Investigations of Humidity Skewness and Variance Profiles in the Convective Boundary Layer and Comparison of the Latter with Large Eddy Simulation Results

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ABSTRACT

Differential absorption lidar (DIAL) measurements of humidity variance and skewness profiles are presented. A detailed error analysis demonstrates that measurements throughout the convective boundary layer can be performed with an accuracy that is mainly limited by sampling errors due to turbulent statistics.

The water vapor variance profiles are compared with results from a large eddy simulation (LES) model to estimate surface and entrainment latent heat fluxes. A promising agreement with surface latent heat fluxes derived by the bulk formula is achieved, but a strong deviation of the variance profiles from the LES results is found in the entrainment zone. Reasons for these discrepancies are discussed. A new approach for the investigation of top-down variance functions using the measured variance profiles is introduced.

It is investigated whether the skewness profiles give further information on the turbulent transport of water vapor. This is accomplished by the analysis of the skewness budget equation and a comparison of the skewness profiles with latent heat flux profiles.

The results demonstrate that DIAL measurements provide unique datasets for studies of turbulent transport processes in the lower troposphere, finally allowing for comprehensive intercomparisons with models.

1. Introduction

In recent years, active remote sensing instruments have reached the accuracy and resolution necessary to resolve turbulent processes in the convective boundary layer. Since these instruments are capable of performing range-resolved measurements, profiles of turbulent variables can be measured directly. This property allows for a more detailed investigation of the turbulence in contrast to single-point in situ measurements. So far, primarily wind components and temperature have been investigated with high resolution. For instance, Doppler radars and radar–Radio Acoustic Sounding Systems (radar–RASS) were used for turbulence profiling (Kropfli 1986) and heat flux measurements (Angevine et al. 1994; Lippmann et al. 1996). Recently, Doppler lidars were also applied to the measurement of momentum flux and turbulence profiles (Eberhard et al. 1989; Gal-Chen et al. 1992).

However, to investigate the transport of atmospheric constituents, an active remote sensing instrument must be added that determines the fluctuations of these constituents with high resolution and accuracy. The measurement of the turbulent transport of the most important trace gas in the atmosphere, namely, of water vapor, is considered essential for the investigation and improvement of models and to extend our understanding of local, regional, and even global water budgets.

Recently, a new water vapor differential absorption lidar (DIAL) system was developed at the Max Planck Institute (MPI) for Meteorology in Hamburg, Germany, which permits the measurement of humidity profiles with unprecedented accuracy and resolution (Wulfmeyer 1998; Wulfmeyer and Boesenberg 1998). The long-term stability of this system and its setup in a transportable container allow for the routine operation in the field. To demonstrate its potential, a field campaign was performed in which the MPI DIAL system was collocated with a radar–RASS of the University of Hamburg. A primary goal of this experiment was to provide a first dataset for model intercomparisons within the scope of the Baltic Sea Experiment (BALTEX) (BALTEX 1995). A detailed introduction to this campaign is found in Wulfmeyer (1999), and a short overview is given in section 2.

Through analysis of a continuous 24-h measurement period, Wulfmeyer (1999) demonstrated that water vapor and vertical wind variance profiles as well as latent heat flux profiles can be determined, the latter according to the eddy correlation technique. The accuracy of these measurements was mainly limited by the sampling er-
errors using 1-h time series with a time resolution of 10 s and a vertical resolution of 60 m.

In this publication, a further step forward is taken to close the gap between high-resolution measurements and model results. Water vapor variance profiles measured under four different meteorological conditions are compared with published results of a large eddy simulation (LES) model (Moeng and Wyngaard 1984). Davis (1992) proposed that these intercomparisons could be used to determine surface and entrainment latent heat fluxes. This technique is introduced in section 4 and is referred to as the variance method. Whereas a first comparison of humidity variance profiles achieved with an airborne DIAL system has already been performed (Kiemle et al. 1997), an incomplete error analysis led to difficulties in the interpretation of the results (see section 3).

The analysis of variance profiles performed here is the first one performed with a ground-based DIAL system as well as the first one to include a comprehensive error analysis with respect to errors due to system noise (noise errors) and atmospheric statistics (sampling errors). The error analysis is described in section 3. The errors are now small enough to allow a detailed intercomparison of the variance profiles with LES results. In contrast to a good agreement in the mixed layer, strong discrepancies between the experimental and model results are found in the entrainment zone. This questions the application of the variance method for the determination of the entrainment flux as well as previous results achieved with the variance method (Kiemle et al. 1997). The suggested reasons for these discrepancies and consequences for future applications of the variance method are discussed in section 4. A new approach for the investigation of the top–down variance function using the measured water vapor variance profiles is introduced in section 4d. This method can be used to verify LES runs, which simulated the same meteorological conditions as the respective measurement.

The accuracy and resolution of the MPI DIAL system also allows for the investigation of profiles of third-order humidity fluctuations. Skewness profiles, which were again measured under four different meteorological conditions, are presented in section 5. Significant structures in the profiles are found, which is clearly shown by an extended error analysis performed in section 3. Skewness profiles measured with an airborne DIAL system were presented in Kiemle et al. (1997), albeit without an error analysis.

