Relative Dispersion: Local and Nonlocal Dynamics

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ABSTRACT

Observations of relative dispersion statistics for stratospheric balloons, including velocity structure functions, relative diffusivities and separation kurtoses, are shown to be consistent with nonlocal dynamics rather than local, self-similar dynamics.

1. Introduction

Observations of relative dispersion of balloons in the stratosphere (Morel and Larcheveque, 1974; Er-El and Peskin, 1981) have been interpreted as evidence for the existence of enstrophy-cascading, self-similar inertial subranges of isotropic and stationary two-dimensional turbulence. The altitudes of the balloons were, on average, 200 mb (Morel and Larcheveque) or 150 mb (Er-El and Peskin). The proposed subranges were inferred from balloon separations between 100 and 3000 km, and elapsed times of 15 days or less from "launch" (either as an "original" or a "chance" pair). The evidence in the studies mentioned above comprised structure functions and relative diffusivities which were approximately proportional to the square of the separation, and separation variances which grew approximately exponentially in time. The balloons were constrained to surfaces of constant density, and thus they could not have behaved strictly as quasi-geostrophic Lagrangian particles. Nevertheless, the evidently reasonable success of two-dimensional local self-similarity theory, in describing the observed particle dispersion, suggests that the identification of balloon paths with particle paths was not a gross oversimplification.

In fact, it has been known for some time (Kraichnan, 1967, 1971; Peskin, 1973; Kowalski and Peskin, 1981; Salmon, 1980; Rhines, 1983), that relative dispersion, at separations lying in an enstrophy-cascading subrange, is weakly nonlocal. That is, relative dispersion is not controlled predominantly by that part of the relative motion field which has the scale of the separation. Rather, it is controlled more by the rate-of-strain field associated with the much larger energy-containing eddies. The purpose of this note is to show that the balloon observations are also consistent with the hypothesis of general nonlocal dynamics, that is, any subrange in which the wavenumber spectrum of enstrophy is not integrable in the low wavenumber limit.

An especially interesting aspect of the analysis of Er-El and Peskin is their estimation of probability distribution functions (pdfs) for zonal and meridional components of separation, five days after launch. The estimated kurtoses have values of 7.5 and 7.0, respectively. The assumption of self-similarity implies that the kurtosis, being a shape factor, must be constant. Estimates of the kurtoses at two times after launch would have provided a decisive test of the self-similarity theory. The Richardson–Kraichnan equation for the temporal evolution of the separation pdf is used here to derive self-similar pdfs for local dynamics, and, for nonlocal dynamics, pdfs which initially are Dirac functions corresponding to statistically sharp initial separations. The kurtoses are constant for local dynamics, and exponentially growing for nonlocal dynamics. These results for kurtoses may be derived from the Richardson–Kraichnan equation without assuming special forms for the separation pdfs.

A feature of the locally and nonlocally derived estimates for structure functions, diffusivities and kurtoses is the continuity of the estimates as functions of spectral slope, in particular at the slope of −3 which marks the transition from local to nonlocal dynamics.

It is shown that the random-straining model (Batchelor, 1953; Kraichnan, 1974) contains the essentials of nonlocal relative dispersion, and is consistent with the observations.

Some implications for the relative dispersal of tracers, and for numerical modeling of relative dispersion, are drawn in Section 5.

2. Relative dispersion: Enstrophy subranges

The self-similarity theory of relative dispersion in an enstrophy subrange appears to be due to Lin (1972). He deduced that the relative diffusivity is given by

$$\frac{d}{dt} \overline{D^2} = c_\eta \overline{\theta^2} D^2,$$

where $\overline{D^2}$ is the mean square of the magnitude $D$ of
the two-dimensional separation vector \( \mathbf{D} \); \( \eta \) is the enstrophy dissipation rate, \( t \) is the elapsed time from launch and \( c_1 \) is a dimensionless constant of order unity. Equation (2.1) is dimensionally consistent, but it cannot be deduced by dimensional arguments alone. It must be assumed that the relative diffusivity is independent of the initial separation \( D_0 \), and the elapsed time, leaving only a self-similar dependence on \( D^2 \) and \( \eta \). Lin pointed out that the time scale need not be that associated with an enstrophy cascade; the crucial assumption is the existence of a single time scale. Equation (2.1) is readily integrated, giving

\[
\frac{d}{dt} \mathbf{D} = D_0^2 \exp(c_1 \eta^{1/3} t),
\]

