Extension of the Stochastic Mixing Model to Cumulonimbus Clouds

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ABSTRACT

The stochastic mixing model of cumulus clouds is extended to the case in which ice and precipitation form. A simple cloud microphysical model is adopted in which ice crystals and aggregates are carried along with the updraft, whereas raindrops, graupel, and hail are assumed to immediately fall out. The model is then applied to the 2 August 1984 case study of convection over the Magdalena Mountains of central New Mexico, with excellent results. The formation of ice and precipitation can explain the transition of this system from a cumulus congestus cloud to a thunderstorm.

1. Introduction

Raymond and Blyth (1986, hereafter RB1) developed a stochastic mixing model of nonprecipitating cumulus clouds that gives good predictions for detrained mass fluxes in observed clouds. The model is based on the notions that entrainment and mixing in cumulus clouds is a random, episodic process, and that parcels seek their level of neutral buoyancy after mixing. The model was an outgrowth of Telford’s (1975) cloud-top entrainment model, with the extension that mixing events could occur at all levels in a cloud. This extension was based on the observation of Blyth et al. (1988) that entrainment was not confined to cloud top, but could occur at any level. Mixing events in the model were assumed to take place with equal probability between cloud base and the level of undilute neutral buoyancy. In addition, mixing of cloud material with the environment was postulated to occur with a uniform distribution of cloud-base fractions, from nearly all cloud material to nearly all environment. Blyth and Raymond (1988) (hereafter BR) subsequently used the model to successfully simulate the slopes of mixing lines in thermodynamic diagrams (Paluch 1979) in a number of Montana cumulus clouds.

The major theoretical simplification in the model is the assumption that each air parcel ascending from cloud base undergoes one and only one mixing event with environmental air. As Taylor and Baker (1990) have shown, it is often difficult to distinguish observationally whether multiple mixing events have occurred, due to the buoyancy sorting principle. Basically, at any given level one is likely to see only a subset of possible air parcels. The distribution of these parcels on a Paluch diagram closely mimics the mixing line structure that is generally taken as evidence of a single mixing event.

Bretherton and Smolarkiewicz (1989) used a two-dimensional cloud model to investigate entrainment and detrainment in clouds. They pointed out that clouds tended to entrain in their model where buoyancy was increasing with height and detrain where it is decreasing. This tendency is generally reproduced in the stochastic mixing model. Taylor and Baker (1990) parameterized this effect and obtained reasonably good agreement with observed detrained mass fluxes.

A number of other investigators have invoked the postulate of buoyancy sorting in their cumulus models. Emanuel (1991) developed a variation in which only that cloud-base fraction resulting in the least buoyancy for the mixture is allowed to occur. It is hard to see how the parcel would know ahead of time the appropriate cloud-base fraction for creation of the most negative buoyancy. However, this assumption may be an adequate surrogate for repeated mixing events occurring in a descending parcel, which presumably continue until all condensed water is evaporated. If parcels descend as an entraining plume or thermal, as postulated by Emanuel (1981), then this may indeed take place.

Kain and Fritsch (1990) have developed a plume model in which mixing after entrainment occurs over a variety of cloud-base fractions. Those fractions that are positively buoyant are immediately mixed with the plume, whereas those with negative buoyancy are detrained. However, the negatively buoyant parcels are not assumed to sink to their level of neutral buoyancy. This model seems to reproduce observed cumulonimbus mass flux profiles in the tropics reasonably well. Unlike the stochastic mixing model, the model of Kain...
and Fritsch requires that a length scale (the initial plume radius) be specified.

In this paper we extend the stochastic mixing model to include the formation of ice and precipitation. The primary difficulty in doing this is establishing sensible representations for ice and precipitation processes, and particularly their effects on buoyancy. This problem has been attacked many times before (e.g., Cotton 1972; Orville and Kopp 1977), but we seek a much more abbreviated model of cloud microphysical processes in line with the simple nature of the dynamical model.

The dynamical content of the model is the same as in RB1. Parcels are sent from cloud base to a set of pressure levels uniformly distributed between cloud base and the level of undilute neutral buoyancy. At each level cloud-base parcels are mixed with environmental air with a variety of cloud-base fractions. Each mixture is then allowed to ascend or descend to its level of neutral buoyancy, where it exits the cloud. In order to retain a degree of simplicity, no attempt is made to include multiple mixing events, even though it is likely that this occurs in real clouds. Downdrafts induced by the evaporation of precipitation are not considered at this time, even though they are undoubtedly very important at lower levels.

A test of the model is made on a thunderstorm observed over Langmuir Laboratory, located in the Magdalena Mountains of central New Mexico, on 2 August 1984 using Doppler radar and instrumented aircraft. This case is extensively documented by Raymond and Blyth (1989, hereafter RB2) and Raymond et al. (1991, hereafter RSB). Though we have run the model on many case studies, we believe that a thorough study of this case is sufficient to point out the main characteristics of the model.

In section 2 of this paper a simple model for ice and precipitation physics is presented. Section 3 shows results from tests on the 2 August 1984 case study. Conclusions are presented in section 4.

2. The model

In this section we first present a general overview of our understanding of precipitation in clouds. We then review the dynamical model presented in RB1, paying particular attention to contributions to the buoyancy from latent heat of freezing, condensed phase specific heats, etc. Next, we present a crude parameterization of the precipitation process. Finally, we describe a number of useful diagnostic calculations to help understand what is occurring in the model, and what its effect is on the environment.

a. Overview of precipitation production

Traditionally, development of precipitation is categorized according to whether or not ice is involved in the process. We use a different scheme here. A distinction is made, based on terminal fall speeds, between particles (other than cloud drops) that are carried along with the parcel while it is in the updraft, and particles that fall out of the parcel. Pristine or lightly rimed ice crystals are in the first category. So are aggregates of ice crystals, which form at temperatures lower than 0°C (Heymsfield and Musil 1982) and are known to have low terminal fall speeds (Locatelli and Hobbs 1974). Some of the particles in this first group may eventually fall out to produce light rain (or snow in winter), but we do not allow for this in the model.