Based on the analysis of the skewness tendency equation, the additional insight into transport processes given by the study of skewness profiles is discussed in section 5. For instance, Mahrt (1991) suggested that skewness profiles permit the distinction between the entrainment drying boundary layer (EDBL) and the moistening boundary layer (MBL). This statement is investigated theoretically and experimentally in section 5, the latter by the comparison of the skewness profiles with directly measured latent heat flux profiles.

2. Overview of the experiment

An extended overview of the applied instruments, the setup of the measurement site, and the performed measurements is found in Wulfmeyer (1999) so that only a short summary is given here. The DIAL system and the radar–RASS were deployed 10 m from the eastern coastline of Gotland, Sweden, a small island in the center of the Baltic Sea, at 57.4°N, 18.92°E. In situ surface flux and further surface data were measured on the island of Östergarnsholm at 57.43°N, 19°E, 6 km northeast of our measurement site. The tower was located nearly in the water at the east coast of Östergarnsholm. The results presented here are part of a continuous 24-h measurement, which was performed between 2300 UTC 12 September 1996 and 2300 UTC 13 September 1996.

During the entire measurement period, air was advected over the Baltic Sea to the measurement site so that orographic effects could be neglected. Also, the sea surface temperature of the Baltic Sea could be considered nearly homogeneous and constant during the whole measurement period. Thus, the presented measurements are a good starting point for model intercomparisons.

An analysis of the complete measurement period showed that it could be roughly divided into three sections from about 2300–0300 UTC, 0300–1100 UTC, and 1100–2300 UTC. These sections could be clearly distinguished by mean atmospheric variables of the boundary layer, for example, the absolute humidity ρ, the temperature, and the boundary layer height z_i. Here, z_i is defined by the location of the increase of the potential virtual temperature above the mixed layer.

A synoptic analysis in combination with horizontal wind measurements of the radar–RASS explained the variation of these variables. It was due to different air masses with different initial conditions that did not complete the process of airmass modification during advection over the Baltic Sea to the measurement site.

To investigate these air masses and the relevant turbulent transport processes in detail, four time periods, P1–P4, were selected. The surface data measured on Östergarnsholm are considered applicable for an analysis of the meteorological conditions at our measurement site because only onshore cases were used. This analysis showed that during all four periods an unstable surface layer was present. During P1, from 2320 to 0000 UTC 12 September, an absolute humidity of about 6.8 g m⁻³ and a low variability of the humidity profiles were found. The convective boundary layer was shallow with a top of about z_i ≈ 350 m and nearly no humidity gradient at z_i. The mean horizontal wind velocity U in the mixed layer was ≈11 m s⁻¹.

Periods P2–P4 were characterized by stronger convection, which was mainly driven by buoyancy. No considerable wind shear was observed in the radar–RASS
horizontal wind data. During P2, from 0434 to 0534 UTC, the Obukhov length $L$ was $-100$ m and the convective velocity scale $w_*$ was $1$ m s$^{-1}$. The absolute humidity in the mixed layer decreased to about $5.4$ g m$^{-3}$, $z_i$ was about $700$ m, and $U$ was $12$ m s$^{-1}$. A strong decrease of the humidity by $1.8$ g m$^{-3}$ in the entrainment zone was found, which ranged from about $500$ to $800$ m. Period P2 appeared to be influenced by the transition between two air masses and therefore could be expected to show the strongest effect of advection.

Period P3, from 0659 to 0803 UTC, took place during the most stationary conditions, which were found between P2 and P4 as the mean humidity leveled off in time and $z_i$ was nearly constant at about $750$ m. In P3 $L$, $w_*$, the absolute humidity in the mixed layer, the decrease of the humidity in the entrainment zone as well as the location of the latter, and $U$ had similar values to those during P2.

In contrast to P1–P3, boundary layer clouds were detected during P4, from 1252 to 1403 UTC. Therefore, this period was selected to investigate a possible effect of these clouds on the transport processes. Again, an increase of the humidity in the mixed layer to $6.3$ g m$^{-3}$ was found. The boundary layer top during P4 was estimated to be about $890$ m and $U$ was $8.5$ m s$^{-1}$. During P4 $L$ and $w_*$ decreased to approximately $-200$ m and $0.6$ m s$^{-1}$, respectively.

3. Data analysis

a. Data processing

The humidity data were analyzed from 195 to 840 m using a height resolution of $15$ m with a moving average of $60$ m. The time resolution was $10$ s and the overall averaging time was typically $1$ h. For the investigation of higher-order moments, all water vapor time series were high-pass filtered and corrected for a linear trend, as described in Wulfmeyer (1999). This procedure yielded profiles of the time series of the water vapor fluctuations $\rho(t, z)$, where $z$ is the height above the sea level. Fluctuations of other variables are also indicated by the prime ($'$). The turbulence was considered to be stationary so that the ensemble mean of a turbulent variable could be replaced by the time average at a fixed point in space. This average of a turbulent variable and its higher-order moments is indicated by the angle brackets ($\langle \rangle$).

