat and moisture budgets over an area covering the average cloud cluster. Estimates of the average cloud properties and the heat and moisture balance of the environment, obtained from the 1956 Marshall Islands data, were also presented. Gray (1972) also proposed a method of determining convective vertical circulation patterns of the summertime tropical atmosphere from large-scale considerations and discussed the magnitude and implication of these local patterns. He also indicated that his results are comparable with those obtained independently by Lopez (1972) through modeling of individual cumulus elements.

The present work is in parallel with that by Yanai et al. However, unlike Yanai et al.'s work where only averaged properties of clouds are discussed, we attempt to determine the spectral distribution of vertical mass flux in terms of the height of the cloud top from observed large-scale variables by solving integral equations. In this sense, our problem may be regarded as an inversion problem in cumulus parameterization. A familiar example of inversion problems in meteorology is the determination of the vertical temperature distribution from observed infrared radiative intensity. In our case, the observed apparent heat and moisture sources given as functions of height will be converted to give the size spectrum of clouds.

2. Equations for the large-scale heat and moisture budgets

The equations that govern the heat and moisture budgets in the atmosphere are given by:

\[ \frac{\partial s}{\partial t} + \nabla \cdot sv + \frac{\partial \omega}{\partial p} = Q_R + L(C - e), \]

\[ \frac{\partial q}{\partial t} + \nabla \cdot qv + \frac{\partial \omega}{\partial p} = -C + e, \]

where

\[ s = C_p T + gz \]

is the dry static energy, \( C_p T \) the enthalpy, \( gz \) the potential energy, \( q \) the water vapor mixing ratio, \( L \) the latent heat, \( Q_R \) the radiation heating rate, \( C \) the rate of condensation, and \( e \) the rate of evaporation of the liquid water. The horizontal wind velocity and the vertical \( p \)-velocity are denoted by \( v \) and \( \omega \).

In the following we shall consider a horizontal area which is large enough to contain a large number of cumulus clouds so that the statistical properties of the clouds within the area are meaningful, yet small enough so that the area can still be considered as a small fraction of the large-scale environment. To establish the relationship between the collective properties of the clouds and the large-scale meteorological data, let us take the horizontal average of Eqs. (1) and (2) over an area as described above. We denote all the averaged quantities by the bar symbol and the deviation from an average by the prime. The heat and moisture equations become

\[ \frac{\partial \bar{s}}{\partial t} + \nabla \cdot \bar{sv} + \frac{\partial \bar{\omega}}{\partial \bar{p}} = \bar{Q}_R + L(\bar{C} - \bar{e}) - \frac{\partial \bar{(s' \omega')}}{\partial \bar{p}}, \]

\[ \frac{\partial \bar{q}}{\partial t} + \nabla \cdot \bar{qv} + \frac{\partial \bar{\omega}}{\partial \bar{p}} = \bar{Q}_R + L(\bar{C} - \bar{e}) + \frac{\partial \bar{(q' \omega')}}{\partial \bar{p}}. \]

For simplicity, we have assumed that

\[ \bar{q} = s' = 0. \]

The terms \( \partial s' \omega' / \partial \bar{p} \) and \( \partial q' \omega' / \partial \bar{p} \) represent the effects of sub-scale cumulus clouds. Physically, \( Q_1 \) can be interpreted as the apparent heat source due to radiation, the release of latent heat, and the vertical convergence due to the sub-scale cumulus convection, while \( Q_2 \) can be considered as the apparent moisture sink due to condensation and the vertical divergence due to the sub-scale convection. Combining Eqs. (4) and (5), we obtain

\[ Q_1 - Q_2 - Q_R = - \frac{\partial h' \omega'}{\partial \bar{p}}, \]

where

\[ h = C_p T + gz + Lq \]

is the moist static energy; \( Q_1 - Q_2 - Q_R \) can be considered as a measure of the activity of cumulus convection; it equals zero when the sub-scale cumulus convection vanishes.

3. The cumulus cloud model

To study the mutual interactions between a cumulus ensemble and their environment, one needs a cloud model to represent the dynamic behavior of cumulus clouds. The physical processes of a single cumulus cloud are relatively well understood. Many models with different degrees of sophistication have been developed (Ogura, 1972). Eventually, one should use the best model that is available to represent the effects of cumulus clouds. However, as a first attempt, a simple model seems to be more desirable for the present.

The model which we have chosen in this preliminary study is the one-dimensional steady-state plume with a
height-independent radius entraining air from the boundary of the cloud. This model is similar to that used by Arakawa (1971a) and Yanai et al. (1973). According to the entrainment hypothesis, the entrainment rate $\lambda$ is given by

$$\lambda = \frac{1}{\frac{\partial m}{m}}$$

(9)

where $m$ is the total vertical mass flux (gm cm$^{-2}$ sec$^{-1}$) inside the plume. We shall assume for simplicity that $\lambda$ is an inherent constant for each cloud. In the next section, $\lambda$ will be used as a measure to classify cumulus clouds.

