Early Radar Echoes from Small, Warm Cumulus: Bragg and Hydrometeor Scattering

CHARLES A. KNIGHT AND L. JAY MILLER
National Center for Atmospheric Research,* Boulder, Colorado

(Manuscript received 6 August 1997, in final form 9 January 1998)

ABSTRACT

Studies of small cumulus clouds in Florida using X- and S-band radar (3- and 10-cm wavelengths) reveal both hydrometeor and Bragg scattering signals. Turbulent mixing between cloudy and drier environmental air can produce centimeter-scale variations in refractive index that can lead to strong mantle echoes around the sides and tops of the clouds. When the environmental air is exceptionally dry, the S-band Bragg scattering signals are as strong as 10 dB at cloud boundaries, with weaker echoes in the cloud cores where hydrometeor scattering is also present. The Bragg signal at S-band is typically about 19 dB stronger than that at X-band, as expected from theory. However, there is in many cases an unexplained, Bragg-like return from the clouds at S-band that correlates with the X-band echo but is only about 10 dB stronger. The X-band echo is often dominated by backscattering from the cloud droplets, and shows adiabatic ascent within the cloud cores fairly often up to at least 1 km above cloud base. In these cases, the radar echo profiles can be used to estimate the adiabatic droplet concentration, given rough knowledge of the cloud-base height and temperature. The first precipitation shafts often occur before the cloud tops reach the 0°C level, are narrow, and probably consist of low concentrations of drops several millimeters in diameter.

1. Introduction

The purpose of this paper is to describe and discuss the interpretations of dual-wavelength, early radar echoes from small, warm cumulus. In using the 10- and 3-cm wavelength radars (S- and X-band, respectively) to study the first stages of precipitation formation, it is found that Bragg scattering is an important component of the radar echo along with the particulate scattering from the hydrometeors (Knight and Miller 1993). However, deciphering the dual-wavelength data has turned out not to be entirely straightforward. The systematic aspects of the phenomena that we identify from the observations are described here so that the problems can be considered by others and so that the qualifications involved in interpreting the radar echo in terms of the hydrometeor populations can be recorded in some detail.

The data are from the CP-2 radar, in CaPE and SCMS (the Convection and Precipitation/Electrification Experiment and the Small Cumulus Microphysics Study), north of the Kennedy Space Center, Florida, in July and August 1991 and 1995. The antennas of the 3- and 10-cm wavelength radars were collocated on the same pedestal, adjusted to the same pointing angle, and their beam patterns are described in Keeler et al. (1984). Clouds were scanned in the elevation angle direction (RHIs) at a series of azimuth angles usually spaced about 1°–1.5° apart. Measurements were taken every 100 m in range and about every 0.3°–0.7° in elevation. Each measurement came from a pulse volume with both azimuth and elevation beamwidths of 1° and length of 150 m.

a. Ideal Bragg and Rayleigh (hydrometeor) scattering

The data will be discussed in terms of an ideal interpretation and the deviations from that ideal that the data seem to demand.

Ideal Bragg scattering arises from a concept of turbulent mixing between media with different indices of refraction, in which the wavenumbers \( k = 2\pi/L \) (where \( L \) is scale size or wavelength) associated with spatial variations in the index of refraction are distributed according to the \(-5/3\) power law. This is described briefly in Knight and Miller (1993), and more complete treatments are to be found in Gossard and Strauch (1983) and Doviak and Zrnić (1984). The multiwavelength approach to distinguishing Bragg and Rayleigh scattering was used by Christian and Wakimoto (1989), and observations of the wavelength dependence of clear-air Bragg scattering were analyzed by Rogers et al. (1992).
An ideal Bragg scattering signal expressed in dBZ = 10 log(Z/1 mm$^6$ m$^{-3}$), where Z is the radar reflectivity factor, leads to a wavelength dependence such that a pure Bragg echo at X-band is about 19 dB weaker than at S-band. [See Eqs. (A2)–(A4) and (A10) in appendix A.]

In ideal Rayleigh scattering from clouds, all of the radar return comes from spherical, liquid water drops that are small enough that the Rayleigh approximation applies. [See Eqs. (A1) and (A3) in appendix A.] The drops must have diameters less than about 2 mm ($d \leq \lambda/16$, Doviak and Zrnić (1984)) to avoid the irregularities of Mie scattering at X-band. The dBZ scale is expressed in terms of ideal Rayleigh scattering, so the units of Z, the equivalent reflectivity factor, are only literally correct when ideal Rayleigh scattering actually holds. For the purposes of separating Bragg from Rayleigh components using data at two wavelengths, the assumption that the Rayleigh component is ideal still works if the particles are nonspherical (since both X- and S-band signals are horizontally polarized) and if the dielectric constant is not that of water; but, it does not work when Mie scattering is important.

If both the Rayleigh and the Bragg scattering components are ideal, as defined above, then it is only possible for dBZ at X-band (to be called DZX) to range from 19 dB less than that at S-band (to be called DZS) to equal to it. We will call this difference DZS-X. Figure 1 is a typical scatter diagram of DZS versus DZX from an entire volume scan of a few small clouds, showing that 19 dB is in fact a reasonably good upper bound at the lower values of reflectivity factor where Bragg scattering is important.

The lower, measured values of DZS and DZX appear to have an uncertainty of ±2–3 dB for the purposes of meteorological interpretation. This range of random variability in the measurements is strongly supported by the scatter of points about the DZS-X = 0 line when both DZS and DZX exceed about 5 dBZ in Fig. 1. The trend of DZS-X values toward 0 dB at higher reflectivities is expected from Rayleigh scattering. In the region where the points fill the space between the 0- and 19-dB lines, one does not know the
extents to which these are due to mixtures of Bragg and Rayleigh scattering or to nonideal Bragg and Mie scattering. It appears reasonable, however, that most of this is due to mixtures.