In the error analysis the system noise errors and the sampling errors have to be taken into account. The noise error variance $\sigma_n^2$ was determined here by averaging a certain percentage of the high-frequency end of the variance spectrum (0.8–1.0 Nyquist frequency) (Senff et al. 1994; Wulfmeyer 1999). Figure 1 shows an example of a variance spectrum measured at a height of 480 m during P2, where the determination of the noise error is indicated. Alternatively, an extrapolation of the autocovariance function of $\rho'$ to the zero lag can be calculated to estimate the atmospheric variance $\sigma_s^2$ (Lenschow and Kristensen 1985). In both methods an error in the determination of $\sigma_n^2$ remains. In the first method this is mainly due to the standard deviation of the average value of $\sigma_s^2$, whereas in the second method it is caused by the curve fit error. Mathematically, both methods are probably about equal with respect to their accuracy. Either way, the noise error must be considered by an error propagation in the calculation of higher-order moments.

The sampling errors were estimated by the determination of the integral scale of $\rho'$ and the application of the error propagation formulas found in Lenschow and Kristensen (1985) and in Lenschow and Stankov (1986) for the second-order moment [see Eq. (5)], as well as in Lenschow et al. (1994) and in Mann et al. (1995) for the third-order moment and the skewness [see Eq. (8)]. Furthermore, the influence of horizontal and vertical averaging was investigated by an analysis of the variance spectra $S(\nu)$ of $\rho'$.

b. Humidity variance profiles

The variance spectra $S(z)$ of the humidity fluctuations were calculated by a Fourier transformation of $\rho'$, giving upper limits of the noise variance $\sigma_n^2$ as described above. Additionally, their accuracy was estimated, which is defined here as the standard error $\Delta \sigma_n^2$ of the average value $\sigma_n^2$.

As the noise and the atmospheric fluctuations in the humidity data can be assumed to be uncorrelated, the atmospheric variance $\sigma_s^2$ can be calculated with

$$\sigma_s^2 = \sigma_n^2 - \sigma_0^2,$$

where $\sigma_0^2 = \langle \rho'^2 \rangle$ is the variance of $\rho'$ including the noise contribution. An error propagation yields

![Fig. 1. Absolute humidity variance spectrum measured at 480 m during P2. The spectrum is normalized to the total variance in the time domain and was averaged using a five-point moving window. Additionally indicated is the $-5/3$ slope of the inertial subrange. The shaded area gives an estimation of the noise variance, which led to a noise error of about 2% using a resolution of 10 s and 60 m vertically.](image-url)
\[ \Delta_{\sigma^2} = \frac{4\sigma^4 \sigma^2}{m} + \Delta_{\sigma^2}^2, \]  
where \( m \) is the number of data points in the time series. Here, \( \Delta_{\sigma^2} \) is an important error contribution that must not be neglected in ground-based and airborne active remote sensing systems. For the MPI DIAL system it was found that in a 1-h time series \( \Delta_{\sigma^2} \approx 0.6\sigma^2 \), which alone still caused a noise error of up to 10% in the variance profiles in the entrainment zone. To reduce this error, in all calculations of noise errors 4-h time series were taken, which led to a reduction of \( \Delta_{\sigma^2} \) error of about a factor of 2. Therefore, long time series are required to reduce \( \Delta_{\sigma^2} \) to a low level. Finally, the corresponding error of \( \sigma^2 \) reads
\[ \Delta_{\sigma^2} = \frac{2\sigma_\sigma}{\sqrt{m}} \frac{1 + 0.02m}{\sigma^2} \]  
This equation was applied in all calculations of noise errors in the atmospheric variance of the humidity (\( \rho^9 \)) and led to an overall error of less than 10% in the entire boundary layer.

In the case of using the extrapolation of the autocovariance function to the zero lag for the determination of \( \sigma^2 \), the error due to system noise reads
\[ \Delta_{\sigma^2} = \frac{\sqrt{2}\sigma_\sigma}{\sqrt{m}}. \]  
This noise contribution was not mentioned in Kiemle et al. (1997), although it alone accounted for an error in the variance profiles of \( \approx 20\% \) using a vertical resolution of 134 m. Additionally, errors due to data averaging and the unknown shape of the autocovariance function around the zero lag occur.

The sampling error for the atmospheric variance reads (Lenschow and Kristensen 1985; Lenschow and Stankov 1986)
\[ \Delta_{\sigma^2,\text{samp}} \approx 2\sigma^2 \sqrt{\frac{IS}{T}}, \]  
where IS is the integral scale of \( \rho^9 \) and \( T \) the average time. Whereas this error source was not considered in Kiemle et al. (1997), the data given there permit the estimation that sampling errors of typically 20\% occurred. In the data presented here an integral scale of about 40 s was observed, which led to similar sampling errors of about 20\%. The longer time averaging of the ground-based system roughly compensated for the greater speed of the aircraft.