In the $p$-coordinate system, Eq. (9) can be written as

$$\frac{1}{m} \frac{\partial m}{\partial p} = \frac{\lambda H}{p}$$

(10)

where $H(p) = RT(p)/g$ is the scale height of the atmosphere, $R$ the gas constant for the dry atmosphere, and $m = -\omega_a a$, where $a$ is the area of the horizontal cross section of the cloud. From Eq. (10) one easily obtains that $m$ is given by

$$m(\lambda, p) = m_B(\lambda) \eta(\lambda, p),$$

(11)

where

$$\eta(\lambda, p) = \exp \left( - \int_{p_B}^{p} \frac{\lambda H}{p} dp \right).$$

(12)

$p_B$ is the height of the cloud base in the $p$-coordinate, and $m_B(\lambda)$ the mass flux at the cloud base.

In the following equations we use the subscript $e$ to denote variables inside the cloud and the tilde to denote variables of the atmosphere between clouds. Because $h$ is approximately conserved under the moist adiabatic process, the thermodynamic process inside the cloud may be written as

$$\frac{\partial m h_e}{\partial p} + \frac{\lambda H}{m} = 0.$$  

(13)

Solving Eq. (13), one obtains

$$h_e(\lambda, p) = \frac{1}{\eta(\lambda, p)} \left[ h_B(\lambda) - \int_{p_B}^{p} \frac{\lambda H}{\eta(\lambda, p)} dp \right].$$

(14)

where $h_B(\lambda)$ is the moist static energy at the cloud base.

For simplicity, we assume that the atmosphere is saturated within the cloud. The relation between $q_e$ and $h_e$ is then given by (Arakawa, 1971b)

$$q_e - \tilde{q}^* = \frac{1}{\gamma} (h_e - \tilde{h}^*).$$

(15)

Here $\gamma = (L/C_D)(\partial q^*/\partial T)$, $q^*(p, T)$ is the saturation mixing ratio as a function of pressure and temperature, $\tilde{q}^*$ the saturation mixing ratio of the atmosphere between the clouds, and $\tilde{h}^*$ the corresponding saturation moist static energy.

From the consideration of water balance inside the cloud, the total condensation rate inside a cloud at a height $p$ is given by

$$C(\lambda, p) = \frac{\lambda H}{p} \left[ m(q_e - \tilde{q}) + m \frac{\partial q_e}{\partial p} \right].$$

(16)

After the water vapor is condensed to form cloud droplets, a conversion process begins through which small cloud water droplets are converted into larger rain drops. We assume that the conversion is achieved at a given rate and that the rate (r) is proportional to the mixing ratio $q_{c1}(gm gm^{-1})$ of cloud droplets. The conversion rate is then written as (Ogura and Takahashi, 1971)

$$r = C q_{c1},$$

(17)

where $C_q$ may be regarded as the reciprocal of the conversion time of the cloud droplets. Ogura and Takahashi used 0.005 sec$^{-1}$ as a typical value for $C_q$ which corresponds to 3.5 min of conversion time.

For the sake of simplicity, we assume further that raindrops, once formed, fall instantaneously out of the cloud. Taking all these processes into account, $q_{c1}$ inside the cloud is determined by

$$\frac{\partial m q_{c1}}{\partial p} + C - ar = 0.$$  

(18)

As noted previously, $a$ is the area of the horizontal cross-section of the cloud.

Solving Eq. (18), we find

$$q_{c1}(\lambda, p) = - \int_{p_B}^{p} C'(\lambda, p) \frac{\eta(\lambda, p)}{\eta(\lambda, p)} \exp \left[ \frac{a C_q}{m_B} \int_{p'}^{p} \frac{\eta(\lambda, p)}{\eta(\lambda, p)} dp \right] dp',$$  

(19)

where

$$C'(\lambda, p) = \frac{\lambda H}{p} \left[ (q_e - \tilde{q}) + \frac{\partial q_e}{\partial p} \right].$$

Two more assumptions have been made in this study. Following Arakawa, it is assumed that each cloud has a thin detrainment layer where it loses its buoyancy force, i.e., $h_e = \tilde{h}^*$. The height $p_D$ of the detrainment layer is determined by

$$\tilde{h}^*(p_D) = h_e(\lambda, p_D)$$

$$= \frac{1}{\eta(\lambda, p_D)} \left[ h_B(\lambda) - \int_{p_B}^{p_D} \frac{\lambda H}{\eta(\lambda, p)} dp \right].$$