(2.2)

Er-El and Peskin found reasonable support for (2.2), with \( 2\tau/c_1 \approx 2.6 \) days, where \( \tau = \eta^{-1/3} \). In fact, they considered relative dispersion in zonal and meridional directions; similar results were obtained for each.

Morel and Larcheveque assumed the existence of a kinetic energy spectrum with the self-similar form

\[
E(k) \sim k^{-\alpha},
\]

where \( k \) is the magnitude of the two-dimensional wavenumber vector \( k \). They then deduced, using self-similarity arguments, that the structure function and relative diffusivity should be, respectively,

\[
\frac{d}{dt} \mathbf{D} \sim D^{\alpha-1},
\]

(2.3)

and

\[
\frac{d}{dt} \mathbf{D}^2 \sim D^{(\alpha+1)/2}.
\]

(2.4)

Morel and Larcheveque found good support for (2.2) and (2.4), with \( 2\tau/c_1 \approx 2.7 \) days and \( \alpha \approx 3 \). This value for the spectral slope is consistent with the assumptions of self-similarity and a single time scale. It has also been deduced from detailed calculations by Kraichnan (1967) for an enstrophy subrange. Equation (2.3) was not well supported by the observations. If anything, a value of \( \alpha \) between 2 and 3 was indicated.

Er-El and Peskin estimated \( P_l(D_1, t) \) and \( P_2(D_2, t) \), which were respectively the pdfs for the zonal and meridional components of \( \mathbf{D} = (D_1, D_2) \), where \( t = 5 \) days was the elapsed time from launch. The two pdfs had kurtoses \( g_1 = 7.5 \) and \( g_2 = 7.0 \), respectively. If we attempt to deduce dimensionless functions such as \( g_i \) directly using similarity arguments, we find only that \( g_i = G_i(\eta^{1/3} t) \) where \( G_i \) is an unknown universal function. Alternatively we may use similarity arguments to deduce that the fourth moment of \( D_1 \) has an exponential time dependence like that in (2.2), and so

\[
g_i(t) = g_i(0) \exp[(c_2 - 2c_1)\eta^{1/3} t].
\]

(2.5)

where \( c_2 \) is a dimensionless constant of order unity (and so on for \( c_n \), \( n = 2, 3, 4, \ldots \)). Note that the sign of the exponent in (2.5) is indeterminate. However, the fact that \( g_i \) is a dimensionless measure of the shape of pdf implies that it is constant for a self-similar pdf. That is, the exponent in (2.5) should vanish. No value for \( g_i \) is indicated thus far by self-similarity theory.

3. Relative dispersion: Local and nonlocal dynamics

The functional forms of the statistical measures of relative dispersal may be estimated for local and nonlocal dynamics, using the general spectral representations of the measures.

a. Structure functions

The variance of the separation velocity, or structure function, in isotropic and stationary two-dimensional turbulence is

\[
\frac{d}{dt} \mathbf{D} = \frac{\mathbf{u}(x + \mathbf{D}, t) - \mathbf{u}(x, t)}{2} \int_{0}^{\infty} E(k)[1 - J_0(kD)]dk,
\]

(3.1)

where \( J_0 \) is the zero-order Bessel function of the first kind. The weighting of the energy spectrum \( E(k) \) has the limiting behavior

\[
1 - J_0(kD) \approx \begin{cases} 
\frac{1}{4} k^2 D^2, & kD \ll 1 \\
1 + O(kD)^{-1/2}, & kD \gg 1
\end{cases}
\]

(3.2)

The approximations in (3.2) show that the larger scale eddies elongate lines of length \( D \) by coherent deformation (relative dispersion), while the smaller scale eddies send the ends of the lines on independent random walks (absolute dispersion).

