The Influence of Hadley Circulation Intensity Changes on Extratropical Climate in an Idealized Model

EDMUND K. M. CHANG

Center for Meteorology and Physical Oceanography, Massachusetts Institute of Technology, Cambridge, Massachusetts

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ABSTRACT

Experiments have been performed using a simple global model with idealized physics and zonally symmetrical forcings to investigate the influence of Hadley circulation intensity changes on extratropical climate. The heating within the Tropics is latitudinally concentrated, while the heating in the extratropics is kept unchanged. This leads to an increase in the intensity of the Hadley circulation. As found earlier by Hou, along with the increase in the intensity of the Hadley circulation, there is a statistically significant temperature increase in the winter high latitudes.

Zonal-mean diagnostics have been performed in order to identify the link between the changes in the Tropics and the extratropics. Detailed diagnosis of the heat budget shows that warming in the winter high latitudes is induced by changes in the mean meridional circulation, over the opposing cooling effects caused by changes in the eddy heat fluxes. Such a change is consistent with an equatorward shift of the jet stream and its associated heating pattern. It is suggested that the equatorward shift in jet position is caused by an increase in westerly acceleration within the Tropics associated with the enhancement in the intensity of the Hadley circulation. Limitations of the model are also discussed.

1. Introduction

Studies on the Hadley circulation arising from axisymmetric forcings by Held and Hou (1980), Lindzen and Hou (1988), and Hou and Lindzen (1992) suggested that in the absence of midlatitude eddies the Hadley circulation driven by equator-to-pole differential heating would accelerate a subtropical jet to a strength much stronger than the observed jet, especially for conditions appropriate for wintertime. Hou and Lindzen (1992) also found that concentration in the thermal forcing within the Hadley cell [which can be viewed as a very rough representation of concentration of rainfall by the intertropical convergence zone (ITCZ)] can lead to a substantial increase in the strength of the Hadley circulation—resulting in a much more intense subtropical jet and greatly enhancing the potential vorticity gradients near the edge of the Hadley cell. They suggested that if the potential vorticity gradient was at all indicative of the intensity of baroclinic wave activities in the extratropics, a small change in the tropical heating structure could significantly alter wave transport at middle and high latitudes, leading to changes in extratropical climate.

It is well known that the zonal-mean circulation is driven by eddy fluxes as well as by axisymmetric forcings (e.g., Kuo 1956; Pfeffer 1981; Crawford and Sasanomi 1981; among others). Hence, to fully assess the effects of changes in the Hadley circulation, we must include the actions by midlatitude eddies. In this paper, we will extend the results of Hou and Lindzen (1992) by the inclusion of the effects of eddies.

Recently, Hou (1993) performed a series of experiments using an idealized general circulation model (basically a full GCM except that prescribed heating replaces radiation calculations, no moisture, and turbulence is replaced by diffusion). By shifting the prescribed tropical heating toward the summer pole while leaving extratropical heating unchanged, Hou (1993) found an increase in the intensity of the Hadley circulation, accompanied by warming at the winter middle and high latitudes as a result of increased dynamical heating. More recently, Hou and Molod (1995) found similar changes in experiments using a full GCM. While the association between the low and high latitude changes appeared robust, it is not clear what mechanism is behind such an association.

The perturbation forcings used by Hou (1993) and Hou and Molod (1995) to enhance the intensity of the Hadley circulation is different from the one used in Hou and Lindzen (1992). In this paper, we adopt the forcing used in Hou and Lindzen (1992) to see whether we still observe changes similar to those observed by Hou and colleagues. Hou (1993) also found significant differences in the response between Northern and Southern Hemisphere winter, suggesting that zonal asymmetries in the lower boundary could play an important role in

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the link between the Tropics and high latitudes. Here, we will perform experiments using a model with neither topography nor surface heat fluxes to see whether we can understand the system better without the complications introduced by strong stationary waves. While the results might not be realistically applied to understanding the real atmosphere, we hope that this will serve as a step in understanding the link between the Hadley regime and the midlatitude eddy regime.

In section 2, we will describe the numerical model used in this study and the experimental setup. The results will be shown in section 3. Zonal-mean diagnostics will be discussed in section 4, and in section 5 we will speculate on what is the link between changes in tropical forcings and extratropical climate change. The sensitivity of the results to changes in the forcing will be presented in section 6. The results will be summarized in section 7.

2. Description of experiment setup

a. The numerical model

The model used in this study is adapted from the model of Ross and Orlanski (1982), which is based on the anelastic approximation of Ogura and Phillips (1962). The hydrostatic assumption is also taken for large-scale flows. We have chosen this model because of the simplicity of the model equations (especially the temperature and hydrostatic equations), which greatly eased the computation and interpretation of temperature diagnostics. A few comparison runs have been performed using a dynamics-only version of the GFDL spectral model (programmed by L. Held, based on the GFDL climate model described in Gordon and Stern, 1982). The results are very similar and will not be shown here.

The model equations are as follows:

\[
\frac{d}{dt} \rho_0 \mathbf{V} = -\nabla \rho_0 \mathbf{F} - f \mathbf{k} \times \rho_0 \mathbf{V} - \frac{\rho_0 \mathbf{V}}{\tau_P} \delta(z - z_0) - \kappa \nabla^4 \rho_0 \mathbf{V}, \tag{1}
\]

\[
G \frac{\partial}{\partial \sigma} \rho_0 w = -\nabla \cdot \rho_0 \mathbf{V}, \tag{2}
\]

\[
\frac{d \theta}{dt} = -\frac{\theta - \theta_e}{\tau_H} - \kappa \nabla^4 \theta, \tag{3}
\]

and

\[
G \frac{\partial \Phi}{\partial \sigma} = \frac{g \theta}{\Theta_0}. \tag{4}
\]

In the above equations, \(\sigma\) is a stretched height coordinate and is related to the physical coordinate \(z\) by the transformation \(G(\sigma) = d\sigma/dz\). The operator \(\nabla\) operates only on the horizontal components. Very simple and idealized parameterizations of physical processes only have been included. Surface friction is modeled by Rayleigh friction at the lowest model level, internal dissipation by biharmonic diffusion, and diabatic processes by thermal damping toward an equilibrium temperature profile. Under these assumptions, apart from the advection terms in the momentum and potential temperature equations, all other terms are linear.

