Properties of Tropical and Midlatitude Ice Cloud Particle Ensembles. Part II: Applications for Mesoscale and Climate Models

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ABSTRACT

This is the second part of a study that characterizes several bulk properties of ice particle populations sampled in synoptically generated midlatitude and convectively generated tropical ice clouds, for the purpose of developing empirical and analytical relationships that describe microphysical properties for use in mesoscale and climate models. The purpose of this paper is to examine the interrelationships between the mass, area, and fall velocity properties of the particle size distributions, and the dependence of these properties on temperature.

Gamma distributions of the form $N = N_0 D_0 e^{-D/D_0}$ are fitted to the measured particle size distributions (PSDs) over sizes ($D$) from as small as 10 μm to as large as 1.5 cm. Exponential distributions ($\mu = 0$) are also fitted to the PSD. The intercept parameter $N_0$ and the slope $\lambda$ are directly related, and decrease monotonically with increasing temperature. The $\mu$ values for the gamma fits tend from positive values at large $\lambda$ to negative values at small $\lambda$. The maximum measured diameter $D_{max}$ increases with decreasing $\lambda$.

The $N_0$ values from the midlatitude clouds are about an order of magnitude lower than those for the tropical PSDs at the same temperatures.

Bulk properties are derived from the fitted PSDs. The ice water contents (IWC) are about an order of magnitude higher for the tropical than for the midlatitude clouds. The median mass diameter ($D_m$) and the effective diameter ($D_e$) each increase with temperature, and are found to be related to each other.

Several aspects related to the modeling of ice particle sedimentation in general circulation models (GCMs), and the relationship of these velocities to other bulk properties, are investigated. On average, the median mass-weighted terminal velocity ($V_m$) increases weakly with temperature. Correlations between $V_m$ and IWC are also weak. It is found that for a given particle ensemble, most of the ice mass is contained within a relatively narrow range of fall velocities, although the values of $V_m$ can be appreciable. Calculations reveal that the fallout of particles that dominate the extinction cannot be ignored, except at temperatures below $-50^\circ$C. Also, the effective diameter is found to be strongly related to the ensemble mean $V_m$, perhaps allowing the two variables to be linked in GCMs.

1. Introduction

This is the second part of a study that uses particle size distribution data measured in midlatitude and tropical ice clouds to characterize the properties of ice particle ensembles, an area of study that may lead to improved parameterizations for use in mesoscale models and general circulation models (GCMs). A main emphasis in this paper is to characterize the fall velocities of particle ensembles in terms of bulk properties such as the ice water content, whereas Heymsfield 2003 (hereafter, Part I) examined the factors that influence the fall velocities of the particle ensembles themselves.

Almost all GCMs now use some form of cloud microphysical parameterization to determine the vertical distribution of amounts of ice condensate. The microphysical parameterizations are designed to prognose the mixing ratio of the condensate at a given model time step and grid point from a water mass continuity equation. The prognosed condensate mass in some models is divided into ice and liquid fractions based on temperature: the Goddard Institute for Space Science (GISS) model (del Genio et al. 1996), the National Center for Atmospheric Research (NCAR) model (Rasch and Kristjansson 1998), and the European Centre for Medium-Range Weather Forecasts (ECMWF) model (Klein and Jakob 1999). Increasing numbers of models prognose the ice and liquid water ratios directly, by using separate prognostic variables for liquid and ice: the Colorado State University (CSU) model (Fowler et al. 1996) and the Met Office (UKMO) model (Wilson 2000).

There are two main ways that ice condensate is converted to ice precipitation in GCMs. More than one-half of the models use a Kessler-type autoconversion scheme to remove the ice precipitation from the atmosphere. A
portion of the ice condensate is converted to precipitation according to a rate that is directly proportional to the ice water content (IWC; GISS model); in some instances, the autoconversion threshold IWC must be reached (CSU model). All precipitating ice is then removed from the atmosphere, converted into vapor or precipitation. In the second group of GCMs, ice precipitation is allowed to fall between levels and is retained from one time step to the next [ECMWF; UKMO; Geophysical Fluid Dynamics Laboratory (GFDL; Donner et al. 1999)]. The rate of fall is usually determined by the IWC, using the characteristics of the particle size distributions, or from relationships reported in the literature.