The second category lumps the products of two familiar precipitation mechanisms together. It contains graupel, hail, and raindrops, all of which grow via accretion of cloud drops. A graupel particle forms when an ice crystal, aggregate, or a frozen drop becomes sufficiently large for the collection efficiency to be significant. A similar condition is necessary for the formation of raindrops, but in that case the embryonic particles are large cloud drops.

Accretional growth of precipitation depends on the product of fall distance relative to the rising cloud air and the liquid water content. If there is no updraft, the former is simply the depth of cloud through which the precipitation embryo falls. The existence of an updraft can effectively increase this distance, and can enhance the cloud liquid water content as well. Therefore, particles confined to an updraft will intercept significantly more liquid water than particles in downdrafts. Precipitation particles can have terminal fall speeds of 10 m s⁻¹ or more (Heymsfield 1978), which are comparable to updraft velocities in weak to moderate cumulonimbus clouds. Therefore, these particles rapidly exit the air parcel in which they are initially produced.

In this paper we refer to particles in the first category that are subject to the deposition—aggregation process as “ice,” and the second group of particles resulting from the accretion process as “precipitation.” The essential difference for the purposes of this paper is that ice is assumed to be carried upward by the updraft and ejected into the anvil, whereas we assume that precipitation falls out of the side or base of the updraft.

The simple scheme used to represent ice and precipitation in this model is based on our current understanding of the formation processes. Heymsfield et al. (1979) have shown that significant ice-crystal formation typically begins near −15°C in clouds over the high plains of the United States. Our own observations of New Mexico cumulus clouds have revealed that cloud tops need only be about −12°C for there to be significant concentrations of ice. We expect little or no liquid water to remain at temperatures lower than about −40°C due to homogeneous freezing (Pruppacher and Klett 1978). However, our own analysis of precipitation trajectories from Doppler radar data indicates that graupel particles can originate at temperatures between −10°C and −30°C. There is clearly
competition between precipitation and ice production in cold clouds. The accretion process can be preempted if all the liquid water freezes to produce ice crystals. Many interrelated factors influence these processes.

The production of precipitation embryos is probably the least well understood aspect of the whole rain process. In ice-free clouds, there may be a mechanism involving mixing with the environment that promotes the production of drizzle-sized drops (Baker and Latham 1979; Telford and Chai 1980; Cooper 1989; Blyth and Latham 1990), which could subsequently grow further by accretion. They could also play an important role in the formation of ice. For example, Koenig (1963) found that large liquid drops were extremely important in accelerating the development of precipitation in Missouri cumulus clouds. Our own microphysical observations made in cumulus congestus clouds over Langmuir Laboratory have shown that large drops are often present before the formation of ice. The details of the mechanisms remain unclear.

A final effect of ice in updrafts is to drive the vapor pressure below the saturation vapor pressure of water when no liquid water is present. A test of the enhancement of buoyancy from this effect was made by replacing the liquid adiabat by the ice adiabat for temperatures below 0°C. Even for this extreme case, unlikely to be approached in the updrafts of any real cloud, a maximum enhancement in buoyancy of order 0.02 m s⁻² (~0.5 K) was found, which is significantly less than buoyancy changes caused by other cloud microphysical processes. We therefore ignore this effect.

b. Dynamical model

As in RB1, the level of undilute neutral buoyancy of cloud-base air is taken to be the cloud top. Parcels from cloud base are assumed to rise to some level between cloud base and cloud top, where they mix with ambient air. Defining levels equally spaced in pressure between cloud base and cloud top, with level spacing Δp, equal masses of cloud-base air are mixed with environmental parcels at each pressure level.

Each mixing event is assumed to be a composite of a number of subevents in which different fractions of cloud-base air appear in the final, mixed product. Nine possible cloud-base fractions (i.e., the fraction of cloud-base air in the mixed product), f = 0.1, 0.2, ..., 0.9, are assumed, and equal masses of the mixed product are assumed to be produced for each value of f. Thus, more cloud-base air is used in producing mixtures with large f than with small f. (Note that we erroneously stated in RB1 that equal amounts of cloud-base air rather than mixed product were allocated to each value of f.) After mixing, each subparcel is allowed to rise or sink to its level of neutral buoyancy. It is then assumed to exit the cloud at this level.

The computation of parcel buoyancy is crucial to this model, and has to be done carefully in cases where the environment is nearly neutral to parcel ascent. RB1 used a very simple expression for computing the entropy of air. BR improved upon this by using the wet equivalent potential temperature introduced by Paluch (1979).

In this paper we begin with an accurate expression for the entropy of moist air, including the liquid phase, assumed to be carried along with the air (Iribarne and Godson 1981). This quantity is conserved in reversible adiabatic processes. The entropy per unit mass of dry air is

\[ s = (c_{pd} + q_T c_L) \ln(T/T_R) - R_D \ln(p_D/p_R) \]

\[ + L(T)q_V/T - q_V R_V \ln(q_V/q_S), \]

(1)

where \( c_{pd} \) and \( c_L \) are the specific heat at constant pressure for dry air and the specific heat for liquid water, and \( R_D \) is the gas constant for dry air. The absolute temperature is \( T \), while \( p_D \) is the partial pressure of dry air. Constant reference values of temperature and pressure are denoted by \( T_R = 273.15 \) K and \( p_R = 1000 \) mb, and the reference entropy for \( T = T_R, p = p_R \), and \( q_V = 0 \) is taken to be zero. The mixing ratios of water vapor and total cloud water (water plus cloud drops) are denoted by \( q_V \) and \( q_T \), while the saturation mixing ratio at the given temperature and pressure is \( q_S \). The temperature dependence of the latent heat of vaporization, \( L(T) = L(T_R) - (c_L - c_{pv})(T - T_R) \), is taken into account, assuming specific heats to be constant. Given the pressure, entropy, and total water, (1) can be iteratively solved for the temperature using Newton's method.

