Boundary Layer Effects on Fronts over Topography

MELINDA S. PENG
Naval Research Laboratory, Monterey, California

JOHN H. POWELL, R. T. WILLIAMS, AND BAO-FONG JENG
Naval Postgraduate School, Monterey, California

(Manuscript received 22 November 1999, in final form 26 January 2001)

ABSTRACT

A hydrostatic, primitive equation model with frontogenetical deformation forcing is used to study the effects of surface friction on fronts passing over a two-dimensional ridge. Surface friction is parameterized using a $K$-theory planetary boundary layer (BL) parameterization with implicitly defined diffusion coefficients, following Keyser and Anthes. Previous studies without surface friction, such as Williams et al., show that a cold front weakens on the upwind slope and intensifies on the lee slope. This is in part due to a superposition effect of mountain flow where colder temperatures exist over the crest and in part due to the divergence pattern caused by the basic flow over the mountain (divergence on the upwind slope and convergence on the lee slope). In Williams et al., the final intensity of a front after passing a symmetric mountain is the same as a front moving over flat land. For no-mountain simulations, the inclusion of the BL results in a more realistic frontal structure and the frontal intensity is weaker than for the frictionless front because weaker temperature gradients are created through vertical mixing. The same type of mixing acts to strengthen a cold front on the upwind slope and weaken it on the downwind slope. The divergence forcing is also frontogenetic on the upwind slope and frontolytic on the lee slope within the BL. The vertical mixing forcing is strongest near the top of BL and weaker within the BL due to weak temperature gradient within the BL. The divergent forcing is strongest within the BL and weak at the top. When BL effects are included, the final intensity of a front passing over a mountain is weaker than the front over flat topography.

The translation of the front is slightly slower with the BL because of the overall reduced cross-frontal speed by surface friction. When moving over a mountain, a front with the BL has a more uniform speed than the frictionless front due to a more uniform flow within the BL.

1. Introduction

Mountainous topography introduces dynamical complications to meteorological phenomena on all scales, from submesoscale systems to the global circulation. Many important synoptic and mesoscale effects are related directly to the dynamics of atmospheric flow over mountains. These effects include lee cyclogenesis, mountain wave generation, and down slope wind storms, such as the Mistral in the southern French Alps. Equally important to forecasters is the effect of mountainous topography on atmospheric frontal intensity, such as those shown in the analyses of the high-resolution Alpine Experiment (ALPEX) data of Hartsough and Blumen (1990) and Radinovic (1986).

Analytical solutions to the problem of fronts moving over mountain ridges are difficult because of the importance of ageostrophic circulations and the resulting nonlinearity of the equations of motion. Bannon (1983) derives frontal solutions for a quasigeostrophic front passing over a two-dimensional ridge. He shows that a cold front experiences frontolysis on the upwind slope and frontogenesis on the lee slope due to a superposition effect. The superposition effect comes as a result of flow over a mountain that generates colder air over the mountain crest so that a front encounters a warm-front type of temperature gradient when moving up a mountain and a cold-front type of gradient on the lee side. Blumen and Gross (1987) examine the effect of mountain dynamics by treating the frontal potential temperature as a passive scalar field that is advected over a two-dimensional ridge by semigeostrophic steady-state flows. Due to mountain-forced acceleration over the crest, the mean flow is divergent on the upwind slope and convergent on the lee slope. This divergent/convergent forcing has a frontolytical effect on a cold front on the upwind slope and a frontogenetical effect on the lee slope. Zehnder and Bannon (1988) use a semigeostrophic numerical model to study the effect of a front moving...
over a two-dimensional ridge through a stationary downstream deformation field. They find that the mountain-forced convergence/divergence pattern causes a similar frontogenesis distribution to that found by Blumen and Gross (1987).

Williams et al. (1992) perform similar simulations using a two-dimensional primitive equation (PE) Boussinesq model in which fronts are forced by a horizontal deformation field moving with a background flow (hereafter this paper is referred to as W92). The frontal and nonfrontal simulations are performed over a range of mountain widths where the semigeostrophic assumption does not apply. The mountain-forced convergence/divergence fields produce frontogenetic variations similar to those found by Zehrnd and Bannon (1988). In both the passive and forced frontal experiments, the mountain effect on frontal intensity is largely symmetrical (except for lee side gravity wave activity and a tendency for the upwind effect to be carried past the crest). In the passive simulations without deformation forcing, the final downstream intensity is nearly equal to the initial intensity. In the forced simulations, the final intensity of a front in the mountain cases is similar to the one traveling over flat topography.

A similar PE model is used by Gross (1994) to simulate a three-dimensional front passing over a finite, isolated synoptic-scale ridge. The front is produced by a developing nonlinear baroclinic wave moving over the ridge. In addition to demonstrating anticyclonic deformation of the frontal zone and horizontal flow diversion around the ridge, the study reveals three-dimensional effects relevant to cross-ridge flow. Down-gradient acceleration of the lee slope winds causes increased convergence and frontogenesis downwind of the ridge. Horizontal shear deformation is shown to have a small effect on frontal forcing over the ridge. Generally, however, the cross-mountain effects on the front agree well with the two-dimensional study of W92 and the long-ridge simulation in the three-dimensional study of Li et al. (1996).