Suppose there is a scale subrange in which \( E \) has the form \( k^{-\alpha} \), and suppose that \( D \) lies in this subrange. If \( \alpha \ll 1 \) then the integral in (3.1) diverges as \( k \to \infty \). Hence the structure function is dominated by the energy on smallest scales: Dispersion is absolute rather than relative. If \( 1 < \alpha < 3 \), then the integral in (3.1) converges as \( k \to 0 \) and \( k \to \infty \), so the structure function is dominated by the relative kinetic energy on scales of order \( D \). That is, dispersion is relative and the local self-similarity arguments of Morel and Larcheveque are valid. Simple rescaling of (3.1) shows that

\[
\frac{d}{dt} \mathbf{D} \sim D^{\alpha-1},
\]

(3.3)

which is in agreement with (2.3). If \( \alpha \ll 3 \) then the integral in (3.1) is divergent as \( k \to 0 \), and the structure function is dominated by the variance of the rate-of-strain of the large-scale energy-containing eddies. Dispersion is relative, and is controlled by nonlocal dynamics. If \( \alpha = 3 \), the divergence as \( k \to 0 \) is only logarithmic. There is a significant contribution from eddies as small as \( D \) and the dynamics are only weakly
nonlocal. If $\alpha \gg 3$, the large scale eddies dominate, and we have for strongly nonlocal dynamics
\[
\left| \frac{d\mathbf{D}}{dt} \right|^2 \approx c_3 \Omega^2 D^2, \tag{3.4}
\]
where
\[
\Omega^2 = \int_0^\infty k^2 E(k)dk \tag{3.5}
\]
is the total enstrophy. The latter provides a well-defined time scale $T = \Omega^{-1}$. The local form (3.3) matches the nonlocal form (3.4) as $\alpha \to 3$. It is to be expected that (3.4) holds also for weakly nonlocal dynamics ($\alpha = 3$), provided that the total enstrophy time scale $T$ is replaced by the enstrophy cascade time scale $\tau = \eta^{-1/3}$.

b. Relative diffusivities

The time rate of change of the separation variance, or relative diffusivity, in isotropic and stationary two-dimensional turbulence is
\[
\frac{d}{dt} \frac{\mathbf{D}^2}{D^2} = \mathbf{D}_0 \cdot \frac{d\mathbf{D}}{dt} + 4 \int_0^\infty E(k)[1 - J_0(kD)] \int_0^t R(k, s)dsdk, \tag{3.6}
\]
where $\mathbf{D}_0$ is the initial separation vector, and $R$ is a normalized Lagrangian energy spectrum:
\[
R(k, s) = U(k, s)/U(k, 0), \tag{3.7}
\]
where
\[
U(k, s) = (2\pi)^{-2} \int \int \exp(ik \cdot \mathbf{D}) \times u(x + \mathbf{D}, t)u(x, t + t - s)d^2\mathbf{D}, \tag{3.8}
\]
and
\[
U(k, 0) = (2\pi k)^{-1} E(k). \tag{3.9}
\]
In (3.8), $u(x, t + t - s)$ is the velocity at time $t - s$ of a particle which passes through the point $x$ at time $t$. Equation (3.6) is the two-dimensional analog of (4.5) in Kraichnan (1966b). The first term on the right-hand side of (3.6) vanishes if the direction of $\mathbf{D}$ at launch is independent of the wind field. Of course, the effective launch occurs once the balloon pair reaches its mean altitude, and so $\mathbf{D}_0$ and $(d\mathbf{D}/dt)_0$ will have some correlation. In this regard, there is no clear distinction between “original” and “chance” pairs. In either case the correlation vanishes after several integral time scales (say $t > 2T$). Morel and Larchevèque found no significant differences in relative diffusivities for original and chance pairs.