Recent mesoscale model and GCM studies have demonstrated the impact that the assumed fall velocities have on predictions of cloudiness and radiative forcing. For example, Manning and Davis (1997) showed that the fifth-generation Pennsylvania State University (PSU)—NCAR Mesoscale Model (MM5) have a cloudy bias and upper-level moist bias that are traced to an inappropriate assumption in the microphysical parameterization; these biases are removed with proper treatment of ice particle sedimentation velocities. Klein and Jakob (1999) compared prognoses of cloudiness from modeling studies to those derived from satellite data and concluded that there needs to be a careful evaluation of the treatment of the ice phase in large-scale models, especially the gravitational settling term. Petch et al. (1997) found in a single-column GCM that large differences in cloud ice water path resulted from differences in the ways that ice fallout is treated. Wu et al. (1999) showed that the modification of ice fall velocities can significantly affect the cloud-resolving model simulation of cloud and radiative properties during the Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE). Heymsfield and Iaquinta (2000) used ECMWF modeling results to show that the earth’s net radiative balance is a strong function of ensemble mean fall velocities. And for those GCMs that require a threshold IWC to be reached before precipitating ice develops, there are many instances at low temperatures when the autoconversion threshold may not be reached. Because all of the particles in this situation have zero fall velocity, particles can only be removed through sublimation, and at low temperatures sublimation rates are usually very low. Because of this effect, these models may depict an area of radiatively active cirrus cloud that is often too large as compared with climatological data (Manning and Davis 1997; L. Donner 2001, personal communication).

This study is concerned primarily with characterizing the properties of midlatitude and tropical ice particle size distributions (PSDs), determining how the PSDs relate to their cloud bulk properties, including the ice water content, the median mass diameter, and the median mass-weighted terminal velocity. This study adds to the results from earlier studies through the use of data from both midlatitude and tropical ice clouds, as well as the use of new instrumentation to provide better habit, size spectra, and IWC information. This article is organized in the following way. Section 2 briefly describes the dataset and the procedures used to fit the PSDs. The results are presented in section 3. Section 4 is a summary.

2. Datasets and instrumentation

Data used in this study are described in detail in Part I. Briefly, the data were acquired from aircraft Lagrangian spiral descents and balloon-borne ascents through ice cloud layers that averaged more than 3 km in depth. All 16 spiral descents commenced at cloud top and ended at cloud base or below the melting layer. With reference to Fig. 1, eight spirals were conducted in synoptically generated cirrus layers in Wisconsin in the fall of 1986 during the First International Satellite Cloud Climatology Project (ISCCP) Research Experiment (FIRE I), with cloud-top temperatures in the $-35^\circ$ to $-40^\circ$C range, and base temperatures of about $-20^\circ$C. Two spirals were conducted in synoptically produced cirrus during the Atmospheric Radiation Measurement (ARM) field campaigns in Oklahoma during March 2000. Cloud-top temperatures were close to $-50^\circ$C. Six spirals were conducted in anvils or stratiform regions associated with convection during the Tropical Rain Measuring Mission (TRMM) campaign near Kwajalein [Kwajalein Experiment (KWAJEX), summer 1999]. Cloud-top temperatures ranged from $-50^\circ$ to $-15^\circ$C and cloud-base temperatures in three cases reached $0^\circ$C. Data were also acquired during three balloon-borne ascents by the NCAR ice crystal replicator through synoptically generated cirrus during FIRE II in Kansas. Cloud-top temperatures ranged between $-55^\circ$ and $-63^\circ$, and base temperatures were around $-30^\circ$C.

