The Two-Dimensional Response of a Tropical Jet to Propagating Lines of Convection

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ABSTRACT

This paper uses simple one- and two-dimensional models to investigate the influence of a propagating line of convective forcing on a tropical jet, representative of the African easterly jet. The results are used to infer changes in the environment of the forcing region, which would in reality tend to influence the evolution of the storm through convective mechanisms, which are not resolved here.

From linear analytical solutions with a rigid lid it is found that the influence of the propagation of the forcing region is to intensify the response on the upstream side of the forcing: this result is confirmed in two-dimensional nonlinear simulations. When linear wave modes are computed in a basic state that includes the jet structure, small sensitivities to the basic-state jet are found.

The two-dimensional nonlinear model has been used further to compute the change in the environmental structure as a result of the forcing. Principally, it is found that the modes of response to the forcing may be associated with characteristic changes in the basic-state shear, convective available potential energy (CAPE), and convective inhibition (CIN), which would be expected to have significant influence on the convective system itself.

1. Introduction

It is an observed characteristic of atmospheric moist convection to organize into coherent structures. Commonly, quasi-two-dimensional forms are observed, and such "squall lines" may propagate rapidly, resulting in severe weather over relatively large areas. From a perspective of studying the storm internal dynamics, it has been shown that the influence of a squall line on the larger scales of the atmosphere has a feedback on the storm itself. For instance, Lafore and Moncrieff (1990) and Garner and Thorpe (1992) have argued that the mesoscale "storm flows" induced by a squall line lead to circulations that have a significant contribution to the thermal and dynamical environment of the storm. Furthermore, Pandya and Durran (1996) have demonstrated that the evolution of the storm flow, created by propagation of gravity waves away from the forcing region of the storm, can account for much of the larger, meso-β-scale storm structure. One important conclusion from such work is to demonstrate that the squall line, in modifying its environment, acts to precondition its own inflow conditions. While mechanistic discussions [such as those of Thorpe et al. (1982) or Rotunno et al. (1988)] help to explain the mechanisms of maintenance of squall lines in terms of environmental conditions, the basic parameters of wind shear and stability [specifically convective available potential energy (CAPE)] are not external to the storm dynamics but are internal to the system, a consequence of the storm flow, and depend on the history of the storm's evolution. It is the aim of this paper to further explain the mechanisms whereby the genesis of a squall line may influence its inflow conditions.

Several authors have discussed the response of a quiescent atmosphere to squall line thermal forcing using simple models (Bretherton and Smolarkiewicz 1989; Nicholls et al. 1991; Mapes 1993; Mapes and Houze 1995). When a band of heating is initiated, it will generate a fast-moving compression wave (Nicholls and Pielke 2000): following this, the heating tends to induce ascent at the heating region and transient "bores" in the form of regions of compensating descent, which propagate away from the source, acting to transfer the heating into thermal anomalies at greater distances. When the transient regions of compensating descent have propagated through the domain, the resulting atmospheric state resembles the large-scale paradigms as represented, for example, by the Matsuno–Gill model (Mapes 1998).

One of the well-known areas in which squall lines are important is that of west Africa, where these systems provide the bulk of the annual rainfall to a region of marginal climate. West African squall lines are known to propagate toward the west, in a mean flow that is dominated by the African easterly jet (AEJ), of a

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strength of around 12 m s$^{-1}$ at an altitude of about 4 km (Reed et al. 1977). The propagation speeds of these squall lines are of an order of 15 m s$^{-1}$, and some are observed to propagate relative to the flow at all levels. It is generally thought that the evolution of these storms is dominated by the internal interaction between the intensity of the gust front and the wind shear, moderated by the storm flow. With this in mind, it is reasonable to consider a storm in terms of the way in which it acts as a forcing for the larger-scale flows, and to regard the feedback of the environment on the storm as a second order effect: parallel with previous authors (Nicholls et al. 1991; Hertenstein and Schubert 1991), the forcing due to a squall line will here be considered as a relatively simple thermal forcing structure.

In considering these west African squall lines and their influence on the larger scales of the atmosphere, there are a number of ways in which the established models come into question.

- The propagation of the systems means that they may move faster than some of the modes of transient response, which would otherwise propagate ahead of the storm—How does this asymmetry influence the resulting flow?
- There is significant vertical wind shear in the basic state—How does this influence the transient response and the long-term response?

