Wave Propagation inside an Inertia Wave. Part II: Wave Breaking

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ABSTRACT

By launching monochromatic gravity wave packets of different frequencies, wavenumbers, and amplitudes below a localized inertia “background” wave, several assumptions and issues related to gravity wave dissipation and parameterizations are investigated: the influence of the time dependence of the background wave on wave breaking; the dependence of wave breaking on the initial wave packet frequency, vertical wavenumber, and amplitude; and the existence of a high-vertical-wavenumber cutoff beyond which all wave packets are dissipated into turbulence. An intermediate model is used that is two-dimensional, linear, and that models wave breaking with a mixed shear and convective adjustment scheme. Wave breaking is found to be reduced by the time dependence of the background wave, which has negative phase velocity, when the gravity waves are initially convectively stable. Large-scale dissipation, that is, dissipation associated with layers deeper than 400 m, occurs for gravity waves of initial vertical wavenumbers close to or smaller than $2\pi/5$ rad km$^{-1}$. Large-scale dissipation is not enhanced by the time dependence of the background wave. However, large-scale dissipation might be enhanced by the time dependence of the background wave if the initial amplitudes of the gravity waves are larger, for example, if the gravity waves are initially convectively unstable.

1. Introduction

Gravity wave parameterizations that are commonly used in global circulation models rely on the following mechanisms of wave dissipation: wave breaking or nonlinear wave–wave interactions. All the parameterizations assume a scale separation between the background quantities and the wave field. Parameterizations can be classified into three groups, depending on the underlying mechanism/theory responsible for wave dissipation: linear saturation theory (wave breaking), diffusive theory (diffusive nonlinearities), and Doppler-spreading theory (advective nonlinearities). In Doppler-spreading theory (Hines 1997a, b), a wave is spread under the effects of advection by horizontal background winds, which “contain” all the other waves of the spectrum. In other words, all the other waves of the spectrum make a contribution to the background winds. However, the background winds involved in Doppler spreading are assumed to be time independent. Furthermore, wave breaking is ignored or assumed to occur for only high-vertical-wavenumber waves.

To study the different hypotheses made in parameterizations, such as the importance of the time dependence of the background winds, ray tracing models are often used (e.g., Eckermann 1997). However, ray theory is based on the hypothesis that there is a scale separation between gravity waves and background winds. Part I of this paper (Sartelet 2003b, hereafter S22b) shows that ray theory performs remarkably well even when the scale separation hypothesis (SSH) is not far off the conditions for Bragg reflection (elastic scattering) where ray theory finally breaks down.

However, it is difficult to study the importance of wave breaking on wave dissipation using ray theory. Wave amplitudes are difficult to compute in the vicinity of caustics using ray theory, especially for time varying backgrounds. Using a 2D numerical model, S22b shows that wave breaking occurs at caustics, that is, where rays cross, as would be expected from the large wave amplitudes predicted by ray theory at caustics. Using ray tracing (Sonmor and Klaassen 2000, hereafter SK20) show that caustics are more frequent when the background wave is time varying than when it is time independent. When the background wave is time independent, wave breaking occurs at caustics that are associated with critical layers for arbitrary initial amplitude of the short wave. When the background wave is time varying, the occurrence of wave breaking at caustics depends on the initial amplitude of the short wave; and for sufficiently small initial amplitude, wave breaking does not occur. Therefore, it is not clear a priori whether wave breaking is more frequent when the background wave is time varying than when it is time independent. When the background wave is time independent and for caustics that are associated with critical
levels, the vertical wavenumbers \( m \) of the waves become very high and the waves break (at caustics or in their vicinities). Because of this high \( m \) wave breaking at critical caustics, some parameterizations, such as the Doppler-spreading parameterization, assume the existence of a high-vertical-wavenumber cutoff after which all waves are dissipated into turbulence. However, when background winds are time varying, waves may escape critical caustics (S22b, and references within), and caustics may not be associated with critical levels at all. At these “noncritical” caustics, the vertical wavenumber of a wave may decrease (Broutman and Young 1986; Bruhwiler and Kaper 1995). Whether a wave breaks or not at a noncritical caustic depends on its initial amplitude (SK20). Therefore, if wave breaking occurs, it may involve waves of low vertical wavenumbers, that is, large-scale dissipation, in contradiction with the existence of a high-vertical-wavenumber cutoff.