(20)

where $p_D$ is a function of $\lambda$, i.e., $p_D = p_D(\lambda)$. Its inverse
function will be written as

$$\lambda_D = \lambda_D (p), \quad (21)$$

where $\lambda_D (p)$ can be interpreted as the entrainment rate of the cloud which detains at the level $p_D$. Usually, $\lambda_D$ is an increasing function of $p$. In other words, the shorter clouds have greater rate of entrainment. Note that at the detrainment layer

$$q_e (\lambda, p_D) = q^* (p_D), \quad (22)$$

$$s_e (\lambda, p_D) = s (p_D). \quad (23)$$

In addition, we assume that the height of the cloud base is common for all the clouds.

4. Spectrum representation of cloud groups

Adopting the cloud model just described, we find that for a given environmental condition, the properties of a single cloud are uniquely determined by the entrainment parameter $\lambda$. We shall assume that a cloud group is composed of a spectrum of clouds with different values of $\lambda$. In the previous section, we have considered the area of the cross section of an individual cloud. To discuss the cloud population, it is more convenient to consider the fractional area occupied by each category of clouds, rather than the cross-sectional areas of individual clouds and the number of clouds in each category. Let $\sigma (\lambda)$ be the fractional area density function for the cloud spectrum, that is, $\sigma (\lambda_0) d\lambda$ gives the total fractional area occupied by the clouds with $\lambda_0 - \Delta \lambda / 2 < \lambda < \lambda_0 + \Delta \lambda / 2$. Let $a$ be a quantity of interest. As before, the values of $a$ inside a cloud and in the region between clouds are denoted by $a_e$ and $a$, respectively. Then the large-scale average of $a$, $a$, is related to $a_e$ and $\tilde{a}$ by

$$\tilde{a} = \int \sigma (\lambda) a_e (\lambda) d\lambda + \left[ 1 - \int \sigma (\lambda) d\lambda \right] \bar{a}. \quad (24)$$

Usually

$$\int \sigma (\lambda) d\lambda \ll 1.$$

If the condition

$$|a_e - \tilde{a}| \ll \tilde{a} \quad (25)$$

is satisfied, the following relation holds approximately:

$$\tilde{a} = \tilde{a} + \int \sigma (\lambda) \left[ a_e (\lambda) - \tilde{a} \right] d\lambda \approx \tilde{a}. \quad (26)$$

Condition (25) is usually satisfied for $h$, $q$ and $s$. Therefore $h$, $\tilde{s}$ and $\tilde{q}$ in the proceeding section may be replaced by $h$, $\tilde{q}$ and $s$, respectively. This is not the case for $\omega$, however.

We now define

$$m (\lambda) = -\sigma (\lambda) \omega_e (\lambda) \quad (27)$$

as the cloud mass flux spectral distribution in $\lambda$-space. As defined, $m (\lambda)$ has units of mb day$^{-1}$ km if $\lambda$ has units of km$^{-1}$. The total mass flux due to all clouds at a height $p$ is then defined as

$$M_e (p) = \int_0^{\lambda_D} m (\lambda) d\lambda = \int_0^{\lambda_D} m (\lambda) \eta (\lambda, p) d\lambda, \quad (28)$$

where Eq. (11) has been used.

The total amount of entrainment due to all clouds at each level $p$ is given by

$$E (p) = \int_0^{\lambda_D} \frac{\lambda H}{p} m (\lambda) d\lambda. \quad (29)$$

By mass continuity, the total detrainment at each level is given by

$$\delta (p) = \frac{m (\lambda_0) d\lambda_D}{d\lambda} = \frac{m (\lambda_D) \eta (\lambda_D, p) d\lambda_D}{d\lambda}. \quad (30)$$

Using Eq. (24), one finds readily that

$$\frac{s \omega'}{\tilde{s}} = \int \sigma (\lambda) (s_e - \tilde{s}) (\omega_e - \tilde{\omega}) d\lambda$

$$= -\left[ \int \sigma (\lambda) (s_e - \tilde{s}) d\lambda \right] \left[ \int \sigma (\lambda) (\omega_e - \tilde{\omega}) d\lambda \right], \quad (31)$$

$$\frac{q \omega'}{\tilde{q}} = \int \sigma (\lambda) (q_e - \tilde{q}) (\omega_e - \tilde{\omega}) d\lambda$

$$= -\left[ \int \sigma (\lambda) (q_e - \tilde{q}) d\lambda \right] \left[ \int \sigma (\lambda) (\omega_e - \tilde{\omega}) d\lambda \right]. \quad (32)$$