This paper makes progress in addressing such questions by looking at the response of the atmosphere to forcing by a propagating band of latent heating in a hierarchy of simple two-dimensional models. The second section discusses the influence of storm propagation on the response, through consideration of linear solutions in a quiescent environment with a rigid lid. Section 3 explores departures from such behavior, which result from a realistic jet structure. Section 4 demonstrates how these results explain the behavior of simulations obtained with a nonlinear model, linking the results to a more explicit thermodynamic response; and the final section is a summary of these results.

2. Linear response to propagating forcing in a quiescent environment

The linear, two-dimensional, incompressible Boussinesq equations of motion may be written

\[
\begin{align*}
\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + w \frac{\partial \pi}{\partial z} - f v &= - \frac{\partial \phi}{\partial z}, \\
\frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} + u \frac{\partial \pi}{\partial z} + w &= 0,
\end{align*}
\]

\[
\begin{align*}
\frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} - b &= - \frac{\partial \phi}{\partial z}, \\
\frac{\partial b}{\partial t} + \frac{\partial b}{\partial x} + v \frac{\partial \phi}{\partial y} + N^2 w &= S,
\end{align*}
\]

where \((u, v, w)\) is the perturbation vector wind, \(\phi = \rho'/\rho_0\) is the dynamic pressure (the pressure perturbation scaled by reference density), \(f\) is the Coriolis parameter, \(b = g \theta'/\theta_0\) is a buoyancy, with \(\theta'\) a perturbation and \(\theta_0\) a reference potential temperature,

\[
N^2 = \frac{\partial b}{\partial z},
\]

defines the Brunt–Väisälä frequency \(N\), and \(S\) is a thermodynamic source term. Terms with overlines represent basic-state fields, which are time-independent solutions of the nonlinear equations. The integer \(n\) is a marker for the hydrostatic approximation: \(n = 0\) indicates hydrostatic flow while \(n = 1\) is nonhydrostatic. For the case of a quiescent atmosphere (\(\pi = 0\)), these equations can be condensed to a single equation for \(w\):

\[
\frac{\partial^2}{\partial t^2} \left( \frac{n \partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right) + N^2 \frac{\partial^2 w}{\partial x^2} + f^2 \frac{\partial^2 w}{\partial z^2} = \frac{\partial^2 S}{\partial x^2}.
\]

For solutions between rigid upper and lower boundary conditions with constant \(N\), this equation can be generalized to a modal form in the vertical (taking, for \(w\), a Fourier series of elements \(a_j \sin mz\) where \(m = \frac{j}{H}\) \(H\), with \(j\) an integer and \(H\) the domain depth, and a cosine form for \(u\), which satisfies boundary conditions of \(w = 0\) at \(z = 0\) and \(z = H\).

In considering sustained propagating convection, a form,

\[
S(x, t) = Q \pi(x - st)H(t),
\]

is assumed here, in the first instance, where \(H(t)\) is the Heaviside function, \(Q\) is a positive constant, and \(s\) is the propagation velocity. It is useful to take a Fourier transform in space [denoted, for a function \(h(x, t)\), by \(\tilde{h}(k, p)\)] and a Laplace transform in time [the combined transform denoted by \(\hat{h}(k, p)\)]. From (8), the combined transform of the forcing is

\[
\hat{S}(k, p) = \frac{Q \tilde{F}(k)}{p - ik s},
\]

so (7) can be transformed to give

\[
\hat{w}(k, p) = \frac{Q k^2 \tilde{F}(k)}{(nk^2 + m^2)(p - ik s)(p^2 + \omega^2)},
\]

where

\[
\omega^2 = \frac{N^2 k^2 + f^2 m^2}{nk^2 + m^2} = k^2 c^2 > 0,
\]
in which $\omega$ is the frequency of neutral waves and $c$ their phase speed. Inverting the Laplace transform leads to

$$\tilde{w}(k, t) = \frac{Q\tilde{F}(k)}{nk^2 + m^2} \left( \frac{e^{ikct}}{c^2 - s^2} - \frac{e^{-ikct}}{2c(c - s)} \right).$$

(12)

For $f = 0$ and $n = 0$ the free modes composing the response to the forcing are nondispersive, of fixed phase speed $c = N/m$ (equal to group speed, $c_g$), and the solution can be obtained analytically:

$$w(x, t) = \frac{Q}{m^2} \left( \frac{F(x - st)}{c^2 - s^2} - \frac{1}{2c(c - s)} \right).$$