An equation for saturation vapor pressure of water consistent with the Clausius–Clapeyron equation is used:

\[ e_s(T) = e_R \left( \frac{T}{T_R} \right)^{(c_L - c_{pv})/R_V} \exp \left( \frac{L(T_R)}{R_V T_R} - \frac{L(T)}{R_V T} \right), \]

(2)

where \( e_R \) is the saturated vapor pressure at the reference temperature of 273.15 K. Inaccuracies in this equation due to the temperature dependencies of \( c_{pv} \) and \( c_L \) are negligible in the present context. All constants are obtained from Iribarne and Godson (1981).

For fully reversible processes in which condensed water is carried along with the air in liquid form, the above entropy is conserved. However, when freezing occurs, additional heat is added to the parcel, with a resulting increase in the entropy. In addition, if liquid water or ice falls out of the parcel as precipitation, the entropy of this component must be deducted.

The change in entropy per unit mass of dry air due to the loss of liquid cloud drops to precipitation or ice is

\[ \Delta s = c_L \ln(T/T_R) \Delta q_T, \]

(3)

where \( \Delta q_T \) is the change in total cloud water mixing ratio, and the entropy of the liquid phase is assumed
to be zero at the reference temperature. An additional entropy change accounts for the presence of ice and the freezing of cloud drops:

$$\Delta s = c_i q_i \Delta T / T + L_f q_f / T,$$  \hspace{1cm} (4)

where $c_i$ is the specific heat of ice per unit mass, $q_i$ is the mixing ratio of ice, $L_f$ is the latent heat of fusion, and $\Delta q_f$ is the mixing ratio of liquid water that freezes. The temperature variation in $L_f$ is accounted for, but a constant value of $c_i$ (at $-20^\circ C$) is used. The first term of (4) is due to the heat capacity of the ice, and the second arises from heat released during freezing. At each step in the evolution of a parcel, the sum of (3) and (4) must be added to the entropy.

Note that in general, $\Delta q_i \neq \Delta q_f$, since part or all of the liquid that freezes may also fall out of the parcel as precipitation. However, the overall conservation equation for water substance

$$\Delta q_p + \Delta q_i + \Delta q_f = 0$$  \hspace{1cm} (5)

must hold, where $\Delta q_p$ is the mixing ratio of precipitation that falls out in the interval under consideration.

Finally, the parcel buoyancy is calculated as

$$b = g [(T - T_E) / T_E + \epsilon (q_v - q_{vE}) - q_L - q_f],$$  \hspace{1cm} (6)

where $\epsilon$ is the ratio of molecular weights of water vapor and dry air, $T_E$ and $q_{vE}$ are, respectively, the environmental temperature and water vapor mixing ratio at the level of interest, $g$ is the acceleration of gravity, and $q_L$ is the mixing ratio of liquid cloud drops. This formula takes into account the virtual temperature effect and the weight of condensed and frozen water as well as the temperature difference from the environment.

c. Cloud microphysics model

We elected to implement a very simple model for cloud microphysics. Two idealized processes are considered, namely, formation of ice and formation of precipitation. Ice is assumed to be in the form of unrimed or lightly rimed crystals or aggregates of crystals, and is further assumed to be carried along with the air parcel in which it originate. This assumption is roughly justified in the convective region of a cloud, but is, of course, not valid in the anvil region. Ice is formed according to the equation

$$\Delta q_l = -R_l q_l \Delta T, \hspace{1cm} T < T_l, \hspace{1cm} \Delta T < 0,$$  \hspace{1cm} (7)

where $q_l$ is the mixing ratio of cloud drops. Ice forms only when the temperature is below a threshold temperature, $T_l$, and only when the parcel temperature change, $\Delta T$, is negative. The rate of ice formation is proportional to the mixing ratio of cloud drops, and is governed by an empirical rate constant $R_l$. (Note that this is a temperature-based rather than a time-based rate. The latter may be more realistic, but isn’t feasible in the context of the present dynamical model.)

Precipitation consists of rain, graupel, and hail, and its formation obeys a similar equation,

$$\Delta q_p = -R_p q_p \Delta T, \hspace{1cm} T < T_p, \hspace{1cm} \Delta T < 0,$$  \hspace{1cm} (8)

but with different threshold and rate constants, $T_p$ and $R_p$. This process is assumed to account for both accretional growth of graupel and hail and growth of precipitation by the warm rain process. Precipitation is immediately removed from the air parcel. Liquid water lost to precipitation is assumed to freeze if the temperature is below freezing.

Equations (7) and (8) are implicit, in that precipitation and ice formation must both be known before the parcel temperature change, $\Delta T$, can be predicted. However, the temperature change obtained by ignoring these processes is sufficiently accurate for evaluating (7) and (8).

Both precipitation production and ice formation are assumed to operate only in ascent. Although ice has been observed first in downdrafts, and typically more ice is observed in downdrafts than in updrafts [perhaps because of ingested ice nuclei (Heymsfield et al. 1979)], it is likely that the effect upon parcel buoyancy of ice formed solely in downdrafts is small. We therefore choose to ignore it in this simple model. The only transformation of water substance that occurs in descent is the melting of ice at the freezing level. When this happens, the water is no longer assumed to move with the air parcel, since melting greatly increases the particle terminal velocity. The melted ice is therefore transferred to the precipitation category.