While it is widely recognized that the boundary layer (BL) can have important effects on flow over topography, few studies address the problem quantitatively. The problem associated with flow over topography in the presence of surface friction is addressed by Richard et al. (1989) and Olafsson and Bougeault (1997). Richard et al. find that, in their two-dimensional study, the genesis of strong down-slope winds is delayed and the wave drag is diminished when surface friction is included. Olafsson and Bougeault’s results show decreased mountain wave amplitude in the presence of surface friction and a complete elimination of wave breaking. Peng and Thompson (2001, manuscript submitted to J. Atmos. Sci., hereafter PT) demonstrate that the mountain wave activity depends on the profile of the boundary layer height over the mountain instead of the actual underlying terrain profile.

Williams (1974) obtains steady-state solutions for deformation forced fronts by adding heat and momentum diffusion terms with constant diffusion coefficients. Keyser and Anthes (1982, hereafter KA82) investigate BL effects on frontogenesis using a two-dimensional hydrostatic PE model incorporating a simple one-layer bulk drag BL parameterization and a multilayer first-order K-theory BL parameterization. Detailed realistic frontal features not produced in frictionless simulations are evident in the BL model results. The multilayer K-theory BL model is shown to produce more realistic results than either the bulk parameterization or the Ekman formulation used by Blumen (1980) in a similar model. The major features in their results include a narrow updraft at the top of the BL at the warm edge of the front, a stable layer capping the BL to the rear of the frontal zone, and slightly unstable or neutral lapse rates in the BL behind the front. The frontal intensity, however, is reduced by the frictional effect. Dunst and Rhodin (1990) study the influence of surface friction on a frontal disturbance using a high-resolution first-order parameterization similar to the one used by KA82 but with a different diffusion coefficient formulation. Dunst and Rhodin show that an initial frontal zone with surface friction intensifies rapidly to a front that is similar to a density current, and it then weakens after 6 h because no external frontogenetic forcing is present.

In more recent studies, Braun et al. (1999a,b) investigate the effect of steep orography on land-falling fronts where the terrain has a very broad, one-side plateau shape similar to orography in the western United States. In the frictionless case, they find strong deceleration of fronts, weak upstream frontogenesis, and strong frontolysis on the windward slope (Braun et al. 1999a), corresponding to the terrain-forced convergence/divergence fields. With the inclusion of boundary layer effects, the fronts experience stronger retardation, stronger upstream frontogenesis, and slightly stronger upwind-slope frontolysis (Braun et al. 1999b). The large change in the surface roughness along the coast results in strong frictional convergence and enhances the upstream frontogenesis.

This study investigates the boundary layer influence on a front passing over a symmetric ridge. The basic model is the two-dimensional PE model employed by W92 with a first order K-theory BL parameterization following KA82. The KA82 formulation is similar to the modified Djolov (1973) mixing length parameterization. Holt and Raman (1988) evaluate the various BL closure schemes and find that, while the overall turbulence structure of the atmosphere is better modeled by more complex turbulent kinetic energy closure schemes, the mean structure of the BL is fairly insensitive to the closure scheme employed. Peng and Williams (1994) compare the results of frontal simulations over flat topography using the KA82 formulation to those used in the Dunst and Rhodin (1990) approach. The two BL parameterizations produce very similar results.
Section 2 contains a brief description of the basic model (details can be found in W92) and the formulation of the BL parameterization. In most cases, model parameters are selected to correspond to those of W92. The effect of the BL on frontal development without topography is given in section 3. The influence of friction on flow over a mountain is given in section 4. The combined effect of topography and the BL on a front is discussed in section 5. The summary and conclusions are given in section 6.

2. Description of the model and boundary layer formulation

a. The model

The two-dimensional hydrostatic primitive equation model, which employs terrain following coordinates, is described in W92. The model equations are solved numerically using finite differencing on an Arakawa B grid (Arakawa and Lamb 1977). Finite differences are centered in space and time, with an Euler backward time step inserted every fourth time step to control solution separation. The periodic model domain extends 3600 km in the east–west (x) direction and 12 km vertically. The horizontal grid spacing is 40 km and there are 50 vertical levels uniformly spaced in z, corresponding to 240-m spacing over flat topography. The time increment is 90 s. The fourth-order diffusion constants in the frictionless model are given the value $0.5 \times 10^{-4}$ at the surface, and they increase linearly in the vertical to a factor of 10 at the upper boundary. The linear increase is included to maximize the damping of small-scale numerical noise throughout the domain without including excessive surface diffusion. A hyperbolic sponge layer is also included near the top of the domain. The output fields show little damping of resolved waves and the sponge layer effectively eliminates reflection off the upper boundary.

Initial fields are the same as those in W92. The initial fields of the nonfrontal mountain flow are defined by the semigeostrophic solutions following Merkine (1975) with constant mean flow and stratification away from the ridge. Periodic boundary conditions are applied to these analytic solutions, and they exhibit symmetric structure with respect to the mountain crest. In the frontal simulations, a perturbation limited to the lower part of the domain is added to the basic fields. The frontogenesis is driven by the following horizontal wind deformation field:

$$U_x = (D/\mu) \cosh \mu y \cos[\mu(x - Ut) + x_1],$$  

$$V_y = (D/\mu) \sinh \mu y \sin[\mu(x - Ut) + x_1],$$

where $U$ is a constant mean flow of 10 m s$^{-1}$ that carries the deformation field with it, $D$ equals $10^{-5}$ s$^{-1}$, $x_1$ equals $-\pi$ to shift the deformation with the disturbance, and $\mu$ equals $2\pi/W$, where $W$ is the model domain.