The model used for this study is global, bounded by rigid horizontal boundaries at the top and bottom. The spectral transform method is used in the horizontal and centered differencing in the vertical. The method of solution closely parallels that of Gordon and Stern (1982), except that time integration is entirely explicit here. The leapfrog scheme is used for forward integration in time (except for the damping terms, for which the Euler forward scheme is used). An Euler backward time step is applied periodically to eliminate the computational mode.

b. The equilibrium temperature profile

The main forcing in the experiments in this study is thermal damping toward an equilibrium temperature profile \(\theta_e\), which sort of mimics the equator-to-pole differential heating due to solar radiation. The form of \(\theta_e\) follows that of Lindzen and Hou (1988) and Hou and Lindzen (1992). For the control experiment, the following profile is used,

\[
\frac{\theta_e - \theta_0}{\Theta_0} = 1 + \frac{\Delta \theta}{3} \left[ 1 - 3(y - y_0)^2 \right] + \Delta \theta \left( \frac{z - 1}{H} \right). \tag{5}
\]

Here, \(y\) is sine of the latitude, \(y_0\) is the latitude of maximum temperature, \(\Delta \theta\) represents equator-to-pole temperature difference, and \(\Delta \theta\) represents vertical stability; \(z\) is the height, and \(H\) is the height of the top boundary. A typical profile is shown in Fig. 1a. (For motivation of such a simple profile see North et al. 1981.)

The \(\theta_e\) profile for the perturbation experiments represents concentration of heating in the Tropics (Hou and Lindzen 1992):

\[
\frac{\theta_e - \theta_0}{\Theta_0} = \begin{cases} 
\theta_e(y_0 + L_t, z)/\Theta_0 + (A + \Delta \theta L_t^2) \\
\times \left\{ \cos \left[ \frac{\pi}{2 \delta_0^2} (y - y_0) \right] \right\}^2 & \text{(6a)} \\
\theta_e(y_0 + L_t, z)/\Theta_0 & \text{(6b)} \\
\theta_e(y, z)/\Theta_0 & \text{(6c)}
\end{cases}
\]

Equation (6a) applies when \(y\) is between \(y_0, y_0 + \delta_0\), (6b) when \(y\) is between \(y_0 + \delta_0, y_0 + L_t\), and (6c) when \(y\) is poleward of \(y_0 + L_t\). The subscript \(t\) denotes ‘‘+’’ or ‘‘−’’ to differentiate quantities on the side of the summer pole from those on the winter pole side (see Hou and Lindzen 1992 for details).

\[
1 \text{Note that there are a few typographical errors in Eq. (1) of Hou and Lindzen (1992). Some subscripts 0 have been left out. Anyway, (6) above is the correct version of the equation.}
\]
δ_ε = 4Δθ_L^2 / 3(A + Δθ_L^2), L is the redistribution width for concentration, and A is a dimensionless "perturbation amplitude" at the heating maximum. A typical profile of θ_ε is shown in Fig. 1b. Note that the total heat input into the system with (6) is unchanged from (5). For the perturbation experiments, if one writes θ_ε = θ_e + Δθ_e, the simple thermal damping used here allows one to rewrite (3) as

\[ \frac{dθ}{dt} = -\frac{θ - θ_e}{τ_H} + \frac{Δθ_e}{τ_H} - κ\nabla^2 θ. \]

Hence, the perturbation in the equilibrium temperature profile Δθ_e is equivalent to a perturbation heating with a heating rate Δθ_e/τ_H. In the rest of this paper we will refer to Δθ_e as perturbation heating.

Hou and Lindzen (1992) showed that increasing the perturbation amplitude A resulted in an increase in the strength of the Hadley circulation. However, in the absence of eddies, the middle and high latitudes remained in thermal equilibrium, and changes in tropical forcing did not lead to any changes in the middle and high latitudes. Here we will examine whether the inclusion of eddies offers a "link" between the Tropics and high latitudes and allows changes in the Tropics to influence the climate in the higher latitudes.

c. Values of parameters used in the numerical experiments

For the experiments performed in this study, we use rhomboidal truncation for spectral decomposition in the horizontal to attain higher resolution in the meridional direction (compared to triangular truncation). The resolution chosen here is R21, using 54 latitudinal and 64 longitudinal points for the transformed Gaussian grid to eliminate nonlinear aliasing. In the vertical direction, a rigid lid is placed at H = 15 km. Ten un-
evenly spaced levels are used in the vertical, with the spacing increasing with \( z \) such that each level represents approximately equal mass (\( \rho_0 \) decreases nearly exponentially with \( z \)). As such, the vertical coordinate is very similar to a pressure coordinate. Since all applied forcings and boundary conditions are axially symmetric, the rigid lid used here does not introduce problems in the form of stationary wave resonances.

For the experiments described here, the Rayleigh friction timescale \( \tau_F \) equals 1 day, radiative damping timescale \( \tau_H \) equals 20 days, and \( \kappa \) is set such that the damping timescale for the highest total wave number (42) is 1 day. We have performed some sensitivity studies by varying the values of \( \tau_F \) and \( \tau_H \) by factors of 2. We have also tested sensitivity of the results to diffusion by changing the strength of the diffusion (to damping timescale between 10 days and 1/2 day) and the form of the diffusion (using \( \nabla^2 \) instead of biharmonic diffusion). The results are qualitatively similar and will not be shown here.

For the radiative equilibrium temperature profile (5) for the control experiment, we have chosen the parameters to roughly represent Southern Hemisphere winter conditions; \( y_0 \) corresponds to a latitude of 10°N, \( \Delta_H \) equals 1/6(1 – \( z^2/H^2 \)) (equivalent to a temperature difference of about 80° between 10°N and the winter pole at the surface), and \( \Delta_V \) is set to be 0.2 (equivalent to a vertical \( \theta \) gradient of approximately 4 K km\(^{-1} \)). Such a profile has been shown in Fig. 1a.