The effect of data averaging was investigated using the variance spectra. The example of Fig. 1 shows clearly that the onset of the inertial subrange was resolved. The extrapolation of the variance spectrum beyond the Nyquist frequency yielded that, at most, 5\% of the total variance was lost. Similar results were also found at other height levels including the entrainment zone.

c. Humidity skewness profiles

The humidity skewness reads \( S = \langle \rho^9 \rangle / (\langle \rho^3 \rangle)^{3/2} \), where \( \langle \rho^3 \rangle \) is the third-order moment of the atmospheric humidity fluctuations. An analysis of the autocovariance function of \( \rho^9 \) and \( \rho^1 \) showed no evidence for a contribution of the noise third-order moment to \( \langle \rho^9 \rangle \), so that the noise fluctuations were effectively symmetric. Furthermore, it was assumed, as in the determination of the atmospheric variance, that higher-order humidity and noise fluctuations are uncorrelated. Under these conditions,
\[ \langle \rho^9 \rangle = \langle \rho^1 \rangle. \]  
An error propagation yields for the noise error \( \Delta_\epsilon \) of the skewness
\[ \Delta_\epsilon = \frac{\rho^3}{\sqrt{m}} \left( \frac{\sigma^2}{\sigma^2} + \frac{\langle \rho^9 \rangle^{1/2}}{(\langle \rho^3 \rangle)^{3/2}} \right), \]  
where \( \langle \rho^9 \rangle \) is the fourth-order moment of the humidity fluctuations including the noise contribution and \( m \) is the number of data points in the time series.

The sampling error is given by
\[ \Delta_{\epsilon,\text{samp}} = f(a) \sqrt{\frac{\text{IS}}{T}}, \]  
where the parameter \( a \) was derived by the inversion of the function \( S(a) \) and inserted in \( f(a) \). Both functions are given in Lenschow et al. (1994) and Mann et al. (1995).

4. Comparisons of humidity variance profiles with LES model results

a. Theory

Two methods have been proposed to derive quantitative information on transport processes by comparing water vapor profiles with LES results. In the first method, humidity gradient profiles are used for comparisons with LES models to estimate surface and entrainment latent heat fluxes (Wyngaard and Brost 1984; Moeng and Wyngaard 1984; Moeng and Wyngaard 1989). This approach is called the gradient method. First attempts using active remote sensing instruments have already been made (Davis 1992; Kiemle et al. 1997). However, Davis (1992) stated that the comparison was unsuccessful due to a too-low signal-to-noise ratio in the airborne data. Kiemle et al. (1997) showed strong deviations of the measured and fitted gradient profiles in the entrainment zone. The estimated entrainment latent heat flux deviated by about 40\% and 160\%, respectively, from in situ data. They did not investigate whether this discrepancy was due to errors in the DIAL data or if the applicability of the LES results should be questioned.

Also, the results achieved with the MPI DIAL system with respect to mean humidity gradient profiles must be carefully interpreted because, up to a height of 500 m,
errors in the absolute humidity profiles are in the range of 5% and can be height dependent due to systematic effects (Wulfmeyer 1999). Since an error propagation shows that a smaller error in the range of 1% is required for the humidity profiles to achieve gradient profiles with an accuracy of 10%–20%, no attempt was made to apply the gradient method for comparisons with models.

In the second method, humidity variance profiles are compared with LES results. This method is referred to as the variance method. In contrast to the gradient method, high absolute accuracy of the water vapor profiling is less important. Since the time series $\rho'$ is high-pass filtered, and it can be assumed that systematic errors are time independent, these are eliminated and mainly a small noise error in the determination of $\rho'$ is important. Consequently, the variance method appears to be more suitable for the application performed here. It can be used to investigate model results and, if the model results are applicable to the specific measurement, to estimate surface and entrainment latent heat fluxes.

In the first comparison of variance profiles achieved with active remote sensing instruments, Kiemle et al. (1997) likewise found strong deviations between the measured and fitted LES variance profiles in the entrainment zone. Again, no conclusion was drawn as to whether this was due to errors in the variance profiles or to the inapplicability of the LES results. However, it is shown below that the significantly lower noise level of the MPI DIAL system now allows comprehensive comparisons with models so that these issues can be addressed.

b. LES model results

A deeper insight into the dependence of the variance of a scalar on scaling parameters in the convective boundary layer became possible with the development of LES models (Wyngaard and Brost 1984; Moeng and Wyngaard 1984; Moeng and Wyngaard 1989). The modeled scalar variance is described by a superposition of top–down and bottom–up diffusion processes. Therefore, the total variance is the sum of the contributions of a bottom–up variance, $f_b$; a top–down variance, $f_t$; and a bottom–up/top–down correlation variance function, $f_{bt}$; thus it reads

$$
\left\langle \rho'^2 \left( \frac{z}{z_{i, LES}} \right) \right\rangle = c_b f_b \left( \frac{z}{z_{i, LES}} \right) + c_t f_t \left( \frac{z}{z_{i, LES}} \right) + c_{bt} f_{bt} \left( \frac{z}{z_{i, LES}} \right),
$$