Assume again that $E(k) \sim k^{-\alpha}$, and suppose $R(0, s) \neq 0$. Then the transition from local to nonlocal dynamics in (3.6) occurs again at $\alpha = 3$. Given local dynamics ($1 < \alpha < 3$) and sufficiently large $t$, the integral over $s$ in (3.6) becomes an integral time scale which may be estimated using self-similarity arguments:
\[
\int_0^\infty R(k, s)ds \sim E^{-1/2}k^{-3/2} \sim k^{(\alpha - 3)/2}. \tag{3.10}
\]
The integral over $k$ in (3.6) is dominated by $k \sim D^{-1}$, leading to
\[
\frac{d\mathbf{D}^2}{dt} \sim D^{(\alpha + 1)/2}, \tag{3.11}
\]
in agreement with (2.4). It is not clear that (3.10) holds long before $(D^2)^{1/2}$ is as big as the energy-containing eddies, by which time the dispersion should be absolute. Nevertheless, (3.11) is well supported by the data ($\alpha = 3$). For weakly nonlocal dynamics exemplified by the enstrophy subrange ($\alpha = 3$), the Lagrangian correlation can only have the form
\[
R(k, s) = S(s/\tau),
\]
where $S$ is an unknown universal function. The double integral in (3.6) is then separable, yielding the integral over $k$ in (3.1) which is logarithmically divergent as $k \to 0$. For strongly nonlocal dynamics ($\alpha > 3$) we may again argue that the Lagrangian correlation $R(k, s)$ is insensitive to $k$. The argument (Kraichnan, 1971) is that the decorrelation time $T(k)$ at wavenumber $k$ is given by the cumulative enstrophy in larger scales:
\[
T(k)^{-2} = \Omega(k)^2 = \int_0^k l^2 E(l)dl, \tag{3.12}
\]
which converges rapidly ($\alpha \gg 3$) for large $k$ to the total enstrophy $\Omega^2$. Thus $R$ is approximately of the form
\[
R(k, s) \approx S(t/T). \tag{3.13}
\]
This leads to
\[
\frac{d}{dt} \frac{\mathbf{D}^2}{D^2} \approx \mathbf{D}_0 \cdot \frac{d\mathbf{D}}{dt} + c_4 D^2 T^{-1} A(t/T), \tag{3.14}
\]
where
\[
A(\alpha) = \int_0^\infty S(b)db. \tag{3.15}
\]
In particular, $A \approx a$ for $a \ll 1$ and $A \to c_5$ as $a \to \infty$. Thus, for $(t/T)$ small we have
\[
\frac{d\mathbf{D}^2}{dt} \approx \mathbf{D}_0 \cdot \frac{d\mathbf{D}}{dt} + c_4 D^2 T^{-2} t, \tag{3.16}
\]
while for $(t/T)$ large we have
\[
\frac{d\mathbf{D}^2}{dt} \approx c_5 D^2 T^{-1}. \tag{3.17}
\]
So far, $D$ has been a conditioned random variable. If we average over all $D$ we find that, for $\alpha > 3$,
\[ \frac{d D^2}{dt} = \begin{cases} D_0 \left( \frac{d D}{dt} \right)_0 + c_4 D^2 T^{-1} t, & t \ll T \quad (3.18) \\ c_5 D^2 T^{-1}, & t \gg T. \end{cases} \]

The match between (3.19) and (3.11) indicates that (3.19) holds also for \( \alpha = 3 \), if we replace \( T \) with \( \tau \). Thus the exponential law (2.2) is approximately valid for all \( \alpha > 3 \). The midlatitude observations of \( D^2 \) reported by Er-El and Peskin indicated that (3.19) is valid only for \( t \gg T \). There are insufficient data to test (3.18).

c. Probability distribution functions

Let \( P_2(D, t) = P_2(D_1, D_2, t) = P_2(D, \theta, t) \) be the bivariate probability distribution function for the two-dimensional separation vector \( D \). The angle \( \theta \) is the azimuth of \( D \) measured anticlockwise from east. For isotropic turbulence \( P_2 \) must have the form

\[ P_2(D, t) = (2\pi)^{-1} D P(D, t), \]

where

\[ \int_0^\infty D P(D, t) dD = 1. \]