An approach that can be used to separate DZS or DZX into their Bragg or Rayleigh components is detailed in appendix A. Assuming that each measurement is a linear combination of ideal Bragg and Rayleigh reflectivity factors, (A14) and Fig. 2 give the corrections needed to deduce DZSb and DZR, the Bragg component of DZS, and the Rayleigh echo DZR. The corrections depend only upon the difference between DZS and DZX, not their magnitudes. Figure 2 shows that for DZS-X less than about 16 dB, the correction to DZX to obtain DZR is less than 3 dB. Likewise for DZS-X greater than 3 dB, DZS may be considered to be DZSb, within 3 dB. This holds, of course, only when DZX is far enough above the noise level for its value to be relied upon within 3 dB. As can be seen from the minimum reflectivity factors listed in Table 1,

![Diagram](https://via.placeholder.com/150)

**Fig. 2.** The correction factors from Eq. (A14) plotted versus DZS-X for the spectral slopes marked. The curves can be used to obtain DZSb, the Bragg component of DZS, and the Rayleigh echo DZR from measured DZS and DZX, under the assumptions discussed in the text. As the signal becomes more Bragg like (large values of DZS-X), the amount of correction to be applied to DZX to obtain DZR becomes increasingly larger. On the other hand, as the signal becomes more Rayleigh like (small values of DZS-X), the amount of correction to be applied to DZS to obtain DZSb becomes increasingly larger. The widely spread curves for the Rayleigh corrections indicate their sensitive to the assumed Bragg scattering law. However, the S-band Bragg correction is almost independent of the assumed law as seen by the very small displacement between the −5/3 and 0 laws, while the −7/3 and −3/3 curves fall essentially on top of the −5/3 curve and the −1/3 curve is about half way between the −5/3 and 0 curves.

Table 1. Minimum radar reflectivity factors at a range of 5 km for the CP2 S- and X-band radars in CaPE (1991) and SCMS (1995). The X-band reflectivities were further adjusted* by adding 1.5 dB in CaPE and 1.0 dB in SCMS to make the S- and X-band signals agree when the echoes were clearly only from hydrometeors.

<table>
<thead>
<tr>
<th>Study</th>
<th>Radar</th>
<th>Noise (dB)</th>
<th>Radar Constant</th>
<th>Minimum dBZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaPE S-band</td>
<td>−107.9</td>
<td>74.2</td>
<td>−19.7 (−18.2*)</td>
<td></td>
</tr>
<tr>
<td>CaPE X-band</td>
<td>−113.7</td>
<td>74.8</td>
<td>−24.9</td>
<td></td>
</tr>
<tr>
<td>SCMS S-band</td>
<td>−113.8</td>
<td>68.7</td>
<td>−31.1 (−30.1*)</td>
<td></td>
</tr>
<tr>
<td>SCMS X-band</td>
<td>−113.8</td>
<td>68.7</td>
<td>−31.1 (−30.1*)</td>
<td></td>
</tr>
</tbody>
</table>

very near the DZX noise level, DZS-X should approach about 0.5 dB in CaPE and 5.2 dB in SCMS.

The main purpose of the field work has been to measure the very low values of DZR that are expected before droplet coalescence begins. Experience in using the correction suggests that the best course is simply to use DZX for DZR, as long as DZS-X < 16 dB, and accept the bias. When DZS-X > 16 dB, it is found that results corrected according to Fig. 2 generally contain considerable variation, and the conservative course is to consider DZR to be irretrievable.

**b. Further qualifications to the radar data**

The descriptions that follow will use the clearest examples showing the features commonly found, selected from the two field programs. In looking at radar echo patterns at low dBZ there are many nonmeteorological influences that need to be kept in mind: primarily sidelobe effects, beam mismatches (in shape, range, azimuth, and elevation angles as well as sidelobe patterns), birds, and insects. Some of these produce effects that are subtle and not easily separated from the signals that one wishes to interpret. The primary means of checking the meteorological reality of what is to be described has been the examination for 1) bias between top and bottom, right and left, or front and back sides of clouds to identify possible angle and range misalignments, 2) differences in the shapes of point targets such as birds and airplanes in data where the spacings between samples were smaller than the nominal pulselength and beamwidth to identify pulse-volume mismatches, 3) the presence of particular ground targets such as towers in the sidelobes, and 4) continuity in space and time of the echo features.

Since the S- and X-band radar datasets are compared on a (range) gate-by-gate basis, any effects of beam misalignments or pulse-volume mismatches were either corrected for to the extent possible or at least assessed. Measurements from those range gates that were obviously

---

1 Point targets were smeared out by the pulse and beam shapes and could be seen at about five to eight elevation angles and in about three to six range gates at the sampling intervals used.
contaminated by ground targets in the sidelobes (near-zero radial velocity and strong return power) were deleted, and then filled in with a linear interpolation in range. Range differences of about 50 m were corrected. No differences were found between the S- and X-band elevation angles, but there did appear to be about 0.1–0.2° misalignment in their azimuths. Because of the relative coarseness of azimuthal sampling, rather than try to correct for this small misalignment, we have chosen to interpret the S- and X-band datasets based on the original RHI slices. Some amount of pulse-volume mismatch was evident in the observed shapes of point targets. We estimate, though, that pulse-volume mismatch and the small azimuth angle misalignment led to S- and X-band reflectivity differences of no more than about 1–3 dB. Some of this difference was likely accounted for by the X-band reflectivity adjustments listed in Table 1.

The presence of particles other than hydrometeors, especially those in the subcloud layer and near clouds, complicate the interpretations of the radar data. Boundary layer, clear-air echo is strong in Florida and has been studied in some detail by Wilson et al. (1994), who found its source to be primarily insects. Insects are plentiful in Florida and can be large enough to produce appreciable Mie deviations from Rayleigh scattering, especially at X-band. In their Fig. 10 Wilson et al. (1994) show that DZS-X is about 5 dB within the Florida boundary layer. It may be that insects are rarely ingested into the clouds, but that cannot be assumed with confidence. They remain a bothersome source of uncertainty in some of the interpretations, and the problem will be discussed briefly after the observations of the systematic features relating to clouds have been presented.

2. Mantle echoes and the transition from Bragg to hydrometeor scattering as clouds age and grow

Small cumulus clouds in Florida in their early, growing stages usually have an S-band “mantle echo” that is dominated by Bragg scattering. It is also fairly common for the very early X-band echoes to show a nearly identical pattern roughly 19 dB weaker that is the ideal Bragg value. Figure 3a shows a particularly good example of this. Because of the great similarity of the echo patterns and the fact that this occurs fairly often, it appears that not only the S-band echo but also the X-band echo must be dominated by Bragg scattering. Several minutes later the patterns are much less similar, as shown in Fig. 3b, where DZS-X in the center of the echo has decreased to less than 13 dB. Scattering from the hydrometeors is dominating in the central part of the X-band echo pattern while Bragg scattering is still dominating the S-band echo pattern throughout.