If $\sigma \ll 1$ and $|\tilde{\omega}| \ll |\omega_e|$, the above equations can be reduced to

$$\frac{s \omega'}{\tilde{s}} = -\int m (\lambda) (s_e - \tilde{s}) d\lambda, \quad (33)$$

$$\frac{q \omega'}{\tilde{q}} = -\int m (\lambda) (q_e - \tilde{q}) d\lambda. \quad (34)$$

Combining (28)–(34), one obtains

$$\frac{\partial \tilde{s}}{\partial \lambda} = M_e (p) \frac{\partial s}{\partial \xi} - L \tilde{C}, \quad (35)$$

$$\frac{\partial \tilde{q}}{\partial \lambda} = M_e (p) \frac{\partial q}{\partial \xi} - \tilde{C} - \frac{\partial (\tilde{q}^* - \tilde{q})}{\partial \lambda}. \quad (36)$$

Substituting (35) and (36) into (4) and (5), we finally
have

\[ Q_1 - Q_R = -L \delta (\bar{q}^* - \bar{q}) - \frac{\partial h}{\partial p} \int_0^{\lambda_D} m_B(\lambda) \eta(\lambda, p) d\lambda. \]  

(40)

This equation contains no terms involving cloud drops. We may therefore use Eq. (40) to determine \( m_B(\lambda) \) without going into any parameterization scheme for the rain formation process. Once \( m_B(\lambda) \) is determined, we may use either one of the two equations (37) and (38) to determine \( \delta \) as a function of \( p \). With Eq. (39), \( q_{el}(\phi_D, \lambda) \) can then be determined as a function of \( \lambda \).

In this way, we may treat Eqs. (40) and (38) [or Eqs. (40) and (37)] as two equations for the two unknown functions \( m_B(\lambda) \) and \( q_{el}(\phi_D, \lambda) \), without knowing any microphysical aspect of rain formation. The total rate of rainfall per unit time can then be computed from

\[ R = - \int_{p_B}^{p_T} \left( \bar{C} - \bar{c} \right) \frac{dp}{\bar{g}} \bigg|_{p=p_B} \]  

(41)

where \( p_T \) is the pressure at the tropopause.

5. Results and discussions

To apply our method of determining cumulus cloud properties from large-scale heat and moisture balance considerations, information concerning \( Q_1 \) (apparent heat source), \( Q_2 \) (apparent moisture sink) and \( Q_R \) (radiative heating rate) are required as input data. The main objective of this article is to test the feasibility of the proposed method rather than to discuss the contribution of convective elements in various weather conditions in detail. As such, we use the average \( Q_1 \) and \( Q_2 \) computed by Nitta (1972) for the period from March through July, 1958, over the Marshall Islands. Because no directly measured \( Q_R \) is available, we shall use the climatological profile of \( Q_R \) computed by Doplick (1970); and since the large-scale temperature \( T \) and water vapor mixing ratio \( \bar{q} \) are also not available from Nitta’s paper, we shall use \( T \) and \( q \) (or \( \delta \) and \( h \)) as given by Yanai et al. (1973) for summer 1956 over the same area.

Fig. 1 shows the vertical profiles of \( Q_1, Q_2 \) and \( Q_R \) used as input data in our study. \( Q_1 \) is greater than \( Q_2 \) above 700 mb and smaller below this level; \( Q_1 \) reaches its maximum of 3.6C day\(^{-1}\) at about 500 mb while \( Q_2 \) reaches its maximum of 2.8C day\(^{-1}\) at about 730 mb. Since data for \( Q_2 \) above 400 mb are not given in Nitta’s paper, the value shown in Fig. 1 represents an arbitrary extrapolation. In general, \( Q_1 \) is remarkably different from \( Q_2 \). As pointed out earlier, \( Q_1 - Q_2 - Q_R \) represents the effect of cumulus clouds in large-scale heat and moisture balance. Although \( Q_R \) is smaller than \( Q_1 \) and \( Q_2 \) by a factor of 2 or 3, \( Q_R \) values are about the same.
as $Q_1 - Q_2$. Therefore, an accurate determination of cloud properties requires accurate values not only for $Q_1$ and $Q_2$, but also for $Q_B$.

Generally speaking, the vertical velocity inside the cloud at the cloud base is of the order of $1 \text{ m sec}^{-1}$. Its variation for different clouds does not seem likely to be more than a factor of 2 or 3. Therefore, as our first attempt, we assume that $C_0/\omega_B(\lambda)$ in Eq. (39) is a constant for all clouds and solve Eq. (38) for $m_B(\lambda)$ using information on $Q_2$ only. Once $m_B(\lambda)$ is determined, we shall use Eq. (37) to predict the values of $Q_1$. A comparison between the predicted and observed values of $Q_1$ shall be made as a test of our assumptions.

