The Mesoscale and Microscale Structure and Organization of Clouds and Precipitation in Midlatitude Cyclones. Part XV: A Numerical Modeling Study of Frontogenesis and Cold-Frontal Rainbands

DAVID J. KNIGHT AND PETER V. HOBBS

Atmospheric Science Department, University of Washington, Seattle, Washington

(Manuscript received 8 April 1987, in final form 7 September 1987)

ABSTRACT

A two-dimensional, hydrostatic, primitive-equation model is used to investigate the dynamics of frontogenesis in a moist atmosphere. The development of a cold front is simulated through shear-deformation associated with the non-linear evolution of an Eady wave. Simulations are performed with 5, 10, 40 and 80 km horizontal resolutions and 14 levels in the vertical (four in the boundary layer).

Compared to the dry case, the inclusion of moisture in the model produces a stronger low-level jet ahead of the front and a stronger upper-level jet. Moisture also produces a stronger ageostrophic circulation across the front and a more concentrated updraft just ahead of the surface front. The updraft develops a banded structure above and behind the surface front, with a wavelength of about 70 km. Bands form near the back edge of the cloud shield and move toward the surface front with a relative velocity of \( \sim 1 \text{ m s}^{-1} \). These characteristics agree with observations of wide cold-frontal rainbands.

The banded structures form in a convectively stable region. The first band that appears in the numerical simulation forms and intensifies in a region of negative equivalent potential vorticity. Subsequent bands form behind the first and intensify as they move into the region of negative equivalent potential vorticity, indicating that conditional symmetric instability (CSI) may play an important role in their formation and intensification. Many of the characteristics of the bands agree with the theory of CSI. The bands disappear when equivalent potential vorticity is everywhere positive. The bands are poorly resolved when the horizontal resolution (\( \Delta x \)) of the model is 40 km, and they are absent with \( \Delta x = 80 \text{ km} \). However, the strength and horizontal scale of the bands is about the same with \( \Delta x = 5 \text{ km} \) and \( \Delta x = 10 \text{ km} \). This indicates that the banded structure is not an artifact of the model.

Frictional convergence in the boundary layer forces a narrow cold-frontal rainbow (NCFR) just above the surface front. The horizontal dimension of this band is greater than that for observed NCFR, presumably because of limited resolution in the model.

1. Introduction

In previous papers in this series we have documented the existence of various types of mesoscale rainbands in midlatitude cyclones (Hobbs et al., 1980; Herzegh and Hobbs, 1980, 1981; Hobbs and Persson, 1982), we have reviewed various mechanisms that have been proposed for their formation (Parsons and Hobbs, 1983a), and we have developed kinematic, numerical models for several of the rainbands (Rutledge and Hobbs, 1983, 1984).

The purpose of the numerical modeling study described in this paper is to increase understanding of the mesoscale circulations associated with cold fronts and the rainbands that can accompany them. Field studies have revealed three types of rainbands associated with cold fronts, each of which is oriented nearly parallel to the front: wide cold-frontal, narrow cold-frontal, and warm-sector rainbands (e.g. Browning and Harrold, 1970; Houze et al., 1976; Hobbs, 1978; Matjeja et al., 1980; Hobbs et al., 1980; Testud et al., 1980; Ogura and Portis, 1982; Parsons and Hobbs, 1983a,b). Narrow cold-frontal (NCF) rainbands are situated above the surface cold front (SCF) and move with the front. Wide cold-frontal (WCF) rainbands form behind the SCF and above the cold-frontal zone and they move toward the SCF (Hobbs, 1978). Warm-sector rainbands are situated ahead of the SCF, they are often convective and generally move away from the cold front (Hobbs, 1978).

In this study, particular attention is paid to the dynamics of WCF rainbands. In particular, we wish to determine if WCF rainbands can be simulated in a numerical model, and if so, determine under what conditions they form in the model. Parsons and Hobbs (1983a) suggested that conditional symmetric instability is the most likely mechanism for the formation of WCF rainbands. Although SI and CSI have received considerable attention, CSI forced by the vertical motions due to frontogenesis has not been treated quan-
titatively. A primitive equation numerical model is required to address this problem, and that is the purpose of the present study. We investigate CSI forced by frontogenesis and its role in producing rainbands in the vicinity of a cold front. The numerical model that we have used for this study is a 2-D, primitive equation, mesoscale model that includes moisture (see section 4). Because upright convection complicates analysis of the results, care is taken to avoid convective instability.

2. A short review of recent theoretical studies of frontogenesis and rainbands

The dynamics of adiabatic, inviscid frontogenesis is now fairly well understood in terms of quasi-geostrophic and semigeostrophic theory (e.g., Hoskins, 1982). The major discrepancy between inviscid, semigeostrophic theory and observed fronts occurs in the vertical velocity field (Blumen, 1980; Ogura and Portis, 1982). Mak and Bannon (1984) and Bannon and Mak (1986) included diabatic effects in a semigeostrophic model by using a wave-CISK parameterization for latent heating. They found that latent heating increased the temperature gradient at mid-levels but had little effect near the surface. Also, including latent heating increased the strength and decreased the horizontal scale of the vertical motions. The semigeostrophic models cannot be used when the potential vorticity is zero or negative, because the Sawyer (1956)–Eliassen (1962) equation for the ageostrophic circulation is then no-longer elliptic and has no unique solution. This means that semigeostrophic models cannot be used to simulate “slantwise” or “upright” convection.

Although some features of fronts can be simulated in adiabatic, inviscid models, the details of actual fronts seem to depend on diabatic processes. Keyser and Anthes (1982) included the effects of “diffusion” (i.e., parameterized turbulent mixing) in a primitive equation model and found that including a realistic treatment of the PBL in the model resulted in a narrow intense updraft at the front, forced by frictional convergence in the boundary layer. They also simulated a stable layer capping an unstable boundary layer behind the front and stable lapse rates in the boundary layer ahead of the front, even when there was no surface heating.

