SHORTER CONTRIBUTIONS

QUANTITATIVE ANALYSIS OF VERTICAL STRUCTURE IN PRECIPITATION

By R. C. Langille

Defence Research Board, Ottawa

and K. L. S. Gunn

McGill University, Montreal

(Manuscript received 7 May 1948)

1. Introduction.—Previous work by Marshall, Langille, and Palmer (1947) has shown a consistent correlation between \( R \), the rate of rainfall, \( M \), the mass of free rain water per unit volume of atmosphere measured at ground level, and the radar echo returned from precipitation. In the present work, it is assumed that the drop-size distribution is of the same form aloft as at the ground. If this assumption is valid, it is possible to interpret the radar presentation as seen on a height-position-indicator (HPI) display (degrees elevation against slant range) in terms of \( M \).

2. Theoretical development.—Assuming that the cross-sectional area of the radar beam is filled uniformly with rain, the theoretical formula developed by Ryle, and modified by Marshall et al. (1947), for the back-scattered power from raindrops for wave lengths not less than ten centimeters, is

\[
P_r = P_0 \frac{\pi A h (\eta^2 - 1)^2}{8 \eta^4 (\eta^2 + 2)^2} Z,
\]

where

- \( P_r \) = back-scattered power received at the radar
- \( P_0 \) = peak output power of the radar
- \( h \) = pulse length
- \( A \) = effective area of the antenna
- \( \lambda \) = wave length (not less than 10 cm)
- \( r \) = range of rain from radar
- \( \eta \) = refractive index of water
- \( Z \) = sum of the sixth powers of diameters of the drops in unit volume.

It is seen that

\[
P_r \propto 1/r^2 \tag{1}
\]

and

\[
P_r \propto Z. \tag{2}
\]

Equation (2) has been checked experimentally by Marshall et al. (1947), who also obtained the empirical relation

\[
Z \propto M^2 \text{ (approximately)} \tag{3}
\]

where \( M \) is mass of free rain water per unit volume of atmosphere (free rain water being used here to describe all those drops in the atmosphere over 0.5 mm in diameter, \textit{i.e.}, excluding the cloud droplets).

Combining relations (1), (2), and (3),

\[
P_r \propto M^2/r^2.
\]

In the present experiment, for purposes of measurement, only a fraction \( P_{rn} \), of the received signal \( P_r \) was amplified:

\[
P_{rn} = 10^{-0.1n} P_r,
\]

where \( n \) is the reduction in receiver gain (\textit{i.e.}, the level of \( P_{rn} \) below \( P_r \)), in decibels. Then

\[
P_{rn} \propto 10^{-0.1n} M^2/r^2.
\]

Let \( M_{min} \) be the least detectable free rain water at a given range \( r \), \textit{i.e.}, the free rain water which from that range provides the smallest detectable power \( P_{rn(min)} \). Then

\[
P_{rn(min)} = \text{constant} = 10^{-0.1n}(M_{min})^2/r^2.
\]

By comparative radar and rain-gauge measurements, for this particular radar operating at full power \((n = 0)\), it has been found that within a factor of two, 40 milligrams per cubic meter \((\text{mg m}^{-3})\) is the least detectable density of free rain water at a range of 5.5 miles. Therefore

\[
10^{-0.1n}(M_{min})^2/r^2 = (40/5.5)^2,
\]

and

\[
M_{min} = (40/5.5) \times 10^{0.06n} r, \tag{4}
\]

where \( M_{min} \) is in mg m\(^{-3}\) and \( r \) is in miles. Hence, the value of the mass \((M)\) of free rain water per unit volume of atmosphere, giving an echo just detectable on the HPI display varies linearly with range.

It is to be kept in mind that this relation (4) has been based upon measurements of the free water content of falling rain at the ground. In applying the values of \( M \) in this work, this relation has been assumed to hold aloft. When this assumption is dubious, the \( M \) values, represented by contours as in fig. 3, can be interpreted merely as reflection coefficients.

3. Experimental methods.—A. The radar. The radar set used for all observations was a microwave-height-finder (MHF), a Royal Canadian Air Force prototype equipment, slightly modified for this research. The wave length employed was 10.7 cm, and the antenna...
produced a horizontally polarized beam, 8° in horizontal, and 1.5° in vertical extent (to half power). Two displays were used, the plan-position-indicator (PPI) and the height-position-indicator (HPI).