To summarize, cloud microphysical processes are parameterized in this model by four constants, namely, the ice and precipitation rate constants, $R_l$ and $R_p$, and by threshold temperatures, $T_l$ and $T_p$.

d. Diagnostics

We now describe a number of diagnostic calculations designed to help us understand the model results. We assume that there are $l$ pressure levels between cloud base and the highest level reached by a parcel, spaced by the constant pressure interval $\Delta p$. There are $n$ different cloud-base fractions, nominally $9$ in the range $0.1, 0.2, \ldots, 0.9$. Level $B$ indicates the cloud-base level, $F$ indicates the freezing level, and $D = D(j, f)$ is the detrainment level for parcels mixing at level $j$ with cloud-base fraction $f$.

In what follows, it is advantageous to be able to pass freely between continuous and discrete representations of functions in the vertical. For any continuous function of pressure, $X(p)$, its value at pressure level $p_i$ is denoted $X_i$. Likewise, any discrete function of pressure is assumed to have a smooth, continuous analog.

It is helpful to define the test functions $u_i(j, f)$, $a_i(j, f)$, and $d_i(j, f)$. The function $u_i$ is 1 if an ascending cloud-base parcel destined to mix at level $j$ passed through level $i$ before mixing, and 0 otherwise. The $a_i$
and \( d_i \) functions play a similar role after parcel mixing. If a parcel that mixed at level \( j \) with cloud-base fraction \( f \) ascends through level \( i \), then \( a_i \) is 1. Otherwise it is 0. Similarly, \( d_i \) is 1 if a parcel descends through level \( i \) after mixing at level \( j \) with cloud-base fraction \( f \), and 0 otherwise.

Given these definitions, we compute separate updraft and downdraft mass fluxes at level \( i \) as

\[
M_{ui} = m \sum_{j,f} [f u_i(j) + a_i(j,f)], \tag{9}
\]

\[
M_{di} = -m \sum_{j,f} d_i(j,f), \tag{10}
\]

where \( m \) is the mass of a parcel after mixing with the environment. The cloud-base fraction, \( f \), appears in the first term on the right side of (9) because the parcel mass before mixing is \( fm \).

The total vertical mass flux at level \( i \) is

\[
M_i = M_{ui} + M_{di}. \tag{11}
\]

This is related to the detrained mass flux, \( F(p) \), by the condition

\[
F(p) = \frac{\partial M(p)}{\partial p}, \tag{12}
\]

where \( M \) and \( F \) are here considered to be continuous functions of the pressure. (Note that the sign convention is such that upward mass flux is positive.)

The detrained mass flux can be computed directly from the simulation results as follows:

\[
F_i = \frac{1}{\Delta p} \left[ \sum_{j,f} m \delta_{i,j}(j,f) - E - lE \delta_{i,b} \right], \tag{13}
\]

where \( E \) is the mass of air entrained at each level (assumed to be independent of level) and \( \delta \) is the Kronecker delta function. The first term on the right expresses the detrainment of mixed parcels, while the second term represents the entrainment of environmental air for mixing. The last term is the inflow of air from cloud base, and equals the total amount of entrained air by hypothesis.

By mass continuity, the total outflow from a cloud (including the entrainment and inflow at cloud base as negative contributions) must be zero, that is,

\[
\sum_i F_i = (nlm - 2El) / \Delta p = 0, \tag{14}
\]

which leads to the condition \( m = 2E / n \). (Recall that \( l \) is the total number of pressure levels between cloud base and cloud top, and \( n \) is the total number of cloud-base fractions.) We insist that the upward mass flux at cloud base \( M_{ub} = 1 \), which serves to normalize the results. This condition implies that \( m = 2l / (nl) \) and \( E = 1 / l \). We verified that \( M_i \) computed using (9)–(11) equals that computed using (12) and (13).

The net convective sources of total cloud water (i.e., vapor plus cloud droplets) and entropy at level \( i \) can be written

\[
Q_i = \frac{1}{\Delta p} \left[ \sum_{j,f} m q_{TijD}(j,f) - E q_{Eij} - lE q_{Bj}\delta_{i,b} \right] - \left[ \frac{\partial}{\partial p} (q_{E,i} M) \right]_i, \tag{15}
\]

\[
S_i = \frac{1}{\Delta p} \left[ \sum_{j,f} m s_{D}(j,f) - E s_{Eij} - lE s_{Bj}\delta_{i,b} \right] - \left[ \frac{\partial}{\partial p} (s_{E,i} M) \right]_i, \tag{16}
\]

where \( q_{TijD} \) and \( s_{Dj} \) are, respectively, the total cloud water and the specific entropy of the air detrained at level \( p_i \) as a result of the mixing event \( (j,f) \), \( q_{Eij} = q_{E}(p_i) \) and \( s_{Eij} = s_{E}(p_i) \) are the environmental mixing ratio and entropy at this level, and \( q_{Bj} \) and \( s_{Bj} \) are cloud-base values of these parameters. These are comparable to the conventional expressions for these quantities (e.g., Ooyama 1971), except that a slight rearrangement of terms makes them more numerically robust. [See the Appendix for the relationship of (15) and (16) to the conventional expressions.] The analogous source term for the ice, \( I(p) \), is simpler than that for total water substance, since there is no source in the environment:

\[
I_i = \frac{1}{\Delta p} \sum_{j,f} m q_{ijD} \delta_{i,j}(j,f), \tag{17}
\]

where \( q_{ijD} \) is the ice mixing ratio of the air detrained at level \( p_i \) as a result of the mixing event \( (j,f) \).

The approximate convective heating at pressure \( p \) is

\[
H(p) = -M(p) \frac{\partial \theta}{\partial p}, \tag{18}
\]

where \( \theta(p) \) is the environmental profile of potential temperature. The approximations involved in (18) include assuming that no liquid water or ice in the outflow evaporates, and that the detrained air exits the cloud with the temperature of the environment. Since by hypothesis, air exits the cloud with neutral buoyancy, the latter approximation is valid if the virtual temperature and precipitation drag corrections can be neglected. We remind the reader that (15)–(18) do not include contributions from the evaporation of precipitation.