The surface topography is defined by

$$z_s = h \cos^2 \left( \frac{x - W/2}{eW/2} \right), \quad \left| x - \frac{W}{2} \right| \leq eW/2. \quad (2.2)$$

The mountain height $h$ equals 2 km and the mountain width $e$ (the fraction of the horizontal domain $W$ occupied by the mountain) equals 0.6.

b. Boundary layer formulation

The boundary layer parameterization employed follows KA82, which is based on the high-resolution nocturnal BL parameterization of Blackadar (1978). Turbulent fluxes are represented in the $K$-theory by

$$\overline{u'w'} = -K_{mc} \frac{\partial u}{\partial z}, \quad (2.3)$$

$$\overline{v'w'} = -K_{mc} \frac{\partial v}{\partial z}, \quad (2.4)$$

$$\overline{\theta'w'} = -K_{mc} \frac{\partial \theta}{\partial z}. \quad (2.5)$$

The diffusion coefficients $K_{uc}$ and $K_{uc}$ are calculated implicitly from model shear and stability fields to minimize a priori assumptions about the boundary layer structure. Blackadar (1978) derives a closed system of equations for calculating $K_{uc}$ and $K_{uc}$ based on the second-order closure scheme of Mellor and Yamada (1974). He adapts the Mellor and Yamada level-two approximation, which neglects advective and diffusive terms in the second moment turbulence equations so that the equations obey Monin–Obuhkov similarity. The equations are solved in terms of the mixing length $l$, vertical shear $s$, and the Richardson number $Ri$. Following KA82, the coefficients are divided into a small, constant diffusive part, $K_{co}$, and a variable part,

$$K_{mc} = K_{uc} = \begin{cases} K_{co} + k_o \theta^2 \left( \frac{R_i - R_i}{R_i} \right), & R_i < R_i, \\ K_{co}, & R_i \geq R_i, \end{cases} \quad (2.6)$$

where $K_{co}$ equals 1 m$^2$ s$^{-1}$, $l$ equals 100 m, and $k_o$ is the von Kármán constant. This formulation defines the vertical fluxes of heat and momentum above the surface layer, which is assumed to be contained within the first model layer. Finite differencing tends to give systematically greater values of $R_i$ than the mean $R_i$ within the layer (Blackadar 1978), so $R_i = 1.0$ is chosen instead of the theoretical value of 0.25 to account for the finite grid resolution (following KA82). The surface layer fluxes are modeled after Monin–Obuhkov similarity theory, which gives
where $z_0$ is the surface roughness length (the height at which velocity decreases to zero). In this study, the constant value $z_0$ equals 0.5 m.

Within the BL, the coefficients for the horizontal fourth-order diffusion terms are defined, following KA82, as

$$K_H = (\Delta x)^2 \left\{ K_{H0} + \frac{1}{2} k_0^2 (\Delta x)^2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right]^{1/2} \right\} ,$$

(2.11)

where $K_{H0}$ equals $10 \times 10^4$ m$^2$s$^{-1}$. The coefficients for $u$, $v$, and $\theta$ are given equal values.

3. Frontal solutions without topography

The effect of surface friction on frontal development is first examined without topography to serve as the control case for the mountain simulations. The initial conditions are given in Fig. 1. Figure 2 contains the frontal solutions without surface friction after 48 h of integration. Only the lower part of the model domain is shown to highlight near-surface effects. The front has intensified and all fields have been advected downstream with the background current. These fields clearly show the maximum thermal gradient and associated vorticity maxima at the surface. These values decrease rapidly with height. The vertical circulation associated with the front extends to about a third of the total domain, i.e., roughly 4 km.

The corresponding BL solutions are shown in Fig. 3. Strong vertical mixing is evident as shown by the near-vertical potential temperature contours throughout the 1.0–1.2-km well-mixed layer. The frontal zone is weaker than that shown in Fig. 2 but has a more realistic structure. The well-mixed BL behind the cold front, capped by a thin stable layer, is similar to the structure obtained by KA82 (their Fig. 12). The cross-front and along-front wind maxima are elevated jets near the top of the BL as winds are frictionally forced to zero at the surface. The northerly flow of the frictionless front with jet maximum of 35 m s$^{-1}$ is replaced by a southerly flow within the boundary layer, topped by a weak northerly of only 5 m s$^{-1}$ (Fig. 3a). The difference between the frictionless case and the BL case seems to be large in Figs. 2a and 3a. However, the frictional effect on a mean westerly flow induces an Ekman-like southerly flow. Therefore, when the spatial average of $v$ is removed from Fig. 3a, it would be more similar to the inviscid case in Fig. 2a. The lifting of the along-front jet to the top of the BL corresponds well with the cold
Fig. 2. Frictionless frontal solution at 48 h: (a) along-front speed, (b) cross-front speed, (c) vertical velocity, and (d) potential temperature.