The X-band mantle echoes are not commonly a prominent feature of the data, because they are so much weaker than those at S-band. They are seen only at very short range, only on very shallow clouds, and only in the very earliest stages of cloud development. Most clouds attain a Rayleigh echo in their interiors that soon dominates the Bragg echo at X-band.

On the most humid days the Bragg echo tends to fill the cloud, with mantles being poorly defined and the maximum DZSb being relatively weak, 0 to 5 dBZ. On the driest days very well-defined mantle echoes exist in the early stages with maximum DZSb up to a little greater than 10 dBZ. Figure 4a shows DZS and DZX from a cloud on 26 July 1995, when the relative humidity in the environment was above 80% from 2 to 4 km. In contrast, Fig. 4b shows a case on 6 August 1995 when the relative humidity at 2, 2.5, and 3 km was 36%, 18%, and 21%, according to local soundings. The mantle echoes in this latter case are the most intense seen. The predominance of S-band mantle echoes on dry days is consistent with the understanding that specific humidity variations can contribute more strongly to index of refraction differences, and hence to Bragg scattering, than variations of liquid water content or temperature (Gossard and Strauch 1983).

The mantle echo patterns fit well with the model of the entrainment and mixing at the tops and sides of thermals and plumes (Scorer 1978). Those are the locations where dry environmental air is actively mingling with the saturated, cloudy air and, therefore, the maximum values of Bragg scattering should be expected. The intensity of DZSb sometimes decreases down the sides of the cloud as the mixing becomes more complete, and is usually weaker within the cloud interior.

Figure 5 shows a sequence of vertical slices of DZS, DZX, and DZS-X through the center of a vigorous, small cumulus that produced a brief burst of precipitation with equivalent reflectivity factor attaining about 6–10 dBZ at the last time shown. This shows the typical development of DZS and DZX patterns for an active, small cumulus, after the very early stage typified by Fig. 3a, but which was not recorded in this case.

In Fig. 5a the cloud top is a little above 3 km above mean sea level (MSL) and the S-band pattern is dominated by the mantle and the prominent weak-echo region just above cloud base. In this and all other cases the echo tops correspond with the sharp, visual tops of the clouds. The visual base of this cloud was at about 1 km MSL, as judged from photographs and from the exceptionally even top of the boundary layer echo on this day. When there are prominent weak-echo regions, the boundary layer echoes are usually “humped up” directly beneath the cloud, and they are relatively flat outside this region. The DZS-X is greater than about 17 dB within the S-band mantle echo, while it is less than about 9 dB in the cloud interior. The X-band pattern is flatter, more horizontal, and is dominated by Rayleigh scattering in its central portion, with Bragg influence at the very top and at the sides.

Three min later the cloud top has risen to about 4 km (Fig. 5b), the S-band mantle is thicker especially at the top of the cloud, and the weak-echo region remains at
Fig. 3. Vertical sections of DZS, DZX, and DZS-X as labeled, showing two stages in the growth of a small cumulus. In (a) nearly identical shapes of mantle echoes exist at both X- and S-band, with DZS-X approximately 19 dB. In (b) the Rayleigh echo dominates most of the X-band pattern. In this and subsequent figures, gray tones with marked contour lines follow the grayscale shown to the right of the panels. A thicker, black contour line at DZX = −18 dBZ is shown for reference on all panels. The date (YYMMDD), universal time (HHMMSS, LT = UT − 4), and azimuth angle (NNN dg) for the RHI slices are shown at the top of the figures. The horizontal axis is range (km) from the radar, while the vertical axis is height (km MSL). The center of the S-band radar antenna was located at a height of 9 m MSL.

cloud base. The DZX is more intense and now more vertically elongated, and DZS-X ≤ 9 dB shows that DZX from the central part of the cloud is dominated by Rayleigh scattering as larger drops form. Note that in both Figs. 5a,b, a weak-echo region is also present in DZX, but is much less pronounced than the one at S-band. A column of DZS-X ≤ 9 dB extends downward into this weak-echo region just inside (to the right of) the tail of S-band mantle echo, suggesting larger hydrometeors are falling down through an updraft column. The cloud top is well below the freezing level at about 4.6 km on this day.

Figures 5c,d, 2 and 4.5 min after Fig. 5b, show the cloud top at nearly 5 km MSL. A precipitation core has developed at about 3 km in Fig. 5c and has fallen to the ground in Fig. 5d, while weaker echo remains aloft. The DZS-X field indicates that S-band Bragg scattering fills the cloud top in Fig. 5c but its intensity is less in Fig. 5d. Vestiges of weak-echo regions remain in both S- and X-band patterns, to the left of the precipitation column. The convection remains active at the lower levels according to the photographic record, but not at the upper levels.

The qualitative interpretations of the radar echo patterns are quite straightforward: the Bragg echo is from mixing between drier, unsaturated environmen-
tal air and saturated, cloudy air and the Rayleigh echo is from hydrometeors, showing the development and fallout of precipitation. The main ambiguities involve the sizes of the hydrometeors that give rise to the Rayleigh echo, the degree to which the Bragg scattering really conforms to the ideal (and, as will be seen, the origins of the Bragg scattering itself), and the significance of the weak-echo region. These subjects are discussed below.

3. The flat echo bases: Unmixed ascent

A striking feature in the X-band data is the occasional presence of flat and horizontal bottoms on the echo contours, usually at $-20$ or $-15$ dBZ, very rarely at $-25$ or $-10$. (We used 5-dB contour intervals in surveying the data.) Examples of this are shown in Figs. 6a,b (but see also Figs. 4b, 5a, 10a,b and 12 for other examples). The interpretation is that the X-band radar is "seeing" adiabatic cloud-water contents: the Rayleigh echoes from the condensation size spectra of cloud droplets within unmixed cloud regions, before coalescence acts to produce enough larger drops to influence the radar return. It is the expression on radar of the fact that the condensation level—the visual cloud base—tends to be flat and horizontal. Figure 7 is a time–height diagram of DZX from the same case shown in Fig. 6b, showing that the echo bottoms are not only flat in vertical cross section (at the times, heights, and dBZ values shown by “FB” in Fig. 7, but not in all of the radar scans), but also constant in time until the cloud develops precipitation that overwhelms the echo from the cloud droplets. This result is the usual case, for vigorous clouds on which the radar scanning starts early enough.