The precipitation itself has a source term,

\[
P_i = \frac{m}{\Delta p} \sum_{j,f} [u_i(j,f) \Delta q_{p} + a_i(j,f) \Delta q_{p}] + d_i(j,f) q_{ijD}, \tag{19}
\]

where the three terms on the right represent the formation of precipitation in unmixed updrafts and mixed
updrafts, and the melting of ice in downdrafts. The symbols $\Delta q_{pu}$ and $\Delta q_{ps}$, respectively, represent (8) evaluated in the unmixed and mixed updrafts, and $q_i(j, f)$ is the mixing ratio of ice in a downdraft created by the mixing event $(j, f)$. The level $i = F$ is the freezing level. In the context of the present model, $P_i$ represents the ejection of precipitation from its point of formation into the environment. Total water substance is subject to the conservation law

$$\sum_i (Q_i + P_i + I_i) = 0. \quad (20)$$

Since the model computation follows the trajectories of all distinct parcels through their ascents and descents, we are able to compute thermodynamic diagrams (Paluch 1979) at each level in the model. We actually employ an extension to the Paluch analysis, wherein the vertical mass fluxes associated with each vertical subcurrent are used to build up a two-dimensional histogram in the entropy–total cloud water plane at each level, updrafts contributing positively to the histogram, while downdrafts contribute negatively. Contour plots of this histogram highlight the thermodynamic characteristics of the upward and downward currents within the simulated cloud. Such extended Paluch diagrams should be useful diagnostic tools on real clouds as well.

In a final diagnostic, we compute the ascent or descent of a parcel after it has mixed with the environment. This variable is then contoured in the cloud-base fraction–mixing level plane. It is useful in determining levels where the motion of parcels is especially sensitive to entrainment and mixing.

3. Case study of 2 August 1984

Figure 1 shows a skew $T$–log$p$ chart for 0900 (all times are reported in MST = UTC – 7 h) sounding from Socorro Airport, which is approximately 25 km east of the storm location. (See RSB for a description of the locale and a discussion of the mass flux in this storm.) Note particularly the stable layer and the decrease in dewpoint that occur near 400 mb. These features in the environment play a key role in the evolution of the storm. The cloud base on this day over the Magdalena Mountains was near 660 mb just before and during the storm, though some clouds with lower bases were observed earlier. Cumulus congestus clouds with light precipitation and gradually increasing cloud-top heights existed from about 1030 to 1050. After 1050, the system rapidly developed into a strong cumulonimbus cloud, with highest cloud tops occurring around 1115. Radar tops ascended from 8 km (350 mb) at 1033, to 9 km (300 mb) at 1050, to 14 km (150 mb) at 1115. By 1130 the main cell of the system was decaying.

The 73°C equivalent potential temperature curve in Fig. 1 approximately indicates cloud-base conditions. Unfortunately, accurate measurements of cloud-base temperature are unavailable for this case. Thus, indirect means have to be employed to infer the temperature there.

Previous studies of storm development over the Magdalena Mountains show that storms are forced by upslope flow induced by solar heating of the mountain slope. As the surrounding boundary layer warms and deepens, the rising column of air over the mountain

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**Fig. 1.** Skew $T$–log$p$ chart of sounding taken at Socorro Airport near 0900 MST 2 August 1984. The observed cloud base was 660 mb. The 73°C moist adiabat shows approximate cloud-base conditions. The right panel shows the westerly (solid) and southerly (dashed) wind components from the sounding.
(called the convective core by Braham and Dragnis 1960) does the same (Raymond and Wilkening 1982). Since the convective core is forcing boundary-layer air upward into stable air, temperatures at its top are cooler and moister than the surrounding environment. Thus, clouds forced by this process have a certain amount of negative buoyancy at cloud base. As diurnal warming progresses, this negative buoyancy typically decreases, and cloud-top height increases. The potential temperature of the convective core increased at a rate of \( \approx 0.7 \text{ K h}^{-1} \) over the Magdalena Mountains in a case study of a thunderstorm on 6 August 1980 (Raymond and Wilkening 1982). As convective development on this day was quite similar to that on 2 August 1984, we assume that a similar rate of warming occurred at cloud base on the latter day.

Intense precipitation typically begins in New Mexico summer clouds when cloud tops exceed 400 mb, at which level the environmental temperature is characteristically \(-15^\circ\text{C}\). This suggests the following procedure for estimating the cloud-base temperature at the onset of precipitation. Ice- and precipitation-free simulations \((R_I = R_p = 0)\) are made using the observed cloud base, with a range of cloud-base temperatures. The lowest cloud-base temperature at which the simulated cloud top exceeds 400 mb is then taken as the value appropriate for the mature phase of the storm. This seems justified, as storms over Langmuir Laboratory typically develop very quickly once precipitation starts, so that additional boundary layer warming that occurs during the maturation process should be insignificant.

Figure 2 shows the vertical mass flux predicted by the model for no ice and precipitation, and with cloud-base temperatures ranging from 0 to 3 K cooler than the Socorro sounding at the 660-mb cloud base. Cloud top exceeds 400 mb only when the cloud-base temperature excess exceeds \(-2.25 \text{ K}\).

We now study the results of the ice- and precipitation-free simulation with this cloud-base temperature excess. Figures 3 and 4 show the simulated vertical and detrained mass flux for this case. The net mass flux is upward below 400 mb and slightly downward above this level. The downward motion is caused by mixing of updrafts with environmental air, with the mixture sinking as a result of evaporation of cloud liquid water. A small amount of mixing, evaporation, and sinking also occurs just above cloud base, resulting in some detrainment of mixed material below cloud base. This occurs because the cloud-base inflow is negatively buoyant. Little downward motion occurs between 600 and 500 mb in the model results.