For the standard perturbation experiment, we have chosen values of \( L_i \) [see (6)] that correspond to concentration from 30°S to 25°N. The amplitude \( A \) is chosen such that the maximum temperature perturbation \( (\Delta \theta_L) \) is 20°, which is equivalent to a maximum perturbation heating rate \( (\Delta \theta_L/\tau_H) \) of 1° day\(^{-1} \). For the standard experiment, \( \theta_L^\prime \) is shown in Fig. 1b, and the temperature perturbation is shown in Fig. 1c. The wiggles seen in Fig. 1b are due to the misrepresentation of the heating concentration by spectral representation, but the errors are small except at the grid point closest to the pole, and the spatial scale is of much higher frequency compared to the changes we will see later. In the next section, we will discuss in detail the results from the control and standard perturbation experiments. We have also performed sensitivity experiments by varying \( L_i \) and \( A \). The results from those experiments will be discussed in section 6.

3. Results

The experiments are initialized by first running a zonally symmetric version of the model until the climate reaches a steady state that resembles the Hadley cell solutions of Lindzen and Hou (1988) and Hou and Lindzen (1992). Then small amplitude noise is added to the eddy components, and the experiment is run for 800 days. Once-daily results from the final 700 days of the runs have been taken to represent the model climate.

\( a. \) Control experiment

The climate of the control experiment, as represented by the zonal-mean zonal wind, potential temperature, and mass flux by the mean meridional circulation (in \( 10^{16} \) kg s\(^{-1} \)), is shown in Fig. 2. Comparing with observed climate in the atmosphere (e.g., Oort 1983), we find that the winter jet is a bit too strong and is too far poleward, while the summer jet is too weak. The Hadley circulation is too weak (observed strength is \( \sim20 \) in the units used here) mainly due to the lack of moisture in the model, but the winter Ferrel cell has about the right strength. These features agree well with the "dry" simulation of Williams (1988).

In the hypothetical case of an atmosphere with no eddies, angular momentum is only transported by the zonal-mean circulation, and the Held and Hou (1980) model predicted that in that case the jet core should be at the edge of the Hadley circulation. In Fig. 2, we see that the jet core in our experiment is poleward of the edge of the Hadley circulation [so are observed jets in the atmosphere, though their poleward displacements w.r.t. the edge of the Hadley circulation are less than those shown in Fig. 2 (see Oort and Peixoto 1983)], and eddy momentum fluxes must be responsible for the poleward transport of angular momentum across the edge of the Hadley cell to maintain the jet. Hence, one can argue that the eddies act to displace the subtropical jet poleward.

As to other quantities, the eddy heat and momentum fluxes (not shown) also have about the right magnitude for the winter side, but everything is too weak in the summer hemisphere. As for the temperature, the stratification near the ground is too strong. This is because we have not put in a realistic boundary layer. We have performed several experiments with vertical diffusion near the lower boundary, modeling the vertical mixing of momentum and heat inside the planetary boundary layer near the surface. The resulting temperature structure near the ground bears closer resemblance to observations, but otherwise the results are very similar and will not be shown here. Since we have used an extremely simple representation for the physical processes, we do not expect our model to be able to reproduce observed climate accurately, thus we have made no attempt to tune the model climate to agree better with observations.

\( b. \) Standard perturbation experiment

The climate of the standard perturbation experiment is shown in Fig. 3. Compared to the climate of the control, we see that the winter Hadley cell is nearly doubled in strength. The winter jet is shifted equatorward by about 3° latitude. The difference in zonal wind and potential temperature between the perturbation and control experiments is shown in Fig.
4. The equatorward shift of the winter jet is clearly visible. For the temperature field, associated with the heating perturbation applied, there is warming near the equator and cooling between 30° and 50°S. However, as in Hou (1993), there is significant warming between 50° and 80°S. The shaded regions indicate a t-test significance level of over 95%. More will be said about statistical significance in the appendix.

Recall that the heating perturbation is applied between 30°S and 25°N. As discussed earlier, if we consider the zonally symmetric circulation without eddies, the “climate” in the higher latitudes outside the reaches of the Hadley circulation corresponds simply to radiative equilibrium. Hence, the control and perturbation climate must be the same in the higher latitudes when eddies are absent, since the radiative forcing there is the same for both cases. Thus, the changes in the winter extratropics we observe in Fig. 4 must have been brought about by changes in eddy transport. In the next section, we will present some zonal-mean diagnostics to examine how these changes have been brought about.

4. Zonal-mean diagnostics

To understand the link between the changes in the middle and higher latitudes and changes in forcing in the Tropics, we have performed a series of zonal-mean diagnostics. Denoting zonal-mean quantities by an overbar and eddies by a prime, we have

\[ x = \bar{x} + x'. \]
Taking the zonal mean of (1) to (4), we get
\[
\frac{\partial}{\partial t} (\rho_0 \overline{\nabla}) = -\nabla \rho_0 \overline{\nabla} - \mathbf{f} \times \rho_0 \overline{\nabla} \\
- \frac{\rho_0 \overline{\nabla}}{\tau_F} \delta(z - z_0) - \kappa \nabla^4 \rho_0 \overline{\nabla} - \nabla \cdot \rho_0 \overline{\nabla} \overline{\nabla}' \\
- G \frac{\partial}{\partial \sigma} \rho_0 \overline{\nabla} \overline{\nabla}' ,
\] (8)

The continuity (2) and hydrostatic (4) equations remain the same since they are linear. Here,
\[
\frac{\partial}{\partial t} x = \frac{\partial x}{\partial t} + \nabla \cdot \overline{\nabla} + G \frac{\partial}{\partial \sigma} \overline{\nabla} ,
\] (10)

We see that apart from the applied zonal-mean forcing, zonal-mean quantities are now also forced by the divergences of eddy fluxes of momentum and heat. We then take the time mean of (8) to (10). Each of the nonlinear terms in (10) can be split up into two terms, one associated with advection by the time and zonal mean, the other advection by transients in the zonal mean, which is small. The advection by transients is dropped, and the equations remain unchanged if we now interpret the overbar as a zonal and time mean. In a quasi-steady state (i.e., no long-term secular trend), \( \partial / \partial t \) also vanishes.