To investigate the role of the time dependence of background winds on wave breaking, the numerical studies of S22b are analyzed in more details. S22b considers the propagation of “short” gravity waves of vertical wavenumber \( m \) inside an inertia “background” wave of vertical wavenumber \( M \). The importance of the time dependence of the background wave on shortwave propagation is stressed and quantified.

In this paper, section 2 recalls the setting of the numerical studies. Section 3 investigates the frequency of occurrence of wave breaking as a function of wavenumbers and frequency of the short wave, and section 4 investigates the importance of large-scale dissipation. Finally in section 5 the difference in terms of wave breaking between initially convectively stable and unstable waves, as defined in section 2, is stressed.

2. Previous numerical studies

To investigate the role of wave breaking on gravity wave propagation, the simulations of S22b are analyzed in more detail. The numerical model, the so-called intermediate model, used is described in the companion paper (S22b). The horizontal domain, discretized with 16 points, is periodic. In the vertical, which is discretized with 1024 points, waves escape the domain at the upper boundary, while the lower boundary is rigid. The intermediate model is two-dimensional, linear, and it avoids the criticized SSH. It represents gravity wave breaking with a mixed shear and convective adjustment (MSCA) scheme, which models breaking via convective and Kelvin–Helmholtz instabilities by locating, with a Richardson number criterion, the layer(s) where the wave field is convective and Kelvin–Helmholtz unstable. In the unstable layer(s), momentum and potential temperature are first assumed to be perfectly mixed. Second, a relaxation scheme is applied, in which potential temperature and momentum are relaxed toward the notional perfectly mixed profiles with a relaxation timescale \( \tau = 300 \) s (which corresponds to about a buoyancy period).

The intermediate model is computationally inexpensive compared to 3D direct numerical simulations. Furthermore, it is successfully tested against the 3D direct numerical simulations (DNS) of wave breaking of Andreassen et al. (1994), Fritts et al. (1994), and Lelong and Dunkerton (1998) in Sartelet (2002a), where it is found that the intermediate model that uses the MSCA scheme models wave breaking via convective instabilities more realistically than 2D DNS in comparison to 3D DNS. For example, the MSCA scheme allows wave overturning to be reduced more efficiently than in 2D DNS, where a strong and unrealistic wave overturning is observed. Furthermore, the MSCA scheme models realistically the occurrence of Kelvin–Helmholtz instabilities for low-frequency waves, even without the simultaneous occurrence of convective instabilities.

S22b studies and quantifies the role of the time dependence of an inertia background wave for the propagation of so-called short gravity waves of different wavenumbers and frequencies. The background wave is described by its horizontal velocity

\[
\pi(z, t) = \pi_0 e^{-(z-z_0)^2/(2L^2)} \cos(Mz - ft),
\]

where \( M \) is the vertical wavenumber; \( L \) determines the dimension of the envelope of the background wave; \( f \) is the Coriolis parameter, \( z_0 \) and \( \pi_0 \) are the center and the amplitude of the background wave, respectively; and \( N \) is the buoyancy frequency. In this paper, \( M \) is negative and the background wave has negative phase velocity. At the initialization, the horizontal velocity of the short wave varies as

\[
u(x, z) = e^{-i(z-z_0)^2/2\sigma^2} \text{Re}[u_0 e^{ikx+mo}],
\]

where \( u_0 \) is the amplitude of the short wave; \( k \) and \( m \) are the horizontal and the vertical wavenumbers, respectively; \( l \) determines the dimension of the envelope of the short wave; and \( z_0 \) is the center of the short wave or launch altitude (\( z_0 < z_0 \)). In all the simulations performed in this paper, the short wave propagates upward, that is, its phase velocity is negative while its group velocity is positive. The short wave is initially convectively stable and its initial amplitude is 0.8 times the threshold \( u_c \) for convective instabilities to occur according to linear saturation theory. Parameters typical of the stratosphere are used (\( N/f = 200 \)). Physical parameters are summarized in Table 1.