Figure 5 shows the average detrained mass flux as measured by radar during a 17-min period in which the system was in the form of weakly precipitating cumulus congestus. The two curves indicate detrained mass flux directly measured by radar and inferred by mass continuity from the measured vertical mass flux. The difference apparently arises because of an uncertainty in the upper boundary condition on vertical wind in regions with downdrafts (see RSB). The simulation results with a cloud-base temperature excess of \(-2.25 \text{ K}\) agree more closely with the measured than with the inferred detrained mass flux in this case—both show
strong detrainment near 420 mb. Since aircraft measurements of detrained mass flux in similar situations also show strong detrainment near these levels (Raymond and Wilkening 1982), the preponderance of the evidence indicates that the inferred detrained mass flux is in error. As discussed by RSB, precipitation particles with large terminal velocities must therefore be found where downdrafts exist at cloud top during this stage of the storm’s development. We conclude that the 420-mb detrainment layer is real.

The simulation with a cloud-base excess of −2.25 K does not show a weak detrainment layer that is observed near 500 mb in the measured detrained mass flux. However, as may be seen in Fig. 4, significant detrainment is predicted near 500 mb when the cloud-base temperature excess is −3 K. This suggests that air flowing into the storm may not be completely homogeneous—relatively small variations in cloud-base conditions can explain the vertical spread in detrainment seen in Fig. 5.

Figure 6 shows the extended Paluch diagram at 380 mb for the preceding simulation. There is a mixing line between cloud-base air and air at about 320 mb. The horizontal hatching indicates that all mixed air is descending at 380 mb. By way of contrast, all mixed air passing through 520 mb is ascending (see Fig. 7). Figure 8 gives a clearer picture of ascent and descent after mixing. Shown are contours of the vertical displacement in pressure units after mixing, as a function of the cloud-base fraction and the pressure level at which the mixing event occurred. From Fig. 8 it is clear that mixing at low levels, especially with small cloud-base fractions, results in descent of the mixture. This is a consequence of the negative buoyancy at cloud base. Between 600 and 420 mb, virtually all mixtures ascend near the 420-mb detrainment layer, while above 420 mb, all mixtures descend to this level. The detrainment layer in the simulation thus consists of mixtures of cloud-base air with environmental air originating between 600 and 300 mb.

As noted above, the cloud-top height of the 2 August 1984 storm rose from 300 mb to 150 mb in approximately 25 min. The simulation results shown in Fig. 2 indicate that with no ice or precipitation, cloud tops
of 150 mb are approached only with cloud-base temperature excesses of $-0.25 \text{ K}$ or greater. Assuming that the 0.7 K h$^{-1}$ warming rate of the convective core found over the mountain for the 6 August 1980 case is applicable here, it would take approximately 3 h of additional boundary-layer warming at this rate to increase the cloud-top height to this level. This is far in excess of the 25 min actually taken for this growth, and suggests that the buoyancy enhancements caused by ice formation and removal of liquid water by accretion are essential to the transformation of the cumulus congestus cloud into the thunderstorm in this case.

To test this hypothesis, four simulations were done with increasingly strong ice and precipitation formation, but retaining the $-2.25 \text{ K}$ cloud-base temperature excess. In these simulations, precipitation was allowed to form at all levels in the cloud, but ice formation was restricted to levels colder than $-15^\circ\text{C}$. The rate parameters are shown in Table 1.

Figure 9 shows unmixed parcel buoyancies for the four different simulations. The differences between the four are dramatic, and are strongest in the upper troposphere, where an increase in buoyancy of 0.1 m s$^{-2}$

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**Table 1.** Simulations of 2 August 1984 case with various settings of the cloud microphysics parameters. In the case with strong precipitation and no freezing, the freezing of precipitation is turned off as well.

<table>
<thead>
<tr>
<th>$R_p$ (K s$^{-1}$)</th>
<th>$T_p$ (C)</th>
<th>$R_f$ (K s$^{-1}$)</th>
<th>$T_f$ (C)</th>
<th>Descriptive title</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td></td>
<td>no ice or precipitation</td>
</tr>
<tr>
<td>0.01</td>
<td>8</td>
<td>0.01</td>
<td>$-15^\circ\text{C}$</td>
<td>weak ice, weak precipitation</td>
</tr>
<tr>
<td>0.05</td>
<td>8</td>
<td>0.05</td>
<td>$-15^\circ\text{C}$</td>
<td>moderate ice, moderate precipitation</td>
</tr>
<tr>
<td>0.2</td>
<td>8</td>
<td>0.2</td>
<td>$-15^\circ\text{C}$</td>
<td>strong ice, strong precipitation</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>0</td>
<td>$-15^\circ\text{C}$</td>
<td>no ice, no precipitation</td>
</tr>
<tr>
<td>0.2</td>
<td>8</td>
<td>0</td>
<td></td>
<td>strong precipitation, no freezing</td>
</tr>
<tr>
<td>0.2</td>
<td>8</td>
<td>0</td>
<td></td>
<td>warm ice, strong precipitation</td>
</tr>
<tr>
<td>0.2</td>
<td>8</td>
<td>0.2</td>
<td>$-5^\circ\text{C}$</td>
<td></td>
</tr>
</tbody>
</table>
(~3 K) is found between the cases with no ice and precipitation and strong ice and precipitation formation. The updraft and downdraft mass fluxes computed by the model in these cases are shown in Fig. 10. Note how the updraft mass flux increases above 440 mb, while the downdraft mass flux decreases there, as ice and precipitation formation intensifies. The maximum simulated cloud top is 200 mb, which is still less than
the observed top of 150 mb, but recall that the model does not take into account the overshooting of cloud tops beyond their level of neutral buoyancy.