(13)

This solution consists of three zones of response in the vertical motion: the first at the location of the moving region of forcing and the others propagating away from the initiation point with the speed, $c$, of neutral waves. Each zone of response has the same horizontal form as the forcing (i.e., the function $F$): Fig. 1 shows the possible forms of the solution for a forcing function with the same horizontal structure and maximum heating rate of 2 J kg$^{-1}$ s$^{-1}$ as used by Nicholls et al. (1991). In the special case of zero source propagation, $s = 0$ (see Fig. 1a), the central zone acts to compensate the diabatic source (centred on the inverted triangle) exactly, through adiabatic cooling or warming, as can be inferred by comparing (8) with the first term of (13). In this case the propagating zones of downward motion act to communicate the diabatic heating into its environment through an expanding circulation, which translates upward motion at the source into descent of the region that has been influenced. Similarly a region of forced cooling would cause ascent and adiabatic cooling of its environment.

This solution, for stationary forcing, has been documented elsewhere (Nicholls et al. 1991). When the source propagates ($s \neq 0$), however, there are significant differences in the results, which depend critically on the relative magnitudes of $c$ and $|s|$. For the sake of argument, and without loss of generality, the following discussion will refer to a single mode of heating corresponding to $m = \pi/H$, so that we consider the response to heating of positive sign in the troposphere, and with
\( s < 0 \) as is typical for west Africa. Now it is helpful to consider two cases separately.

First, \(|s| < c\) (Fig. 1b).

Here, the signs of the denominators in (13) remain positive, so the solution has some qualitative resemblance to the \( s = 0 \) case. However, the downward branch propagating in the same direction as the forcing experiences an enhancement due to the denominator \( c - s \) while the response propagating away from the source is diminished. The upward motion at the source region (centered on the inverted triangle) no longer exactly compensates the diabatic heating, and its magnitude is increased to give a net cooling in this zone. The consequences of these changes are an increased thermal anomaly and an increased horizontal wind anomaly ahead of the forcing; the faster the propagation speed of the forcing, the more intense the response in the relative inflow to the forcing.

Second, \(|s| > c\) (Fig. 1c).

In this case, which corresponds to rapid propagation of the forcing and/or a high vertical wavenumber \( m \), the signs of the two forward-propagating branches are reversed. The region of forcing is accompanied by downward motion while the wave response propagating in the same direction from the source is upward.

A number of authors have commented on the possibility of a (perhaps counterintuitive) downward response to positive thermal forcing in a cross flow (Thorpe et al. 1980; Lin and Smith 1986): here it appears that this response can be understood by the propagation of the forcing faster than the hydrostatic group speed at the given wavenumber (which is the upper limit of the group speed for the nonhydrostatic response). The response can also be seen in terms of a Froude number, \( Fr = \frac{|s|m}{N} = \frac{|s|}{c} \), (14)

relating the system speed \( s \) to the natural wave speed. When \( Fr > 1 \), there is no wave response ahead of the system and descent is the response to forcing. Since shallower modes (higher \( m \)) involve lower phase speeds, the high Froude number situation is more likely for these modes. This observation has implications for the atmospheric response to shallow forcing features (such as the melting layer in cumulonimbus convection) in a propagating system. Thorpe et al. (1980) and Parker (1998) have also used similar Froude number arguments to characterise the transition between "symmetric" and "jump" downdraughts.

Bretherton (1988) also discussed the response to a thermal forcing in terms of wave propagation characteristics, finding conditions for the response to be steady and showing how a stationary group velocity relative to the forcing leads to growing solutions. In the case described here, this occurs for \( s = c \) (with nondispersive waves), in which case the solution has a component which grows linearly in time. For \( s > 0 \), this growing component has the form

\[
w = -\frac{Qt}{2mN} \frac{d}{dx} f(x - ct),
\]

so that there is ascent in the inflow to positive forcing and descent behind: this is a situation that is potentially very favorable for resonance of a gravity wave with deep convective forcing.

It is worth noting that the discrete form of the response in the linear solutions, occurring as distinct bores propagating away from the source, arises as a result of the given form of the startup function \( H(t) \). A smoother initiation of the forcing, such as a linear ramp for the interval \( 0 < t < a \), gives a more smoothed response [this can be demonstrated analytically for the case \( F(x) = \delta(x) \)]. Further possible refinements to this analytical model include oscillatory forcing, which leads to an oscillatory response, and mass or momentum flux convergence terms in the forcing, which lead to solutions for \( w \) with the same propagating form as a thermal forcing, but with different balances between \( w \) and \( u \) or \( \theta \).