Ross and Orlanski (1978), Williams et al. (1981), Hsie and Anthes (1984) and Hsie et al. (1984) included the effects of moisture in primitive equation models of frontogenesis. All four studies showed increased frontogenesis at midlevels in the atmosphere due to condensation, which agrees with semigeostrophic studies. They also showed that latent heating increased the strength and decreased the horizontal scale of the vertical motions, but the strength of the surface front was not significantly affected by the inclusion of water.

The models of both Ross and Orlanski (1978) and Hsie et al. (1984) showed convective rainbands forming in the warm sector of a cyclone. However, they started with conditionally unstable conditions and the lifting due to frontal circulations was more than enough to release this instability. The explicit simulation of upright convection is questionable in a hydrostatic model, where vertical accelerations are assumed small. The details of the convective rainbands in Hsie et al.'s case (more rainbands, smaller scale, with higher resolution and/or less horizontal diffusion) indicate the importance of numerical truncation and the amount of diffusion in determining the structure of the bands (Hsie, 1983). Following Lilly (1960), it can be shown that the wavelength of the most unstable mode of convection in a hydrostatic, convectively unstable atmosphere increases with increasing diffusion (Hsie, 1983). Williams et al. (1981) started with stable initial conditions, but their study was aimed at finding the effect of moisture on steady-state fronts. Steady-state solutions depend more on the boundary conditions and forcing than on initial conditions. Since the interest here is in the development of frontal circulations in a convectively stable atmosphere, none of the above studies are particularly relevant.

3. Symmetric instability

Due to the importance of symmetric baroclinic instability (or simply symmetric instability, hereafter called SI) in the subsequent discussion, we will review SI theory here.

Hoskins (1974) showed that for 2-D, inviscid, adiabatic flows, instability to symmetric disturbances is possible only if the potential vorticity \( q \) is negative. Potential vorticity and equivalent potential vorticity \( q_e \) can be defined by

\[
\begin{align*}
q &= -(f k + \nabla \times \mathbf{V}) \cdot \nabla \theta \\
q_e &= -(f k + \nabla \times \mathbf{V}) \cdot \nabla \theta_e
\end{align*}
\]

where \( \mathbf{V} \) is the total vector wind, \( \nabla \) is the gradient operator in \( x, y, p \) coordinates, \( k \) a unit vector in the vertical, \( \theta \) the potential temperature, \( \theta_e \) the equivalent potential temperature and \( f \) the Coriolis parameter. The term “symmetric” means that the perturbations do not vary along the basic flow (Stone, 1966). Symmetric instability is a result of an unstable balance between buoyancy and the pressure gradient and Coriolis forces. For inviscid, adiabatic motions, \( q \) is conserved (Hoskins, 1974). In an atmosphere that is initially symmetrically stable, friction or diabatic effects are required to produce unstable conditions. When the atmosphere is made symmetrically unstable by the release of latent heat, it is said to be conditionally symmetrically unstable. If the equivalent potential vorticity is negative, the atmosphere is subject to conditional symmetric instability (CSI) (Bennetts and Hoskins, 1979).

In the 2-D primitive-equation model employed here, potential vorticity is given by
\[ q = \frac{1}{p^*} \left\{ \frac{\partial v}{\partial \theta} \frac{\partial \theta}{\partial x} - \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial x} - \frac{\partial v}{\partial x} + f \right\} \frac{\partial \theta}{\partial \sigma} \]  

(2)

where \( p^* \) and \( \sigma \) are defined in section 4, \( v \) is the alongfront wind and \( u \) the across-front wind. The equivalent potential vorticity is defined in the same way, but with \( \theta \) replaced by \( \theta_e \). (Equivalent) potential vorticity is conserved in (moist) dry, adiabatic, inviscid flow, neglecting the effect of the fallout of rain. Hence, in the present model, the only sources of equivalent potential vorticity are friction and turbulent vertical fluxes in the PBL and numerical diffusion.

The rate of change of potential vorticity for the dry model is (Baldwin et al., 1984):

\[
\frac{dq}{dt} = \frac{1}{p^*} \left\{ \frac{\partial v}{\partial \sigma} \frac{\partial F_o}{\partial x} - \left( \frac{\partial v}{\partial x} + f \right) \frac{\partial F_o}{\partial \sigma} + \frac{\partial \theta}{\partial \sigma} \frac{\partial F_v}{\partial x} - \frac{\partial \theta}{\partial x} \frac{\partial F_v}{\partial \sigma} - \frac{\partial \theta}{\partial \sigma} \frac{\partial F_v}{\partial x} - \frac{\partial \theta}{\partial x} \frac{\partial F_v}{\partial \sigma} \right\}
\]

(3)

where \( F_o \) is the diffusion of a general variable \( \alpha \), \( F_v = F_{h\alpha} + F_{e\alpha} \), and \( F_h \) and \( F_v \) are the horizontal diffusion and vertical turbulent transfer, respectively (see section 4). The change of \( q_e \) in the moist model is given by (3) but with \( \theta \) replaced by \( \theta_e \). Equivalent potential vorticity can also change through the term \( f(g/\theta_0)k \cdot (\nabla \theta_0 \times \nabla \theta) \) (Bennetts and Hoskins, 1979), however, this term is zero in a 2-D model, or if the atmosphere is uniformly dry or saturated. The model used here is not really two-dimensional because of the N–S temperature gradient associated with the Eady wave. However, a diagnosis of the change of potential vorticity in the model shows that it is dominated by vertical mixing of heat and momentum in the boundary layer (Knight, 1987).