Here we want to diagnose what gives rise to the changes in temperature in the higher latitudes. From (9), we see that temperature changes give rise to a change in radiative damping (assuming \( \theta \) is the same, which is the case in the higher latitudes). Assuming that the effect of diffusion is negligible (see section 5a, Fig. 8b), this change must be balanced by changes in the dynamic heating. In Fig. 5a, we show the differ-
Fig. 4. Difference between the standard perturbation climate and control climate. (a) Zonal-mean zonal wind (contour interval 2 m s\(^{-1}\)). (b) Potential temperature (contour interval 0.5 K). The shaded regions represent >95% confidence level based on the \( t \) test.

ences in dynamic heating between the standard perturbation experiment and the control experiment. [Note that dynamic heating has dimensions of time rate of change of temperature. However, (3) and (3') suggest that we can convert it into an equivalent temperature change by multiplication by \( \tau_H \) (20 days here). This is what is shown in Fig. 5 and most subsequent plots of heating rates for easier comparison to the temperature changes.] We see that indeed the changes in dynamic heating resembles the temperature changes shown in Fig. 4b outside the region where the perturbation heating is applied. The small difference between Figs. 5a and 4b in the higher latitudes is due to the presence of diffusion and the misrepresentation of the perturbation heating profile by spectral expansion due to the Gibb's phenomenon.

Total dynamic heating can be further decomposed into heating due to convergence of eddy heat fluxes and heating due to transport by the mean meridional circulation (MMC). Their respective contributions to the change in total dynamic heating are shown in Figs. 5b,c. In the extratropics, both are much larger than the total dynamic heating, and to first order they cancel the effects of each other. However, we see that changes in heating due to changes in eddy heat fluxes are mostly negatively correlated with changes in total dynamic heating (and hence also negatively correlated to changes in temperature), while changes in heating due to changes in the MMC are positively correlated to changes in total dynamic heating. Figures 6a,b show the eddy meridional heat fluxes in the control and standard perturbation experiments, respectively. We see that while the heat flux in the perturbation experiment is slightly stronger than that in the control, the main difference is an equatorward shift in the latitudinal position of the heat flux in the perturbation experiment that gives rise to the "cool-warm-cool" pattern seen in Fig. 5b (see also Fig. 9b). This shift is consistent with the equatorward shift in the position of the jet discussed in the previous section. Figure 5 suggests that the increase in temperature that we see in the high southern latitudes is mainly due to changes in dynamic heating due to changes in the MMC.

Studies of the forcing of the MMC (e.g., Kuo 1956; Pfeffer 1981; Crawford and Sasamori 1981; and others) show that the MMC is forced by eddy fluxes, diabatic forcing, and dissipation. From (8) to (10), we see that in our case, while we have only imposed a change in the diabatic heating, changes in the eddy fluxes in response to the change in diabatic heating will also act to force changes in the MMC. We have taken the following approach to assess the individual effect of changes in heat fluxes, momentum fluxes, and diabatic heating (\( \Delta \theta_s \)). First, we linearize (8) and (9), using a basic state representing the average of the control and standard perturbation climates (quantities represented by \( \langle \cdot \rangle_m \), see Schneider 1988):
\[ \frac{d}{dt} (\rho_0 \Delta \vec{v}) + \nabla \cdot \Delta \vec{v} \bar{V}_m + G \frac{\partial}{\partial \sigma} \Delta \vec{w} \bar{V}_m \]

\[ = -\nabla \rho_0 \Delta \vec{b} - f k \times \rho_0 \Delta \vec{v} - \rho_0 \frac{\Delta \vec{v}}{\tau_F} \delta (z - z_0), \]

\[ - \kappa \nabla^4 \rho_0 \Delta \vec{v} - \nabla_3 \cdot \Delta M \]

(11)

\[ \frac{d}{dt} (\Delta \bar{\theta}) + \nabla \cdot \Delta \vec{v} \bar{\theta}_m + G \frac{\partial}{\partial \sigma} \Delta \vec{w} \bar{\theta}_m \]

\[ = -\frac{\Delta \bar{\theta}}{\tau_H} + \frac{\Delta \bar{\theta}_c}{\tau_H} - \kappa \nabla^4 \Delta \bar{\theta} - \frac{1}{\rho_0} \nabla_3 \cdot \Delta H, \]

(12)

where

\[ \frac{d}{dt} x = \frac{\partial x}{\partial t} + \nabla \cdot \bar{V}_m x + G \frac{\partial}{\partial \sigma} \bar{W}_m x. \]

(13)

Note that the divergence signs are largely symbolic, since the derivative in the zonal direction is zero for zonal-mean quantities. These equations, together with (2) and (4) (now applied to perturbation quantities), are a full set of linear prognostic equations, with perturbation \( \Delta \bar{u}, \Delta \bar{v}, \) and \( \Delta \bar{\theta} \) as the prognostic variables, forced by perturbation (standard perturbation experiment minus control) in the eddy heat fluxes (\( \Delta H \)), eddy momentum fluxes (\( \Delta M \)), and diabatic heating (\( \Delta \theta_c \)). We have highlighted these forcing terms in (11) and (12) by bracketing underneath. With these equations, we can separately assess the individual effect of each of the forcing terms by putting in one term at a time. The model described in section 2a can be easily modified to solve this set of zonal-mean equations. The individual effects of changes in eddy heat fluxes, eddy momentum fluxes, and diabatic heating on the steady-
e. Why are changes in the heat budget dominated by changes in the MMC?

Both observational and theoretical studies (e.g., Oort and Peixoto 1983; Stone and Branscombe 1992) suggest that in the current climate, the heat budget in the high latitudes is dominated by convergence of eddy heat fluxes, with heat transport by the MMC playing only a minor role. So why are the changes in the heat budget in our experiments not dominated by changes in the eddy heat fluxes? In this section, we will examine the heat budget of both the control and perturbation experiments in more detail.

The vertically and zonally averaged heat budget for the winter hemisphere of the control experiment is displayed in Fig. 8. Figure 8a shows the heating rate due to convergence of heat fluxes due to heat transport by the MMC (curve A) and eddies (curve B). The three curves in Fig. 8b show total dynamic heating (curve A equals the sum of the two curves in Fig. 8a), diabatic heating (curve B equals \(-\frac{\Theta - \Theta_0}{\tau_0}\)), and diffusion (curve C). The three curves in Fig. 8b should add up to zero for long time averages [see (9)].

From Fig. 8b, we see that dynamic heating is balanced by radiative cooling over much of the winter hemi-
sphere. The contributions from diffusion are negligible. The distribution of radiative heating/cooling qualitatively resembles that observed in the atmosphere, but the dynamic heating differs significantly from observed heating due to atmospheric energy transport (see Oort and Peixoto 1983). In section 5c, we will discuss this difference in more detail and discuss whether we expect this difference to affect the applicability of our results to the atmosphere. Here, we will concentrate on examining the results from the experiments.
eddy heat flux convergence is larger than that due to the MMC, the magnitude of the meridional gradient of heating due to the MMC is generally larger in the higher latitudes (see Fig. 8a); hence, the changes in heating are dominated by changes in the MMC.