The explanation of the flat echo bases as showing adiabatic ascent fits the observations both qualitatively and quantitatively, as far as it has been possible to check. Figure 8 is a scatter diagram of the X-band reflectivity factor versus height from Fig. 6a, combined with a very similar-looking RHI taken about 1.5 min later at an azimuth angle separated by $1.5^\circ$. The data points with reflectivity values increasing downward below about 0.9 km are mostly from insects in the subcloud boundary layer. Superimposed upon this scatter diagram are lines representing reflectivity factor from adiabatic ascent of an air parcel from a cloud base with three different sets of properties. (The method used to calculate adiabatic cloud-water content and the associated radar reflectivity factor is outlined in appendix B.) One, the solid-line curve with approximate droplet diameters marked
Fig. 5. Vertical sections of a vigorously growing cumulus seen at S- and X-band, and DZS-X, at four times as indicated. The sequence of four times illustrates several features discussed in the text. The freezing level is about 4.6 km.
alongside at five heights, activates 100 droplets cm$^{-3}$ at a cloud base that is at 0.8 km MSL (933 mb) and 25°C. It matches the upper bound of dBZ values quite well up to about 1.7 km if the lower part that is obviously contaminated by boundary layer echo is ignored. This corresponds to the interior portion of the cloud in Fig. 6b, delineated by the edges of the flat echo bases. The droplet concentration at cloud base could be adjusted to fit the lower part of the data near cloud base to about 1.7 km or the upper part above 1.7 km, but not both with any plausible combination of cloud-base height and temperature. The cloud-base height and temperature were estimated independently (and necessarily roughly) from soundings, the radar pattern, and photographs. If the cloud-base altitude and temperature are correct and this curve is the correct assignment of droplet concentration, the data above about 1.7 km presumably show drizzle forming. The two dashed-line curves (A and B) represent a range of uncertainty in cloud-base conditions and droplet concentrations.

Data to resolve the ambiguities in this case are not available. Nevertheless, scatter diagrams much like that in Fig. 8 are common in both the CaPE and SCMS data and can be used to estimate cloud-droplet concentration and size if DZX is dominated by Rayleigh scattering from cloud droplets alone. Figure 9 shows the sensitivity of the calculated reflectivity factor versus height lines to the cloud-base height and temperature (the narrow, gray-shaded region to the left of curve 1 shows the very small change in this curve when the temperature at 944 mb is decreased from 27°C to 23°C) and to the droplet
concentration. Under the assumptions of single-size and constant droplet mixing ratio, by far the greatest sensitivity is to the droplet concentration. (This is in terms of the expected magnitudes of the uncertainties, given only indirect information from sounding and surface data.) Thus, this kind of data may sometimes provide useful estimates of the continentality of clouds, using X-band or shorter-wavelength radar, when conditions provide for the presence of clouds with these kinds of profiles.

If this interpretation of the radar data is correct, that the cores of the small cumulus with these radar characteristics are unmixed, then quantitative estimates of the liquid water content in the mixed regions probably may be made as well, using the analysis of Paluch et al. (1996). The key conditions for these interpretations to hold are first that DZX is predominantly Rayleigh scattering (the discussion below is section 4b raises some doubt here) and that drops produced by coalescence have negligible influence on Z.

4. The Bragg echo
   
   a. The mantle echoes

The mantle echoes have been described above, and it was noted that the most intense, well-defined ones occur when clouds penetrate the driest air. This might be explainable in at least two ways. The first is that the index-of-refraction variations are expected to be greatest in this circumstance. The water vapor content may vary locally up to several g m\(^{-3}\) at cloud boundaries and the liquid water content may vary by similar amounts, except close to cloud base. These variations reinforce each other’s effect upon index of refraction fluctuations, because the filaments of undersaturated, entrained air contain no liquid water, and the cloudy air is saturated. The humidity variations may be the most important because water vapor content variations produce about 30 times more Bragg scattering than do the same variations of water content in the form of cloud droplets (Gossard and Strauch 1983), a difference of 15 dB in reflectivity factor. Filaments of dry air of the scale needed for the Bragg scattering—a few centimeters—would not stay either dry or droplet free very long because of the appreciable fall velocities of the cloud droplets (about 0.3, 1.2, and 2.7 cm s\(^{-1}\) for drops 10, 20, and 30 microns in diameter), but they can be created continually in the mixing process close to the cloud edges, and so there might be strong contributions to the Bragg scattering from them.

It is striking that the mantle echoes grow to fill the entire cloud tops, 1 km or more wide and nearly 1 km deep, with little decrease of echo intensity (Fig. 5b). This seems to imply that blobs or filaments of environmental air that are small compared to the radar resolution (which is about 150 m) but much larger than several cm, are transported throughout these regions so that the very small-scale variations responsible for the intense Bragg scattering can be produced. This fits with the observation that these cloud turrets commonly evaporate completely within a very few minutes after the buoyant impulse reaches its maximum height, in spite of there being little downward transport of the decaying cloud according to time-lapse photography. The strong Bragg echo decays rapidly as the visual cloud disappears.

A second potential reason for the observed correlation between environmental humidity and mantle echo intensity may be invigoration of small-scale turbulence from local buoyancy differences caused by the strong evaporative cooling at cloud edge.
b. Similar echo patterns at the two wavelengths, with different echo intensities

The argument was used in section 2 (Fig. 3a) that the existence of mantle echo patterns at both wavelengths with about 19-dB difference in intensity shows that the echo at both wavelengths is dominated by Bragg scattering. This argument seems quite strong, since a mantle echo pattern is not to be expected from hydrometeors, and the 19-dB value is unlikely to be a coincidence. The patterns of the S- and X-band echoes are also the same at values of reflectivity factor above about 5 dBZ, where their intensities become approximately equal. Here both DZX and DZS are dominated by scattering from hydrometeors (e.g., Figs. 1, 5d, 10e, and 11e). Between these two regimes with similar patterns at X- and S-band is the region in which DZS is dominated by Bragg and DZX by Rayleigh scattering: where DZS-X is between about 3 and 16 dB. Different echo patterns are expected here, and when the mantle echoes are present the overall patterns are different. However, in this region it is also common to find instances in which the patterns of echo are very similar but close to neither 0 nor 19 dB, but rather about 10 dB. This implies a close correlation between the intensities of the Bragg and the Rayleigh scattering, a phenomenon that was not anticipated.