Using the parcel method, Emanuel (1983a) showed that SI can be evaluated from the relative slopes of \( M \) and \( S \) surfaces, where \( M = f x + V_g \), \( V_g \) is the geostrophic wind and \( x \) the horizontal coordinate perpendicular to the thermal wind vector, and \( S \) surfaces are defined as those along which the temperature of a displaced parcel is the same as the environment (i.e., the parcel is neutrally buoyant). In a (dry) saturated atmosphere, \( S \) surfaces correspond to \( (\theta) \theta_e \) surfaces. It should be noted that unlike \( M \) surfaces, if the atmosphere is not uniformly dry or saturated, \( S \) surfaces are defined only in terms of a particular air parcel. If \( S \) surfaces are more vertical than \( M \) surfaces then the atmosphere is symmetrically unstable. Here SI can be viewed as either inertial instability on \( \theta \) surfaces \( (f + \frac{\partial v}{\partial x} < 0) \) (Hoskins, 1974) or gravitational instability on \( M \) surfaces \( (\partial \theta/\partial z < 0) \) (Emanuel, 1983a). Traditional inertial and convective instabilities are special cases of SI.

Hydrostatic SI results in a series of slanted roll circulations with their axes oriented along the thermal wind vector, and with slopes comparable to isentropic surfaces. The horizontal wavelength of the rolls depends more on the depth of the unstable region and the slope of the moist isentropic surfaces than on the diffusive properties of the fluid (Emanuel, 1979). The energy source for SI can be either available potential energy (buoyancy) or kinetic energy (inertial) (Stone, 1972; Tokioka, 1970; Miller, 1985). For a Prandtl number of unity and hydrostatic SI, the energy source is a combination of the above two mechanisms. Busse and Chen (1981) and Miller and Antar (1986) considered nearly symmetric instability, where the unstable motions are aligned nearly along the thermal wind vector. They found that in some cases these modes are more unstable than pure SI. Yanai and Tokioka (1969) performed numerical experiments with a limited region of SI. They found that the unstable perturbations were confined mostly within the region of instability.

In an atmosphere that is symmetrically unstable, linear theory predicts that perturbation motions grow exponentially [i.e. \( w = w_0 \exp(\sigma t) \)] (e.g., Bennetts and Hoskins, 1979). The square of the growth rate \( \sigma \) is

\[ \sigma^2 = -f \left( f + \frac{\partial v}{\partial x} \right) + \frac{\left( g \frac{\partial \theta}{\partial x} \right)^2}{\left( \frac{g \frac{\partial \theta}{\partial x}}{\theta_0} \right)^2}. \]

(4)

This equation is strictly valid only for dry SI, however, since the banded updrafts form in a saturated region, and there is no equivalent relationship for CSI, we will use (4) with \( \theta \) replaced by \( \theta_e \).

It has been hypothesized that some rainbands may be due to CSI (e.g., Hoskins, 1974; Bennetts and Hoskins, 1979; Bennetts and Sharp, 1982; Emanuel, 1979, 1983b; Parsons and Hobbs, 1983a; Sun, 1984). However, there are a number of other possible mechanisms for rainbands, these include boundary (Ekman) layer instability (Faller, 1965), gravity waves [either ducted (Lindzen and Tung, 1976) or not (Chimonas et al., 1980)], wave–CISK (Lindzen, 1974; Raymond, 1975), and differential advection with a preferred scale for convection (Browning et al., 1973). For the case of wide cold-frontal rainbands, SI appears to be the most likely mechanism (Parsons and Hobbs, 1983a).

Several observational studies have attempted to show that CSI may be the cause of some rainbands (Bennetts and Sharp, 1982; Bennetts and Ryder, 1984; Selzler et al., 1985; Wolfsberg et al., 1986). These studies have shown that when the precipitation is banded, there is general agreement between the theory of CSI and observations: the rainbands are nearly parallel to the thermal wind, there is strong vertical wind shear and near-neutral static stability, and the wavelength of the rainbands is related to the depth of the unstable region and the slope of the moist isentropic surfaces. Observational studies are difficult because the widths of most rainbands (~50 km) are too small to be adequately resolved by the standard network of upper-air observations.
The atmosphere is frequently not in geostrophic balance on the mesoscale, and 3-D effects are often important. However, the theories of SI and CSI have been developed only for a 2-D basic state with unidirectional vertical wind shear in thermal wind balance. When frontogenesis is occurring, the basic assumptions of the existing theory are violated. Because of the above problems, definite conclusions regarding the role of SI in the formation of rainbands could not be reached by the observational studies. However, they demonstrated that the theory of SI may still be useful. Since frontogenesis occurs in the model simulations described in this paper, the assumptions used to develop the theory of CSI are also violated in the simulations described here. However, we will assume the theory is still useful and proceed accordingly.

Sanders and Bosart (1985) described a persistent rainband that was nearly neutral to symmetric motions, and was apparently forced by frontogenesis. Thorpe and Nash (1984) and Emanuel (1985) showed diagnostically that frontogenesis can force stronger and more concentrated vertical motions in the warm air if it is less symmetrically stable than the cold air. This is because potential vorticity \( q \) acts like stability in the diagnostic Sawyer–Eliassen circulation equation. Thorpe and Nash and Emanuel used the semigeostrophic approximation, and so could not consider the case of neutral or unstable motions (i.e. \( q < 0 \)). Thorpe and Emanuel (1985) obtained similar results using a prognostic semigeostrophic model.

4. Description of the model

The model we have used is a 2-D version of the NCAR/Pennsylvania State University (PSU) primitive equation, mesoscale model, first described by Anthes and Warner (1978). The 2-D version of this model was adapted by Keyser (1981) to study the evolution of a finite-amplitude baroclinic wave, including the effects of PBL physics. Hsie (1983) further modified the model to include the effects of moisture.