Figure 11 shows the detrained mass fluxes for these simulations. As the strength of ice and precipitation production increases, the detrainment layer near 420 mb diminishes and high-level detrainment increases. Figure 12 shows radar measurements of detrained mass flux for the mature thunderstorm phase of the 2 August 1984 case. The simulation results with strong ice and precipitation production agree with the observed anvil-level detrainment near 220 mb. Thus, many mixed parcels that descend in the ice- and precipitation-free simulation move upward in this case due to the increased buoyancy. However, some detrainment at 420 mb still exists in this simulation, which is not seen in the observations.

Figure 13 shows the vertical motion of parcels after mixing for the simulation with strong ice and precipitation. As conjectured, no significant downward motion of mixed parcels is shown above 600 mb.

It is interesting to see how ice and precipitation change the simulated thermodynamic diagrams. Figures 14 and 15 show these diagrams at 380 mb and 520 mb, respectively, for the strong ice and precipitation case. The main effect at 520 mb is to decrease the total water content, as would be expected if cloud water were converted to precipitation. At 380 mb the entropy of unmixed air is also increased, due to freezing.

Since our results are so sensitive to the development of ice and precipitation, we now present calculations showing the importance of individual cloud physical effects. Figure 16 shows the buoyancy of unmixed parcels for three additional situations. The short-dashed line shows the case in which all precipitation is suppressed but ice is allowed to form, while the case in which ice formation is suppressed but precipitation is allowed to form (and freeze above the freezing level) is shown by the long-short-dashed line. The case in which all freezing (including that of precipitation) is suppressed is given by the long-dashed line. Also shown

Fig. 11. Detrained mass fluxes for simulations illustrated in Fig. 9.

Fig. 12. Radar-determined detrained mass flux profiles for the mature thunderstorm stage (averaged from 1110 to 1130) for the 2 August 1984 storm. The solid line (measured) is obtained by horizontally integrating the horizontal divergence, while the dashed line (inferred) results from applying mass continuity to the vertical mass flux.
of freezing in ice formation (i.e., the short-dashed case in Fig. 16) causes a buoyancy enhancement comparable to the unloading of precipitation, but only above 400 mb, due to the assumed −15°C threshold temperature for ice formation. When the ice process is added to the strong precipitation process, a slight decrease in buoyancy occurs at high levels (compare the long-short-dashed curve with the right solid curve in Fig. 16), due to the fact that ice, as opposed to precipitation, is carried along with the parcel in our model.

A test was also made in which the parameters of the strong ice and strong precipitation simulation were modified so that the ice formation was allowed to begin at −5°C rather than −15°C (warm ice). Updrafts had slightly less buoyancy at high levels, due to the increase in the ice loading, and a larger fraction of cloud water turned into ice. As these changes are relatively small, the results are not illustrated.

The main conclusion from these sensitivity tests is that no single cloud physical process dominates in this situation, but that all of them taken together cause an increase in buoyancy that peaks at \( \sim 0.1 \, \text{m s}^{-2} \), or \( \sim 3 \, \text{K} \) near 300 mb. As Fig. 17 shows, the vertical mass fluxes for the three sensitivity tests are very different at high levels, showing that the associated changes in buoyancies have significant dynamical consequences.

Cumulus clouds affect their environment through the convective sources of total cloud water, entropy, ice, and precipitation. Vertical profiles of these quantities are shown for the cases with no ice and precipitation and with strong ice and precipitation in Figs. 18–20. These figures show how cloud microphysical processes modify the various convective source terms in the model.

Both the total cloud water and entropy source terms show strong sinks at cloud base, which simply reflects the fact that air flowing into cloud base is typically

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**Fig. 13.** Contours of the ascent or descent (in pressure units) of parcels after mixing as a function of cloud-base fraction and mixing level for the 2 August 1984 simulation with strong ice and precipitation. The contour interval is 40 mb.

for reference by solid lines are the buoyancy profiles for strong ice and precipitation, and for no ice and precipitation.

It is clear from considering the cases with no ice, but with freezing of precipitation turned on and off (i.e., compare the long-short-dashed and long-dashed lines, respectively, in Fig. 16 with the left-hand solid line), that the additional latent heat from freezing and the unloading of liquid water from the updraft are comparable factors in increasing the buoyancy of unmixed parcels, though the latter has more effect at lower levels. If no precipitation is allowed to form, the latent heat

**Fig. 14.** Extended Paluch diagram as in Fig. 6 except simulation with strong ice and precipitation formation, 380 mb. The vertical hatching indicates that mixed parcels are ascending.

**Fig. 15.** As in Fig. 14 except at 520 mb.
Fig. 16. Buoyancy of unmixed cloud-base parcels with a cloud-base temperature excess of 
-2.25 K. The left and right solid lines show the cases with no ice and precipitation and strong 
ice and precipitation. The long-short-dashed line shows the case in which ice formation is suppressed 
(but precipitation still can freeze), the long-dashed line shows the case in which all freezing 
(including that of precipitation) is suppressed, while the short-dashed line shows only the effects 
of ice formation with no precipitation.

Fig. 17. Updraft and downdraft mass fluxes for simulations with strong ice but no precipitation 
(short-dashed), strong precipitation but no freezing (long-dashed), and strong precipitation but 
no ice (long-short-dashed).
more moist and unstable than the surrounding environmental air. [See (A4) and (A5).]

The two cases with and without ice and precipitation show similar profiles of cloud water and entropy source up to about 500 mb. Above this level, the case with no precipitation shows strong moistening peaked in the 420-mb detrainment layer, while the case with precipitation shows drying there. Both cases show a tendency to increase the entropy above 500 mb. In the nonprecipitating case, the entropy source term is sharply peaked in the detrainment layer, while the distribution is broader and weighted toward higher altitudes in the precipitating case.

The nonprecipitating case, of course, exhibits no source of ice or precipitation. In the precipitating case, ice is detrained primarily at the anvil detrainment level as expected, while the precipitation source term peaks near 420 mb. Competition between ice and precipitation processes reduces the precipitation source term above this level.