(9)

with

$$
c_b = \frac{F_0}{W_0},
$$

(10)

and

$$
c_t = \frac{F_E}{W_0}.
$$

(11)

The surface and entrainment latent heat fluxes are $F_0$ and $F_E$, respectively. In the left term of Eq. (9) the measured variance profile is inserted normalized in height by the $z_{*, LES}$ observed during the measurement period. The three terms at the right-hand side are adapted to the data by applying a least squares fit. As $f_b$, $f_t$, and $f_{bt}$ are known from LES model results (see below), the results of the fit procedure are best estimates of the variables $c_b$ and $c_t$. Consequently, if $W_0$ is known during the measurement period, which was the case here (see section 2), so that Eqs. (10) and (11) can be used, the variance method opens the possibility of estimating $F_0$ and $F_E$.

The measured variance profile has to be normalized with the boundary layer height $z_{*, LES}$ defined in the same way as in the LES model. This definition does not necessarily agree with the conventional definition of $z_*$ as mentioned in section 2 since it is defined by the minimum of the sensible heat flux in the entrainment zone. However, LES results show that $z_{*, LES}$ agrees within about 50 m with the location of the maximum of the humidity variance profile (Sullivan et al. 1998). Consequently, the locations of the maxima of the humidity variance profiles were used to determine $z_{*, LES}$ whenever this approach was applicable. This was the case during P1–P3. During P4 a maximum in the variance profile could not be found due to the occurrence of clouds. As during P2 and P3 the conventional $z_*$ was about 80 m higher than $z_{*, LES}$; therefore, it was set $z_{*, LES} = z_* - 80$ m during P4. Thus estimations of $z_{*, LES}$ during all periods P1–P4 were available. These values were used to normalize the height $z$ of the measured variance profiles for the comparison with the LES results.

The variance functions have to be determined by LES model runs, which have been carried out for only a few cases (Moeng and Wyngaard 1984; Moeng and Wyngaard 1989). In the following, results from a model run are taken that covered a 5 km × 5 km × 2 km (height) domain with a 40 × 40 × 40 grid and a time step of 3 s (Moeng and Wyngaard 1984). This model run was chosen since its resolution agrees well with the resolution of the DIAL system. After the model run the most important results were $L = 100$ m, $z_{*, LES} = 950$ m, $w_0$ = 2 m s$^{-1}$, and a strong inversion of 7 K between 950 and 1150 m as well as a moderate wind shear of about 2 m s$^{-1}$ at the boundary layer top. The turbulence Richardson number $Ri = g\Delta s \Delta z / (T_0 w_0^2)$ (Deardorff 1979) at the top of the boundary layer was about 54 (Moeng and Wyngaard 1984), where $g$ is the acceleration due to gravity, $\Delta s$ is the jump of the virtual dry...
static energy across the inversion layer, and $T_0$ is the surface temperature. Under these conditions,

\[ f_o = 0.47 \left( \frac{z}{z_{i,LES}} \right)^{-5/4} \quad \text{for} \quad \frac{z}{z_{i,LES}} > 0.1, \quad (12) \]

\[ f_{bt} = 1.4 \quad \text{for} \quad \frac{z}{z_{i,LES}} < 0.9, \quad (13) \]

\[ f_i = 2.1 \left( 1 - \frac{z}{z_{i,LES}} \right)^{-3/2} \quad \text{for} \quad \frac{z}{z_{i,LES}} < 0.9. \quad (14) \]

It is these nondimensional, best-fit variance functions from the LES that are used for comparison with the DIAL-measured variance profiles in the next section.

c. Results of the comparison

The application of the variance method is demonstrated in Fig. 2. All variance profiles are normalized separately by $z_{i,LES}$ estimated during the measurement period. During P1, the variance was essentially zero and the boundary layer top was too low to achieve sufficient overlap between the DIAL data and the model functions. Therefore, a comparison was not performed here. During P2–P4, variance profiles could be measured with high accuracy, which was mainly limited by the sampling errors. During P4 the profile was truncated at about 600 m to avoid conditional sampling errors due to clouds.