The gradient in heating due to the MMC can be larger than that due to eddy heat fluxes because of the difference in scale between the two heatings. The scale of the MMC is related to that of the momentum fluxes, which is generally narrower in meridional scale compared to that of the heat fluxes, in the real atmosphere as well as in our experiments. Heating due to eddy heat fluxes is basically a dipole, while that due to the three-cell pattern of the MMC consists of one and a half "waves" in a hemisphere. Hence, even though the amplitude of heating due to MMC is smaller, the gradient in the heating can be larger.

If we examine the patterns shown in Figs. 8a and 9 more closely, we see that the peaks in heating in the Southern Hemisphere extratropics due to the heat fluxes and MMC are not exactly in phase, with the peak in MMC heating (actually cooling) occurring around 55°S, and the peak due to the heat fluxes a few degrees poleward. Hence, even though changes in heating due to changes in MMC dominate over much of the winter extratropics, around 55°S (close to the peak of the MMC heating) the changes due to heating by the MMC are near zero and changes in the eddy heat fluxes dominate near that latitude. If we examine Figs. 5 and 7 closely, we can also see that around 55°S, changes in heating due to changes in eddy heat fluxes dominate,
while nearly everywhere else changes in heating due to changes in MMC (or eddy momentum fluxes, which only affect the temperature through the MMC) dominate.

**b. Equatorward shift of jet position**

In the previous paragraphs, we have seen that the climate changes observed in the winter extratropics in the perturbation experiment are consistent with an equatorward shift in the heating pattern. This shift is consistent with the equatorward shift in jet position discussed in section 3b. In this section, we will try to speculate on what causes the shift of the wind and heating pattern.

Let us first recall what effects concentrated heating have on the climate in the absence of eddies. The equal area solution of Hou and Lindzen (1992) suggested that if the perturbation heating was entirely confined within the original Hadley cell, there would be no change in either the zonal wind or temperature. The Hadley cell would become stronger with concentration, but since the zonal wind inside the Hadley cell had the profile given by “angular momentum conservation,” the absolute vorticity within the Hadley cell would be zero, and the increased meridional wind experienced no net Coriolis acceleration (in the Eulerian framework). Hence, the only effect of the heating perturbation is to drive a stronger MMC to transport heat within the Hadley cell to compensate for the heating perturbation when no eddies are present.

The situation is quite different in the presence of eddies. In the atmosphere, absolute vorticity in the Tropics is not exactly zero and is in fact not significantly smaller than $f$, the Coriolis parameter. Held and Phillips (1990) discussed how a Rossby wave could “invade” the interior of the Hadley cell to reestablish part of the vorticity gradient within the Hadley cell. In Fig. 10, we show the meridional profile of the absolute vorticity at a height of 8 km for the control experiment. The profiles for $f$ and for the axisymmetric case without eddies are also shown for comparison. We see that for the axisymmetric case, the absolute vorticity in the Tropics, while not exactly zero due to diffusion and
finite spatial resolution, is indeed significantly less than \( \bar{f} \). However, in the experiment with eddies, the eddies have effectively mixed nonzero vorticity air back into the Tropics, and the absolute vorticity inside the Hadley cell is closer to \( \bar{f} \) than to zero.

In the presence of nonzero absolute vorticity, enhancement in the meridional wind component will lead to acceleration of the zonal-mean wind. The change in zonal wind acceleration \( \Delta \bar{u}/\Delta t \) due to changes in the MMC between the standard perturbation and control experiments is shown in Fig. 11a. In the Tropics in the Southern Hemisphere, the increase in strength in the Hadley circulation gives rise to an increase in \( \Delta \bar{u}/\Delta t \) of up to 0.7 m s\(^{-1}\) day\(^{-1}\). (The changes outside of the Tropics can be interpreted as response of the MMC to changes in eddy momentum fluxes and are not part of the “original” change in Hadley forcing.) Is this increase in westerly acceleration within the Tropics responsible for the changes observed in the extratropics in the perturbation experiment?

We performed an additional experiment to address this question. For this experiment, which we will call the westerly acceleration experiment, we use the same temperature forcing as in the control experiment. The only difference is that on the lhs of the zonal component of the momentum equation (1) we have added an additional acceleration term. This extra forcing has peak amplitude of 1 m s\(^{-1}\) day\(^{-1}\) at the top, and its vertical structure is similar to that seen in Fig. 11a. The applied forcing is shown in Fig. 11b. Horizontally, it has the shape of a Gaussian distribution centered at 18°S, with a width of 8° in latitude. As such, this forcing is stronger and wider than that seen in Fig. 11a (with the resolution used here narrowing down the forcing further will give rise to rather noisy results).

The results of the westerly acceleration experiment, as represented by changes in \( \bar{U} \) and \( \bar{\theta} \) from the control experiment, are shown in Fig. 12. Compared to the results of the standard perturbation experiment shown in Fig. 4, we see that the structure of the changes in the Southern Hemisphere is very similar. The observed changes are stronger than those shown in Fig. 4 because the applied forcing is stronger than that inferred from Fig. 11a. Analysis of the heat budget in the winter extratropics also agrees well with that discussed in section 4. Hence, the results of this experiment support the hypothesis that the increase in westerly acceleration in the Tropics could be responsible for the changes observed in the extratropics.