Before presenting the result, it is noted that $Q_2$, $\delta q_{v1}$ and $\delta (q^* - \bar{q})$ are all positive quantities in (38). This implies that the integral term is the largest one in (38), especially near the region of $\lambda = 0$ where $Q_2$, $q_{v1}$, $q^*$ and $\bar{q}$ are all nearly zero. Accurate numerical solutions of integral equations of this type are rather difficult to obtain for the first few steps, that is, near the region of $\lambda = 0$ in our case (Noble, 1964). Furthermore, values of moisture in the top portion of the troposphere are generally subject to large observational errors. For these reasons, we use an averaged form of Eq. (38) to determine the average mass flux above 200 mb, instead of trying to get a finer resolution. The proper choice of the value of $C_0$ is difficult to decide. Ogura and Takahashi's result (1971) indicates that it would be of the order of $10^{-3} \text{ sec}^{-1}$. We use $C_0/\omega_B = 0.0416 \text{ mb}^{-1}$. If $w_B$ (vertical velocity at the cloud base) is $1 \text{ m sec}^{-1}$, this value gives roughly $C_0 = 0.004 \text{ sec}^{-1}$.

Fig. 2 shows the variation of the mass flux at cloud base, $m_B(\lambda)$, thus determined. It has a minimum at $\lambda = 0.3$ which corresponds to a cloud with the cloud top roughly at the 500-mb level. In Fig. 3, the flux distribution at cloud base, $m_B(p)$, is shown as a function of the height of the cloud top: $m_B(p) = m_B(\lambda)(d\lambda/dp)$ in units of day$^{-1}$. A bi-modal distribution has emerged in the figures. There are two groups of clouds—one group penetrates up to 200–300 mb and another group stays below 600 mb, while relatively few cloud tops lie between 400 and 600 mb. In other words, the entire mass flux distribution appears to consist of two distinctly different distributions, one having a maximum
at the cloud base which decreases as the cloud top height increases, and the other a sharp maximum around 300 mb.

The input data used in this study correspond to the mean summertime tropical atmosphere and do not represent any particular weather conditions. Nevertheless, it is of some interest to refer to the following observed fact concerning the size distribution of clouds in different weather conditions: "When this ingredient (low-level convergence) is absent, the size spectrum of clouds exhibits a logarithmic distribution, with very few, if any, clouds reaching radii of 1.5-2.0 km (which appears to be the minimum size required to penetrate, undiluted, through the middle troposphere). In the absence of low-level convergence, clouds reach heights of only 3-6 km, and very little precipitation reaches the ground. However, when low-level convergence is present, the size distribution becomes highly bimodal, with just the very large and very small clouds present." (Plan for U. S. Participation in the GARP Atlantic Tropical Experiment, National Academy of Sciences, 1971).

Fig. 4 shows a comparison between the predicted and observed $Q_1$. The agreement is not too bad both in the magnitude and the general behavior of their vertical variations. The predicted $Q_1$ is smaller than the observed between 300 and 600 mb and larger than the observed below 730 mb. The difference can be attributed to several reasons. The most obvious one is the assumption of $\lambda$-independence of $\omega_N$.

Before passing, it is noted that the determination of $m_\beta(\lambda)$ from Eq. (38) is rather sensitive to the value we choose for $C_0/\omega_N$.

In order to remove the ambiguity in $C_0/\omega_N$, we find it most convenient to determine $m_\beta(\lambda)$ from Eq. (40) and then solve for $q_{c1}(\rho, \lambda)$ from either one of the two equations (37) and (38) and the assumption equation (39). As indicated in the last part of the preceding section, such a scheme does not depend on the value of $C_0/\omega_N$ at all. Figs. 5 and 6 show $m_\beta(\lambda)$ and $m_\beta(\rho)$ thus determined. These results generally agree with those obtained by the previous method. One obvious improvement is that we have a better resolution for clouds with small $\lambda$. The bi-modal structure in $m_\beta(\rho)$ is more evident than in Fig. 3.

Fig. 7 shows the vertical distribution of the total detrainment rate $\delta$. We observe that $\delta$ has a minimum at 500 mb and a local maximum at 300 mb. The large $\delta$ in the layer below indicates that a large number of shallow clouds are detrained immediately above the cloud base. Thus, the bi-modal structure of a cumulus ensemble is again clearly shown in Fig. 7. Fig. 8 gives the vertical distributions of the total mass flux $M_e$ due to all clouds as defined in Eq. (28), the large-scale mass flux $\overline{M}$ and the residual mass flux in the environment $\overline{M} - \overline{M_e}$. Starting from a large value near the cloud base, $M_e$ decreases as height increases until a local minimum is reached at 550 mb; a local maximum then occurs at around 300 mb. The general behavior of $M_e$ and the magnitude of $\delta$ and $M_e$ are quite comparable with those obtained by Yanai et al. and Gray, despite the difference in various assumptions and methods applied. The only noticeable difference may be that the heights of the local maxima for $\delta$ and $M_e$ are about 100 mb lower than those in Yanai et al.
Fig. 9. Vertical distributions of the contributions of various physical processes to the large-scale moisture balance.