The NCAR/PSU model is hydrostatic and compressible. It is bounded above by a constant pressure surface \( p_0 \) (see Table 1 for further details) and below by a rigid surface of variable pressure \( p_t \). The model is formulated in sigma coordinates, where \( \sigma \) is defined as

\[
\sigma = \frac{p - p_t}{p^*},
\]

and \( p^* = p_t - p_i \). The vertical velocity \( \dot{\sigma} = d\sigma/dt \) is zero at the top and bottom boundaries of the model. With the flat terrain used in this study, \( \sigma = 0 \) corresponds to zero vertical velocity \( \dot{z} = 0 \) at the surface \( \sigma = 1 \). At the top \( \sigma = 0 \) \( \dot{\sigma} = 0 \), which implies that \( \omega = dp/dt = 0 \), so \( p_t \) is constant.

Horizontal diffusion is required for the model to remain stable, and to simulate diffusion in the atmosphere. Fourth-order diffusion is used because it is more scale selective than the usual second-order scheme (Williamson, 1978; Anthes and Keyser, 1979). Horizontal diffusion of a variable \( \alpha \) is given by

\[
F_H = -K_H \dot{\alpha} \partial^2 \alpha / \partial x^4,
\]

where \( K_H = K_{H0} + \frac{1}{2} K_0^2 (\partial x^0) \left[ (\partial u / \partial x)^2 + (\partial v / \partial x)^2 \right] \right]^{1/2} \), \( K_0 = 0.4 \) is the von Kármán constant, and \( K_{H0} \) is a background diffusion (which is made just large enough to control numerical noise in the model, see Table 1). In this study diffusion is handled in the same way for all variables, so the Prandtl number is unity. The boundary layer is parameterized in a medium-resolution form following Blackadar (1978) and Zhang and Anthes (1982). Turbulent transfer in the vertical is based on \( K \)-theory, with the coefficients \( K_0 \) depending on a local Richardson number:

\[
\text{Ri} = \left( \frac{N^2}{(\partial w / \partial z)^2} \right) = \frac{(g/\theta)(\partial \theta / \partial z)}{(\partial \theta / \partial z)^2},
\]

where \( w \) is the horizontal wind speed, \( N \) the buoyancy frequency, \( g \) the gravitational constant and \( z \) the vertical coordinate. The mixing coefficients are given by

\[
K_0 = \left\{ \begin{array}{ll}
K_0 + \frac{1}{2} L^2 \left( \frac{\partial w}{\partial z} \right) \left( \text{Ri}_c - \text{Ri} \right), & \text{when } \text{Ri} < \text{Ri}_c, \\
K_0, & \text{when } \text{Ri} \geq \text{Ri}_c
\end{array} \right.
\]

where \( L \) is an assumed constant mixing length (100 m), \( \text{Ri}_c = 1 \), and \( K_0 = 1 \text{ m}^2 \text{ s}^{-1} \), which allows for weak background mixing.

The 3-D system of primitive equations (given in Anthes and Warner, 1978) is reduced to 2-D \( (x - \sigma) \) by specifying the \( y \) dependence of the dependent variables \( u, v, T, p^* \) and \( q_0 \). The 2-D cross section is perpendicular to a straight front oriented in the north–south direction. The positive x-axis and the warm air are toward the east. The coordinate system moves toward the east with the constant phase speed \( c \) of the analytic wave (Keyser, 1981), so that the front stays nearly stationary in the model. Frontal motion has no effect in inviscid models, but it does affect the solution.
when friction is included (Williams et al., 1981; Keyser and Anthes, 1982).

In this study the 2-D model is used to simulate frontal development in an Eady wave, so the $y$ (N–S) dependence of the model variables are specified as follows. The gradients of $u$ and $v$ are zero on constant height surfaces. The gradient of $p^*$ is defined in terms of the geostrophic wind associated with the Eady wave. The gradient of $T$ is derived from the expression for $\frac{\partial \theta}{\partial y}$ from the Eady problem. For this study the N–S potential temperature gradient is $-1$°K/100 km. Boundary conditions for water vapor ($q_v$) are derived by assuming that the relative humidity does not change in the $y$ direction on constant height surfaces. This simulates advection of warm, moist air from the south and cold, dry air from the north. Liquid water is assumed to have no variation in the $y$ direction on constant height surfaces. The model is periodic in the east–west direction, with the length of the domain equal to the wavelength of the most unstable mode in the Eady problem. These boundary conditions are substituted into the 3-D set of primitive equations, this results in the 2-D system of equations, which are given by Keyser and Anthes (1982). There are prognostic equations for $u$, $v$, $p^*$ and $T$ in flux form (to ensure that energy is conserved), and diagnostic equations for $\sigma$, $\omega$ and $\phi$ ($\omega$ is required in the thermodynamic equation).

Time integration in the model follows Brown and Campana (1978). This allows a long time step, because of averaging the pressure-gradient term, but it produces results nearly identical to the standard leapfrog scheme. Following Asselin (1972), a time filter is used. The finite difference form of these equations conserves mass and momentum. Standard second-order centered differencing is done in space.

The moist model has additional equations to represent the effects of condensation and evaporation, and the associated latent heats. There are prognostic equations for water vapor ($q_v$), cloud water ($q_c$) (which is assumed to have no fallspeed), and rainwater ($q_r$) (which does fall). The parameterization of moisture follows Kessler (1969), Liu and Orville (1969) and Orville and Kopp (1977), and conserves total water (Hsie and Anthes, 1984). Special care is needed when evaluating the advection and fallout of rain to ensure that no more liquid water is advected away from a grid point than is available there. The inclusion of moisture in the model requires modification of the thermodynamic, hydrostatic and ideal gas equations. The effect of water loading is included in the hydrostatic equation. However, the effects of liquid water are neglected in the thermodynamic equation and in the ideal gas law. Initial relative humidity is specified to be a constant 80%. The model used in this study is very similar to that used by Hsie et al. (1984) and the reader is referred to that publication for further details.