Figure 10 is a series of cross sections through a vigorous, small cumulus showing the development of this phenomenon: Fig. 10c especially, but also Fig. 10b. Especially remarkable in this case is the fact that there is a preference for DZS-X about 10 dB, as shown in the scatter diagrams in Figs. 11b and 11c. This looks highly systematic, and indeed, looking at data from other clouds, this phenomenon is common, though not always
with the preferred difference just halfway between 0 and 19 dB. This phenomenon is mostly confined either to the core reflectivity and its nearby surroundings, for example, that portion of the cloud in Fig. 10c above about 1.8 km, or to the region with relatively flat contours near cloud base (Fig. 10b). Scatterplots show the evolution from more Bragg-like echoes, with mantles (Fig. 10a) and DZS versus DZX points scattering between lines marked 10.5 and 19.2 dB (Fig. 11a), to a substantial portion of the cloud having about 10-dB differences (Figs. 10b, 10c, 11b, and 11c). And finally, the cell reaches the precipitation stage (Figs. 10d, 10e, 11d, and 11e) where the core shows only small differences between S- and X-band reflectivities. However, the S-band echo is still mostly Bragg scattering (about 14 dB differences in Figs. 10e and 11e) along the sides of the cloud even in this late stage.

The origin of the similar DZS and DZX patterns when their values are significantly different, and of the preference for values of DZS-X near 10 dB can be discussed on the basis of two assumptions. First, the Rayleigh and Bragg scattering are essentially independent with one coming from drops ($\Sigma ND^6$) and the other coming from index-of-refraction gradients at one-half the radar wavelengths, but not necessarily conforming to the $-5/3$ law. Second, the two kinds of backscattering add to produce the total radar backscattering. The problematical cases can be explained under these assumptions in one of two ways, since clearly it is not possible for both DZS and DZX to be dominated by Rayleigh scattering. Either 1) both are dominated by Bragg scattering or 2) for some reason there is a strong correlation between the Bragg and Rayleigh components. The first possibility would imply that the scale dependence of the index-of-refraction variations must be about zero, to give the 10-dB value (see appendix A, Table A1, and Fig. 2). If this were true, DZX would not represent Rayleigh scattering up to about 5 dBZ, where the correlation ceases (Figs. 11b,c.). However, an argument against this is that it would invalidate the interpretation of the flat echo bases at X-band and the adiabatic interpretation of the scatterplots like that in Fig. 8. The second possibility would
Fig. 10. Five times in the development of a small cumulus on 10 August 1995. Note the disappearance of the S-band mantle echo in (a) (the environmental air is more humid at 3 km than at 2 km on this day) and especially the very similar echo patterns in (c), with DZS-X about 10 dB. Precipitation is becoming evident in (d) where DZS-X values are less than about 6 dB and reflectivity factors exceed 6 dBZ at both S- and X-band, while in (e) the core of the echo is dominated by precipitation at both S- and X-band. The freezing level is about 4.8 km.
imply a surprisingly good correlation between the cloud water content and Bragg scattering. The presence of some correlation here would not be particularly mysterious, if, for instance, the index of refraction contrasts responsible for the Bragg scattering ran between adiabatic liquid water content and zero. That could explain qualitatively the similar patterns of DZS and DZX. However, one would need a mechanism to generate these contrasts within adiabatic cloud, if the present interpretation of the flat X-band echo bases is correct, and likewise the preference for a particular value of DZS-X would remain problematical. No matter how we look at these results, they remain difficult to explain.

Rogers et al. (1994) and Cohn et al. (1995) documented a case in which clear-air Bragg scattering at an inversion intensified greatly as light rain started to fall through the layer. They found this phenomenon difficult to explain also, after examining several possibilities in some detail. It is possible that their phenomenon and the present one are related, in which case the explanation does not involve the cloud (the cloud droplets themselves) in a fundamental way. Yet another problematical, apparent relationship between Bragg scattering and scattering from particulates was reported recently by Rogers and Brown (1997), concerning radar returns at two wavelengths from the plume of an intense fire. These phenomena make one wonder whether the basic scattering mechanisms are even completely understood in principle.

The most puzzling example of the unexplained correlation between X and S in our small-cloud data is shown in Fig. 12. Here the flat echo bases at X-band are developed to a very exceptional extent, especially from 1.1 to 1.8 km height. Their central parts (near 8-km range) correspond with horizontal contours of the S-band reflectivity factor, but with significantly stronger echo, showing that DZS is primarily Bragg. DZS-X varies here between about 5 and 9 dB. If the flat bases at X-band represent unmixed ascent (an interpretation that is so strong that we consider it nearly certain) then the Bragg scattering must exist within the unmixed region and it must increase with ascent of the air in a regular, systematic way. The Bragg scattering cannot result from initial inhomogeneities in the water vapor mixing ratio of the air feeding the cloud because if that were the case it would decrease markedly from the lowest condensation level of the mixture to the highest, as the fluctuations of vapor content are replaced by fluctuations of liquid water content. Then it would remain nearly constant with further ascent.

Aircraft data within cloud have suggested the existence of centimeter-scale fluctuations of droplet concentration within a larger-scale uniformity (Baker 1992), which is what these radar data also indicate. As far as we can tell, the problematical echo feature described here is likely always present in the radar data, except that often it is obscured by the mantle echo. The cases that show little sign of mantle echoes because the air...
in the cloud environment has high humidity show this phenomenon consistently, as in Fig. 4a. It is found in the range of DZX from about $-20$ to 0 dBZ and DZS from about $-10$ to 10 dBZ.

It is possible of course that the evidence is deceptive in some unrecognized way, and it is also possible that some aspect of scattering theory unknown to us might explain this phenomenon. However, if neither of these is the case, then it appears that there must be some mechanism for cloud that is adiabatic, homogeneous, and presumably not very turbulent to become heterogeneous at small scales. Perhaps the fall of the droplets relative to the air might cause an instability leading to local clumping of droplets. It is well known that the airflow around droplets leads to lateral droplet motions, producing such effects as wake capture. Interactions of this kind between groups of droplets would be needed to produce centimeter-scale inhomogeneities. Such a process has not been anticipated nor observed, to our knowledge.