Fig. 3. Same as in Fig. 2 except for BL front.
front analyses within the BL by Sanders (1955) and Brummer (1988). The cross-front wind speed within the BL is nearly uniform horizontally except near the frontal edge. The vertical scale of the frontal disturbance is reduced in half within the BL, as can be seen by comparing the vertical velocity field in Figs. 2 and 3. Note that the vertical velocity in the BL simulation is almost three times larger than the frictionless value.

Peng and Williams (1994) employ a variable horizontal coordinate and obtain frontal solutions similar to those shown in Fig. 3. A major finding is that the simulated fronts are always weaker when BL effects are included, due primarily to vertical mixing within BL. Since the horizontal temperature gradient of the inviscid front is largest at the surface and decreases with height, vertical mixing will decrease the maximum temperature gradient, as depicted in the schematic diagram in Fig. 4.

To quantify the strength of the frontal zone, the parameter $d$ is defined as

$$d = \frac{\Delta \theta}{\partial \theta / \partial x}_{\text{max}},$$

where $\Delta \theta$ is the maximum horizontal potential temperature variation across the total domain on the lowest model level. The temporal evolution of the $d$-value for the flat topography simulations is shown in Fig. 5. After an initial period of reduced frontogenesis before 12 h, maybe due to adjustment to the surface friction from the initial inviscid condition, the BL front undergoes frontogenesis at a rate nearly equal to that of the frictionless front (indicated by the slopes of the curves in Fig. 5) until about 30 h. Thereafter, the frontogenesis for the BL front becomes less than the rate for the frictionless front, and by 36 h, the BL front reaches its maximum intensity. At this point, the frontogenetical forcing is matched by the diffusive damping and the front is in a quasi-steady state.

The front in the BL case also translates slower than the frictionless front, which is over 200 km further downstream at $t = 48$ h. The difference between the frontal positions (defined by the location of maximum potential temperature gradient) is shown as a function of time in Fig. 6a. The increase in the distance with which the BL front lags behind is correlated to the difference in maximum cross-frontal speeds between the frictional and frictionless cases (Fig. 6b). The velocities shown are vertical averages over the four lowest numerical levels (equivalent to 960 m) and are representative of the average BL cross-frontal wind at the frontal surface. Between the initial time and 12 h, the BL case develops a stronger jet at the frontal edge, causing a greater cross-front speed at the BL top (figure not shown) and partially canceling the deceleration effect of surface friction so that the vertically averaged speed is about the same as in the frictionless case. This may be an initial adjustment effect to surface friction. After 12 h, the velocities diverge and the frontal separation steadily increases. Thus, it appears that the reduction of the cross-frontal wind velocity caused by the BL results in a slower translation speed and a different vertical tilt, causing the BL surface front to lag behind the frictionless front.

4. Effect of surface friction on mountain solution

Before investigating topographic and frictional effects on a front, the effects of the BL on the basic flow over the ridge alone must be examined. In this experiment, the initial fields that are symmetric about the ridge axis take the form of the semigeostrophic mountain solution with no temperature perturbation. The mountain flow without the BL at $t = 48$ h (Fig. 7) shows only a slight departure from the semigeostrophic solutions as weak wave activity is generated and the cross-ridge velocity maximum shifts slightly toward the lee slope. The BL solutions for $t = 48$ h (Fig. 8) are more asymmetric in the lower layers as the elevated cross-frontal jet is displaced significantly to the lee slope.
frictional downstream shifting of the velocity can be reproduced in a simple model (see appendix). The jet centers of the cross-ridge flow and the along-ridge flow are in the middle of the BL. No gravity waves are evident in Fig. 8. The BL effect on the lee side gravity waves is the same as obtained in Olafsson and Bougeault (1997) and PT. Peng and Thompson show that the reduction of mountain wave activity by the BL is due to the reduced slope defined by the BL height over the mountain so that the flow experiences a barrier effect that is smoother than the actual mountain profile. The damping and diminishing of the gravity waves was also noted by Braun et al. (1999b), but in their case, involved the damping of downstream inertia gravity waves.

In Fig. 8d, the horizontal gradient of $\theta$ on a height surface is also very small within the boundary layer. This is very different from the no-BL case (Fig. 7d), which has a large horizontal gradient because of the cold temperatures over the top of the mountain. The weak temperature gradient in Fig. 8d is caused by downward mixing of warmer potential temperature from above. In order to isolate this effect, let us consider stratified flow over a mountain such as in Fig. 8d. If we assume that mixing makes $\theta$ uniform up to the top of BL, then the temperature within the BL can be written as

$$\theta(x, z, t) = \theta(x, z_s + h, t),$$

(4.1)

where $z_s$ is the height of the topography and $h$ is the thickness of the BL. The horizontal temperature gradient within the BL can be written as

$$\frac{\partial \theta(x, z, t)}{\partial x} = \frac{\partial \theta(x, z_s + h, t)}{\partial x} + \frac{\partial \theta(x, z_s + h, t)}{\partial z} \left( \frac{\partial z_s}{\partial x} + \frac{\partial h}{\partial x} \right).$$

(4.2)