The model is initialized with the semigeostrophic solution to the Eady problem at 60 h (for details see: Hoskins and Bretherton, 1972; Keyser, 1981; Reeder and Smith, 1986). The pressure at the top of the model is obtained from the semigeostrophic, analytic model.

Our “control” case uses a horizonal resolution $\Delta x = 10$ km. Fourteen levels are used in the vertical, with $\Delta \sigma = 0.1$ except in the lowest four layers where higher resolution is used to adequately resolve the PBL.

5. Results

The initial conditions used in this study are shown in Fig. 1. (Note: all figures show only a portion of the model domain, which extends from $-1753$ km to $+1753$ km). The position of the frontal surface shown in all figures is determined subjectively as a compromise between the location of maximum vorticity and the leading edge of strong temperature gradients.

Plots of equivalent potential temperature (Fig. 1a) show that initially there is a small region of conditional convective instability near the surface in the warm air (the upper boundary of the unstable region is marked by the heavy dashed line). A small region of negative equivalent potential vorticity (in the lower right-hand corner of Fig. 1b) indicates the region of CSI that is due primarily to the region of conditional convective instability. The dry potential vorticity (not shown) is everywhere positive.

Figure 1c shows the initial component of the wind along the front ($\omega$), which forces frontogenesis by shearing deformation of the temperature gradient along the front. The maximum values of initial absolute vorticity and convergence are $1.35 \times 10^{-4}$ and $0.72 \times 10^{-5}$ s$^{-1}$, respectively (not shown in Fig. 1). Also not shown in Fig. 1 are the weak, broad, initial updraft and downdraft, both of which have a maximum strength of 1.2 cm s$^{-1}$. These initial conditions resemble a weak, dry, cold front. The vertical stability is larger than that used by Hsie et al. (1984) and this results in initial conditions that are conditionally symmetrically unstable, but do not result in convectively unstable motions. This allows investigation of the dynamics of a cold front in which primarily stratiform precipitation is occurring. Increased stability results in a longer wavelength and smaller growth rate of the baroclinic wave.

As the integration proceeds, the frontal characteristics become more pronounced. Increased vorticity and potential temperature gradients are evident in Fig. 2. The atmosphere becomes saturated in the warm sector. Condensation begins to increase the strength and decrease the scale of the updraft (Fig. 3), which is consistent with semigeostrophic and primitive-equation studies (Mak and Bannon, 1984; Williams et al., 1981; Hsie et al., 1984). These trends continue throughout the integration. In fact, the 2-D assumptions made in this study provide an infinite source of potential energy for both cyclone development and frontogenesis.

To facilitate comparison with the earlier work of Hsie et al. (1984) and Baldwin et al. (1984), we now
present the model results at $t = 81$ h (this is after 21 h of primitive-equation simulation, since the model is initialized at 60 h). At this time, the region of CSI is advected toward, then up and along the frontal zone (Fig. 4). The area covered by CSI has increased due to turbulent mixing in the boundary layer. As the region of CSI is lifted up along the frontal zone, the instability is released, resulting in a banded structure in the updraft (Fig. 5). The slope of the ascending air in the bands (Fig. 6) is about equal to that of constant $\theta_e$ surfaces (Fig. 7), agreeing with the theory of CSI. It is important to note that unlike the simulation of convective warm-sector bands by Hsie et al. (1984), the bands described here form in a convectively stable region above and behind the SCF and they closely resemble wide cold-frontal rainbands (Hobbs, 1978).

To better illustrate the development of vertical motions, Fig. 8 shows the updraft velocity at 79, 81, 83, 85 and 87 h in a domain smaller than that used in previous figures. At 79 h, a band is located at $z \approx 3-4$ km and $x \approx 180$ km (we will refer to this band as WCF1), and there is a hint of a second band (WCF2) at $x \approx 130$ km and $z \approx 5$ km (Fig. 8a). At $x \approx 250$ km a NCF rainband begins to become well defined. At 81 h, WCF1 is centered at $x \approx 170$ km, a still weak WCF2 is at $x = 110$ km, and a better defined and stronger NCF is now located at $x \approx 215$ km (Fig. 8b). Note that while the bottom of WCF2 is in the region of negative $q_e$, the maximum vertical velocity is not in this region; this maximum in vertical velocity is forced by convergence caused by the divergence at the top of WCF1 when it reaches the upper boundary of the region of instability (Fig. 9). The rainfall rate at 81 h is shown in Fig. 10. A true banded structure is not evident.

Fig. 1. Initial conditions for the numerical simulations (0 h of integration with primitive equation model, but after 60 h of frontogenesis in the analytic model). In this figure (and also in Figs. 2–18) dashed lines are contours of $\theta$ (in °K) with a contour interval of 2°K. (a) Solid lines show equivalent potential temperature ($\theta_e$, in °K) with a contour interval of 2°K. The region where $\partial \theta_e/\partial z < 0$ is bounded above by the heavy dashed line. (b) Solid lines show equivalent potential vorticity ($q_e$, in units of $10^{-9}$ °K s⁻¹ mb⁻¹) with a contour interval of 10 units. The region of conditional symmetric instability lies below the $q_e = 0$ contour. (c) Solid lines show the along-front wind speed ($u$, in units of m s⁻¹) with a contour interval of 5 m s⁻¹.
in the precipitation rate because WCF1 and NCF partially overlap and the updraft in WCF1 is relatively weak. However, a secondary maximum in rainfall rate due to WCF1 is evident. Inclusion of ice-phase microphysics would no doubt enhance the precipitation associated with WCF1 through the seeder-feeder mechanism (Rutledge and Hobbs, 1983).