When the background wave is artificially made time
independent, there exists a vertical wavenumber cutoff \( m_c = N/\pi u \) and a horizontal wavenumber cutoff \( k_c = M \), between short waves that are dissipated by the interaction with the background wave, and short waves that are not. The wavenumber cutoffs no longer exist when the time dependence of the background wave is taken into account. Transmission and back-reflection of the short waves are quantified in S22b by defining transmitted \( A' \) and back-reflected \( A'' \) wave activity coefficients. The closer to 1 (0) the sum \( A' + A'' \) is, the less (more) dissipation the short wave undergoes.

3. Frequency of occurrence of layers of all depths

The MSCA scheme, which models wave breaking in the intermediate model, computes the vertical depths of the layers that are involved in wave breaking. The frequency of occurrence of layers of various depths is defined as the number of times the various depths are involved in wave breaking during a simulation. Simulations last until the short wave is completely dissipated, that is, until the transmitted wave activity is smaller than 0.02 (see S22b), or until it escapes the background wave (i.e. until it propagates past 40 km). The frequency of occurrence of layers of all depths (FOLD) is obtained by summing over depth the frequency of occurrence of layers of various depths. The FOLD not only measures the frequency of occurrence of wave breaking and short-wave dissipation. The FOLD also measures wave-breaking efficiency. On the one hand, the lower the FOLD is, the less frequent wave breaking is and the less dissipation by wave breaking the short wave undergoes. On the other hand, the lower the FOLD is, the quicker and the more efficient wave breaking is. The FOLD (divided by a factor of 1000) is given by the first number inside the brackets on Figs. 1a and 1b for short waves of different frequencies and wavenumbers. The parameters of the background wave are identical to those of Figs. 4a and 4c of S22b, respectively. The vertical wavenumber of the background wave is \( M = 2 \pi/5 \) rad km\(^{-1} = m_c/2 \). The background wave is time independent in Fig. 1a, and it is time varying (with negative phase velocity) in Fig. 1b. The variation of the FOLD with wavenumber and frequency is first studied when the background wave is time independent. Then the influence of the time dependence of the background wave on the FOLD is investigated.

a. Time-independent background wave

When the background wave is time independent, the largest FOLD is obtained for short waves of vertical wavenumbers \( m = 2 \pi/2.5 \) rad km\(^{-1} = m_c \), the vertical wavenumber cutoff. The FOLD tends to decrease as \( m \) decreases or increases away from \( m = m_c \). As \( m \) decreases away from \( m_c \), the FOLD tends to decrease, because low \( m \) waves do not encounter critical levels, and they do not dissipate much. As seen in S22b, when \( m \) is low, wave dissipation occurs mostly at \( m = M/2 \) because of elastic scattering. As \( m \) increases away from \( m_c \), the FOLD decreases, because high \( m \) waves encounter critical levels and the higher \( m \) is, the more efficient or the quicker wave breaking is at dissipating the waves (SK20). Furthermore, the higher \( m \) is, the more efficiently numerical diffusivity is at dissipating the short wave as it approaches the critical level, that is, as \( m \) increases.

