THE GEOMETRY OF SMOKE SCREENS

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ABSTRACT

Sutton’s expressions for diffusion in the lower atmosphere are applied to the prediction of size and shape of areas screened by smoke generators. The equation for gas concentration is integrated with respect to height from ground level and the integral set equal to the minimum smoke density per unit ground area required for screening. The result is an equation for the coordinates of the screened area. The procedure is extended to lines of generators with overlapping smoke clouds. The results are illustrated by numerical examples.

1. Introduction

One of the important technological achievements of the war was the development of large generators for the production of tactical smoke screens. Prior to the outbreak of the war there had been developed small smoke generators with a capacity of some 6 to 10 gal hr⁻¹ of oil and producing a slightly yellowish grey smoke. When used in large numbers these were capable of screening several buildings so as to make them invisible from the air. The new large generators developed during the war, such as the U. S. Army M-1 mechanical smoke generator had capacities of approximately 100 gal hr⁻¹. These produced an opaque white smoke, and when used in batteries of a dozen or more were capable of screening several square miles.

The effective use of smoke generators depends on knowledge of the behavior of the smoke clouds produced, and the effect on the spread of the cloud of such factors as wind speed, generator capacity, air turbulence, and character of the smoke. Except for the last variable, the problem is one of applying the existing theory and data from the field of micrometeorology. It involves the same basic concepts and variables as the problem of predicting the behavior of toxic gas clouds, or the spread of smoke from chimneys. Toxic gas clouds have been studied intensively over a period of years, especially by the U. S. Chemical Corps, and by the Chemical Defense Experimental Station at Porton, in England. The work at Porton has been summarized in two recent articles by Sutton [2; 3], which provide a basis for estimating the behavior of screening smokes. The present article illustrates how Sutton’s work may be used to estimate the effect of the several principal variables on the shape and size of the area screened from aerial observation by a smoke cloud.

Material is transferred across a turbulent air stream by mechanical mixing, which is very much more rapid than molecular diffusion. Sutton reviews the theories of turbulent or eddy diffusion in the lower atmosphere, and shows how both the “statistical” and the “mixing-length” theories lead to the use of a diffusion equation based on the principle that the rate of diffusion is proportional to the concentration gradient. The theory is developed by Sutton, and will not be described. The starting point for our present purpose is Sutton’s equation for the case of a continuous point source:

\[ c = \frac{2Q}{\pi C_v C_s \bar{u} x^{2-n}} \exp \left\{ -\frac{1}{x^{2-n}} \left( \frac{x^2}{C_v^2} + \frac{z^2}{C_s^2} \right) \right\}, \tag{1} \]

where \( x, y, \) and \( z \) are the distances from the point source downwind, crosswind, and vertical; \( c \) is the concentration, g m⁻³; \( \bar{u} \) is the mean wind velocity, m sec⁻¹; \( Q \) is the source strength, g sec⁻¹; and \( C_v \) and \( C_s \) are the diffusion coefficients in the crosswind and vertical directions. The exponent \( n \) is that appearing in the velocity-profile relation expressed as a power function of the vertical height:

\[ \bar{u} = \bar{u}_1 s^m, \tag{2} \]

Sutton expresses \( C_v \) and \( C_s \) as functions of the deviating velocities, or turbulence components, and of the exponent \( n \), and since \( n \) varies with the degree of turbulence near the ground, it is evident that variations in atmospheric turbulence conditions are allowed for in equation (1) by the variables \( n, C_v, \) and \( C_s \). Sutton’s equation is for a point source in space; the factor 2 is to allow for the plane of the earth.

2. Application to smoke screens

It has long been known that smokes become opaque when the weight of suspended matter per unit of projected area reaches or exceeds some minimum value. This minimum density for screening will be represented by \( N_0 \), g m⁻³. Its numerical value varies with the nature of the smoke, and has been shown to be a function of the particle size [1]. For oil smokes
Fig. 1. Patterns of screened areas for single large smoke generators. Small temperature gradient; \( n = \frac{1}{2}; C_y = 0.15. \)

\( N_0 \) is a minimum when the particles are uniformly in the range 0.4 to 1.0 microns diameter. \( N_0 \) also varies with the manner in which the smoke screen is illuminated: \( N_0 \) is small if the sun shines down on the white cloud in such a way that the reflected light "blinds" the observer above, and \( N_0 \) is large if the object below the cloud is well illuminated.