A least squares fit of Eq. (9) in combination with Eqs. (12)–(14) was applied to the measurements. Figure 2 shows that the contribution of $f_{bt}$ to the variance can be neglected. In all cases the variance profiles could be fitted up to about $0.7z_{i,LES}$. If it is assumed that the LES model results can be applied up to that range, the determination of $c_b$ gives an estimation of the surface latent heat flux $F_0$ by inserting in Eq. (10) the corresponding $w^*$ observed during the measurement period. The results for $F_0$ are summarized in Table 1 along with the values achieved with the bulk formula (Smith 1988) in combination with surface data (Wulfmeyer 1999). The errors of the results achieved with the variance

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**Table 1. Estimated surface latent heat fluxes during P1–P4 derived with the variance method and the bulk formula.**

<table>
<thead>
<tr>
<th>Period number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_0$ [W m$^{-2}$] using LES model</td>
<td>330 ± 60</td>
<td>250 ± 70</td>
<td>150 ± 50</td>
<td></td>
</tr>
<tr>
<td>$F_0$ [W m$^{-2}$] using bulk formula</td>
<td>180 ± 50</td>
<td>260 ± 60</td>
<td>270 ± 60</td>
<td>150 ± 40</td>
</tr>
</tbody>
</table>
method are due to the curve fit errors of $c_6$ and the error for $w_a$, which was assumed to be 0.1 m s$^{-1}$. The latter error was derived by applying error propagation to the equation for determining $w_a$ (Wulfmeyer 1999). Notwithstanding the specified errors, a clear decrease of $F_0$ from P3 to P4 was found. Table 1 demonstrates a promising agreement between the values derived with the LES model and those derived with the bulk formula not only concerning the trends in $F_0$ but also in the absolute values of $F_0$. Therefore, the results give rise to optimism that the variance method can at least be applied to determine $F_0$ using high-resolution water vapor DIAL measurements. An extension of the useful range of the DIAL system down to about 0.1 $z_i$ would further reduce the curve fit error considerably.

During P4, no conclusions could be drawn in the regions between 0.7$z_i$/LES and 0.9$z_i$/LES. During P2 and P3, the fitted profiles did not resemble the shape of the measured profiles over the entire range if values above 0.7 or 0.8$z_i$/LES, respectively, were included. Therefore, fits were only performed up to these heights as demonstrated in Fig. 2. Nevertheless, even under these conditions the predicted $f_t$ was not capable of reproducing the shape of the variance profiles. The extrapolation of the fit results yielded a deviation of about a factor of 3 in the variance at 0.9 $z_i$/LES.

This discrepancy cannot be explained by different resolutions of the LES model and the DIAL system because these were comparable. The occurrence of gravity waves in the region of the boundary layer top also hardly caused this discrepancy because gravity waves would likely increase the humidity variance by additional induced oscillations in the humidity time series (see, e.g., Balaji et al. 1993). Depending on the wavelength of the gravity waves, their influence may be partly removed by a trend correction and high-pass filtering. However, even without trend correction and high-pass filtering the increase in humidity variance was less than 10%, which cannot explain the observed deviation.

Furthermore, advection of variance was most likely not responsible for this deviation. Whereas P2 may still be influenced by advection, during P3 a similarly large discrepancy was found (see Fig. 2). During P3 all mean variables in the boundary layer were fairly constant, such as the humidity, the temperature, and the boundary layer depth. Additionally, it was shown in Wulfmeyer (1999) that the advection of humidity was also very low. Therefore, it can be assumed that the air mass observed during P3 was horizontally nearly homogenous over the homogenous sea surface, which results in horizontally homogenous $F_E$ and $w_a$. Hence, the variance in the entrainment zone can also be considered nearly homogenous [see Eq. 11] so that its advection was low. Therefore, it is very likely that advection of variance cannot be the reason for the deviation of $\approx 300\%$ between experimental and LES results at 0.9$z_i$.

One reason for the observed discrepancy is that not only does $c_6$ vary with $F_E$ and $w_a$ but also the functional dependence $f_t(z_i)/F_E$ changes with the stability at the top of the boundary layer. This was previously not taken into account in LES runs (C.-H. Moeng 1997, personal communication) and comparisons with LES model results (Davis 1992; Kiemle et al. 1997). A first investigation for two different strengths of the capping inversion was performed in Sorbjan (1996) and indeed showed a change of the shape of $f_t$.

Another reason may be a strong wind shear at the top of the boundary layer, which could cause a redistribution of the variance in the entrainment zone. However, no investigation of the influence of wind shear on the top–down variance has been reported yet.

Additionally, Sorbjan (1999) proposed a different scaling of the top–down gradient function dependence on the humidity lapse rate at the top of the mixed layer. He stated that his scaling still works during “evening atrophy,” where $w_a$ is no longer defined. This is a strong indication that the scaling of $f_t$ in Eq. (14) by $F_E$ and $w_a$ has to be modified.

d. *Our approach to approximate $f_t(z_i/LES)$*

The investigation above demonstrates that the functional dependence of $f_t$ on $z_i$/LES has to be modified to explain the measured variance profiles. The DIAL system provides the data required for finding a reasonable approximation of $f_t$. The most likely dependence of $f_t$ during P2 and P3 is shown in Fig. 3. Here the variance profiles are plotted on a double logarithmic scale and are fitted to a new $f_t$ while maintaining the function $f_t^*$ from the LES model. The linear increase of the variance in this scale above about 0.5 $z_i$/LES suggests that
\[ f_i \propto \left( \frac{z}{z_{\text{LES}}} \right)^a \quad \text{with} \quad a > 0 \] (15)

and now explains the variance profiles up to 0.9 \( z/z_{\text{LES}} \). This functional dependence found for \( f_i \) is a completely different behavior from the one derived from the LES results [see Eq. (14)]. For the periods P2 and P3 a least squares fit yielded

\[ a = 2.5 \pm 0.5. \] (16)