An alternative to Bragg scattering producing the relatively flat bases is that Mie scattering from insects is the cause. It is reasonable for insects from the boundary layer that are ingested into cloud to be size sorted in the vertical direction so that the DZS-X values would be fairly uniform in the horizontal direction. The relatively low values of DZS-X (less than about 5 dB) near cloud-base point to the possibility that the echo is being strongly influenced by insects. From the calculations of Wilson et al. (1994), however, this would require cm-sized insects, and these sizes with fall speeds of around 10 m s$^{-1}$ seem unlikely to be ingested into cloud. We do not expect updrafts near cloud base to be able to lift such large insects out of the boundary layer. Beneath the clouds both S- and X-band boundary layer echoes are usually humped up, but in this peculiar case the S-band weak echo region extends nearly to the ground (Fig. 12).

c. Nonideal Bragg scattering

Values of DZS-X above 19 dB are either from spurious effects such as mismatching of the radar beams, or from nonideal Bragg scattering. One rather common, systematic occurrence of DZS-X a few dB greater than 19 appears probably to be nonideal Bragg scattering and is illustrated in Fig. 13. This is the same slice as in Fig. 5a except that the X-band signal is thresholded at a lower level to show this feature and the contour values are changed. [At this range, 6 km, the noise level at X-band is about $-18$ dBZ for CaPE data (Table 1).] Patches of DZS-X exceeding 22 dBZ occur along the curved centerline of the mantle echo (Fig. 13). The presence of DZS-X maxima several dB in excess of 19 within the mantle is fairly common on some days. It might be explained by a steeper than $-5/3$ slope of the index-of-refraction variations in the vicinity of the 1.5-cm scale to which the X-band radar is sensitive. This would be expected at scale sizes where dissipation by molecular diffusion starts to affect the spectral shape (Ottersten 1969). For this case (A2) no longer applies, and the wavelength dependence of the radar reflectivity should be evaluated from (A5) and (A6), and the ratio of measured reflectivities would be given by (A9) rather than (A4). The ratio of reflectivity factors and the DZS-X dB values for several spectral slopes are given in Table A1.
d. The weak-echo regions

A distinct, S-band, weak-echo region is usually found at the bases of the more vigorously growing, small cumulus. This is shown well in Figs. 3a, 5, 6, 12, and 13. The term “weak-echo region” here does not refer to all the region beneath the mantle, but to the much smaller, weaker-echo region near cloud base. These are local minima in the S-band Bragg scattering and are seen in nearly all actively growing clouds. They often have echo minima down to −18 or −22 dBZ with Bragg echo surrounding them as strong as 0 dBZ. The cloud shown in Fig. 10 is unusual in lacking such a weak-echo region (WER).

These weak Bragg-echo regions are different from the familiar WERs of severe thunderstorms, which are features of the hydrometeor echo. They presumably represent nearly laminar flow in the updraft interiors at very low levels. Perhaps they show the small regions of fastest updraft below and just above cloud base where the inhomogeneity that produce the puzzling correlation between the Bragg and hydrometeor scattering has not had time to develop.

5. The precipitation shafts

The first precipitation shafts from the small cumulus clouds studied in CaPE and SCMS are always remarkably narrow. Their echo cores are about 500 m across, considerably narrower than the clouds themselves. Figure 5d is one example, and Fig. 10e shows another. The final precipitation growth that produces these echo columns clearly occurs in the central, strongest-updraft portions of the clouds where the liquid water content is expected to be highest. The drops responsible for the strongest echo are not elevated into the diverging flow at cloud top.

Cloud-top rise rates of 5 to 10 m s$^{-1}$ are the rule, and descent rates of the precipitation echo of 5 m s$^{-1}$ and more (drop diameters greater than about 1.5 mm) also are common. The latter is the descent rate of the bottom of a value of reflectivity factor ascribable to precipitation, such as the −5 or 0 dBZ contours in Fig. 7. It seems likely, therefore, that these first precipitation shafts are visible on radar because of low concentrations of several millimeter-size raindrops, formed either on supergiant nuclei (Johnson 1982) or on drizzle drops recycled into the updraft from a previous ascent. This agrees with Illingworth’s results on small, warm cumulus (1988). The cloud in Fig. 5 did go above the freezing level (about 4.6 km), and the local core of DZX exceeding −2 dBZ above 4 km might have been influenced by ice processes. The cloud top in Fig. 10 did not reach the freezing level (4.8 km) so the precipitation process is entirely coalescence. There is no chance of ice contributing to the precipitation formation in many of the clouds studied.

6. Insects and Mie scattering

The main question regarding insects in the present context is whether backscattering from them might influence DZS–DZX enough to confuse the interpretation of the radar return as being from either Bragg or hydrometeor scattering. In their Fig. 5b, Wilson et al. (1994) show DZS-X for backscattering from simple in-
sect models (prolate water drops) with equivalent spherical diameters up to 25 mm. For particles whose equivalent spherical diameters are from 2 to 3 mm, DZS-X ranges from about –1.0 dB (1:3 axis ratio) to about 0.5 dB (1:1 axis ratio). Equivalent diameters from 3 to about 6 mm lead to DZS-X as small as about –5 dB (X exceeds S-band), while diameters larger than about 6 mm lead to DZS-X from –3 dB to more than 10 dB depending on the axis ratio. This somewhat complicated behavior of DZS-X comes from Mie scattering at X-band, and shows that insects with these sizes could be significant. Insects of centimeter sizes (and even birds, though that is much less likely) may contribute to observations reported herein.

Small insects drawn into cloud base and upward might well constitute supergiant nuclei for drizzle and raindrop formation in Florida. There is no way that radar can be used to rule out this possibility, because developing cloud echoes are nearly always mingled to some extent with the top of the boundary layer echo at dBZ values of –15 to –20. Several of the figures show this (Figs. 3a, 4, 5a, and 6a), and this mingling is not usually accompanied by DZS-X about 19 dB, so that it is not likely due entirely to Bragg scattering.

It appears also that neither the DZS-X values that are systematically higher than 19 dB nor the preferred DZS-X values near 10 dB (Figs. 11b,c) can be due to Mie scattering. The X-band Mie scattering peak (Mie greater than Rayleigh) for 3- to 5-mm water drops can increase the X-band signal by no more than 5 dB over the S-band Rayleigh component (DZR). This is the wrong direction to explain the too-high values of DZS-X and not large enough to help explain the preferred DZS-X values near 10 dB. Because insects (water covered or not) can be nearly all the same size, one cannot say with absolute certainty that any particular radar result is impossible. However, the fact that both of these effects in DZS-X are quite common argues against this kind of very special explanation.