In order to understand how these simple results apply to the more complete model of nonhydrostatic flow on a rotating plane, (12) can be solved numerically. As discussed by Pandya et al. (1993), the effect of nonhydrostatic dynamics is to deplete the propagating response as it moves away from the source, through wave dispersion. Inclusion of a nonzero coriolis parameter \( f \) leads to a significant \( u \)-component of the wind, as the forcing tends to spin up vorticity. Significant long-line winds have been observed by Chalon et al. (1988) and may be significant in the interaction of squall lines with African easterly waves. This part of the wind field can be obtained from the \( u \)-momentum equation and has the same vertical structure as \( w \): solutions for the horizontal structure are plotted in Fig. 2, for the three forcing speeds used in Fig. 1, where a latitude of 12.5°N has been chosen to compute \( f \), as representative of the AEJ. For these solutions with a rigid lid, it is possible to evaluate the long-time limit of the \( u \) field analytically, using the relation

\[
\lim_{s \to 0} v(k, t) = \lim_{p \to 0} p v(k, p);
\]

for \( s \neq 0 \), in the vicinity of \( x = 0 \), and the solution decays exponentially with distance, with an \( e \)-folding scale proportional to the Rossby radius,

\[
L_R = \frac{N}{fm},
\]

as in the classic Rossby adjustment problem (see Gill 1982) or the forced tropical solutions of Mapes (1998) which generate a Matsuno–Gill time response over a period of many hours. Close to the propagating source, the limiting solution can be derived analytically in the hydrostatic \((n = 0)\) case, and there is a similar exponential decay of \( u \) with distance, here with an \( e \)-folding scale of
Fig. 2. The $v$ component of the wind (along the line) associated with the three cases of Fig. 1: $s = 0$ is solid, $s = -10$ m s$^{-1}$ is dotted, and $s = -30$ m s$^{-1}$ is dashed. The vertical structure of the modes is the same as that of $u$, and the field of vertical motion acts to spin up vorticity through stretching.

$$L_s = \frac{\sqrt{\left(N^2/m\right) - s^2}}{f},$$

so that $L_s \to L_R$ for low $m$ or $s$ provided the source speed is lower than the neutral wave speed $N/m$. If this is not the case, the solution is zero ahead of the forcing and wavelike to the rear.

Although the long time limit can be derived analytically for these cases, in more realistic basic states, and in the absence of wave trapping, the horizontal scale of the forced response will be determined by the rate of wave propagation into the stratosphere rather than the Rossby radius; at 12.5°N a typical value of $L_R$ is of the order of 3000 km, while the decay scale of forced perturbations will be much shorter (as seen in section 4).

3. Effects of a more realistic basic state jet

The simple hydrostatic model, employing a rigid lid and a quiescent environment, has some limitations when applied to the real atmosphere: Nicholls et al. (1991) and Pandya et al. (1993) have shown how the combined effects of nonhydrostatic motion and upward propagation of energy act to diminish the propagating bores away from the source. In addition, curvature of the basic state wind field $\mathcal{U}(z)$ influences the nature of the gravity wave response. Here, the effect of a variable $\mathcal{U}(z)$ is investigated through consideration of numerically computed linear gravity wave modes, and, in the next section, comparison of these with two-dimensional initial value solutions is addressed.

For the case of $f = 0$, $n = 1$, reduction of the linearized Eqs. (1)–(5), assuming modal solutions of the form $\exp[i(kx - \omega t)]$, yields the single eigenvalue problem for $w^1$:

$$\frac{\partial^2 w}{\partial z^2} + \left[ \frac{\mathcal{U}_{zz}}{(c - \mathcal{U})} + \frac{N^2}{(c - \mathcal{U})^2} - k^2 \right] w = 0,$$

known as the Taylor–Goldstein equation (Taylor 1931; Goldstein 1931). In this equation, the wavenumber $k$ and frequency $\omega$ are eigenvalues and the eigenfunction is $w(z)$. The first two bracketed terms are conventionally combined into a complex Scorer parameter $\lambda$, defined by

$$\lambda^2(z) = \frac{\mathcal{U}_{zz}}{(c - \mathcal{U})} + \frac{N^2}{(c - \mathcal{U})^2}.$$  

Equation (21) has been solved numerically, employing the following procedure [which follows Mobbs and Darby (1989)]. If a rigid lid is used, the upper boundary condition is $w = 0$ and the choice of $w$ at the next grid level determines the amplitude of the modal solution. For a given choice of $k$, an initial guess at $\omega$ is taken and (21) is integrated downward using a fourth-order Runge–Kutta scheme. The lower boundary condition of $w(0) = 0$ must be satisfied in order for an eigenfunction to exist; a given $(k, \omega)$ will yield an error, which is the amplitude of $w(0)$, and this is used, through Newton relaxation, to obtain a new estimate of $\omega$.