Fig. 9 shows how the large-scale moisture balance is achieved at each level: $M_M(\partial \bar{q}/\partial p)$ represents the drying effect due to the compensating downdraft between clouds, $\delta(q_e-\bar{q})$ the detrainment of water vapor from clouds, and $\delta q_{e1}$ the evaporation of cloud drops which are detrained from clouds. In general, $M_M(\partial \bar{q}/\partial p)$ is the largest term in the large-scale moisture balance. While $M_M(\partial \bar{q}/\partial p)$ and $Q_2$ appear to be the dominant terms between 400 and 600 mb, $M_M(\partial \bar{q}/\partial p)$ and $\delta(q_e-\bar{q})$ become dominant below 800 mb.

Fig. 10 shows the large-scale heat balance in which $-M_M(\partial \bar{q}/\partial p)$ represents the adiabatic heating due to the compensating downward motion in the environment and $\delta q_{e1}$ represents the cooling due to evaporation of liquid drops detrained from clouds. There is no detrainment of sensible heat from clouds because of our assumption that the detrainment takes place only at the cloud top where cloud temperature is the same as that in the environment. Fig. 10 indicates that clouds act as a heating agent for large-scale motions through the induction of the compensating downward motion and act as a cooling agent through evaporation of cloud drops in the environment, but the downdraft warming is much larger than the evaporation cooling. The general behavior of the large-scale moisture and heat balances shown in Fig. 9 and 10 again agree with those obtained by Yanai et al., and Gray.

Fig. 11 and 12 show, respectively, the vertical distributions of the excess temperature $(T_e-\bar{T})$ and excess water vapor mixing ratio $(q_e-\bar{q})$ for two values of $\lambda[0.1, 0.5]$, together with the weighted average profiles $(\bar{T}_e-\bar{T})$ and $\bar{q}_e-\bar{q}$, where $\bar{T}$ is defined as

$$\bar{T} = \int T_e(\lambda) m(\lambda) d\lambda \left[ \int m(\lambda) d\lambda \right]^{-1},$$

and the similar definition applies to $\bar{q}_e$. We observe that the averaged profile of the excess temperature reaches a maximum of 2.2°C at 600 mb while that of the excess water vapor reaches a maximum of 4.8 gm kg$^{-1}$ at 780 mb.

The total rainfall rate calculated from Eq. (41) is about 11 mm day$^{-1}$. This is in good agreement with the observed value of 9.6 mm day$^{-1}$ indicated in Nitta’s paper.

Having determined $m_d(\lambda)$ from Eq. (40), we then determined $q_{e1}$ at the cloud top ($p = p_D$) from Eq. (38).

Fig. 10. Vertical distributions of the contributions of various physical processes to the large-scale heat balance.

Fig. 11. Vertical distributions of excess temperature inside clouds for two different entrainment rates ($\lambda$) and excess temperature averaged over all clouds.

Fig. 12. Same as Fig. 11 but for excess water vapor mixing ratio.
The same quantity was also computed from Eq. (37). These two values of \( q_1(p_0, \lambda) \) are found to agree with each other up to the fourth digit. However, when we proceeded to compute \( \omega_B(\lambda) \) from Eq. (39) using these computed values of \( q_1(p_0, \lambda) \), some inconsistency has been found. First, Eq. (39) indicates that \( q_1(p_0, \lambda) \) increases with increasing \( |\omega_B(\lambda)| \). For a given value of \( \lambda \), there is a maximum for \( q_1(p_0, \lambda) \) for all negative values of \( \omega_B(\lambda) \):

\[
\text{Max}(q_1(p_0, \lambda)) = \lim_{\omega_B \to -\infty} q_1(p_0, \lambda) = -\int_{\omega_B}^{p_D} C'(\lambda) \frac{\eta(\lambda, \rho'_D)}{\eta(\lambda, \rho_D)} d\rho'.
\]  

(42)