By 83 h, WCF1 has moved to $x \approx 145$ km, WCF2 is at $x \approx 80$ km and the NCF rainband is at $x \approx 185$ km (Fig. 8c). The bands continue to gain strength as the integration proceeds. At 85 h, WCF1 is at $x \approx 125$ km, WCF2 at $x \approx 55$ km and the NCF rainband has moved to $x \approx 160$ km (Fig. 8d). Note that closed roll circulations predicted by the theory of CSI do not occur presumably because they are overwhelmed by the vertical motions forced by frontogenesis. Finally, at 87 h, WCF1 is located at $x \approx 105$ km, WCF2 at $x \approx 40$ km, WCF3 at $x \approx -10$ km and the NCF rainband is at $x \approx 140$ km (Fig. 8e). At 87 h, WCF1 has started to straddle the NCF rainband, but since it is no longer in the region where $q_r < 0$ it has weakened considerably. This is consistent with observations that when a WCF

---

**Fig. 2.** Solid lines show absolute vorticity (in units of $10^{-3}$ s$^{-1}$) at 70 h with a contour interval of $2 \times 10^{-3}$ s$^{-1}$. In this figure (and also in Figs. 3–18) the region containing cloud water is shaded.

**Fig. 3.** Solid lines show vertical velocity ($w$, in units of cm s$^{-1}$) at 70 h with a contour interval of 0.5 cm s$^{-1}$.

**Fig. 4.** Solid lines show equivalent potential vorticity ($q_r$, in units of $10^{-6}$ K s$^{-1}$ mb$^{-1}$) at 81 h with a contour interval of 10 units. The region of conditional symmetric instability lies within the $q_r = 0$ contour.

**Fig. 5.** Solid lines show vertical velocity ($w$, in units of cm s$^{-1}$) at 81 h with a contour interval of 0.5 cm s$^{-1}$. 
rainband catches up with the NCF rainband, the former can be destroyed (Parsons and Hobbs, 1983b). However, in some cases a WCF rainband can move ahead of the NCF rainband and become a warm-sector band (Parsons and Hobbs, 1983b). The numerical simulation was halted at 87 h because the atmosphere had become convectively unstable in the warm sector, and this instability was being released.

Between 79 and 87 h, WCF1 moves toward the NCF band with velocity of \(\sim 1 \text{ m s}^{-1}\). This motion compares well with observations of WCF rainbands (Hobbs et al., 1980; Parsons and Hobbs, 1983b). Subsequent bands form toward the rear of the cloud shield and move towards the surface front. Observed WCF rainbands move with the wind at the top of the bands, but move toward the warm air at \(\sim 3 \text{ m s}^{-1}\) relative to the winds at the level of maximum updraft (Hobbs et al., 1980). The WCF bands simulated here move with a velocity about \(3 \text{ m s}^{-1}\) slower than the Eady wave, which was moving at \(14.35 \text{ m s}^{-1}\). Thus, the WCF bands are moving at about the same velocity as the winds near the top of the bands \((z = 5 \text{ km})\). The mean relative cross-frontal velocity at the level of the bands (between \(z = 3-4 \text{ km}\)) is \(\sim 5 \text{ m s}^{-1}\) slower than the Eady wave, so the bands are propagating toward the warm air. It is difficult to determine how observed and numerically predicted motions of rainbands compare with SI theory. This is because the predictions of linear SI theory depend on the boundary conditions used, and the manner in which latent heating is parameterized. Emanuel (1979) showed that dry SI rolls do not propagate with respect to the mean winds in the region of upward vertical air motion. Using a CISK type of moisture parameterization, Emanuel (1982) and Sun (1984) showed that the rainbands propagate toward the warm air with the speed of internal gravity waves. The velocity of the bands simulated here is intermediate between these predictions. Seltzer et al. (1985) also observed that rainbands believed to be due to SI propagate slowly towards the warm air. It appears that the existing theory for SI does not adequately predict the motion of the rainbands.

The wavelength \((\sim 70 \text{ km})\) and the width \((\sim 30 \text{ km})\) of the bands simulated here agree well with observations of WCF bands and CSI theory. In the region of the bands, the depth of the unstable layer is \(\sim 2 \text{ km}\) and the slope of the \(\theta_e\) surfaces is \(\sim 1:40\), so linear CSI theory would predict a wavelength of \(\sim 80 \text{ km}\), which is similar to that modeled here. Figure 11 shows the development of the vertical velocity in WCF1. Between 79 and 85 h, the strength of the updraft increases exponentially with a growth rate \((\sigma)\) of \(1.7 \times 10^{-5} \text{ s}^{-1}\). The growth rate predicted by linear theory [see Eq. (4)] is about \(1.0 \times 10^{-4} \text{ s}^{-1}\), which is much larger than predicted by the present model. It is not clear if nonlinear effects, numerical diffusion or the deviation from the assumed basic state is responsible for the reduced growth rate of the bands in the model. After 85 h, the strength of the updraft in WCF1 begins to decrease, this is consistent with the fact that the band is no longer in the unstable region. The theory of SI predicts that the vertical velocity maxima should slope in the same direction as the vertical motions themselves (i.e., along moist isentropic surfaces). However, Fig. 8 shows the vertical velocity maxima sloping toward the warm air. We attribute this discrepancy to the vertical shear of the across-front wind (which is not included in the current theory of SI) tending to rotate the axis of maximum vertical velocity. One might expect that mixing caused

![Figure 6](image1)

**Fig. 6.** Vector winds in plane of cross section with respect to the moving Eady wave at 81 h.

![Figure 7](image2)

**Fig. 7.** Solid lines show equivalent potential temperature \((\theta_e, \text{ in } ^{\circ}\text{K})\) at 81 h with a contour interval of 2\(^{\circ}\)K. The region where \(\partial \theta_e / \partial z < 0\) is contained within the heavy dashed line.
Fig. 8. Solid lines show vertical air velocity ($w$, in units of cm s$^{-1}$) with a contour interval of 0.5 cm s$^{-1}$. The region of negative potential vorticity is contained within the heavy dashed line. (a) 79 h, (b) 81 h, (c) 83 h, (d) 85 h, and (e) 87 h.
by the rainbands would decrease the degree of SI (Emanuel, 1979; Stone, 1972). However, in our model simulations, the increased vertical motions are not enough to decrease the degree of SI.