For frequencies \( \omega > 4f \), the FOLD decreases as \( \omega \) increases or equivalently as \( k \) increases. This decrease seems at first sight to contradict the results of S22b where the sum \( A' + A'' \) decrease as \( k \) increases. From the decrease in \( A' + A'' \), one would expect the FOLD to increase in parallel with the increase in short-wave dissipation. However the FOLD decreases as \( \omega \) increases, because the lower the frequency of the short wave is, the longer the short wave takes to reach its critical level and to be dissipated. For frequencies \( \omega < 4f \), the FOLD increases as \( \omega \) increases or equivalently as \( k \) increases, partly because of the larger importance of numerical diffusivity.

b. Time-varying versus time-independent background wave

When the background wave is time independent, the vertical wavenumber cutoff \( m = m_c \) is apparent from the FOLD. The FOLD maximum is at \( m = m_c \) and it decreases as \( m \) decreases or increases away from \( m = m_c \). When the background wave is time varying, the FOLD maximum is still located at \( m = m_c \), although the vertical wavenumber cutoff is removed by the time variations of the background wave. Similar to the time-independent case, the FOLD is maximum at \( \omega = 4f \); that is, it decreases as \( \omega \) increases when \( \omega > 4f \) and it increases as \( \omega \) increases when \( \omega < 4f \).

For all shortwave parameters, the FOLD is higher when the background wave is time independent rather than time varying. There are two reasons for this. First, short-wave propagation is enhanced and therefore wave dissipation is reduced by the time variations of background waves with negative phase velocity (S22b). Second, short waves may not break at noncritical caustics when the background wave is time varying, although the short waves do break at critical caustics when the background wave is time independent. In this section, the initial amplitudes of the short waves are below the threshold for convective instabilities \((u_0 = 0.8u_s)\) to occur according to linear saturation theory. The influence of the initial amplitude of the short wave on wave breaking is further discussed in section 5.

At low \( \omega \) and high \( m \), the phase velocity of the background wave is much higher than the group velocity of the short wave (it is the slow limit discussed in S22b). The background wave appears almost as independent of the vertical to the short wave, and the time dependence of the background wave strongly influences the short-
wave propagation. The FOLD is high when the background wave is time independent, because the short wave is dissipated at critical level, that is, critical caustic. On the opposite, the FOLD is low, or even zero, when the background wave is time varying, because the time dependence of the background wave allows the short wave to escape its critical level. Furthermore, the initial amplitude of the short wave is too small for the short wave to break at noncritical caustics.

As \( \omega \) increases, the differences between the FOLD observed when the background wave is time independent and time varying diminish. At high \( \omega \) and low \( m \), the phase velocity of the background wave is much lower than the group velocity of the short wave (it is the fast limit discussed in S22b). The background wave appears almost as time independent to the short wave and the FOLD is very similar when the background wave is time independent and time varying.

Although the FOLD decreases toward low \( m \), high values are observed at \( m = m_/4 \), especially when the background wave is time independent. These high values are a consequence of elastic scattering and the resulting wave breaking. Elastic scattering generates back-reflected waves and causes dissipation. Furthermore, short-wave propagation is enhanced by the time variations of the background wave. In other words, wave dissipation is reduced by the time variations of the background wave. As a consequence, the FOLD is higher when the background wave is time independent than when it is time varying and the back-reflected wave activity coefficient \( A^c \) is smaller.

4. Layer depths involved in wave breaking

a. Order of magnitude

Turbulent layers due to Kelvin–Helmholtz instabilities are typically of the order of tens of meters to hundreds of meters in the stratosphere (e.g., Sato and Woodman 1982). Because convective instabilities involve higher-frequency waves than Kelvin–Helmholtz instabilities, layers associated with convective instabilities are deeper. According to Fritts and Rastogi (1985), typical depths would be about 1–3 km.

The depths of the layers involved in wave breaking are measured by the frequency of occurrence of layers of various depths, which is computed by the MSCA scheme. The MSCA scheme models both Kelvin–Helmholtz and convective instabilities. The order of magnitude of the layer depths computed by the MSCA scheme corresponds well with the observations. Figure 1 indicates the average and maximum layer depths for short waves of different wavenumbers and frequencies (second and third numbers inside the brackets, respectively). The background wave, of vertical wavenumber \( M = m_/2 \), is time independent in Fig. 1a and time varying with negative phase velocity in Fig. 1b. The layer depths range between 83 m, that is, twice the vertical step, and 2531 m.