Therefore, if the values of \( q_1(p_0, \lambda) \) determined from Eq. (38) [or Eq. (37)] and Eq. (40) are larger than \( \text{Max}(q_1(p_0, \lambda)) \) in Eq. (42), no negative solution can be found for \( \omega_B(\lambda) \) from Eq. (39). Indeed, this has been found for clouds which do not penetrate above the 800-mb level (\( \lambda > 0.75 \)). This fact could be due to several reasons. First of all, \( \delta q_1 \) is the smallest term in the large-scale moisture balance (Fig. 9), amounting to only 10–20% of the leading term \( \dot{M}_g(\delta q/\delta p) \). Therefore, the value we have obtained for \( q_1(p_0, \lambda) \) may be subject to large errors which are ultimately traced to errors involved in \( Q_1, Q_2, Q_R \) and \( m_B(\lambda) \). Second, the rain formation process represented in (17) may be oversimplified and (39) may not be very realistic. Third, the assumption equation (39) may be questionable. Evaporation from raindrops can also have some contribution to the total evaporation rate \( \dot{\epsilon} \). We shall make more comments on these points later.

In any case, \( \omega_B(\lambda) \) has been computed for those clouds for which negative solutions exist for Eq. (39). The result is shown in Fig. 13 in terms of \( \omega_B \) rather than \( \omega_B \). A value of 0.001 sec\(^{-1}\) was assigned to \( C_0 \). The variation of \( \omega_B \) as a function of \( \lambda \) does not show any regular behavior. It is again remarked that \( \omega_B \) thus determined is very sensitive to \( q_1(p_0, \lambda) \): a small variation of \( q_1(p_0, \lambda) \) results in a large variation of \( \omega_B(\lambda) \). A local maximum of \( \omega_B \) at \( \lambda = 0.4 \) can be traced to wiggles of small amplitude around 600 mb in the vertical profile of \( \delta q_1 \) (Figs. 9 and 10). For these reasons, all we can say at this stage appears to be that \( \omega_B(\lambda) \) is somewhere near 1 m sec\(^{-1}\) for all clouds. Regarding this value of \( \omega_B \), it is cautioned that \( \omega_B(\lambda) \) appears in Eq. (39) in combination with \( C_0 \) as \( C_0/|\omega_B(\lambda)| \). Consequently, if \( C_0 \) is chosen as 0.005 sec\(^{-1}\), \( \omega_B(\lambda) \) should be somewhere near 5 m sec\(^{-1}\).

If we assume that \( \omega_B \) is independent of \( \lambda \), the variations of \( m_B \) with the height of the cloud top (Fig. 6) can be immediately converted to the variation of \( \sigma \) (fractional area coverage of clouds) with the cloud top height, because of the relation \( m_B = -\sigma \omega_B \). The ordinate on the right side of Fig. 6 was drawn in this way with the assumption of \( \omega_B = 1 \) m sec\(^{-1}\) irrespective of \( \lambda \). Fig. 6 then indicates that the total percentage area covered by clouds (\( \sigma \)) is approximately 1.5%. Gray (1972) noted that rain areas in the tropics take up 10% of the cluster which in turn takes up about 20% of the entire tropical belt. He further noted that only about 10–20% (or 0.2–0.4%) of the entire tropical belt of the rain areas have active towering cumulus or cumulonimbus updrafts in operation. The few percent of \( \sigma \) derived above may then not be too unrealistic for the averaged tropical situation.

Based on the above result, we may depict the schematic mass circulation in the summertime tropical atmosphere in the following way. First, at levels near the cloud base, \( M_e \) and \( \bar{M} \) are of the same order of magnitude while \( \bar{M} \) is one order of magnitude less than \( M_e \) or \( \bar{M} \) (Fig. 8). This indicates that mass one order of magnitude larger than mass converged in the sub-cloud layer enters into ascending clouds. This excess mass is of course compensated by the downward motions in the area between clouds. This is the aspect emphasized by Gray (1972) for the mass circulation in cloud clusters. An implication of this conclusion is that low-level convergence may give little indication of the actual vertical circulation in operation. Another important implication is the interaction between the convective layer and the sub-cloud layer. The large mass entering from the convective layer into the sub-cloud layer in the region between clouds is expected to have a significant effect on the structure of heat, moisture and momentum fields in the subcloud layer.

Second, the total fractional area coverage of clouds (\( \sigma \)) is of the order of 1%. Therefore, if the updraft inside clouds at the cloud base is of the order of 1 m sec\(^{-1}\), \( \bar{w} \) (downdraft in the environment) at that height must be of the order of 1 cm sec\(^{-1}\). Consequently, \( \bar{w} \) (large-scale upward velocity) must be of the order of 0.1 cm sec\(^{-1}\). This value of \( \bar{w} \) is consistent with the observed value of \( \bar{M} \) (Fig. 8) or with the observed average convergence of the order of 10\(^{-4}\) sec\(^{-1}\) at low levels for the mean tropical conditions (Nitta, 1972; Yanai et al., 1973) and for cloud clusters (Williams, 1970).