So how can we interpret this shift? Concentration of heating in the Tropics gives rise to an increase in the strength of the Hadley circulation. Coriolis acceleration acting on the increased meridional velocity leads to anomalous westerly acceleration over the Tropics. Averaged over time, this increase in westerly acceleration must be counteracted by deceleration due to divergence of eddy momentum fluxes. In the control experiment, the eddy momentum fluxes act to displace the subtrop-

![Fig. 11](image_url)

**Fig. 11.** (a) Change in zonal-mean zonal wind acceleration due to changes in the mean meridional circulation (standard perturbation minus control). (b) Zonal wind acceleration forcing applied in the westerly acceleration experiment. Contour interval is 0.2 m s\(^{-1}\) day\(^{-1}\).
ical jet poleward from its position implied by the eddy-free Held and Hou (1980) solutions. Stronger westerly acceleration in the Tropics means that the eddy momentum fluxes will not be as effective in displacing the jet poleward,\(^2\) hence, the jet position in the perturbation and westerly acceleration experiments is more equatorward when compared to that in the control experiment. The eddies and their associated heating/cooling pattern shift equatorward together with the jet, giving rise to the temperature changes in the middle and higher latitudes as discussed in the preceding sections.

c. Applicability to the atmosphere

In this paper, we have used a very idealized model to investigate the changes in extratropical climate associated with a change in Hadley forcing. A legitimate question to ask is whether the changes observed here to be realized in a more realistic (earth-like) climate.

\(^2\) Here we have implicitly assumed that changes in the amplitude of eddy momentum flux are small, at least when compared to the change in intensity of the Hadley circulation. The results of Green (1970) and Stone and Yao (1987) suggest that the magnitude of the eddy momentum flux depends mainly on the latitudinal temperature gradient and on the background potential vorticity gradient. Since neither of these change appreciably between the control and perturbation experiments, we expect that the magnitude of the eddy momentum flux will not change appreciably, which is the case observed in the experiments.

In section 5a, we mentioned that the dynamic heating in our control climate (curve A in Fig. 8b) differs significantly from the observed convergence of energy fluxes due to atmospheric transport. If we examine Fig. 13 (adopted from Figs. 42 and 43 in Oort and Peixoto (1983)) and compare their Northern Hemisphere annual mean (Southern Hemisphere observations are much less reliable) divergence of energy flux to those shown in Fig. 8, we can see a few significant differences. In our control experiment, heating due to MMC dominates equatorward of 45° latitude. However, in the observations, eddy transport dominates poleward of about 20° latitude, and MMC transport dominates only within the deep Tropics. One reason for this difference is the absence of moisture in our model. In the real atmosphere where moisture is present, transport of energy by the MMC is reduced by latent heat transport, whereas for eddies latent heat transport has the same sign as transport of sensible heat. This is especially important in the middle and lower latitudes, such that in the presence of moisture eddy transport of energy dominates everywhere outside of the deep Tropics.

Another difference between our idealization and the real atmosphere is the absence of heat fluxes from the lower boundary in our model simulations. Observations suggest that the atmospheric transport of energy in the winter midlatitudes (not shown), say around 40°N, is dominated by eddy transport, which is divergent and gives rise to a cooling tendency. The top-of-atmosphere radiation budget requires a net convergence of energy into the winter midlatitudes of the atmosphere, which
is actually supplied by oceanic transport of energy and changes in oceanic energy storage, which enter the atmosphere as sensible and latent heat fluxes across the lower boundary of the atmosphere. In our model simulations, we do not have heat fluxes across the lower boundary. The surface heat flux due to changes in oceanic storage averages out to about zero over a year, which is the reason we chose to compare our dynamic heating with the annual mean heating (instead of heating in winter) due to transport in the atmosphere.

In view of such significant differences between the heat budget in our idealized model and the real atmosphere, can we still expect the results obtained here to be applicable to understanding the real atmosphere? The differences in the heat budget due to latent heat and oceanic transport and storage are most important in the midlatitudes and relatively unimportant in the higher latitudes. In Fig. 13, we see that in the higher latitudes, say poleward of 60°N, the annual mean [also true for winter observations, see Oort and Peixoto (1983)] gradient of observed heating is such that the heating rate increases with latitude, and the gradient is generally dominated by that due to heating by the MMC, qualitatively similar to those seen in our experiments (Fig. 8). Hence in the event of an equatorward shift in the heating pattern, the behavior of the winter high latitudes of the atmosphere should be qualitatively the same as that in our idealized experiments, that is an increase in temperature in the winter high latitudes, with the changes being dominated by changes in the MMC. In section 7 we will say more about the applicability of our model results to understanding the atmosphere.

6. Sensitivity of extratropical response to changes in strength of forcing

Assuming that the increase in westerly acceleration in the Tropics due to enhancement in the strength of the Hadley circulation is the driving force behind the changes observed, can we predict how sensitive the temperature increase in the higher latitudes is versus changes in the applied forcing? Here we will try to make a very rough estimate. We make the additional assumption that the forcing is proportional to the integral of the westerly acceleration induced by the enhanced Hadley circulation; that is,

$$F \propto \int_{x_0}^{x} f \Delta v dy.$$  (14)

Here $y_0$ is the southern boundary of the heating perturbation [equal to $y_0 + L_-$ in (6)], $\Delta v$ is the meridional velocity that will transport the right amount of heat within the Hadley circulation to erase the applied heating perturbation, and $f$ is the Coriolis parameter. We have approximated the absolute vorticity by the Coriolis parameter, which is not a bad approximation in the presence of eddies (see Fig. 10). The above integral can be easily computed given the heating perturbation $\Delta \theta$, from (6) to (5).

First we investigate what happens when we change the perturbation amplitude at the heating maximum $A$. Shown in Fig. 14 is the sensitivity to changes in $A$ of the forcing $F$. Note that all other quantities have been fixed, using the parameters we used for the standard perturbation experiment. The forcing plotted is relative to that of the standard experiment, which has a maximum heating rate equivalent to 1.0 K day$^{-1}$. We see that the relative forcing flattens out very quickly and does not increase much with further increase in $A$. This is because with the form of the concentration (6), as $A$ is increased the region of concentrated heating becomes more and more confined to close to the heating maximum, and the increase in the meridional velocity occurs mainly where $f$ is small, leading to insignificant increases in zonal wind acceleration. The points plotted on Fig. 14 represent results from sensitivity experiments performed using different amplitudes of the heating perturbation. We did not go beyond 2 K day$^{-1}$ because by then the heating maximum is so narrow that the resolution used for the experiments began to become insufficient to resolve the peak (see the wiggles in Fig. 1b). The values plotted represent the maximum temperature increase observed in the higher latitudes, normalized by the value found for the standard experiment (0.94 K). The error bars represent 95% confi-
the relative change in perturbation temperature is consistent with the relative change in forcing. Obviously, we still need to establish how the absolute magnitude of the temperature change is related to the change in forcing, which is a rather complicated task since the results here suggest that the final temperature change represents small residues between the effects of changes in heat fluxes and MMC that largely cancel out each other.