b. Dependence on the short-wave parameters

Both when the background wave is time independent and time varying, the average and the maximum layer depths increase as the vertical wavenumber \( m \) of the short wave decreases, that is, as its vertical wavelength increases. Furthermore, the maximum layer depth increases as the frequency of the short wave increases.

c. Dependence on the time variations of the background wave

When \( m = m_/4 \), both the average and the maximum layer depths are deeper when the background wave is time varying rather than time independent. When \( m = m_/4 = M/2 \), wave breaking is mostly a consequence of elastic scattering, a phenomenon that occurs independently of whether the time variations of the background wave are taken into account or not. Because wave breaking occurs at noncritical caustics when the background wave is time varying and at critical caustics when the background wave is time independent, the average and the maximum layer depths involved in wave breaking are deeper when the background wave is time varying rather than time independent.

When \( m \neq m_/4 \) and \( \omega \leq 10f \), both the average and the maximum layer depths are deeper when the background wave is time independent rather than time varying. Effectively, the FOLD is low (or zero) when the background wave is time varying.

When \( m \neq m_/4 \) and \( \omega \geq 50f \), the maximum layer depth tends to be independent of whether the background wave is time varying or not, and the average layer depth is deeper when the background wave is time varying rather than time independent. Although the short wave breaks at noncritical caustics when the background wave is time varying, wave breaking is not as frequent as when the background wave is time independent and it involves deeper layers in average.

d. Large-scale dissipation

The Doppler-spreading theory neglects large-scale dissipation. In other words, the Doppler-spreading theory neglects wave breaking that is associated with vertical wavelengths larger than a vertical cutoff. This cutoff is of the order of 50 or 400 m according to Hines (1999). Large-scale dissipation, that is, dissipation associated with vertical wavelengths larger than 400 m does occur for short waves of low vertical wavenumbers, that is, for \( m \leq m_/2 \). Because the short wave does not break in the absence of background wave, this large-scale dissipation occurs because of the interaction with the background wave. When the background wave is time varying, the maximum layer depth is smaller than
Fig. 1. In brackets: frequency of occurrence of layers of all depths divided by 1000, average and maximum layer depth in meters. (a) The background wave is time independent, $M = m_c / 2$. (b) Background wave with downward phase velocity, $M = m_c / 2$.
when the background is time independent, although it may be deeper when \( m = m_c/4 \) as a result of elastic scattering. In this section the initial amplitudes of the short waves are equal to \( u_0 = 0.8u_s \), and wave breaking and large-scale dissipation are not enhanced but reduced by the time dependence of the background wave. However, wave breaking and large-scale dissipation might be enhanced by the time dependence of the background wave, if the short waves are initially convectively unstable (e.g., \( u_0 = 1.2u_s \)) rather than convectively stable (\( u_0 = 0.8u_s \)).

5. Initially convectively stable or unstable waves

This section aims at comparing the behavior of the MSCA scheme and wave breaking when short waves are initially convectively unstable and convectively stable.

In Fig. 2, a simulation of S22b (Fig. 1, \( m = M, \omega = 5.1f \)) is repeated not only for a short wave that is initially convectively unstable (\( u_0 = 0.8u_s \)), but also for a short wave that is initially convectively unstable (\( u_0 = 1.2u_s \)). The comparison of Fig. 2a with Fig. 2c shows that the trajectories of the short waves are not affected by their initial amplitudes. When the short wave is initially convectively unstable, strong wave breaking occurs at the beginning of the simulations (Fig. 2b). Dissipation occurs quickly, such as stabilizing the wave field, and wave breaking occurs at caustics in the rest of the simulations. When the short wave is initially convectively unstable (Fig. 2b), wave breaking reduces the wave amplitude, so that the short wave becomes convectively stable. However, this amplitude might be larger, that is, closer to neutral stability, than the amplitude of the short wave that is initially convectively stable (Fig. 2d). These larger amplitudes explain why wave breaking is observed to be stronger at caustics and to last slightly longer, when the short wave is initially convectively unstable than when it is initially convectively stable.
For reference, the straight line corresponds to a $-3$ slope. In Fig. 3, the spectra obtained with $m = M$ and $\omega = 10 f$. Dash–dot curve: spectrum at initial time; dashed curve: simulation with wave breaking; solid curve: the breaking scheme is not applied. For reference, the straight line corresponds to a $-3$ slope.