Richardson (1926) proposed that \( P \) satisfy

\[ \frac{\partial P}{\partial t} = D^{-1} \frac{\partial}{\partial D} \left( DK \frac{\partial P}{\partial D} \right) \]

(3.22)

(where \( K \) is the relative diffusivity) for \( 0 < t < \infty \) and \( 0 < D < \infty \), given suitable initial and boundary conditions for \( P \):

\[ P(D, 0) = P_0(D), \quad 0 < D < \infty, \]

where \( P \) satisfies (3.21), and

\[ DK \frac{\partial P}{\partial D} \rightarrow 0 \quad \text{as} \quad D \rightarrow 0, \infty. \]

Kraichnan (1966b) has derived Richardson’s equation (3.22) from the Navier–Stokes equations, using the Lagrangian History Direct Interaction Approximation (Kraichnan, 1965, 1966a).

For local, self-similar dynamics \( (E \sim k^{-\alpha}, 1 < \alpha < 3) \) we have from (3.11) that

\[ K = \frac{d D^2}{dt} = h D^\beta, \]

where \( 1 < \beta = (\alpha + 1)/2 < 2 \), and \( h \) is a dimensional constant. There is a corresponding self-similar solution of (3.22),

\[ P(D, t) = Q t^{-\gamma} \exp(-c D^2 - \beta t)^{-1}, \]

where \( \gamma = 2(2 - \beta)^{-1}, c = h^{-1}(2 - \beta)^{-2} \) and \( Q = Q(\beta, h) \) is a normalization factor such that (3.21) is satisfied. Note that

\[ DP(D, t) \rightarrow \delta(D) \]

as \( t \rightarrow 0 \), where \( \delta \) is the Dirac delta function.

For nonlocal dynamics \( (\alpha \gg 3) \) the relative diffusivity is given by (3.14). It is inappropriate to seek a self-similar solution of (3.22); in fact none exists. Equation (3.27) suggests that we seek instead a solution for which the initial separation is statistically sharp, with value \( D_0 \):

\[ DP_0(D) = \delta(D - D_0). \]

If the launch correlation in the relative diffusivity (3.14) is neglected, then an application of the Laplace transform to (3.22)–(3.24) and (3.28) leads to (Lundgren, 1981)

\[ P(D, t) = (2D_0 D^{-1}(\pi f)^{-1/2}) \times \exp \left\{ -f - \left[ \ln(D/D_0) \right]^2 (4f)^{-1} \right\}, \]

where

\[ f(t) = c_4 \int_0^t A(a) da \approx c_5(t/T) \]

as \( (t/T) \rightarrow \infty \). It is consistent to assume that (3.29) holds also for weakly nonlocal dynamics \( (\alpha = 3) \), provided \( T \) is replaced by \( \tau \).

d. Kurtoses

Er-El and Peskin estimated the kurtosis of \( D_1 \), the zonal component of separation. Since \( D_1 = D \cos \theta \), the moments of \( D_1 \) are readily evaluated using the isotropic joint distribution (3.20) for \( D \) and \( \theta \). In particular, all odd moments of \( D_1 \) vanish identically. For local dynamics \( (1 < \alpha < 3) \), it follows from (3.26) that

\[ \frac{D_1^m}{m!} \propto t^{m(2-\beta)}, \quad m = 0, 1, 2, \ldots. \]

and so the kurtosis \( g_1(t) \) is independent of time (but dependent on \( \alpha \) and dimensionless constants). This is to be expected for a self-similar distribution. For nonlocal dynamics \( (\alpha \gg 3) \) it follows from (3.29) that the kurtosis \( g_1(t) \) is asymptotically exponential in time:

\[ g_1(t) = \frac{3}{2} \exp[8f(t)], \]

where \( f \) is given by (3.30). The factor of \( 3/2 \) is the kurtosis of \( \cos \theta \). This is consistent with a constant kurtosis (of \( 3/2 \)) as \( \alpha \rightarrow 3 \), provided that \( c_4 = c_4(\alpha) \rightarrow 0 \) as \( \alpha \rightarrow 3 \). Since Er-El and Peskin found \( g_1 \) (5 days) \( = 7.5 \), and since the energy spectrum was probably close to the frequently reported \( k^{-3} \) profile (e.g., Boer and Shepherd, 1983) it must be the case that \( c_4 \) is sensitive to values of \( \alpha \) near 3. Alternatively, planetary-scale anisotropic effects may have become important.