Equation (1) gives the concentration per unit volume but the quantity of importance in the case of a smoke screen is the density, or weight per unit of horizontal area. The variable density \( N \) can be obtained by evaluating the definite integral \( (C_y \text{ and } C_z \text{ assumed constant}) \) of concentration with height, as follows:

\[
N = \int_0^\infty c \, dz = \frac{Q}{\sqrt{\pi} \, C_y \, \sqrt{2}} \exp \left( - \frac{y^2}{x^2 \, C_y} \right),
\]

or, solving for \( y \),

\[
y = x^{(3-n)/2} \, C_y \left( \ln \frac{Q}{\sqrt{\pi} \, C_y \, \sqrt{2} \, N_0 x^{(3-n)/2}} \right)^{1/2}.
\]

By the substitution of \( N_0 \) for \( N \), Equation (4) represents the locus of the boundary of a pencil- or cigar-shaped area within which the density is \( N_0 \) or greater, i.e., the boundary of the area screened by a single point source. It is of interest to investigate the way in which the principal variables affect the shape and size of this screened area.

Several numerical cases have been computed using values of \( n \) and \( C_y \) suggested by Sutton. The exponent \( n \) is given as \( \frac{1}{2} \) for conditions of a large lapse rate, \( \frac{3}{8} \) for "average" conditions of zero or small temperature gradient, \( \frac{1}{2} \) for a moderate inversion, and \( \frac{3}{4} \) for a large inversion. \( C_y \) varies with height above ground, being given as \( 0.21 \) for a source at ground level, decreasing to \( 0.07 \) for a source at 100 m, the former value being for conditions of small lapse rate with "gustiness appropriate to winds over undulating downland and \( \bar{u} = 5 \text{ m sec}^{-1} \)" (\( C_y \) has dimensions \( L^{1/8} \)). Sutton indicates the nature of the variation of \( C_y \) with meteorological conditions; for purposes of applying the results to smoke \( C_s \) has been assumed to vary in the same way. Certainly the data provided by Sutton can be considered to be only fragmentary, and inadequate as a basis for reliable calculation of what smoke should do; they are sufficient, however, to suggest that the results to be described may be roughly quantitative.

Fig. 1 shows three examples of screen patterns calculated from equation (4) for conditions of zero or small temperature gradient, using \( n = \frac{1}{2} \) and \( C_y = 0.15 \). Here \( B = Q/\bar{u}N_0 \) in meters. For the 100 gal hr\(^{-1}\) oil smoke generators \( Q \) is about 84 g sec\(^{-1}\) and \( N_0 \) for aerial screening is of the order of 0.33 g m\(^{-3}\) (approximately 250 U. S. gal mi\(^{-2}\)). With these values \( B = 50 \text{ m at 5 m sec}^{-1} \text{ wind speed} \).

It should be noted that the ordinate and abscissa scales of fig. 1 are not the same, and that the true shape of the screen is greatly elongated. The downwind screened length increases faster than \( B \), and the screened area increases faster than \( B^2 \). This explains why a 100 gal hr\(^{-1}\) generator is so much more impressive than one putting out only 25 gal hr\(^{-1}\).

The general shape of the screen may be obtained by the use of simple equations for the length and width. For \( y = 0 \), the length at the center line is obtained from equation (3) as

\[
x_{\text{max}} = \left( \frac{Q}{\sqrt{\pi} \, C_y \, \bar{u} \, N_0} \right)^{3/(3-n)}. \tag{5}
\]

Differentiating equation (4) and setting \( dy/dx = 0 \), the maximum half-width is obtained as

\[
y_{\text{max}} = 0.243(Q/\bar{u}N_0) = 0.243B, \tag{6}
\]

while

\[
x \text{ (at max y) } = (0.606)x_{\text{max}}. \tag{7}
\]

Fig. 2 indicates the variation in screened pattern with meteorological conditions, from moderate inversion to large lapse rate. Although the width of the
screen is not affected, its length varies as much as for a several-fold variation in $B$. War experience confirms the very long screened lengths obtained under inversion conditions.