### 6. Discussion

#### a. Wave breaking

When the background wave is time independent, wave breaking occurs at caustics that are associated to critical layers independently of the initial amplitude of the short wave. When the background wave is time varying, whether wave breaking occurs or not at caustics depends on the initial amplitude of the short wave. The short waves are initially convectively stable in this paper and the frequency of occurrence of wave breaking is reduced by the time dependence of the background wave, which has negative phase velocity.

In this paper, large-scale dissipation, that is, dissipation associated with layers deeper than 400 m, occurs for short waves of initial vertical wavenumbers close to or smaller than $m/2$. Large-scale dissipation is not enhanced by the time dependence of the background wave. However, large-scale dissipation might be enhanced by the time dependence of the background wave if the initial amplitudes of the short waves are larger, for example, if the short waves are initially convectively unstable. Nevertheless a broad spectrum of short waves should be launched before drawing any final conclusions on the importance of large-scale dissipation in the real atmosphere. As argued by Hines (1997b), because wave breaking is less efficient when a superposition of waves is considered, it might have no opportunity to extract wave activity from large-scale waves in the real atmosphere.

As already pointed out by Broutman et al. (1997), it is interesting to note that a broad spectrum of waves in vertical wavenumbers is obtained by launching only one short wave (e.g., Fig. 3). Furthermore, the spectral slope at vertical wavenumbers $m$ higher than $m_{br} = 2.5 \times 10^{-4}$ cycle m$^{-1}$, is similar to the universally observed minus three slope. In Fig. 3, the spectra obtained with and without applying the breaking scheme are compared. At low $m$, the spectrum is not modified by applying the breaking scheme. Wave breaking only affects vertical wavenumbers higher than $m > m_{br} = 3 \times 10^{-3}$ cycle m$^{-1}$, resulting in a slight broadening of the spectrum for $m > m_{br}$.

#### b. Resonant interactions

Three kinds of resonant interactions are commonly distinguished, namely induced diffusion, parametric subharmonic instabilities, and elastic scattering. In resonant interaction theory, waves are assumed to be of infinitely small amplitudes. The occurrence of partial back-reflection via elastic scattering in S22b and in this paper suggests that resonant interactions might be important, even for waves of finite amplitudes. Using Floquet theory (e.g., Klostermeyer 1991; Sonmor and Klaassen 1997) or 2D nonlinear simulations (Dunkerton 1987), parametric instabilities have proven to be relevant not only for small, but also for finite-amplitude waves. However, the importance of resonant interactions for finite-amplitude waves has been restricted to parametric instabilities in these studies. Elastic scattering and induced diffusion have been ignored partly because Floquet theory does not allow background shear to be taken into account. In the presence of a background shear, that is, of a background wave in the simulations reported in S22b and in this paper, elastic scattering and induced diffusion are observed. They are apparent, respectively, in the mentioned partial back-reflection at $m = M/2$ and in the broad spectrum that results from the interaction with the background wave.

The coincidence of the locations of wave breaking and caustics, for example, where Bragg reflection originates, confirms the suggestion of Klostermeyer (1991) and Sonmor and Klaassen (1997) that convective/Kelvin–Helmholtz instabilities and resonant/wave–wave interactions should not be viewed as different mechanisms for wave instability. Instead, convective/Kelvin–Helmholtz instabilities provide informations about the locations of strong turbulence, that is, of the instabilities, whereas resonant/wave–wave instabilities give details about the dynamic nature of the instabilities.

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**REFERENCES**

Andreassen, O., C. E. Wasberg, D. C. Fritts, and J. R. Isler, 1994: Gravity wave breaking in two and three dimensions. 1. Model...


