A method for improving the response of recording instruments

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In the course of an investigation into the characteristics of meteorological measuring systems, I have encountered a simple method of improving the response of certain types of instruments. The method has been used in the past with hot-wire anemometers in wind-tunnel measurements, but apparently has not been applied to general meteorological observations.

The object of measurement of the time variation of a meteorological quantity is to obtain a record which is an exact reproduction of the actual variation of such a quantity. Any measuring system is limited by, among other things, the time required to respond to a change in the input and the manner in which the response takes place. These can be called the "resolution" and "phase distortion" of the system. As in the case of communication theory, the merits of a system can perhaps best be understood by the use of Fourier integral analysis, whereby nonperiodic changes in the measured quantity can be resolved into harmonic oscillations of various frequencies, amplitudes and phases. The merits of a measuring system can then be determined from its frequency response and the phase shift which it introduces. It can be shown that a perfect system would be one which amplified or transmitted to the recording pen all frequencies from zero to infinity without attenuation or discrimination, and in which any phase shift was linearly related to frequency.

As an example of a relatively poor system we may take a temperature element such as a thermocouple, having a time constant $\lambda$. Its response $A$ at any circular frequency $\omega$ is given by:

$$A = \left(1 + \lambda^2 \omega^2\right)^{-1},$$
which shows increasing attenuation (or decreasing transmission) as the frequency increases. The phase is also shifted (retarded) in a nonlinear manner, the phase angle $\phi$ being

$$\phi = \arctan \lambda \omega.$$  

As a result of this attenuation and phase shift, the response of the thermocouple to a sudden change in its environmental temperature is the familiar exponential rise, approaching the true value asymptotically. Consequently the only time such a system indicates the true temperature is when the true temperature happens to "cross" the indicated value.

To make such a system approach the "perfect" one mentioned above, it is possible to insert somewhere in the system a compensating network having characteristics which are the inverse of those of the temperature element. Such a network would then "boost" the higher frequencies and would advance the phase to the extent that the thermocouple retarded it. A parallel capacitor-resistor network has the desired characteristics, its frequency response (or admittance) and its phase shift being

$$A = R^{-1}(1 + R^2C\omega^2)^{1/2},$$

$$\phi = -\arctan RC\omega.$$  

When such a network is actually inserted in a circuit its characteristics cannot be fully realized due to the other circuit elements present. However a close approximation to perfect compensation can be obtained with little difficulty, resulting in considerable improvement in the response characteristics of the system. The accompanying illustration shows the results of the use of such a network placed in the cathode lead of a direct-coupled amplifier. The sudden changes in temperature were produced by quickly transferring the thermocouple from one water bath to another of different temperature. Trace A shows the normal exponential characteristics of the uncompensated thermocouple. It will be noted that some 50 seconds are required for the system to rise to a close approximation to the true value (shown by the dashed lines and arrows). In trace B is shown the response of the same thermocouple with the use of the compensating network. In this case the system rises to its final true value in about 1 second, and the exponential characteristic has been eliminated. Trace C shows the response of the uncompensated thermocouple to square waves of 10 and 5 seconds duration at each temperature value, and trace D shows the response to the same waves with partial compensation. It will be observed that the compensated system gives true values of the temperature in the course of each oscillation.

In practice it is generally desirable to choose a temperature element with as low a time constant as possible, since this will reduce the amount of compensation necessary. Also, in the case of a temperature element exposed to the atmosphere, the time constant will change with the rate of ventilation and with the density of the air. In the case of aircraft measurements the compensating network can be adjusted in accordance with air speed and altitude. In measurements at the ground where the ventilation will vary with turbulent flow, the network can be set at its optimum value for the highest rate of ventilation expected. This will result in somewhat imperfect compensation at lower ventilation rates (as in trace D), but of course the response will be much improved over an uncompensated system. A more constant ventilation rate can also be supplied artificially. In addition, attention is being given to automatic adjustment of the network in accordance with the ventilation rate. It will be necessary, of course, to measure the time constant in order to supply the proper degree of compensation.

Similar compensation can be used in systems for the measurement of wind speed with pressure-plate or briddled anemometers, and in other types of measurements. Sensing elements involving mass will have characteristics somewhat different from those of a temperature element, and the accompanying network will require the addition of an inductive element.

It should be emphasized that the above compensation will hold true only in the case of "linear" systems, and that if nonlinear elements such as thermistors or pressure-operated anemometers are used it will be necessary to insert a linearizing network in the system.

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