In section 5, we suggested that the temperature change in the winter high latitudes is probably related to the equatorward shift in jet position. The sensitivity study shown in Figs. 14 and 15 involves seven independent cases, and it is interesting to see how the temperature change is related to the shift in jet position. The shift in jet position for each case is estimated based on a figure similar to Fig. 9, using the shift in the zero position of MMC and eddy heating near 45°S, which can be estimated fairly accurately by interpolation. The maximum increase in temperature in the winter high latitudes is plotted against the equatorward shift in jet position in Fig. 16. The point for the standard perturbation experiment described in detail in sections 3–5 is plotted with a circle. From Fig. 16, we see that while the correlation is not perfect, in general, larger temperature increase is associated with a larger equatorial shift in jet position. Note that the 95% confidence limit for temperature change is about 0.4° and that the estimation in the shift in jet position is quite rough since the nominal latitudinal resolution of the Gaussian grid for the

The curve represents sensitivity of forcing to change in the amplitude A. The amplitude is expressed as heating rates in K day⁻¹. The forcing shown is normalized by the forcing with amplitude A equals 1.0 K day⁻¹. The points shown are experiment results of maximum temperature change in the higher latitudes, again normalized by the result of the standard experiment. Hence, 1.0 represents 0.94 K in temperature change. The error bars represent 95% confidence limits. The results appear to be consistent with the change in the forcing. However, one must take this apparent agreement with a grain of salt, since it is not at all clear that the maximum temperature change is the most representative quantity to compare, and the extratropical response to changes in tropical forcing is not necessarily linear.

Next, we examine what happens when we change the latitudinal limit of heating perturbation $y_-$ while keeping $A$ the same as in the standard experiment. The resulting change in $F$ is shown in Fig. 15. We see that the forcing is extremely sensitive to $y_-$. From (6), we can estimate that the magnitude of the heat flux necessary to compensate for the heating perturbation goes up roughly as $y_-$; hence, $\Delta y$ is proportional to $y_-$.

The Coriolis parameter $f$ adds in another power, and the integral adds in a further one; hence, $F$ goes up roughly as $y_-$. The points again represent high-latitude temperature changes seen in the sensitivity experiments. Again, they are consistent with the change in forcing. While the response increases dramatically with an increase in $y_-$, for the experiments having $y_-$ south of 30°S, the heating perturbation actually extends outside of the southern boundary of the Hadley circulation, thus rendering the results inconsistent with the original assumption that the heating redistribution occurs entirely within the Hadley cell. Here, we have shown that

![THEORETICAL CHANGE IN FORCING](image)

**Fig. 15.** Same as Fig. 14 except for sensitivity to change in southernmost latitude of concentration. The relative forcing is normalized by the forcing with the southernmost latitude at 30°S.
R21 truncation is about 3.4°, and in all cases the observed shift is less than two grid points.

7. Summary and conclusions

Hou and Lindzen (1992) showed that concentration in thermal forcing within the Tropics, while keeping the net energy input into the atmosphere unchanged, can greatly increase the strength of the Hadley circulation. In this study, we found that in the presence of eddies, the increased intensity of the Hadley circulation is associated with an increase in temperature in the winter high latitudes. Diagnosis of the heat budget suggested that this change in extratropical climate is brought about by changes in the dynamic heating associated with the MMC, rather than changes in poleward heat transport by the eddy heat fluxes. However, as the MMC in the extratropics can be viewed as being entirely driven by eddies (notice the absence of an MMC in the extratropics in the eddy-free Held and Hou model), changes in the MMC transport can also be regarded as part of the eddy response.

In section 5, we saw that the change in the heat budget is consistent with an equatorward shift of the dynamic heating/cooling pattern associated with the eddy heat fluxes and the MMC. We then speculated that the equatorward shift in the position of the jet and its associated eddy activity is caused by enhanced westerly acceleration in the tropical upper troposphere due to the increase in the Hadley intensity. Results from an experiment in which we artificially introduced an anomalous westerly acceleration in the Tropics offer support to this hypothesis. In this picture, apart from transporting more heat toward the higher latitudes, eddies come into play in another important way. Without eddies, the absolute vorticity in the upper troposphere inside the Hadley circulation is homogenized by advection by the mean meridional flow. Hence, increase in the intensity of the Hadley circulation will not lead to any westerly acceleration, as pointed out by Held and Hou (1980). However, the presence of eddies reestablishes the absolute vorticity gradient within the Hadley cell by mixing high PV air equatorward. This enables the increased intensity of the Hadley circulation to be translated into an increase in westerly acceleration. Held and Phillips (1990) had examined certain aspects of this mixing using a barotropic model. We are now working on trying to understand how this happens in the presence of baroclinic effects.

The increase in temperature in the winter high latitudes associated with the increase in Hadley circulation is similar to the results obtained by Hou (1993). However, there are important differences between the results here and his results. Here, the equatorward shift of the winter jet is quite obvious (Figs. 2–4). However, in Hou (1993) there is no obvious shift in the winter jet. In fact, the zonal wind anomalies shown in Hou suggested a concentration instead of an equatorward shift of the winter jet. The difference is probably due to the difference in the applied perturbation forcing. Here, we concentrated the heating within the Hadley cell but kept the latitude of maximum heating unchanged such that the Hadley circulation simply intensifies in place. Hou (1993) displaced the tropical heating maximum by 15° latitude toward the summer pole. Under such a perturbation, the Hadley circulation not only intensifies but the rising branch of the Hadley cell also shifts toward the summer pole by approximately 15°, giving rise to strong easterly anomalies in the tropical upper troposphere and drastically altering the momentum balance within the Tropics. We believe that our experiments here reproduce a more straightforward manifestation of an intensification of the Hadley circulation. Noting the above differences, we do not expect the mechanism suggested here to apply directly to Hou’s results. What gives rise to the extratropical response in Hou’s experiments requires further diagnosis.