The model simulations show a series of downdrafts, spaced about 35 km apart, below the front (see Fig. 8a–e). When evaporation is turned off in the model, these downdrafts disappear, but the banded structure remains above the front. The disappearance of the downdrafts in the absence of evaporation indicates that they are not the result of gravity waves produced by frontogenesis, a mechanism that was hypothesized by Lindzen and Tung (1976) to cause rainbands. Net heating by condensation and evaporation is shown in Fig. 12. As expected, the rate of condensational heating is closely related to updraft strength, and there is evaporation at the cloud boundaries. The apparent periodicity of the downdrafts in Fig. 8 is a result of evaporation at the edge of the cloud boundary, and is an artifact of the model. It is due to limited vertical resolution and the slope of the frontal cloud shield; a vertical resolution of about 0.5 km and a frontal slope of 1:70 gives the spacing of 35 km.

To determine the effects of model resolution on the banded structure, additional simulations were performed with 80, 40 and 5 km horizontal resolution.
Horizontal diffusion was adjusted in accordance with
the change in grid spacing. It should be noted that a
water cycle parameterization that was developed for a
high resolution cloud model may not be equally appli-
cable to simulations with Δx = 5 and 80 km. In
particular, the thresholds for the onset of condensa-
tion and autoconversion may depend on grid size (see
Hsia and Anthes, 1984, for additional discussion on the
applicability of the moisture parameterization used in
this model). No bands appeared with the 80 km res-
solution. Simulations with 40 km resolution produced

only one WCF band, because the area of SI was only
large enough to support one band. Simulations with 5
km resolution resulted in an additional weak WCF
band (Fig. 13), but the horizontal scale and wavelength
of the WCF bands were about the same as in the 10
km case. The fact that when the bands are properly
resolved the wavelength is independent of the model
resolution indicates that the size of the region of SI
and the slope of moist isentropic surfaces determine
the scale of the bands, which is consistent with the

---

**Fig. 13.** Solid lines show vertical velocity (w, in units of cm s⁻¹)
for the simulation using Δx = 5 km at 81 h with a contour interval
of 0.5 cm s⁻¹.

**Fig. 14.** Solid lines show absolute vorticity (in units of 10⁻⁵ s⁻¹)
at 81 h with a contour interval of 2 × 10⁻⁵ s⁻¹.

**Fig. 15.** Solid lines show the total average rate of change of vorticity
following a fluid parcel [(d/dt)(∂ω/∂x), in units of 1 × 10⁻³ s⁻¹ day⁻¹]
between 78 and 84 h with a contour interval of 5 × 10⁻⁵ s⁻¹ day⁻¹.

**Fig. 16.** Solid lines show the total average rate of change of the
horizontal potential temperature gradient following a fluid parcel
[(d/dt)(∂θ/∂x), in units of °K (100 km)⁻¹ day⁻¹] between 78 and 84
h with a contour interval of 2.0 units.
theory of SI. With 5 km model resolution, the bands have slightly stronger vertical motions because more grid points fall within each band so they are better resolved and numerical diffusion effects are reduced.

To help determine if the banded structure simulated in the model is due to CSI, the vertical stability was slowly increased until the bands were no longer evident. As discussed in section 3, potential vorticity increases when the vertical stability is increased. As expected, the strength of the vertical air motions are reduced as the vertical stability is increased. In addition, the bandedness disappears when $q_v$ is everywhere positive. Hence, the banded structure is clearly dependent on having a region of CSI. Unfortunately, the theory of CSI has been developed only for an atmosphere in geostrophic balance, whereas, our numerical simulations show significant ageostrophic winds. The criteria for CSI has not been determined in this case, but it appears that $q_v < 0$ is still a useful criteria for the formation of WCF bands.

Figure 14 shows the vorticity at 81 h. There is a maximum in vorticity of $3.2 \times 10^{-4} \text{s}^{-1}$ at the surface front, and a secondary maximum associated with WCF1. Enhanced vorticity in the region of rainbands is consistent with observations (Hertzman and Hobbs, 1988). Figure 15 shows the Lagrangian vorticity production. The fields shown in Fig. 15 and Fig. 16 are averaged over 6 hours (between 78–84 h) and 100 km in the horizontal to facilitate comparison with diagnosis of observations. The maximum vorticity production near the SCF is due to convergence (Knight, 1987). The enhanced vorticity production in the region of WCF1 is due to convergence at lower levels in the band, and the tilting of horizontal vorticity into the vertical in the upper levels of the WCF band (Knight, 1987). Figure 16 shows the average Lagrangian rate of frontogenesis. There is a maximum near the SCF due to convergence and the rotation of along-front gradients into the cross-front direction. Figure 16 also shows enhanced frontogenesis in the region of WCF1, which is due to tilting of vertical gradients into the horizontal which is only partially canceled by horizontal gradients of condensation (Knight, 1987). Unfortunately observational studies of WCF rainbands have been unable to determine if they result in regions of increased frontogenesis.

The model predicts strong vertical motions above the surface front (Fig. 8), which is similar to a NCF rainband (though the scale is different because of model resolution). The scale of the NCF rainband decreases with increasing resolution. This band is also simulated in the dry model (although it is not as strong as in the moist simulation), and is forced by frictional convergence in the boundary layer.