The first term on the right is the temperature gradient above the BL, or the temperature gradient before the BL mixing, and the second term is the BL term. In order to see the topographic mixing effect more clearly, let us consider the schematic diagram in Fig. 9. In this case, $\partial \theta / \partial z$ is positive and the first term is negative on the upwind slope and positive on the lee slope before vertical mixing takes place. If we neglect the variation of $h$ in the BL term, then $\partial \theta / \partial x$ will be increased on the west side of the mountain and decreased on the east side due to the second term because the mountain slope changes sign from one side to the other. On the slope, the potential temperature is mixed down from higher levels that are warmer, and this increases the horizontal gradient in the boundary layer. Therefore, on the up slope, after mixing in the boundary layer, air ahead would have a warmer potential temperature than air behind. This produces a positive temperature gradient on the upwind slope. The same mechanism produces a negative temperature gradient on the lee side. The temper-
ature gradient generated by the BL effect is opposite to the gradient produced in the no-friction case (Fig. 7d). We call this the topographic BL effect. Once this process takes place, the temperature gradient becomes very weak within the BL. Therefore, after the development of the BL, this mixing mechanism will be significant only near the top of the BL.

In order to understand the mountain BL effect on a
passing front, the frontogenetic forcing function based on the mountain flow only is computed and analyzed. The Lagrangian frontogenesis equation can be written in the \((x, z, t)\) coordinates as follows:

\[
\frac{d}{dt} \left( \frac{\partial \theta}{\partial x} \right) = \frac{\partial U}{\partial x} \frac{\partial \theta}{\partial x} - \frac{\partial u}{\partial x} \frac{\partial \theta}{\partial x} - \frac{\partial w}{\partial x} \frac{\partial \theta}{\partial z} + \frac{\partial}{\partial z} \left[ \frac{\partial}{\partial x} \left( k \frac{\partial \theta}{\partial z} \right) \right].
\] (4.3)

The first term on the right is the deformation term that drives the frontogenesis and is zero in the mountain-only case. The second and the third terms are the divergence and tilting terms, respectively. The last term involves the turbulent diffusion. Since the BL forms quickly within 6 h, it is illustrative to examine how the forcing acts in the initial stage. In order not to be confused with conventional frontogenetic effects on a front, when we refer as frontogenetic in the following discussion, we imply the increase of the absolute value of the temperature gradient. Figure 10 depicts the potential temperature, cross-ridge flow, and forcing terms for the mountain solution with BL at 1 h. The frictional reduction of wind speed toward zero at the surface (Fig. 10a) induces convergence on the upwind slope and divergence on the lee slope within the BL (Fig. 10c). This has a frontogenetic effect on the upwind slope and frontal-tic on the lee slope as convergence increases the temperature gradient and divergence reduces the gradient. This is indicated by the frontogenetic forcing term in Fig. 10d [second term in (4.3)] for the temperature field where \(\partial \theta / \partial x\) is negative on the upwind slope and positive on the down slope (Fig. 10b). As illustrated in the schematic diagram (Fig. 9), vertical mixing brings warmer temperatures from the top of the BL to the surface and also frontal-tic for a cold front gradient on the lee slope. This is clearly shown in the mixing term in Fig. 10f. Furthermore, as the boundary layer forms, the isentrops are pushed up so that \(\partial \theta / \partial z\) has a maximum near the BL top and this increases \(\partial \theta / \partial x\) in that area. Therefore, the mixing term makes \(\partial \theta / \partial x\) more negative on the windward slope and more positive on the lee slope near the top of the BL and therefore is frontogenetic on the slope of both sides. The tilting term (Fig. 10e) has its maximum over the crest, and it is zero within the BL where \(\partial \theta / \partial z\) is zero. When there is no BL, this term converts the temperature gradient from negative upwind to positive downwind. The magnitude of the vertical mixing term is greater than the other terms, indicating a dominant effect.

By 6 h, the potential temperature becomes quite well mixed within the BL (Fig. 11). The divergence field increases but maintains its pattern. Due to the weak temperature gradient within the boundary layer after mixing (Fig. 11b), the maximum divergent forcing shows in hour 1 and is lifted to the top of the boundary layer where the temperature gradient is large (Fig. 11d) even though the divergence itself has similar a pattern as before (Fig. 11c). The lifting of the maximum forcing to the upper part of the BL is also pronounced for the mixing term (Fig. 11f). There is less change in the tilting
term (Fig. 11e). This general pattern then continues with time.

In the frictionless case, the horizontal divergence/convergence forcing plays an important role in the frontogenetic forcing on topographic slopes (Blumen and Gross 1987; Zehrner and Bannon 1988; W92). For comparison, the divergence fields at 24 h for the frictionless case and the BL case are depicted in Fig. 12. In the frictionless case, the flow is divergent on the upwind slope and convergent on the lee slope as the air accelerates over the ridge. The low-high-low pattern on the lee side corresponds to gravity waves that are formed on the lee slope. In the BL, the divergence pattern within the BL is reversed, as discussed in previous paragraph. Therefore, at lower levels, the divergence–convergence forcing associated with the frictional mountain is opposite to the one associated with the frictionless mountain. Above the BL, the across-mountain divergence is similar to the no-BL simulation.