Special consideration must be given to the numerical solution when the integration passes through a critical level, at which $c - \mathcal{U} = 0$ [see Mobbs and Darby (1989)]. However, since the primary aim of the present study is to investigate fast modes propagating ahead of the forcing region and the basic-state jets, critical levels are not considered here. When this methodology is used to explore the extension of the solutions obtained in section 2 to jet structure (retaining the rigid lid), the method converges efficiently and robust results are obtained.

Solutions have been computed for a basic state with constant $N^2 = 1.1 \times 10^{-4}$ s$^{-2}$ and a jet structure (Fig. 3) constructed to resemble that of the Convection Profunde Tropicale 1981 (COT81) case, which occurred on 23–24 June 1981 and has been studied in some detail elsewhere (e.g., Redelsperger and Lafore 1988; Chalon et al. 1988). In particular, this case is of interest in the present discussion since the modification of the envi-
The field of basic state zonal wind with height, derived from the upstream profile for the COPT81 storm. The wind is constant above 20 km.

The dispersion relations for the first three vertical modes (plotted points) and the equivalent, analytical solutions for a quiescent basic state (solid curves), computed for the wind profile of Fig. 3. Modes are shown with wavelengths between 50 and 1000 km.

The environment as a result of the storm forcing was the subject of discussion by Lafore and Moncrieff (1989). It can be seen that for the first three modes, the occurrence of the wind curvature has relatively little influence on the dispersion relation, apart from an apparent Doppler shift of the modes by an amount representing a fraction of the maximum jet strength (Fig. 4). The jet structure leads to a modification of the vertical structures of the eigenmodes: comparison of these solutions with results from a nonlinear initial value simulation are made in the next section.

When the rigid lid condition is relaxed, convergence of solutions becomes quite problematic, as other authors have found (e.g., Mobbs and Darby 1989). The numerical boundary condition employs an assumption that the basic state at the top of the domain is constant with height, in which case, (21) reduces to

\[ w \sim \exp(imz) \]

where \( m^2 = \lambda^2 - k^2 \). Since there are two complex roots for \( m \), the root is chosen according to (a) a trapping condition of \( \text{Im}(m) > 0 \) or, in the special case of \( \text{Im}(m) = 0 \), (b) a condition of upward group velocity. The determination of \( m \) gives a relationship between the top two points in the finite difference integration.

The key phenomenon with an open upper boundary is the upward radiation of waves, leading to a decay of the amplitude of the modes (a significant imaginary part to \( \omega \)). The nature of the solutions to (21) is determined by the Scorer parameter, \( \lambda \): for low \( \lambda \) the solutions are exponential and for high \( \lambda \) they are wavelike. In the absence of curvature of the basic-state wind (\( \tau_{zz} = 0 \)), wavelike modes are anticipated, for all but the shortest waves. Suitable wind fields may modify this behaviour to reduce \( \lambda \) and lead to “trapped” solutions for which the waves are evanescent outside the domain of interest.

The importance of trapping is that relatively little energy is propagated upwards out of the domain when such modes exist, and thus the waves may propagate over large distances with relatively little loss of amplitude. Schmidt and Cotton (1990) have used trapping conditions derived from the Taylor–Goldstein equation to discuss the interaction between upper and lower level gravity waves, and their contribution to squall line maintenance.

Inspection of (21) indicates that regions of trapping are likely to be those of low \( N^2 \), high \( |c - \pi| \), and negative values of the wind curvature term. From simple theory applicable to a quiescent atmosphere [e.g., following Eq. (7)] the wave speed scales as \( c \sim N/m \), with \( m \) the vertical wavenumber, so low \( m \) waves (of deep vertical scale) are more likely to be trapped. In the case of easterly propagating waves, with \( (c - \pi) < 0 \), a negative curvature term occurs around the maxima of easterly jets such as those observed in the west African monsoon. In summary, modes are likely to be trapped when there is (i) high wavenumber \( k \) (short waves); (ii)
low tropospheric $m$ (deep waves); (iii) low $N^2$ aloft (e.g., Tripoli and Cotton 1989; Nance and Durran 1998); and (iv) a strong jet below the tropopause, where $N^2$ is not large.