7. Conclusions

Most of the qualitative features of the X- and S-band radar echoes from these small, warm cumulus before the onset of rapid coalescence are understandable in terms of Rayleigh scattering from cloud droplets and Bragg scattering from mixing of cloud with dry, environmental air. The general transitions of the echo patterns as the clouds grow and age are as follows. First there are Bragg mantle echoes at both wavelengths with DZS-X roughly 19 dB. Then Rayleigh scattering from cloud droplets comes to dominate the X-band return, sometimes showing flat, horizontal bases on closed contours of dBZ. This feature is interpreted as representing the increase in cloud-droplet size in adiabatic ascent. The Bragg mantle echoes at S-band thicken with time, sometimes to fill the cloud, and as coalescence begins and rain forms, the echo intensity at both wavelengths increases and both become dominated by the scattering from the hydrometeors.

There are many cases of DZS-X about 10 dB, with almost identical patterns of the radar echo. These pose a problem because with a difference that great, DZS must be dominated by Bragg scattering, but at the same time DZX must be dominated by scattering from the hydrometeors. There appears to be a strong correlation between the Bragg and Rayleigh scattering that is unexplained at present.

A reviewer of this paper asked the question ‘‘just how long does it take to develop precipitation from cloud formation and can present theories explain this?’’ This question was the main motivation for the SCMS field program, so it deserves a response. In the first place, the question phrased that way is deceptive because it implies that there might be a simple answer. Unfortunately, neither the time of cloud formation nor that of the first precipitation are well defined; and even if they were, the radar reflectivity data alone would not specify them closely. The difficulties can be summarized in relation to Fig. 7, the X-band time–height diagram of a small, growing cloud. There are two ambiguities regarding cloud formation time. One, always present, is that between the Bragg and Rayleigh sources for the echo. The other, almost always present, is that the cloud identity itself becomes ambiguous at the very early stages. Then, when does ‘‘precipitation’’ start? Few would question that drops produced by coalescence are important for the 0-dBZ echo in Fig. 7, but the Bragg–Rayleigh ambiguity prevents any conclusion much stronger than that. More troublesome for testing theories of warm rain formation, however, is the lack of evidence on drop size at these early-echo stages. The echo may be from a few big drops or many small ones.

The development of the X-band radar echo does place constraints on the theories of warm rain development. This paper represents a first attempt to understand the radar echo itself, which is the first step toward defining what those constraints are. Promising sources of other data are shorter-wavelength radar (probably K-band, 8-mm wavelength), polarization-diversity radar, and aircraft. Information from the latter two sources are available from SCMS and are being analyzed, but the answer to the reviewer’s question will certainly remain complicated.

Acknowledgments. We gratefully acknowledge John Lutz and the other engineers and technicians from NCAR ATD/RSF who so ably operated the CP2 radar in CaPE and SCMS. We further thank all staff who very capably participated in the operations control rooms.

APPENDIX A

Separation of Signals into Bragg and Rayleigh Scattering

The strength of the signal, returned from a scattering medium, is related to $\eta$, the average backscattering cross...
section per unit volume. Since the cross section of a small, spherical water drop is proportional to $D^6$, the Rayleigh reflectivity is (Battan 1973)

$$\eta_r = \frac{\pi^2 |K|^2}{\lambda^4} \sum N_D^6,$$

(A1)

where $\lambda$ is radar wavelength, $D$ is drop diameter, and $|K|^2$ is the dielectric factor for water, 0.91 to 0.93. The quantity $\Sigma N_D^6$, a summation over all drops in a unit volume of air, is the familiar radar reflectivity factor $Z_e$.

When scattering comes from turbulent fluctuations in refractive index and these obey the $k^{-3/3}$ law (i.e., the inertial subrange of fully developed, isotropic turbulence), the Bragg reflectivity comes from variations of refractive index at scales near $\lambda/2$ and is expressed by (Ottersten 1969)

$$\eta_b = \frac{0.38}{\lambda^{3/3}} C_n^3,$$

(A2)

where $C_n^3$, the refractive index structure constant, characterizes the strength or variance of the fluctuations. So long as there are any spatial variations in refractive index at the scale of $\lambda/2$, regardless of the relevant law, there will be a returned signal. It is only in the case of the $-5/3$ law that the strength of this signal is predicted by (A2).

The ratio of S- to X-band Rayleigh reflectivities (A1) is $(\eta_r/\eta_s) = (\lambda_s/\lambda_x)^4$ and the ratio of Bragg reflectivities (A2) is $(\eta_b/\eta_s)_s = (\lambda_s/\lambda_x)^{3/3}$. For the CP2 S- and X-band wavelengths of 10.68 and 3.2 cm, the difference in signal strengths is $10 \log(\eta_s/\eta_s) = -20.94$ and $-1.7$ dB, respectively. If the hydrometeors are larger than about $\lambda/16$, the approximate boundary between Rayleigh and Mie scattering (Doviak and Zrnić 1984), their cross sections become very complicated functions of size so that the ratio of reflectivities can take on almost any value, even taking on values from Mie scattering at the shorter wavelength and Rayleigh scattering at the longer one.

It is customary in radar meteorology to transform the measured reflectivity, whether it comes from Rayleigh, Mie, or Bragg scattering, to a “Rayleigh-equivalent” radar reflectivity factor defined as

$$Z_e = \frac{\lambda^4}{\pi^2 |K|^2} \eta.$$  (A3)

If the received signals are both from Rayleigh scattering, $Z_e = Z_s$, is the same at both wavelengths. But for Bragg scattering, the ratio of equivalent reflectivity factors becomes

$$\alpha = \frac{Z_{bs}}{Z_{bs}} = \left(\frac{\lambda_s}{\lambda_x}\right)^{2-s}.$$  (A4)

where $\alpha = 83.0$ for CP2.