The constant \( c_i \) cannot be derived by this procedure, but if it is assumed that it did not change significantly between P2 and P3, the ratio of the entrainment fluxes can be estimated, which yields

\[ \frac{F_{EP2}}{F_{EP3}} = 1.5. \] (17)

### 5. Absolute humidity skewness profiles

#### a. Theory

LES models (Wyngaard and Brost 1984; Moeng and Wyngaard 1984) and analytical studies (Sawford and Guest 1987; Weil 1990) indicate a transport asymmetry in the convective boundary layer that results in different eddy diffusivities for top–down and bottom–up diffusion. This asymmetry is connected with area fractions of updrafts, which differ from 0.5 as observed in laboratory (Dardorff and Willis 1985) and atmospheric studies (Lenschow and Stephens 1980; Young 1988). As this implies a vertical velocity and humidity skewness, the investigation of skewness profiles gives further important insight into the structure of the vertical transport.

Whereas the vertical velocity skewness has already been the subject of several model studies (Moeng and Rotunno 1990; Schumann and Moeng 1991; Wyngaard 1987; Wyngaard and Weil 1991; Schumann 1993) and experiments (Mayor 1995; Mayor et al. 1997), only a few observations are reported concerning the humidity skewness (see Mahrt 1991 and references within). Mahrt (1991) suggested that the humidity skewness \( S \) helps to distinguish between two prototypical boundary layers: EDBL, with a positive divergence of the latent heat flux and a negative skewness, and MBL, with a negative divergence of the latent heat flux and a positive skewness. Correspondingly, observations of the skewness may help understand the evolution of humidity in the boundary layer.

The relationship between these prototypes and the skewness can be investigated using a tendency equation for \( S \) that reads (Wyngaard and Sundararajan 1977)

\[ \frac{\partial}{\partial t} S = \frac{1}{\langle \rho'^2 \rangle} \left[ -3 \frac{dp}{dz} (w' \rho'^2) - \frac{d}{dz} \langle w' \rho'^3 \rangle + 3 \langle \rho'^3 \rangle \frac{d}{dz} (w' \rho') - 3 D_3 \right] + \frac{3}{2} S \left[ 2 \frac{dp}{dz} (w' \rho') + \frac{d}{dz} \langle w' \rho'^3 \rangle + 2 D_2 \right] \] (18)

\[ = A - SB, \]

where, for the sake of simplicity, horizontal homogeneity was assumed and vertical advection by the mean motion as well as phase changes were neglected. In the brackets of the first line of Eq. (18) the tendency of \( \langle \rho'^3 \rangle \) is described by the gradient production term, the flux divergence of \( \langle \rho'^3 \rangle \), the moisture flux divergence term, and the dissipation of \( \langle \rho'^3 \rangle \). In the brackets of the second line the tendency of \( \langle \rho'^2 \rangle \) appears, which is determined by the gradient production, the flux divergence of \( \langle \rho'^2 \rangle \), and the dissipation of \( \langle \rho'^2 \rangle \), respectively.

In Mahrt (1991) only the first line in Eq. (18) was considered, and he stated that in an EDBL the first and the third term in the tendency of \( \langle \rho'^3 \rangle \) led to a negative \( A \), which was considered sufficient to cause a negative tendency of the skewness. However, the moisture flux divergence is positive in EDBL, which corresponds to a positive tendency of \( \langle \rho'^3 \rangle \). Additionally, besides the contribution of the other two terms, the analysis is more complicated due to the tendency of \( \langle \rho'^2 \rangle \) where the skewness appears again.

To estimate the sign of \( S \) in Eq. (18), let the boundary condition at \( t = 0 \) be \( \langle \rho' \rangle \). In this case the solution for \( S \) reads

\[ S = S_0 \exp(-Bt) + \frac{A}{B} \left[ 1 - \exp(-Bt) \right], \] (19)

where it is assumed that the time dependence of the tendencies \( \langle \rho' \rangle \) and \( \langle \rho'^2 \rangle \) is negligible. In the case \( B > 0 \) a negative skewness is achieved if \( A < 0 \). However, in the case \( B < 0 \) and \( S_0 > AB \) a positive skewness can be maintained even if \( A < 0 \). An estimate of the order of magnitudes of \( A \) and \( B \) shows that this case may actually occur in practice. For instance, a slight disturbance \( \delta S/\delta t \ll 1 \) from steady state with \( S(t = 0) \approx 1 \) implies \( A \approx B \).

Consequently, the behavior of the skewness in EDBL and MBL seems to be more complicated, and the budget equation, especially the tendency of \( \langle \rho'^3 \rangle \), has to be analyzed more rigorously, by using LES models for instance. First analyses are currently under way that suggest that indeed a negative skewness occurs in EDBL and a positive skewness in MBL (P. Sullivan 1997, personal communication).