Upwardly propagating waves have been found to have similar phase speeds but much higher decay rates (typically $1 \times 10^{-3} \text{s}^{-1}$) corresponding to a length scale of around 100 km. However, the results obtained using this method are not sufficiently robust to be included in detail here.

4. Two-dimensional simulations

Extension of the linear results into the nonlinear forced problem has been performed using a simple numerical code, based around the model of Rees (1988). The dynamics represent a two-dimensional, nonhydrostatic, incompressible, rotating (at 12.5°N latitude) Boussinesq fluid, with a Richardson number dependent mixing length closure for turbulence and a surface layer boundary condition based on a Monin–Obukhov scaling length. The model is periodic in the horizontal dimension and has a rigid lid. For the simulations described here the lid was set at 50 km and a Rayleigh damping term was used. The model is periodic in the horizontal dimension $x$ and has a rigid lid. For the simulations described here the lid was set at 50 km and a Rayleigh damping term employed in the top 20 km of the domain, increasing linearly up to $10^{-2} \text{s}^{-1}$ on the upper boundary: there was no significant change in the solutions when an upper boundary was set at 100-km altitude. The assumption of incompressibility is not a good one for such deep domains in the atmosphere, but was retained in the model for direct comparison of the results with the linear solutions of sections 2 and 3; indeed, the upper part of the domain in these simulations is only present in order to minimize downward reflection of gravity waves from the upper boundary. The advection scheme has been modified to incorporate Leonard’s “QUICKEST ULTIMATE” scheme (Leonard 1991).

Model simulations have been performed at a uniform horizontal resolution of 5 km with 512 grid points (giving a domain of 2560 km) and a vertical resolution of 100 points on a stretched grid, ranging from 21 m at the surface to 1034 m at the upper boundary. The initial profiles of wind and temperature were obtained by running a one-dimensional version of the model to obtain an approximately steady solution. This procedure was initialized with the COPT81 wind profile shown in Fig. 3, with stability of $N^2 = 1.1 \times 10^{-4} \text{s}^{-2}$ below 15 km and $N^2 = 7.0 \times 10^{-4} \text{s}^{-2}$ in the stratosphere. The lowerboundary condition was of fixed temperature at 298.5 K, leading to a relatively inactive boundary layer of depth approximately 150 m, and the resulting solutions from the one dimensional model only departed noticeably from the initial COPT81 profiles in the lowest levels. The two-dimensional model was then forced by a heating function of cosine horizontal profile, taking the form

$$\sigma(z) = \begin{cases} 40 \sin \left( \frac{\pi (z - z_1)}{z_2 - z_1} \right) \text{K h}^{-1}, & \text{for } z_1 < z < z_2, \\ 0, & \text{otherwise,} \end{cases}$$

with $z_1 = 3$ km, $z_2 = 11$ km, chosen to correspond broadly with the heating structures obtained in cloud resolving simulations of this case (J.-L. Redelsperger 2000, personal communication).

Figure 5a shows vertical velocity in the model after 5 h of simulation, demonstrating the characteristic bores of response in the vertical motion to each mode of the heating, as described in section 2 and by other authors. The vertical structures of the eastward and westward-moving first modes in the model are plotted in Fig. 6, alongside solutions obtained by integration of the Taylor–Goldstein Eq. (21), as described in the previous section. A good agreement between these structures can be seen, despite the differences in the upper-boundary conditions for each method and nonlinearity in the two-dimensional model. These curves also confirm that, as derived in section 2, the mode moving westward, in the same direction as the forcing, is stronger than the eastward mode. The change in the mode vertical structures (relative to the sinusoidal solution for a uniform basic state wind) is most marked at the AEJ level and represents an amplification of the eastward mode and a reduction in the westward mode. The authors do not have a simple explanation for this phenomenon, but some insight can be gained by considering the mode curvature, $- (\lambda^2 - k^2) w$. At the AEJ level, the $\pi w$ term increases the magnitude of the curvature for the eastward propagating mode and reduces it for the westward mode, which is consistent with the forms observed.