The overall effect of moisture on frontogenesis that has been revealed by the present study agrees qualitatively with the results obtained by Hsie et al. (1984). Compared with the dry run (not shown), the moist model exhibits: increased vorticity and convergence at low levels, stronger vertical velocities, lower minimum surface pressure and increased temperature gradients at midlevels. These characteristics were also observed in the semigeostrophic studies of Mak and Bannon (1984) and Bannon and Mak (1986). Figure 17 shows the along-front wind in the moist simulation at 81 h. Compared to the dry case, the along-front winds are stronger in the moist case. The strength of low-level along-front jets are important not only because they
lead to increased across-front gradients but also because
the southerly jet provides a source of warm moist air.
Figure 18 shows the ageostrophic component of the
along-front wind at 81 h. In addition to the strong
ageostrophic along-front wind in the PBL, there are
significant ageostrophic winds in the region of the rain-
bands and the upper level jet. Figure 18 shows that the
common assumption of across-front geostrophic bal-
ance in 2-D models of frontogenesis (e.g., Hoskins and
Bretherton, 1972) is not valid for the simulations de-
scribed here.
Thorpe and Emanuel (1985) showed that the wave-
length of the most unstable Eady wave decreases and
its growth rate increases with decreased potential vor-
ticity. Therefore, since our model is periodic in the x-
direction, with a length equal to the wavelength of the
most unstable dry Eady wave, when condensation be-
gins the baroclinic wave becomes “detuned”. It is not

---

**Fig. 19.** Schematic diagram of the processes leading to the for-
maton of cold-frontal rainbands. The cross hatching shows the region
of negative equivalent potential vorticity, shading shows the region
containing cloud water. Vertical hatching below cloud base represents
precipitation. The geostrophic wind in the plane of the cross section
($U_g$) is indicated on the left hand side of the figure. Broad arrows
show ageostrophic air motions in the plane of the cross section. All
winds shown are relative to the motion of the Eady wave. (a) A region
of negative equivalent potential vorticity, which forms in the warm
air, is advected toward then up along the frontal surface by the ageo-
strphic motions. (b) When the region of conditional symmetric in-
stability (CSI) becomes saturated the instability is realized, resulting
in a wide cold-frontal rainband (WCF1). A second rainband (WCF2)
is forced by convergence behind WCF1. Convergence in the planetary
boundary layer forces a narrow cold-frontal rainband (NCF). (c)
WCF1 moves toward the warm air, WCF2 moves into the region of
CSI and intensifies, a third band (WCF3) is forced by convergence
behind WCF2. See text for more detailed discussion.
clear what effect, if any, this has on the simulations described here.

6. Summary and conclusions

A cold front forced by shear deformation has been simulated in a 2-D primitive equation model that contains moisture and PBL physics.

The model simulates a narrow cold-frontal rainband, forced by convergence in the PBL. The inclusion of moisture in the model results in stronger temperature gradients at midlevels and stronger low-level and upper-level jets along the front.

When there is a region of negative $q_e$, the inclusion of moisture also leads to updrafts with a banded structure above and behind the surface cold front (SCF). These bands have a wavelength of $\sim 70$ km and move toward the SCF. The location, movement and spacing of the bands agree well with observations of wide cold-frontal (WCF) rainbands. The first band that appears in the numerical simulation forms and intensifies in a region of conditional symmetric instability (CSI). Subsequent bands form behind the first and intensify as they move into the region of CSI. The slope of the updrafts and the spacing of the bands agree well with the theory of CSI. While in the region of CSI, the vertical air velocity in the bands decreases exponentially in time, although the growth rate is smaller than that predicted by the linear SI theory. The wavelength of the bands is about the same in simulations using horizontal resolutions of 5 and 10 km. The WCF bands disappear when the vertical stability is increased to the point that equivalent potential vorticity is everywhere positive.

In the simulations described here, there is initially a region of CSI near the surface in the warm sector of the cyclone. This conditionally unstable region is advected toward, then up along the frontal surface. When the region of CSI becomes saturated, the instability is released, resulting in a WCF band. In the present study, a region of $q_e < 0$ was prescribed by the specified initial conditions. In an actual cyclone, negative $q_e$ could be caused by surface heating or radiative processes, along with turbulent effects in the PBL. An angle between $\theta$ and $q_e$ surfaces can also create or destroy $q_e$ (Bennetts and Hoskins, 1979).

We are left with the following picture for the formation of WCF rainbands, which is shown schematically in Fig. 19. During the early stages of cyclone development, a region of CSI is produced in the warm air near the surface. This region of CSI is lifted by the ageostrophic circulation associated with frontogenesis (Fig. 19a). When the air in the region of CSI becomes saturated, the instability is released and this results in the formation of a WCF rainband (WCF1 in Fig. 19b). Subsequent rainbands form behind the first and intensify as they move into the region of CSI (WCF2 and WCF3 in Fig. 19c). Note that WCF2 and WCF3 in Fig. 19 form in a region that is not symmetrically unstable. These incipient bands are forced by convergence behind the rearmost rainband that is in the region of CSI (see WCF1 in Fig. 19b and WCF2 in Fig. 19c). This convergence is a result of divergent outflow at the top of the rainband when it reaches the top of the unstable region (see Fig. 9). Upward air motions in the rainbands are nearly along moist isentropic surfaces. The spacing of the rainbands depends on the slope of the moist isentropic surfaces and the depth of the region of CSI. The rainbands move with the winds at the top of the rainband, and hence propagate toward the warm air relative to the surface front.

Some outstanding problems include determination of the mechanisms that produce negative $q_e$ in a cyclone, and the effects of the heat, moisture and momentum fluxes produced by rainbands. The effects of topography and land–sea contrast on frontal development and rainbands also need investigation. Finally, the role of ice phase microphysics on the dynamics of rainbands, and the stability of rainbands to perturbations along the front, should be considered.

Acknowledgments: Eih-Yu Hsie provided some of the computer programs used in this study. This research was supported by the National Science Foundation (NSF) under a series of grants, the most recent of which was ATM83-11147. Computer support was provided by NCAR, which is funded by the NSF.

REFERENCES


Seltzer, M. A., R. E. Passarelli and K. A. Emanuel, 1985: The possible