5. Front over topography with boundary layer

In this section, we compare the frictionless frontal solutions with the BL frontal solutions over a mountain. The initial potential temperature field has a maximum temperature gradient upstream from the mountain (see Fig. 9a in W92) and it is placed at the point of maximum confluence in the deformation field. Figures 13 and 14 contain the time evolution of the potential temperature field at 6-h intervals for the frontal simulation over the mountain without the BL and with the BL, respectively. The frictionless case basically reproduces the broad mountain case in W92. By \( t = 6 \) h, a well-mixed layer is already present in the BL case, and the trend continues through \( t = 12 \) h as the well-mixed layer continues to develop and deepen. The vertical mixing has produced a significant strengthening of the surface potential temperature gradient compared to the frictionless potential temperature field. Comparison of Figs. 13 and 14 indicates that, while the front weakens moving up-slope in the frictionless case, it intensifies when surface friction is present. As suggested by Eq. (4.2) and Fig. 9, the BL mountain effect enhances a cold front on the upwind slope and the effect is greatest where the slope is largest. This effect tends to move the frontal edge forward from the no-BL position until 12 h, and then it holds the front back from its no-BL position until it moves over the crest. This effect causes the front to move more uniformly over the slope compared with the no-BL case. Between 12 and 24 h, the front moves over the mountaintop where the terrain profile is much flatter.
Fig. 13. Potential temperature fields for front moving over frictionless mountain at 6-h intervals.

Fig. 14. Potential temperature fields for front moving over mountain with BL at 6-h intervals.
and concave downward. During this period, the BL front experiences frontolysis. At about $t = 18$ h, the frictionless front passes the crest of the ridge and moves down the lee slope in the next 18 h, undergoing rapid frontogenesis. Comparing with the frictionless case, the BL front lags behind by about 6–9 h and intensifies at a much lower rate as it moves down the lee slope and then gradually weakens on the lower half of the slope. By the end of the simulation at $t = 48$ h, the BL front is much weaker than the frictionless front.

The frontogenetic forcing is computed from Eq. (4.3). Since the divergence term associated with the deformation mean flow [first term on the rhs of (4.3)] is the same in frictionless and frictional cases, it is not displayed. In the case without the BL (Fig. 15), the divergent term [second term on the rhs of (4.3)] is frontolytic on the upwind slope and frontogenetic on the lee slope as the mountain-forced flow is divergent on the upwind slope and convergent on the lee slope. The temperature gradient associated with a cold front is positive so that positive value from this term is frontogenetic. This upwind effect shows up more clearly at 12 and 18 h but not at 6 h because of the nearly horizontal isentrops on the mountain slope at that time. The large positive forcing associated with the convergence on the lee slope leads to a steep increase of the frontal intensity. The tilting term (Fig. 16) is important on the mountain top to convert the horizontal temperature gradient from negative upwind to positive downstream that is associated with the mountain temperature disturbance. It is frontolytic on the slope on both sides of the mountain, and the frontolytic effect enhances as the front moves down the slope.

In the BL case, the forcing function exhibits a more complicated pattern, as the structure within the BL is very different. The divergent forcing term (Fig. 17) is significant only within the BL, and it is frontogenetic for a cold front from the BL convergence. As the front moves to the lee side, it experiences frontolysis within the BL due to divergence and frontogenesis above the BL. This is consistent with the analysis carried out for the mountain-only case in section 4. The front weakens compared with the frictionless front in the BL and intensifies at the top of the BL. The forcing term from vertical mixing (Fig. 18) is frontogenetic on the upwind slope and frontolytic on the lee slope, but the effect is significant only near the BL top where the vertical temperature gradient is large. Therefore, both divergent forcing and the mixing term act in the same direction toward frontogenesis, but their relative contribution is different at different heights. The forcing from vertical tilting (Fig. 19) is basically the same as in the frictionless case except the pattern is lifted above the BL and there is little contribution to the frontal intensity within the BL.

The frontal strength variations are summarized by the $d$-value in Fig. 20, which contains the temporal evolution of the $d$-values for both frictionless and BL simulations. The inviscid mountain curve is somewhat different from the curve given in Fig. 4 of W92. This difference is due to an error in W92 but does not change the general conclusions in that paper. Since a deformation is imposed on the front all the time, the front moving over a flat land intensifies with time. The $d$-value curve for the BL front shows a strengthening on the upwind slope and weakening on the mountain top and strengthening again on the lee slope. The frontogenetic rate on the lee slope is about the same as the front over a flat land, indicating the frontolytic mixing effect almost cancels the convergence forcing by the mountain on the lee slope. The two frictionless solutions end up at the same strength at 48 h, while the mountain BL solution finishes with the largest frontal scale (weakest gradient) at 48 h.

In summary, a frictionless cold front weakens on the upwind slope and intensifies on the downwind slope, while a frictional cold front intensifies on the way up, weakens on the top portion of the mountain, and intensifies somewhat on the lee slope.

The six-hourly frontal positions for all four cases are shown in Fig. 21. As discussed previously, the BL front moving over flat land translates slower than the frictionless front due to the reduced cross-front speed by surface friction. The frictionless front is slowed down at the foot of the hill both up slope and down slope. Its speed is fastest when moving over the crest, corresponding to maximum cross-ridge speed as shown by the mountain-only case in Fig. 7b. On the other hand, the translating speed for the BL front is more uniform during the passage over the mountain. This is due to the more uniform cross-front speed within the boundary layer (Fig. 8b) and the BL topographic effect. After moving over the mountain, the final position of the BL front at 48 h is 180 km behind the frictionless front. This displacement is close to the difference of the frontal positions for fronts translating over flat land (Fig. 6a). The detailed movement of the front over the mountain with and without the BL is shown more clearly in the Hovmoller diagrams (variation of $\partial\theta/\partial x$ with time on the lowest model level) in Fig. 22. The upslope intensification of the BL front is evident compared with the frictionless front.