The bores diminish in amplitude as they propagate away from the source region: this decay is confirmed in the perturbation potential temperature shown in Fig. 5b. This field can be compared with that obtained in a simulation with a rigid lid at the tropopause height of 15 km, as shown in Fig. 7. When the rigid lid is included, waves in the troposphere are trapped perfectly and the thermal anomalies produced by the forcing are almost uniform with distance away from the forcing, as in the analytical solutions. This uniformity occurs in the face of waviness in the vertical velocity field, due to dispersion of nonhydrostatic modes, as a result of the integral nature of the thermal anomaly induced by adia-
Fig. 5. Results from the two-dimensional simulation with simple, prescribed forcing, after 5 h of simulation: (a) vertical velocity, contoured at 2 cm s$^{-1}$; (b) perturbation potential temperature contoured at 0.5 K; (c) storm relative and (d) perturbation horizontal wind, at 2 m s$^{-1}$; (e) perturbation potential vorticity (relative to the initial tropospheric value), contoured at 0.02 PVU (1 PVU = 10$^{-6}$ km$^2$ kg$^{-1}$ s$^{-1}$); and (f) meridional (along-line) wind, contoured at 0.5 m s$^{-1}$; In all cases, values of zero or less are represented by dashed contours. The storm initial position was at 2000 km and the propagation velocity was $-14$ m s$^{-1}$, so that at the time shown, the forcing was centered at 1748 km, as indicated by the inverted triangle.
FIG. 6. Comparison of modal solutions to the Taylor–Goldstein problem for a non-trivial basic state jet structure (dashed lines), computed for a wavenumber of $2 \pi/1000$ km; with vertical velocity fields obtained from the two-dimensional nonlinear simulations of Fig. 5 (solid lines). The westward mode is the stronger, and was taken at 1000 km in the two-dimensional domain, while the weaker eastward-propagating mode was taken at 130 km. The normal mode solutions have been normalized to have the same maximum amplitude as the profiles from the nonlinear model.

FIG. 7. As in Fig. 5b, but for a simulation with a rigid lid at the tropopause.

The nonlinear model confirms that the only modes propagating ahead of the forcing region are those of deep enough vertical extent to outrun the forcing: evidence of modes 1–4 can be seen in Fig. 5a. From the solutions we can infer changes in the thermodynamic environment of importance to storm dynamics.

- The influence of the forced response on the shear below the AEJ amounts to a reduction in the magnitude of the shear by 50%, close to the storm, which would lead to a doubling in a bulk Richardson number estimate for storm characteristics (Weisman and Klemp 1982).
- CAPE and convective inhibition (CIN) have been computed from pseudoadiabatic moist ascent calculations (Emanuel 1994). Since the model does not carry moisture explicitly, the calculations have been based on a range of assumed dewpoint depressions, $T - T_d$ (assumed constant with depth in the atmosphere; the moisture content at the initial parcel height will be of greater significance to the resulting CAPE and CIN than the moisture in the profile above). Although there are known to be significant discrepancies between different methods of calculating CAPE and CIN, in the present study we are interested primarily in the sensitivity of these values to system-scale forcing rather than the absolute values obtained. It turns out (not shown) that a good estimate for the CAPE anomaly due to a change in the $\theta$ profile of $\theta'(z)$ can be obtained using

\[
\frac{\text{CAPE}_{\text{anomaly}}}{\text{CAPE}_{\text{base}}} = \frac{\text{CAPE}_{\text{new}}}{\text{CAPE}_{\text{new}}} = \frac{\theta'(z)}{\theta(z)}
\]
In addition to the direct responses of CAPE and CIN, where $z_b$ is the level of free convection and neutral buoyancy, and this form is independent of the parcel evolution. Figure 8 shows the CAPE and CIN computed as a function of horizontal distance in the model, using the lowest positive model level as the parcel origin and a dew point depression of 8 K. The principle (fastest) mode of descent warms the troposphere and hence reduces CAPE, but has little influence on CIN. The changes in CIN are maximized by higher wavenumber modes around 250 km ahead of the forcing, which project more strongly onto the region of negative parcel buoyancy. As the forcing advances, the CIN drops as a response to low level ascent due to the wavenumber-3 and -4 modes, and the CAPE climbs—a situation conducive to triggering at the gust front. This kind of behavior, in which a higher-order mode of response may favor convection in the vicinity of "gregarious" thermal forcing, has been described by Mapes (1993).