#### b. Measurements

The skewness profiles measured with the water vapor DIAL system are shown in Fig. 4. A previous attempt
to measure the humidity skewness using a DIAL system was reported in Kiemle et al. (1997). However, to our knowledge the profiles shown here are the first skewness profiles measured with an active remote sensing system, which contains a detailed error analysis (see section 3b), proving that after 1-h averaging the errors are sufficiently low to identify significant structures.

Positive values of the skewness were generally found near the top of the boundary layer. A similar behavior is observed in preliminary results of LES models (P. Sullivan 1997, personal communication). During P1 the skewness was mainly positive up to 500 m with a minimum at about 350 m. During P2 and P3 the skewness could be investigated up to 800 m. Whereas $S$ decreased from slightly positive values at 250 m to negative values in the region of 500 m during P2, it was essentially zero during P3 with an evidence of slightly negative values at 300 m. Significant negative values of $S$ were found below the clouds during P4.

The absolute value of $S$ depends on the cutoff frequency chosen for high-pass filtering of the time series but not on the exact shape of the filter function. Interestingly, a higher cutoff frequency caused a decrease of the negative as well as the positive values, whereas the whole structure of the skewness profile was nearly maintained. It is the subject of ongoing studies to investigate these dependences in detail.

Using directly measured flux profiles, which were published in Wulfmeyer (1999), the relation between the skewness and the flux divergence can be investigated. A negative skewness was found during P2 between 400 and 500 m where a negative flux divergence was observed. In constrast, during P4 a slightly positive flux divergence in the lower boundary layer was found, while the skewness was negative. In the entrainment zones where the flux divergences were negative, the skewness generally increased to positive values. Consequently, the measurements presented here do not allow the conclusion that a relationship between the flux divergence and the sign of the skewness exists. Thus, the statement that third-order moments of humidity can contribute to the distinction between entrainment drying and moistening in the boundary layer (Mahrt 1991) needs further investigation both by LES runs and high-resolution measurements.

6. Discussion and outlook

In this study, two current issues in boundary layer research were addressed that could until recently hardly be investigated by means of measurements: the comparison of water vapor variance profiles with LES results and the relevance of humidity skewness profiles for transport processes. A humidity sensor capable of providing new insight into these issues is now available. As demonstrated by an extended error analysis, the MPI water vapor DIAL system allows absolute humidity measurement with high resolution and accuracy so that profiles of humidity moments up to the third order in the convective boundary layer can be determined.
Powerful applications of the variance method, the comparison of measured humidity variance profiles with LES results, were demonstrated. On the one hand, LES top–down and bottom–up variance functions can be verified. For instance, the top–down variance function \( f_j \) showed a completely different dependence of \( z_i \) than expected from LES results. This discrepancy was most likely due to a different wind shear and stability at the boundary layer top used in the LES run. This effect makes the comparison of experimental data with LES more complicated. A large database of LES results is considered essential to take full advantage of the variance method. A parameterization of \( f_j \) in the form \( f_j(z/z_{\text{LES}}, R_i, R_b) \) is suggested, where \( R_i \) is the bulk Richardson number (Stull 1988) and thus a measure of the wind shear at the boundary layer top. Furthermore, the investigation of the correct scaling of \( f_j \) is proposed. This will allow for a comprehensive comparison of LES with experimental data as well as for the determination of \( F_j \) using the variance method.

On the other hand, if applicable LES model functions were available, quantitative results like surface and entrainment latent heat fluxes could be derived. The relative sampling errors in the variance profiles are typically lower than the relative sampling errors made in directly measured flux profiles. Provided that an unambiguous relationship between measured and modeled variance profiles is found, the accuracy of the derived entrainment and surface fluxes can consequently be presumed to be at least as reliable as the results achieved with the flux profiles alone. Table 1 demonstrates already a promising agreement of surface latent heat flux data determined using the variance method with results achieved with the bulk formula.

The low noise error of the water vapor DIAL system allowed, for the first time with specified accuracy, to our knowledge, the determination of profiles of the normalized third-order moment of the humidity fluctuations—the skewness. Due to the low errors in the skewness profiles presented here, DIAL measurements can contribute to the investigation of the relationship between the skewness and the latent heat flux divergence. The findings of Mahrt (1991) in convective continental boundary layers appear not to apply to the present marine boundary layer data. In his observations and citations therein, negative moisture skewness was generated mainly in cases of rapidly growing morning boundary layers or cases of weak surface moisture flux, whereas positive skewness was observed in moistening boundary layers. In contrast, we found a positive skewness in the entrainment zone of a marine boundary layer with nearly zero latent heat flux divergence. Consequently, further investigations are needed to develop a general relationship between the sign of the skewness and the flux divergence.

The results presented here show the potential of water vapor DIAL for boundary layer research. Through further field campaigns and in-depth comparisons with models, this technique is capable of acquiring invaluable information on one of the vital issues for atmospheric science: the closing of gaps in our knowledge of the global water cycle.

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