The gain of the intermediate-frequency amplifier of the radar could be reduced by controlling the bias on the grids of the second and third tubes. This bias could be changed automatically in steps by switching a series of potentiometers into the circuit. The potentiometers were calibrated to reduce the gain, in 10-decibel steps from full gain, at the end of each upward and downward elevation sweep of the array, and hence of the HPI display.

These successive reductions in receiver gain, equivalent to increases in power received to give a comparable signal, were carried on until the echo was eliminated. The calibration of the potentiometers was performed by inserting the signal of a Boonton S-Band signal generator (type 102E) into the wave guide between the TR switch and the antenna. The calibration was checked approximately every two hours.

B. Recording observations. The HPI and PPI displays were photographed with sixteen-millimeter motion picture cameras, modified so that one frame of the film was exposed for each complete sweep of the display. Each frame also recorded the time, date and sequence number. In each case of the HPI display, the bearing of the echo being investigated and the state of the receiver gain were also recorded.

The observations were taken in sequences. A sequence consisted of at least ten successive sets showing the HPI display when the gain of the receiver was reduced in 10-db steps from full gain, one frame per step, until the echo was eliminated. Before and after each sequence, at least three frames of PPI display were taken to record the changes in shape and disposition of the echoes being examined, over the duration of the sequence.

To study the structure of the showers, information was easily obtained from the individual frames. To study motion within the shower, the length of film making up one sequence was spliced into a loop for continuous running through a standard sixteen-millimeter projector. The time necessary to record one sequence was of the order of ten minutes. Effects produced by the motion of the shower in range in this interval, particularly across the beam, were reduced to a minimum by choosing a favorable bearing for the vertical sections from a study of the PPI display (fig. 1). Thus the film record, in the form of a loop, contained the instantaneous picture desired for the study of shower structure, and also the moving picture for studying the internal motion and development.

4. Method of analysis.—A. Shower structure. The outline of the echo on each frame was traced from the projected photographs of the HPI display. Generally three or four frames (i.e., steps below full gain) were necessary to eliminate the echo. This gave a set of four or five contours of power received, each contour indicating the echo just detectable at that particular setting of the receiver gain. This picture is effectively instantaneous, there being a total duration of forty seconds or less from the outer to the inner contour.

Fig. 2. The HPI display for 1525 local time, 11 July 1947, showing the four stages of receiver gain: (a) full gain, (b) reduced 10 db, (c) reduced 20 db, (d) reduced 30 db. The original tracings referred to in section 4 (A) were made from these photographs.
These contours of power received were then transferred to a graph of height in thousands of feet against horizontal range in miles. The scales were of nearly equal magnitude (5000 ft against one mile). This graph showed, then, a section in the vertical plane of a rain shower, on a particular bearing.

To translate the contours of power received to contours of $M$, use has been made of equation (4). From it, appropriate values of $M$ were calculated and applied to the power-received contours at each mile of the range scale. Then, by joining points of equal $M$ value, the final contours of $M$ were obtained. Since these $M$ contours are lines of equal rain-water concentration in the atmosphere they afford a convenient means of studying the vertical structure of showers (fig. 3). Several such analyses separated by two or three minutes were taken from each sequence.

**B. Internal motion and development.** The analysis of the internal motion and development of the showers has been carried out by intensive visual study of the film loops, projected continuously. The upward sweep of the HPI takes 9 seconds, the downward sweep 7 seconds—16 seconds for two frames of film. The standard speed of silent motion picture film is 16 frames per second. Therefore the passage of events will be speeded up, on projection, 128 times. This speeded-up effect combined with the continuous repetition of the sequence being viewed provides an excellent method for studying motion and development of precipitation.

Verification of any motion seen by this method can usually be obtained from the movements of the associated $M$ contours over the period of a sequence.

**5. Results.**—**A. The diagrams.** The result of the analysis of one sequence, taken on 11 July 1947, a day of widely scattered convective showers, is presented in figs. 1–3. Fig. 1 shows the PPI display with the disposition of echoes along the bearing examined just before and after the sequence. Fig. 2 shows the HPI display at the four settings of receiver gain, for 1525 local time. Fig. 3 shows the $M$ contours corresponding to those HPI displays, as well as two others (at 1528 and 1530 local time) in the same sequence.

**B. Limitations of the method.** The appearance of a contour of high $M$ value in the final part of a shower-structure analysis depends on the individual judgment used to decide, when tracing the echo outline from the original HPI display photographs, (1) if there is a just detectable echo, and (2) the extent of its boundaries at, say, gain reduced 30 db. As may be seen from the small HPI-display photographs, which are reproduced in fig. 2, this is no easy task. Inaccuracies arise from this source, from successive graphical transformations, and of course, from the fact that displays at different stages of receiver gain are not quite simultaneous. Discrepancies in the over-all result can be observed when the $M$ contours, converted to appropriate axes, are drawn on the original photograph. Such discrepancies should not prevent the use of the $M$ contours in studying shower structures. Conclusions should only be accepted, however, when the accuracy of the contours involved has been checked against the HPI-display photographs.