We can get some idea of how $\alpha$ might vary for slopes, $s$, other than $-5/3$ following the derivations in Ottersten (1969) and Hardy (1972). In general, radar reflectivity is

![Table A1. The ratio of S- to X-band Bragg reflectivity factors (A10) for CP2 and DZS-X in dB for different slopes of the index-of-refraction variance spectrum.](image)

$\eta_b(k) = -\frac{\pi}{8} k \frac{dS(k)}{dk}$.  (A5)

where the one-dimensional refractive index variance spectrum $S(k)$ is isotropic for wavenumber $k = 4\pi \lambda$. For the moment, assume that $S(k)$ is proportional to $k^4$, then $dS(k)/dk = sk^{-1}S(k) \propto (s k^{-1})$ and with (A5)

$$\eta_b(k) \propto -\frac{\pi}{8} s \left(\frac{4\pi}{\lambda}\right)^{s+2}.$$  (A6)

Radar reflectivity is transformed to radar reflectivity factor by multiplying (A6) by $\lambda^4$ according to (A3),

$$Z \propto -\frac{\pi}{8} s \left(\frac{4\pi}{\lambda}\right)^{s+2} \lambda^{2-s}.$$  (A7)

Then (A4) becomes

$$\alpha = \frac{Z_{bs}}{Z_{bs}} = \left(\frac{\lambda_s}{\lambda_x}\right)^{2-s},$$  (A9)

and the difference between the dB $Z_e$ values (the ratio of $Z_e$ values) is

$$\text{dB}Z_{bs} - \text{dB}Z_{bs} = 10 \log \left(\frac{\lambda_s}{\lambda_x}\right)^{2-s}.$$  (A10)

The ratio of S- to X-band radar reflectivity factors for CP2 for different spectral slopes is given in Table A1. For $s = -5/3$, the dB-difference in (A10) is 19.2 dB, while in the limit as the slope of the variance spectrum in (A5) approaches zero, $a$ approaches $(\lambda_s/\lambda_x)^{2-s} = 11.1$ or a dB difference of 10.5 dB.

We assume that the measured equivalent radar reflectivity factor at any wavelength is a simple linear combination of reflectivities from Bragg and hydrometeor scattering. For S- and X-band wavelengths, the two measured equivalent reflectivity factors are

$$Z_{es} = Z_{bs} + Z_r,$$

and

$$Z_{es} = Z_{bs} + Z_r.$$  (A11)
From (A4) or (A9) and (A11), the deduced Rayleigh reflectivity factor is

$$Z_r = \frac{aZ_{ex} - Z_{es}}{a - 1}, \quad \text{(A12)}$$

and the Bragg equivalent reflectivity factors for the two wavelengths are

$$Z_{bs} = \frac{a(Z_{ex} - Z_{es})}{a - 1}$$

and

$$Z_{bs} = \frac{Z_{es} - Z_{ex}}{a - 1}. \quad \text{(A13)}$$

We can rearrange (A12) and (A13), and put them in a logarithmic form to see how the Rayleigh and Bragg solutions relate to the original measurements, the assumed inertial subrange law [i.e., the value of $a$ in (A9) for other than the $-5/3$ law], and $(b = Z_{es}/Z_{ex})$, the ratio of the measured S- and X-band equivalent reflectivity factors,

$$\text{d}BZ_r = \text{d}BZ_{ex} - 10 \log(a - 1) + 10 \log(a - b),$$

$$\text{d}BZ_{bs} = \text{d}BZ_{ex} + 10 \log\left(\frac{a}{a - 1}\right) - 10 \log\left(\frac{b}{b - 1}\right),$$

and

$$\text{d}BZ_{bs} = \text{d}BZ_{ex} - 10 \log(a - 1) + 10 \log(b - 1). \quad \text{(A14)}$$

The sums of the last two terms on the right of each equation in (A14) are the corrections (Fig. 2) to be applied to the original measurements (the first term on the right) to deduce S-band Bragg and Rayleigh scattering in terms of the assumed law (middle term on the right) and the measured ratio of equivalent reflectivity factors. If $b = 1$, neither $\text{d}BZ_{bs}$ nor $\text{d}BZ_{es}$ can be determined, and both X- and S-band signals are entirely from Rayleigh scattering. If $b = a$, both X- and S-band signals are entirely Bragg scattering that obeys the assumed law, and Rayleigh scattering cannot be determined. All other values (1 $< b < a$) must be from mixtures of Bragg and Rayleigh scattering.

**APPENDIX B**

**Adiabatic Cloud Liquid Water Content and Radar Reflectivity Factor**

Radar reflectivity factor ($Z$) can be calculated as a function of the cloud liquid water content ($Q$) following the procedure outlined by Atlas (1954). The moments of any cloud drop size distribution $N(D)$ are

$$M_n = \frac{1}{N_r} \int_0^{D_m} D^n n(D) \, dD, \quad \text{(B1)}$$

where the integral is over a unit volume, $D_m$ is the maximum diameter, and $N_r$ is the total number of cloud droplets in the volume. The radar reflectivity factor is given by

$$Z = \int_0^{D_m} D^n n(D) \, dD = N_r M_6, \quad \text{(B2)}$$

where $M_6$ is the sixth moment of the drop size distribution. The liquid water content is

$$Q = \frac{\pi \rho_a}{6} \int_0^{D_m} D^5 n(D) \, dD = \frac{\pi \rho_a}{6} N_r M_3, \quad \text{(B3)}$$

where $M_3$ is the third moment. Equations (B2) and (B3) can be combined to give reflectivity in terms of liquid water content,

$$Z = \left(\frac{6}{\pi \rho_a}\right)^{2/3} \frac{Q^2 M_6}{N_r M_3^3}. \quad \text{(B4)}$$

The height variation of the total number of cloud droplets per unit volume can be written in terms of the number of droplets, $N_o$, activated near cloud base and air density $\rho$,

$$N_r(z) = N_o \frac{\rho(z)}{\rho_0}, \quad \text{(B5)}$$

where $\rho_0$ is air density at cloud base. The moments of narrow, symmetric size distributions are all nearly the same so that $(M_3/M_2^3) \approx 1$, and adiabatic radar reflectivity factor becomes

$$Z_a = \left(\frac{6}{\pi \rho_a}\right)^{2/3} \frac{Q^2}{N_r}. \quad \text{(B6)}$$

The vertical profile of adiabatic liquid water content, $Q_a$, is calculated assuming unmixed, parcel ascent from cloud-base conditions of pressure and temperature, and $N_r$ is calculated from (B5).

**REFERENCES**