Even closer to the forcing, the response in vertical motion (Fig. 5a) seems to be confined to the lower part of the domain, with ascent below around 5 km. The modal Taylor–Goldstein solutions in this regime of speeds close to the jet maximum exhibit a close spectrum of eigenmodes, which appear to be differentiated principally by their structure near the jet maximum, where $|c - \pi|$ is small and the Scorer parameter becomes large. As a result of this, the forced response, involving a combination of such modes, is difficult to interpret in these modal terms.

The CAPE changes induced by the storm flows, of the order of 40% of the magnitude of typical CAPE values, would have a similar percentage influence on an upstream bulk Richardson number.

- In addition to the direct responses of CAPE and CIN to the thermal changes in the troposphere, it is reasonable to expect some indirect changes in the CAPE and CIN as $\theta_e$ in the boundary layer evolves over a diurnal cycle. Ahead of the storm, CIN increases (as in Fig. 8): this will tend to suppress convection, reducing both the downdraught and the entrainment fluxes of low $\theta_e$ air into the boundary layer from the free troposphere, and thereby allowing CAPE to accumulate. A simulation with surface heating and a growing convective boundary layer (not shown) did indeed yield a shallower and warmer boundary layer (suggesting enhanced CAPE) in the region of higher CIN. However, an accurate treatment of this behaviour would require modelling of moist processes (particularly surface fluxes, entrainment and shallow convection) and radiative flux convergence (e.g., Parsons et al. 2000), and is not attempted here. It is hoped that the details of such interactions may be illuminated by future cloud modelling studies.

In addition to these influences on the flow and thermodynamics in the squall line’s upstream environment, there are important effects on the potential vorticity of the environment. As discussed previously, the storm propagation means that the influence on the upstream flow is through wave propagation, and is thereby a "reversible" process. The "irreversible" change in the flow is through potential vorticity (PV) modification and this is principally restricted to the air that has been materially modified by moving through the region of diabatic sources. As discussed by Hertenstein and Schubert (1991), the PV response to a propagating squall line is generally composed of a negative anomaly of PV aloft and a positive anomaly at lower levels (in contrast to a region of stationary heating for which the air ascending through the heating region successively gains and then loses PV, leading to a single positive anomaly at mid-levels). This PV dipole can be seen in the results of the two dimensional simulation (Fig. 5e): the structure is modified by the basic state wind variation with height, so that the PV anomalies are extended horizontally in regions where the front-to-rear flow relative to the squall line is greatest. Near the lower levels of the heating, where the PV source is greatest, the forcing is propagating at a speed close to that of the jet and the maximum positive PV anomaly is advected with the forcing.

Inspection of the along-line wind component, $v$, in the model results (Fig. 5f) confirms that the PV anomalies seen in Fig. 5e do correspond to appropriate anomalies of vertical vorticity, despite the fact that the PV has its origin in a thermal tendency: this suggests that some degree of thermal wind balance is achieved over the 5 h of the model run. The presence of such along-line winds is observed in west African squall lines such as the COPT81 case (Chalon et al. 1988), and their role in the full evolution of squall line systems must be addressed. Questions such as the importance of these meridional winds in forcing of African easterly waves, and in the full three dimensional PV budget, are currently being explored.

5. Summary

These results, based on a two-dimensional view of storm flows, have given an indication of the significance of storm propagation and nontrivial wind structures on
the evolution of storm systems. Linear results have shown that propagation of the source intensifies the inflow response, while basic-state flow curvature has a relatively weak influence on the principal modes of response. In a two-dimensional nonlinear model, realistic forcings give rise to feedbacks, which amount to factor 2 changes in a bulk Richardson number estimate for the storm inflow characteristics.

The importance of these processes is in assessing the feedback of the evolving storm flow on the storm itself. However, in considering the upscale influence—the influence of the storm diabatic heating on synoptic flows and the climatological basic state—a three-dimensional perspective would be needed. By definition, two-dimensional models cannot accommodate along-line gradients of potential vorticity, so the along-line winds demonstrated in Fig. 5f cannot advect potential vorticity, only potential temperature. Thus, such models do not demonstrate in Fig. 5f cannot advect potential vorticity, only potential temperature. Thus, such models do not describe the barotropic instability mechanisms that are central to African easterly wave generation, for example. This kind of three-dimensional behavior is the subject of parallel work currently being undertaken.

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