In our frontal development, the frontogenetic deformation moves with a constant mean speed of 10 m s$^{-1}$. Since the front does not move with a constant speed, it must move out of optimum phase with the deformation field. Thus, the overall slower speed of the frictional front could have an effect on the frontal intensity. We computed the frontal speed every 6 h for fronts moving over the flat land. The mean speed of the frictionless front is slightly greater than the mean wind of 10 m s$^{-1}$, and the speed of the frictional front is less than the mean wind. Since the sinusoidal deformation field has a synoptic-scale variation, we believe the small difference in frontal position would have a small effect on frontal intensity. The detailed movement of the front over the
Fig. 15. Divergent term in the frontogenetic forcing function (term 2 on the rhs of (4.3)) for the front over a frictionless mountain ($10 \times 10$ Km$^2$, $1$ S$^2$, $1$). Contour interval is 0.2 in the upper panels, 2 in the middle panels, and 4 in the lower panels.

Fig. 16. Tilting term in the frontogenetic forcing function (term 3 on the rhs of (4.3)) for the front over a frictionless mountain ($10 \times 10$ Km$^2$, $1$ S$^2$, $1$). Contour interval is 2 from 6 to 24 h, 6 at 6 h, and 12 from 36 to 48 h.
Fig. 17. Divergent term in the frontgenetic forcing function [term 2 on the rhs of (4.3)] for the front over a frictional mountain ($10^{-10}$ K m$^{-1}$S$^{-1}$). Contour interval is 1.5.

Fig. 18. Vertical mixing term in the frontgenetic forcing function [term 4 on the rhs of (4.3)] for the front over a frictional mountain ($10^{-10}$ K m$^{-1}$S$^{-1}$). Contour interval is 4. This term is zero at $t = 0$ because the mixing coefficient is zero initially.
FIG. 19. Tilting term in the frontogenetic forcing function [term 3 on the rhs of (4.3)] for the front over a frictional mountain ($10^{-18}$ K m$^{-3}$S$^{-1}$). Contour interval is 2 from 6 to 24 h and 6 from 30 to 48 h.

FIG. 20. Frontal strength parameter $d$ (km). nm-nb: Front with mountain and no BL; m-nb: with mountain and no BL; nm-b: without mountain but with BL; m-b: with mountain and BL. The mountain width parameter $\varepsilon = 0.6$.

Mountain with and without the BL is shown more clearly in the Hovmoller diagrams (variation of $\partial \theta / \partial x$ with time on the lowest model level) in Fig. 22. The upslope intensification of the BL front is evident compared with the frictionless front.

To further extend our study, we reduce the mountain width by changing the mountain width parameter $\varepsilon$ in Eq. (2.2). Since the frontal disturbance is placed at the same location, narrowing the mountain width also has the effect of moving the initial frontal position away from the mountain. In the case where $\varepsilon$ is reduced from 0.6 to 0.4, the mountain width is reduced a third and the front does not start climbing over the upstream slope until 6 h, at which time the BL has already established over the mountain (section 4). The variations of the frontal scales over mountain with and without BL as measured by the $d$-value are depicted in Fig. 23. The two cases without a mountain are also included for comparison. Comparison of the wide-mountain and the narrow-mountain cases in Figs. 23 and 20 show the same
Fig. 21. Six-hourly frontal positions: (a) BL over mountain, (b) frictionless over mountain, (c) frictionless over flat land, and (d) BL over flat land.

trend in the evolutions of the frontal scales under the influence of the mountain except in the early stage. The difference in the early stage is due to the different positions of the fronts relative to the mountain. Before 6 h, the front in the narrow-mountain case has not reached the mountain. As noted by Williams et al. (1992), there is a small region of convergence forced by the mountain flow at the foot of the mountain and it has a small frontogenetic effect on the front. Therefore, the front in the narrow-mountain case intensifies faster than the front in the wide-mountain case before 6 h. When it starts climbing the upwind slope, its intensity is much stronger than its counterpart in the wider mountain case at the same time, and the weakening effect by the mountain on the upwind slope appears to be less. Beyond this period, the mountain and/or the boundary layer effects on the front are almost the same. Examination of the frontogenetic forcing functions indicates the same mechanism discussed in the wide-mountain case also applies in the narrow-mountain case. An interesting and important point is that the final frontal scales are almost the same. Since the BL is already established at 6 h when the front reaches the mountain, this result also indicates that the BL spin-up effect has little influence on our analyses. In cases with mountain width $\epsilon$ further reduced to 0.3 and 0.2, the mountain/BL effects remain but, due to the decreased time the front spends on traveling over the mountains, the effect is less pronounced.

Based on our analysis, the BL height will have an effect on the frontal intensity variation. The higher the BL is, the stronger the mixing effect is as the frontal horizontal gradient decreases with height. This is tested with the variation of surface roughness. As the surface roughness decreases from 50 to 10 cm, the BL depth decreases and the overall reduction of the frontal intensity after passing the mountain also decreases. However, the sensitivity of roughness on the BL depth and the BL effect is weak. In our study, the surface roughness is kept uniform in the domain. An abrupt change of the roughness may have a stronger effect, as suggested in Braun et al. (1999b).