Here we have suggested one possible scenario on how changes in the Tropics may affect the higher latitudes. We obviously do not claim that this scenario can be applied to the understanding of all instances of climate change. For example, if the climate change is driven by a change in the (radiative equilibrium) temperature gradient in the midlatitudes, we would expect that changes in the eddy heat fluxes will dominate over the effects of changes in the MMC (this is confirmed by results of numerical experiments not shown here). In fact, even in our control climate, poleward heat transport by eddy heat fluxes dominates that by the MMC in the middle and higher latitudes (see Stone and Branscome 1992). It is only the perturbation (i.e., perturbation experiment minus control) heat transports that have approximately the same magnitude, with the
heat carried by the perturbation MMC being slightly larger.

The concentration in thermal forcing (6) was introduced by Hou and Lindzen (1992) as a crude representation for concentration of latent heating associated with the ITCZ. It is somewhat reassuring that some of the changes discussed above, namely the equatorward shift in the position of the jet and storm track, can also be observed in comparisons of moist and dry runs of more complicated GCMs (Williams 1988). One might argue that the omission of moisture in our model may seem to be too drastic an approximation that renders our results totally inapplicable to understanding the dynamics of the real atmosphere. However, the recent full GCM results of Hou and Molod (1995) suggest that it is dynamic heating, rather than latent heating, that changed in the winter extratropics in association with changes in intensity of the winter Hadley circulation. Hence, we believe that studies with dry models can still be useful.

We have described results of experiments being forced by persistent winter conditions. We have not attempted to incorporate a realistic seasonal cycle, but we did conduct some transient experiments similar to those discussed in Hou (1993) to examine how fast the observed changes are established. The results from the transient experiments suggest that 50 days after the forcing is changed, the magnitude of the temperature change realized in the higher latitudes is about half that of the "equilibrium" changes discussed above.

The results shown in Figs. 14 and 15 suggest that if we confine the heating perturbation entirely within the Hadley cell, with the parameters used here given in section 2c, it is difficult to get extratropical temperature increases of significantly more than about 1 K by concentration of heating in the Tropics. While this temperature increase may seem small, a few degrees shift in the mean position of the storm track could affect local climate significantly. We expect the exact quantitative changes to depend on the values of the parameters chosen, but performing experiments to cover the entire parameter space is beyond our current computational capabilities. Moreover, our intention here is to establish a link between changes in tropical forcings and climate in the extratropics rather than to estimate the absolute magnitude of such changes. For realistic estimates applicable to the atmosphere, more complicated GCMs having more realistic representations of the physical processes would have to be used.

Finally, we need to emphasize again the idealized nature of the model. As discussed in the introduction, we have chosen to use zonally symmetrical forcing and lower boundary conditions in order to gain some insight into a system simpler and hopefully easier to understand than the atmosphere. Zonally asymmetrical forcing is definitely very important in the atmosphere and is probably the reason why the observed winter jet appears to be positioned right at the edge of the Hadley circulation [see section 6.5 of Grotjahn (1993)] instead of being displaced poleward w.r.t. the edge of the Hadley circulation, as in our control climate shown in Fig. 2. So one may legitimately question whether the shift in jet position associated with the intensification of the Hadley circulation seen in the model results may be realized in the real atmosphere. The results shown in Fig. 16 suggest that if the original jet position had started out to be a few degrees equatorward of where it is in the control experiment, increase in the Hadley forcing would still result in an equatorward shift in the position of the jet as well as the associated change in extratropical climate. Obviously, we cannot extrapolate the results with confidence to a situation in which the original jet position is at 30°S. However, we believe that as long as the increase in westerly acceleration due to increase in the Hadley intensity is equatorward of the original jet position, the jet will respond by shifting equatorward. Whether this is true has to be resolved by studies using more realistic GCMs and observational analyses, which will be a future step in our research. There are hints that our results may not be entirely inapplicable to the real atmosphere. Analyses of observations suggest that during El Niño years, the intensity of the Hadley circulation is enhanced. At the same time, the zonal-mean subtropical jet is also observed to be displaced equatorward of its normal position (Rosen et al. 1984) together with its associated high-frequency eddy activity. Hence, it is tempting to associate what we find in our model results with the above-observed phenomena. As discussed earlier, to apply our results to the real atmosphere we need to address the issues of zonal asymmetry and the mean position of the subtropical jet. We will try to examine such complications in future experiments and analyses. This paper is not intended to be the final word on this topic but a next step in the increase in complexity from the works of Lindzen and Hou (1988) and Hou and Lindzen (1992).

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3 In a paper presented at the 19th Annual Climate Diagnostic Workshop, Oort and Yienger (1994) suggested from analyses of radiosonde data that the intensity of the Hadley circulation during El Niño years is about 10% stronger than normal and about 20% higher than that during La Niña years.
APPENDIX

Statistical Significance of Temperature Changes

Even though we have applied time-independent forcing, the climate itself does experience low-frequency variability driven by variations in synoptic timescale eddy activity (e.g., Robinson 1993). Hence, we need to make sure that the changes we observe between the perturbation and control experiments are real and not just a manifestation of climate fluctuations. The issue gains more importance when we note the fact that if we perform a principal component analysis on the internal climate fluctuations of either the control experiment or the perturbation experiment, the leading eigenvector bears a large resemblance to the pattern shown in Fig. 4, basically a north–south shift of the westerly jet. A similar pattern has also been found in observational studies of climate variations (e.g., Karoly 1990).

To judge whether the observed changes shown in Fig. 4 are statistically significant under the t test, we need to know the variance of the mean climate. We have a time series of 700 data points taken one day apart, but the data points are obviously not all independent; hence, the degree of freedom is less than 700. We have used two methods to estimate the variance of the mean climate. For the first method, we computed the time-lagged autocorrelation of the perturbations and obtained a one-day lagged correlation of 0.9 for the temperature fluctuations in the winter middle and higher latitudes, corresponding to an e-folding memory of 10 days (a reasonable value given a radiative timescale of 20 days with dynamics helping to wipe out anomalies a bit faster). The discussions in Trenberth (1984) suggested that with such an autocorrelation, the effective degrees of freedom would decrease by a factor of 20. We tested this estimate using a second method. The 700 days are chopped into 14 chunks of 50 days each, which are assumed to be independent (a fair assumption given a memory of 10 days). The variance of the 50-day mean is then compared to the variance of the dataset, and the ratio comes out to be close to the result predicted by the first method. Thus, in estimating the variance of the 700-day mean from the variance of the dataset, we have used 35 as the number of degrees of freedom instead of 700. This memory in climate fluctuations is the reason why we have to run the experiments for such long durations to get statistically significant results.

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