3. Smoke generators placed in cross-wind line

When several generators are placed along a crosswind line, the smoke from one generator reinforces that from others and the screened area is greatly enlarged. If smoke issued uniformly along a line of length $y$, then $Q/\bar{u}N_s y$ would have to be greater than unity to obtain any screening. This suggests that the spacing $y$, between generators might be as large as $B$. Although this is approximately true, the downwind distance to the point where screens from two generators merge may be too great to be practical. Because of the unscreened V-shaped regions between generators, the line must be placed a considerable distance upwind, which means an excessive time interval between starting the generators and full development of the screen, particularly at low wind speeds. Because of this, generators must be placed closer together than required for economical screen development.

The smoke density obtained at the midpoint between two generators resulting from the overlapping of the two screens is given by

$$N = \frac{2Q}{\sqrt{\pi} \bar{u} C_v x^{(3-n)/2}} \exp \left( -\frac{2y^2}{4x^n C_v^2} \right).$$

(8)

If $N$ is set equal to $N_0$, the downwind length of the unscreened "V" may be calculated. Fig. 3 shows the result of such calculations for three values of $B$ with $n = \frac{3}{4}$ and $C_v = 0.15$.

When several generators are placed on a cross-wind line, the base of the screen is widened, but its length is not greatly different than if the same total smoke

![Fig. 3. Length of unscreened "V" between adjacent smoke generators on a cross-wind line.](image)

issued from a point source. With three generators spaced 100 m apart, and $n = \frac{3}{4}$, the outer boundary of the screen is given by

$$N_0 = \frac{Q}{\sqrt{\pi} \bar{u} x^{0.875} C_v^2} \left[ \exp \left( -\frac{y^2}{C_v^2 x^{1.75}} \right) \right. $$

$$+ \exp \left( -\frac{(y + 100)^2}{C_v^2 x^{1.75}} \right)$$

$$+ \exp \left( -\frac{(y - 100)^2}{C_v^2 x^{1.75}} \right) \right] .$$

(9)

where the origin is placed at the middle generator. Values of $y$ and $x$ calculated from this relation for $B = 200$ are shown on fig. 4 by the solid line. For comparison, the dotted curves illustrate the screens produced by single generators, and the dashed curve the area screened by one triple-size generator ($B = 600$). The principal effect of using three generators instead of one very large one is the widening of the base of the screen.

The maximum downwind screened length obtained at $y = 0$ from equations similar to equation (9) is shown in fig. 5 to increase with the number of generators in line, especially when $B$ is several times the spacing.

4. Use of smoke in micrometeorology

If data on turbulent diffusion are applicable to the prediction of screening smoke behavior, it would seem obvious that the reverse might be true, and that smoke screens may prove helpful in experimental studies of turbulence in the lower atmosphere. If $N_0$ is known, photographs of smoke plumes from very small generators would give $x_{\text{max}}$, from which $C_v$ could be obtained from equation (5). Such photographs would have to

![Fig. 4. Screened pattern obtained with three smoke generators on a cross-wind line compared with pattern for a single triple-size generator.](image)
be statistical, taken from above using multiple exposures and a very small lens aperture. \( N_0 \) for the experimental smoke might be obtained by letting it issue from closely-spaced ports in a pipe 5 to 20 m long placed cross-wind. By adjusting the total smoke rate until screening was just obtained immediately downwind from the center of the pipe, \( N_0 \) could be calculated from the measured value of source strength per meter of pipe length and of the mean wind speed.

Screening in this case would also be judged by multiple-exposure photographs.

**Table of Symbols**

\[
\begin{align*}
B &= Q / \bar{u} N_0, \text{ m} \\
\bar{c} &= \text{concentration, g m}^{-2} \\
C_v &= \text{diffusion coefficient in cross-wind direction, m}^{1/3} \\
C_x &= \text{diffusion coefficient in vertical direction, m}^{1/3} \\
N &= \text{smoke density, g m}^{-2} \\
N_0 &= \text{minimum smoke density for screening, g m}^{-2} \\
Q &= \text{source strength, g sec}^{-1} \\
\bar{u} &= \text{mean wind speed, m sec}^{-1} \\
x &= \text{downwind distance from source, m} \\
x_{\text{max}} &= \text{maximum downwind length of screened area, m} \\
y &= \text{crosswind distance, m} \\
y_{\text{max}} &= \text{maximum half-width of screened area, m} \\
z &= \text{spacing between generators, m} \\
z &= \text{elevation above ground, m}.
\end{align*}
\]

**References**