4. Discussion and conclusions

In this paper the original stochastic mixing model of nonprecipitating cumulus clouds is extended to the case in which hydrometeors form. The simplest possible cloud microphysics model with an element of realism is used. Hydrometeors are divided into two classes. Ice, which actually includes pristine ice crystals and aggregates, and which typically has terminal fall speeds much less than convective updraft velocities, is assumed to be carried along with the parcel in which it forms. Precipitation, consisting of raindrops, graupel, and hail, that is, all hydrometeors formed by accretion, are assumed to have larger terminal fall speeds that allow them to fall out. All accretion of cloud droplets occurring at temperatures lower than 0°C is assumed to be accompanied by freezing, while ice creation is assumed to have a lower threshold temperature, typically −15°C. Both freezing and the removal of precipitation from the parcel of interest affect parcel buoyancy.

A flaw in the model is its inability to account for buoyancy effects (weight and evaporation) of precipitation that falls into a parcel from above. In addition, the fallout, melting, and evaporation of ice ejected into the high atmosphere by the model must be accounted for externally. However, to the extent that mesoscale updrafts are simply the remnants of convective updrafts rising slowly in a weakly unstable environment, this model accounts for them, since all parcels are assumed to move vertically to their level of neutral buoyancy. One cloud microphysical effect not included due to its
small size and uncertain magnitude is the change from
a water to an ice moist adiabat in updrafts at high
altitudes.

We have applied the model to many case studies,
but decided to restrict the current presentation to a
single, well-documented storm that occurred over
Langmuir Laboratory in the Magdalena Mountains of
central New Mexico on 2 August 1984. Though each
case study yields different results, the changes induced
by adding ice and precipitation are typically rather
similar. Also, since ice and precipitation significantly
complicate the results, we considered it impractical to
present more than one case in detail.

It is well known that freezing of liquid water and
unloading of precipitation can significantly increase the
buoyancy of updrafts, up to the equivalent of 3 K at
the $-15^\circ$C level in our model. The novel result is the
effect of this buoyancy enhancement on the stochastic
mixing model. For the 2 August 1984 case, the buoy-
ancy increase due to these effects is sufficient to explain
most of the observed differences in the vertical profiles
of mass flux between the cumulus congestus and ma-
ture thunderstorm stages. Thus, no additional dynam-
ic mechanism is required to explain the transition
from the former to the latter. This result should gen-
eralize to many other situations, as the majority of
the globe's thunderstorms occur in marginally unstable
environments where small buoyancy increments can
be crucial to storm development.

Comparison with the results of one or a few case
studies is insufficient to test the general validity of a
cloud model. Thus, a great deal of work remains to be
done before we can be confident that this model ade-
quately represents the behavior of moist convection
under a wide range of conditions. Unfortunately, ade-
quate observations against which to test this or any
other convective model are difficult to obtain. What is
required is the measurement of detrained mass, mois-
ture, and entropy fluxes, as well as equivalent mea-
surements of ice and precipitation production through
the life cycle of a wide variety of cumulus clouds and
thunderstorms.

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8914116.

APPENDIX

Relation of Convective Sources to Usual Forms

Equations (15) and (16) for the convective sources
of moisture and entropy are in a somewhat unfamiliar
form. In this appendix we show that they are equivalent
to the usual expressions (e.g., Ooyama 1971). From
(9)–(11) we find that

$$M_t = m \sum_{j, f} [f u_t(j) + a_t(j, f) - d_t(j, f)].$$

If $M$ were a continuous function of pressure, derivatives
of $u$, $a$, and $d$ with respect to $p$ would yield Dirac
delta functions. For a finite grid, the delta functions are
replaced by Kronecker deltas over the level spacing in
pressure, $\Delta p$. Thus,

$$\left[ \frac{\partial M}{\partial p} \right]_t = \frac{m}{\Delta p} \sum_{j, f} [\delta_t, (1 - f) - f \delta_t, B + \delta_t, D(j, f)] = \frac{1}{\Delta p} \sum_{j, f} [\delta_t, D(j, f) - E - lE \delta_t, B], \quad (A2)$$

where

$$\sum_f f = \sum_f (1 - f) = n/2 \quad (A3)$$

and $m = 2/(nl) = 2E/n$ have been used. Physically,
the three terms on the right side of (A2) represent de-
trainment of mixed parcels, entrainment of environ-
mental air, and flow of air into cloud base.

Substitution of (A2) into (15) and (16), respectively,
yields

$$Q_t = \frac{1}{\Delta p} \left[ \sum_{j, f} m(q_{Td} - q_{E}) \delta_t, D(j, f) - lE(q_t - q_{EB}) \delta_t, B \right]$$

$$\left[ \frac{\partial q_t}{\partial p} \right]_t, \quad (A4)$$

and

$$S_t = \frac{1}{\Delta p} \left[ \sum_{j, f} m(s_t - s_E) \delta_t, D(j, f) - lE(s_t - s_{EB}) \delta_t, B \right]$$

$$\left[ \frac{\partial s_t}{\partial p} \right]_t, \quad (A5)$$

where cloud-base values of mixing ratio and entropy,
$q_B$ and $s_B$, are not necessarily equal to environmental
values at cloud base, $q_{EB}$ and $s_{EB}$. These differences are
allowed because air entering cloud base is often lifted
from well below cloud base. They result in a net sink
of moisture and entropy at cloud base that must be
counteracted by processes that lift moist air from lower
levels.

Aside from the cloud-base term, the rest of (A4) and
(A5) simply represent the familiar "detrainment" and
"compensating subsidence" terms common to most
cumulus parameterizations. (The minus sign before
the compensating subsidence term results from $M$ being
defined positive upward.) The advantage of the forms
presented in the main text is that the pressure integrals
of $Q(p)$ and $S(p)$ come out closer to zero in their finite-
difference forms when total cloud water and entropy
are conserved. This has significant numerical advan-
tages.
REFERENCES