We remark that the mass circulation and the typical values of \( \sigma \), \( \bar{w} \) and \( \bar{w} \) given above may not be true for
tropical cyclones. For the core region of tropical cyclones, it is more likely that $\delta \approx 10\%$ and $\bar{M} \approx M_s = \bar{M}$ at the cloud base.

6. Concluding remarks and future work

In this article a method is proposed whereby some properties of cumulus cloud populations are determined from observed large-scale variables. Some encouraging results are obtained by applying this method to the mean conditions of the summertime tropical atmosphere.

In order to discuss in detail the role of convective clouds in large-scale heat and moisture balances, this method should be applied to individual cases in different weather conditions, such as clear and cloudy conditions with variable clouds or cloud clusters as classified by Gray (1972) or weather conditions as observed in different segments of propagating easterly waves by Reed and Recker (1971). We are planning to do so in order to better understand the relationships between cloud activity and the large-scale flows. It is expected that the projected GARP Atlantic Tropical Experiment (GATE), particularly its B-scale observational network, will provide much needed data. Upper air soundings, observations of the radiative heat flux, planetary boundary layer by tethered balloon systems, and vertical fluxes of heat and water vapor at the sea surface will make it possible to calculate the input data $Q_h, Q_s$ and $Q_M$ accurately. The cloud population will then be predicted. This prediction will be compared with high-resolution image data from the GOES and radar data. Aircraft data will be utilized also.

The proposed method requires the information of the time derivatives of some large-scale variables such as static energy and moisture. These time derivatives are the ones which are unknown in a prognostic problem. It is hoped that the cumulus parameterization scheme used for predicting large-scale circulations can be improved by establishing relationships between large-scale flows and cumulus ensembles.

Another direction we may take in improving the present diagnostic approach will be to incorporate a more sophisticated cloud model. In this article we incorporated a simple steady-state model. Even though in reality all clouds undergo life cycles, the steady-state assumption may be acceptable. This is because we are not concerned with the life cycle of an individual cloud but with the statistical properties of many clouds averaged over the wide domain.

The question is whether the steady-state model provides the vertical distributions of relevant cloud variables with sufficient accuracy. One of the authors has developed with Takahashi a fairly sophisticated time-dependent one-dimensional model (Ogura and Takahashi, 1973). Fig. 14 compares the vertical distributions of the mass flux inside the individual cloud computed by the two different models. The solid line marked with $\bar{m}$ is the mass flux at $t=40$ min and the line marked with $\langle \bar{m} \rangle$ is the mean vertical flux averaged over the entire life cycle ($\rho \bar{m}$). Dashed lines are from the present cloud model for two different values of $\lambda$.

![Fig. 14. Vertical distributions of the vertical mass flux inside an individual cloud. Solid lines are from Ogura and Takahashi’s cloud model at 40 min ($\rho \bar{m}$) and averaged over the entire life cycle ($\langle \rho \bar{m} \rangle$). Dashed lines are from the present cloud model for two different values of $\lambda$.](image-url)
(1971). Subsequently, Ogura and Takahashi (1973) extended their work so that the conversion from cloud droplets to precipitating raindrops was not parameterized. A total of 61 mass categories corresponding to radii from 4 μm to 4 mm were used to determine the size distribution of drops. The size distribution of water drops evolves with time as a result of condensation, evaporation, stochastic coalescence, sedimentation, and breakup. It is therefore interesting to compare the height-time variation of the mixing ratios of cloud droplets and raindrops shown in Fig. 3 of Ogura and Takahashi’s 1971 paper with these shown in Figs. 19 and 20 of the 1973 paper. The experimental conditions (environmental and initial conditions) for those two experiments are different. Nevertheless, the qualitative differences between these two experiments are evident. Fig. 20 of the 1973 paper shows that raindrops are produced only in the upper portion of the clouds and at a later stage of the development of the cloud. This behavior is not well simulated in the 1971 model with the parameterization scheme of Eq. (17).

We may also mention that in this article we have considered only active clouds and assumed that evaporation of liquid water takes place only in the region outside the cloud. On the other hand, Ogura and Takahashi’s result (1973) shows that an appreciable portion of the entire evaporation process also takes place inside the cloud during the decaying stage of the cloud. Moreover, as pointed out in the preceding section, we have considered only evaporation from small cloud drops. This again is not completely satisfactory.

For these reasons, it will be very interesting to incorporate a more sophisticated time-dependent cloud model into our diagnostic study and see how the resulting cloud population differs from that obtained with the steady-state entraining cloud model. In this approach, the vertical distributions of relevant cloud variables used here will become those of time-dependent clouds averaged over the life cycles for each category of clouds.

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