Finally, we note that (3.31) and (3.32) may be derived directly from the Richardson–Kraichnan equation (3.22), given only the relative diffusivities (3.25) and (3.14). That is, special forms for \( P \) need not be assumed. See the Appendix for details.

4. Relative dispersion: The random-strain model

There is an elegant model of relative dispersion controlled by nonlocal dynamics. The model was proposed
by Batchelor (1953) and developed in detail by Kraichnan (1974). In this model, the separation $D$ is controlled by the random differential equations
\[
\frac{d}{dt} D_1 = \rho D_1, \quad (4.1)
\]
\[
\frac{d}{dt} D_2 = -\rho D_2. \quad (4.2)
\]
In (4.1) and (4.2), $D_1$ and $D_2$ are not the zonal meridional components of $D$, but instead are the components in the direction of the principal axes of the symmetric part of the local rate-of-strain tensor. These axes, and the separation vector itself, rotate as the motion is followed. The principal rates of dilatation and compression are $\rho$ and $-\rho$, respectively. They sum to zero since the flow is incompressible. In general, $\rho$ is a stationary random function of time, with a positive mean $\bar{\rho}$ of order $T^{-1}$, and a decorrelation time scale $T_0$ of order $T$. Following Kraichnan (1974), however, we shall eventually assume $T_0 \ll T$ in order to obtain “clean results”.

Several general results may be drawn from (4.1) and (4.2). First, the structure function is given by
\[
\left| \frac{dD}{dt} \right|^2 = \rho^2 D^2, \quad (4.3)
\]
in agreement with (3.4). Second, the relative diffusivity is
\[
\frac{dD^2}{dt} = 2\rho(D_1^2 - D_2^2), \quad (4.4)
\]
which will be seen to be $\sim \bar{D}^2$ for $t \gg T$, in agreement with (3.19). The general solutions of (4.1) and (4.2) are
\[
D_1 = D_{10} \exp(q), \quad (4.5)
\]
\[
D_2 = D_{20} \exp(-q), \quad (4.6)
\]
where
\[
q = q(t) = \int_0^t p(s)ds. \quad (4.7)
\]

Cocke (1972) has shown that if $T_0 \ll t$, then $q$ is a normal process with mean $\bar{\rho}t$ and variance $2\nu^2 T_0 t$, where $\nu^2 (\sim T^{-2})$ is the variance of $\nu$. It follows that
\[
\ln D^2 \approx \ln D^2_1 \approx (\bar{\rho} + \nu^2 T_0)t \quad (4.8)
\]
for $t \gg T$, while the kurtosis of the zonal component of $D$ is
\[
g_1 \approx (\gamma_2) \exp(8\nu^2 T_0 t). \quad (4.9)
\]

It has been assumed that at “launch”, the separation $D$ lies along the dilatation axis. An extreme alternative is to assume that their directions are independent initially; this would raise $g_1$ by a further factor of $^{3/2}$.

The relative diffusivity time scale (2.6 days) and kurtosis (7.5) reported by Er-El and Peskin are consistent with (4.8) and (4.9), if, for example, $\bar{\rho}^{-1} = \nu^{-1} = 2.9$ days, and $T_0 = 0.34$ days. This estimate for $T_0$ is unrealistically small, but consistent with (4.8).

It is possible to derive an evolution equation for the pdf of $D$ governed by (4.1) and (4.2). Kraichnan (1974) has derived an analogous equation in wavenumber space for the case of $\rho(t)$ being white noise ($T_0 \sim 0$, $\nu^2 T_0$ finite); see also Rhines (1983). In any case, the moments of $D$ may be estimated directly from the exact solutions (4.5) and (4.6). The moments of $D$ and $k$ grow exponentially as $|t| \rightarrow \infty$. Neighboring particles were, in expectation, further apart at earlier times and so gradients of passive scalars may be expected to intensify.