6. Summary and conclusions

In this study, we investigate the boundary layer effect on a front passing over a mountain. The initial synoptic-scale disturbance intensifies through an imposed horizontal deformation field that moves with the mean wind. The boundary layer effect is parameterized using the multilayer $K$-theory (KA82) in which the mixing coefficients are determined by the local Richardson number. The frontal development with surface friction over a flat land produces similar results to those in KA82. The strengthened jet at the warm edge of the front is located at the top of the boundary layer, and the vertical mixing produces a nontilting frontal zone. The BL front is more realistic than the frictionless front and compares favorably with analyses of real cases (Sanders 1955).
Frontal intensity is decreased with the inclusion of surface friction, and the translation speed is slightly slower than the frictionless front. The reduction in the frontal intensity is mainly due to strong vertical mixing within the BL that brings down air from the top of the BL where the horizontal potential temperature gradient is weaker to lower levels. The higher the boundary layer depth, the greater the reduction of the frontal gradient because a deeper boundary layer brings down air from higher levels where the gradients are weaker. This is confirmed by experiments in which the mean speed is reduced so that a shallower BL develops.

The frictionless cold front tends to weaken on the upwind slope and strengthen on the lee slope. This behavior is caused by divergence (convergence) forcing on the upwind slope (lee slope) and superposition of the mountain forced temperature field on the front as it travels over the mountain. After moving over the mountain, the final intensity is the same as a front that moves over flat land in the same period (W92). With surface friction, the behavior of the cold front as it moves over the mountain is approximately reversed. On the upwind slope, turbulent mixing brings down warmer air from upper levels ahead of the front, which builds up the positive temperature gradient, while on the lee slope, the same process builds up to the negative temperature gradient. Thus, the vertical mixing enhances a cold front on the upwind slope and a warm front on the lee slope. Furthermore, with surface friction, flow over mountains forces convergence within the boundary layer topped by much weaker divergence on the upwind slope and vice versa on the downwind slope. Therefore, both divergent forcing and vertical mixing are frontogenetic for a cold front on the upwind slope and frontolytic on the lee slope. Once the BL is established, the mixing forcing has its maximum near the top of the BL and the divergent forcing has its maximum within the BL due to a weak temperature gradient and a larger wind divergence within the BL. After moving over a mountain with surface friction, the final frontal intensity is weaker than for a frictional front moving over a flat land.

The frontal movement is also influenced by surface friction. In the frictionless case, the front is retarded on the upwind slope and is accelerated on the lee slope due to a maximum cross-ridge speed at the mountaintop. With surface friction, the cross-ridge flow within the boundary layer is reduced and is more uniform in the horizontal direction following the terrain. The BL topographic effect also tends to pull the front toward the midslope point where the effect is a maximum. Therefore, the frictional front moves at a much more uniform and slower speed across the mountain.

Further studies on cases with narrower mountains show similar mountain/BL effects on a front as in the wide mountain case, although the effects are less pronounced as the front spends less time traveling over a narrower mountain. An interesting and important point is that the frontal intensities after passing over mountains with different width are about the same.
The dramatic reversal of frontal behavior with the BL verses no BL demonstrates the importance of BL effects in mountain dynamics. The vertical mixing and mountain-forced divergence field in the BL on sloping terrain produces unique effects, and it is clear that the inclusion of a realistic BL parameterization is even more critical than in frontal simulations over flat topography. The vertical mixing effect depends heavily on the initial potential temperature distribution, and future studies should also address this sensitivity. Spatial variations in the roughness are likely to be important (Braun et al. 1999b). In particular, the inclusion of diurnal surface heating should have a major effect on frontal structure and dynamics. Also, if the initial temperature state is not realistic in the BL simulation, the development of the BL may introduce extraneous frontal effects in the first stages of the integration. The experiments should be expanded to a three-dimensional domain to evaluate the BL effects on the along-ridge variations found by Gross (1994). Finally, the moisture/precipitation is another effect not considered in this study that would have a large impact on the BL structure and frontal evolution.

Acknowledgments. This research was in part supported by the National Science Foundation. Division of Atmospheric Science, under Grant ATM-9208751 and by the Naval Research Laboratory program element 0601153N, sponsored by the Office of Naval Research.

APPENDIX

We can show that friction causes the wind maximum to be shifted downstream from the mountain top with a simple model. The steady-state linearized shallow water equations with the Coriolis force set to zero can be written as

\begin{align*}
U \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} &= -C_d u, \quad (A1) \\
U \frac{\partial h}{\partial x} - U \frac{\partial u}{\partial x} + h \frac{\partial u}{\partial x} &= 0, \quad (A2)
\end{align*}

where \( C_d \) is the drag coefficient, \( h \) is the mountain height, \( H \) is the mean depth, and \( U \) is the mean flow. After elimination of \( h \), we introduce \( u = |u| \exp[i(kx - \phi)] \) and \( \phi = \phi_0 \exp(ikx) \), which leads to

\[ \tan \phi = \frac{C_d U}{gH - U^2}. \quad (A3) \]

This expression shows that the maximum \( u \) is shifted downstream from the mountain top as long as \( gh > U^2 \).

REFERENCES


