Comments on “Impact of Horizontal Diffusion on T21, T42, and T63 Singular Vectors”

P. L. HOUTEKAMER

Direction de la Recherche en Météorologie, Atmospheric Environment Service, Dorval, Quebec, Canada

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1. Introduction

In a recent paper (Buizza 1998, hereafter B98), some experiments are performed to evaluate the impact of horizontal diffusion on singular vectors computed using a total energy norm. It is found that the horizontal diffusion needs to be rather strong; however, it is noted that “in numerical weather prediction, horizontal diffusion is a numerical artifact used to control the noise in the model, and to allow longer time steps to be used.” The paper then goes on to show “how the horizontal diffusion can be used to control the singular vectors’ characteristics.” In Fig. 3 of B98 it is shown how the spectrum at initial and final time depends on the spectral truncation and on the diffusion damping time. The best results are obtained with horizontal diffusion damping times on the smallest scale (of divergence) of 3 h at T42 and T63 resolution, and of 12 h at T21. The purpose of this note is to hypothesize on the origin of the “noise problem” that necessitates the rather strong damping in this application.

2. Interpretation

In B98 a total energy inner product is used to constrain the initial perturbations. It will now be argued that if the analysis error probability distribution is isotropic in phase space with a given inner product, the spectrum of the associated norm of the spherical harmonics projections will increase with the two-dimensional wavenumber (i.e., be blue). To this end, one may imagine a hypersphere in a spectral space of which the radius squared corresponds to the prescribed initial perturbation energy. Note that this space has as many axes as there are spectral coordinates. Looking at the intersections of the sphere with the axes, one then observes the same amount of energy for each coordinate (this is simply the definition of a sphere). In the case of triangular truncation, one has a total of $2n + 1$ coordinates for each two-dimensional wavenumber $n$. For random fields the expected perturbation energy at the two-dimensional wavenumber $n$ will thus be proportional to $2n + 1$. Such a spectrum is called “blue.” For the complete set of singular vectors, one will find the same spectral dependence. For a smaller set of singular vectors one may obtain somewhat different results due to the ordering effect of the model dynamics and the cost function selected at the forecast time. It is noted that the initial spectra shown in Fig. 3 of B98 (for the case with very little horizontal diffusion) display a nearly linear increase with the two-dimensional wavenumber as would be expected from the above argument. One may now ask if the analysis errors do indeed have a blue spectrum with respect to a total energy norm. If this is the case, the total energy norm may be an appropriate norm for the construction of the singular vectors.

Some evidence to the contrary is given by Molteni et al. (1996, hereafter M96). In Fig. 4a of M96 it is shown that the spectrum of analysis errors is white (or even red) if it is computed using a total energy norm. One may draw the conclusion that the analysis error is not isotropic in phase space with a total energy norm. In fact, if one has to use a norm of the same simple structure, it would seem from Fig. 4b of M96 that an enstrophy norm would be more appropriate. Singular vectors constructed with an enstrophy norm will have much larger scales at the initial time due to the bigger penalty against the selection of small-scale features. For the evolution of such singular vectors one may then be able to use a tangent linear model with more realistic, less diffusive physics.

It is suspected that the origin of the discrepancy is in the following statement that appears without proof in M96: “If the analysis error probability distribution is isotropic in phase space with a given inner product, the spectrum of the associated norm of the spherical harmonics projections will be white.” If this statement
were true, it would follow from Fig. 4a of M96 that a total energy norm would be appropriate. One would then have found a white spectrum in Fig. 3 of B98.

3. Conclusions

It has been shown that the use of a total energy product for the construction of singular vectors leads to initial perturbations with a (blue) spectrum that is rather different from what is currently known about analysis error. While this problem may be controlled by means of the damping time, as done in B98, it might also be worthwhile to consider different constraints for the initial perturbations. One may then be able to use less diffusive linear dynamics.

REFERENCES