5. Summary

The dependencies of separation statistics upon separation and time may be estimated using the general spectral representations of the statistics, together with the concepts of local and nonlocal dynamics. The essential feature of local dynamics is the validity of self-similarity arguments. Nonlocal dynamics are characterized by a unique time scale ($\eta^{-1/3}$ for weakly nonlocal dynamics where $\eta$ is the energy transfer cascade rate; $\Omega^{-1}$ for strongly nonlocal dynamics where $\Omega$ is the total energy). Structure functions and relative diffusivities for local and nonlocal dynamics match at the weakly nonlocal transition. The Richardson-Kraichnan equation leads to self-similar separation pdfs with constant kurtoses for local dynamics, and log-normal pdfs with exponentially growing kurtoses for nonlocal dynamics. The Batchelor-Kraichnan random strain model captures the essentials of nonlocal dynamics.

The balloon observations are consistent with nonlocal dynamics. The large kurtoses indicate strong nonlocality. On the other hand, Eulerian estimates of kinetic energy spectra frequently indicate weak nonlocalness (the $-3$ subrange), which is supported by evidence of a broad maximum in the energy transfer cascade rate (Boer and Shepherd, 1983).

Numerical simulations of two-dimensional turbulence usually produce steep spectra indicative of nonlocal dynamics; spectra slopes of $-4.5$ are common (see, e.g., Bennett and Middleton, 1983; Bennett and Haidvogel, 1983, and references therein). Statistically reliable estimations of separation pdfs would be costly but instructive.

Local dynamics should cause a small blob of passive scalar to become highly convoluted. Major cross-diffusion should occur before the blob envelope is comparable with the large-scale eddies (Garrett, 1983). Nonlocal dynamics should cause such a blob to be drawn into a few long streaks. Minor cross-diffusion should occur before the streak lengths are comparable with the large-scale eddies. Thereafter, the ends of streaks will be sent on independent random walks, with probabilities of $O(1/T)^{1/2}$ of being as close ever again. Only loose winding of streaks and hence little cross-diffusion should be expected: “streakiness” should persist. This scenario for nonlocal dynamics is
realized in the numerical simulations of Holloway (1982), Holloway and Kristmansonsson (1984), and Haidvogel and Kefler (1984). The inability of numerical models, with moderate resolution, to simulate the weakly nonlocal enstrophy subrange is apparently not serious as far as relative dispersion is concerned. At worst, the strongly nonlocal dynamics of the models may tend to exaggerate the persistence of streakiness.

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APPENDIX
Separation Moments

Assume that the relative diffusivity $K$ in the Richardson–Kraichnan equation (3.22) has the form in (3.25)

$$K = D^\beta. \quad \text{(A1)}$$

This includes both local dynamics ($1 < \beta < 2$) and also nonlocal dynamics ($\beta = 2$), if we neglect the launch correlation [see (3.14)]. Any scale factor or separable time dependence in $K$ may be removed by redefining the time scale. Substitution of (A1) into (3.22), multiplication of (3.22) by $D^{m+1}$ for any integer $m$, integration by parts, and remembering that the distribution of $D$ is $DP(D, t)$, leads to

$$\frac{d}{dt} \bar{D}^m = m(m + \beta)\bar{D}^{m-2+\beta}. \quad \text{(A2)}$$

If $\beta \neq 2$, this is a system of equations with a solution

$$\bar{D}^m = l_m^{m(2-\beta)}, \quad \text{(A3)}$$

where

$$l_m = (2 - \beta)(m + \beta)l_{m-2+\beta}. \quad \text{(A4)}$$

The similarity solution (3.26) exemplifies (A3). If $\beta = 2$ then (A2) has the simple solution

$$\bar{D}^m = \bar{D}_0^m \exp[m(m + 2)t], \quad \text{(A5)}$$

which holds also for the initially-sharp distribution given by (3.23).

REFERENCES