**C. Some observations concerning structure and development of showers.** During the summer of 1947, six sequences were analysed. Though this small number of analyses cannot lead to any firm conclusions as to structure and development, the following tendencies have been noted:

1. Within a shower, the $M$ values may vary by as much as a factor of 100 in a horizontal distance of one mile.
2. At any instant within a given shower, $M$ values may be greater aloft than near the ground.
3. The portions of maximum $M$ value within a given shower tend to be elongated in the vertical.
4. Where motion of a dense portion has occurred, its center of gravity has always moved towards the ground.

5. The contours of lowest \( M \) value (100 mg m\(^{-2}\)) have stayed at a reasonably constant height over the period of a sequence, while below them, a contour of higher \( M \) value may have fallen through a considerable distance.

6. The detailed vertical pattern of a shower changes rapidly with time, and may change out of all recognition in as little as five minutes.

D. Future plans. A considerable set of film records was obtained during the summer of 1947. Work is proceeding on their analysis according to the general method described here. More specific attention is being given to mass movement within the showers and to correlation of the 'kinematics' with the physical mechanisms involved.

Acknowledgments.—The research described was carried out in the Radio Propagation Laboratory of the Defence Research Board as part of their Radar Meteorology Project. The collaboration of Mr. W. McK. Palmer and the advice and assistance of Dr. J. S. Marshall, both of McGill University, is gratefully acknowledged.

REFERENCES


EXPERIMENTAL VERIFICATION OF ENTRAINMENT OF AIR INTO CUMULUS

By E. W. Barrett and H. Riehl

University of Chicago

(Manuscript received 27 May 1948)

During the summer and fall of 1947, a group of four meteorologists from the University of Chicago, including the writers, engaged in a study of typhoons and other weather phenomena of the tropical Pacific Ocean. The study was carried out under the sponsorship of the Office of Naval Research. Project headquarters were located at Fleet Weather Central, Guam.

It was decided during the planning of the project to install a number of special aircraft weather instruments in weather reconnaissance aircraft for the purpose of making detailed aerological ascents and cross sections in selected weather situations. It should be possible to obtain detailed information about the mechanisms of low-latitude disturbances in this way; in particular, the ascents might uncover some interesting facts concerning the process of convection.

Since the measurement problems involved were of a micrometeorological character, the conventional weather-reconnaissance instrument, the wet- and dry-bulb psychrometer, was much too slow in response to serve adequately. Accordingly, fine-wire thermocouples were chosen to effect the measurement of temperature. One thermocouple was designed to be mounted on a strut below the nose of the aircraft so as to be completely exposed to the free flow of air and water. The other was mounted in a specially constructed housing, pictured in cross section in fig. 1 and shown mounted under the nose of the aircraft in fig. 2. The housing was constructed to allow free circulation of air about the thermojunction while deflecting the water droplets. In this manner the temperature of the air might be determined without errors due to contact of the thermocouple with large drops falling from a colder region higher in the cloud.

Because of the uselessness of the wet-bulb thermometer under rapidly varying humidity conditions, moisture measurements were made by means of an electronic dewpoint hygrometer developed in the Instrument Laboratory of the Department of Meteorology of the University of Chicago.\(^1\) A portable lightweight model capable of being powered from the aircraft electrical system was designed and constructed especially for the project.

The voltages developed by the hygrometer thermocouple and those mounted outside the aircraft were measured and recorded on a Brown strip-chart Potentiometer Recorder. The couples were connected sequentially to the recorder terminals by means of a solenoid-actuated rotary stepping switch impelled by a cam-driven microswitch in series with the solenoid and the direct-current aircraft supply. The cam was mounted directly on the shaft of one of the gears in the paper-drive mechanism of the Brown Recorder and was arranged to close the microswitch for one-half second every five seconds. Thus the output of each thermocouple was recorded for five seconds. For convenience in studying the record, one couple was switched in twice in succession during each sequence.

\(^1\) University of Chicago, Department of Meteorology, "On the development of instruments for accurate pressure, temperature, and humidity measurements in the upper atmosphere." Final progress report for the period April 1, 1946, to June 30, 1947 (mimeographed).