As discussed in detail in Part I, size-spectra measurements were obtained over the size range from about 50 $\mu$m to larger than 1 cm with several imaging probes, depending on the field program. There are basic questions regarding the presence and size distributions of particles below the detection limits of the imaging probes, some of which were addressed by our balloon-borne ice crystal replicator. The replicator observations provided information on the concentrations of ice particles beginning at about 10-$\mu$m diameter (Miloshevich and Heymsfield 1997). Nevertheless, there remains substantial questions about the size distributions for sizes below 50 $\mu$m. Two-dimensional images of the particles were obtained at low and high resolution, depending upon the probes used. Average size distributions were measured over 5- to 7-s intervals, or about 1 km of horizontal flight distance. Average spectra were obtained from the replicator in 300-m increments of height.

The assumptions used to derive and parameterize the ice particle cross-sectional areas, the densities, the masses, and the ice particle terminal velocities are presented...
Temperature measurements during the Lagrangian spiral descents. Each (one) data point represents the temperature for one size distribution. Each size distribution represents about 5 to 7 s of aircraft sampling times or about 1 km of the horizontal flight path. Horizontal lines indicate the field program from which the data were collected. Periods with bad temperature or other data during the spirals are not plotted in the figures.

In Part I, based on direct measurements of the ice water content available for a limited part of the dataset (from the ARM campaigns) and from comparisons of measured and calculated radar reflectivity and fall velocities from the TRMM part of the dataset, the accuracy of the calculations of IWC is estimated to be ±25%.

Parameterized forms of the PSD are used here to represent the measured PSDs as trends in the PSD properties with temperature or IWC can be more clearly ascertained then by using the measured, bin-averaged data. The use of parameterized PSDs partially compensates for the absence of aircraft PSD measurements below 50 μm, although the functional form used for particles below 50 μm represents an extrapolation that is not known explicitly from the data.

Gamma distributions of order \(m\), intercept \(N_0\), and slope \(\lambda\) are derived in this part of the study to represent the relationship between the concentration \(N\) and maximum particle dimension \(D\):

\[
N = N_0 D^m e^{-\lambda D}.
\]

Symbols used in this paper are defined in the text that follows and are summarized in Part I, appendix A. We begin with the measured concentrations in approximately 50 nonequally spaced size bins. The fitting method is based on matching three moments of the PSD, as in Kozu and Nakamura (1991). Following Heymsfield et al. (2002, hereafter H02), the three moments used here are the first (diameter), second (area), and sixth (reflectivity). Although bimodal size distributions were often observed in the FIRE and ARM datasets, these are not considered differently than monomodal spectra because their effects on IWC and bulk properties are generally small compared with the effects of other uncertainties. A small percentage of the fitted PSD is omitted from the analysis for those cases where the correlation coefficients between the fit and the data (\(r^2\)) were below 0.6. This process selectively omitted bimodal spectra with extremely peaked distributions; this constituted 14% of the FIRE spectra, 2% of the TRMM spectra, 30% of the ARM spectra, and 23% of the replicator spectra, or an average of 12% of all spectra. Had these been included, the results would have been minimally different. For a full discussion of analytic treatments for bimodal PSD, see Ivanova et al. (2001).

For the calculations reported in this article, the parameterized size distributions are used to calculate the concentrations in 100 equally spaced size bins that span the range from 0 cm to the largest measured particle size. This is illustrated in Fig. 2a, which shows a broad size distribution and the gamma PSD fitted to the data. The bulk properties are then derived using the mass and area parameterizations described in Part I. The bulk properties from measured (bin averaged) size distributions were also derived for evaluating the accuracy of the bin-averaged estimates.

The median mass diameter \((D_m)\) is used to characterize the mass-weighted size of the PSDs. The \(D_m\) values are derived as illustrated in Fig. 2b. The diameter at the 50th percentile of the fractional cumulative total IWC defines \(D_m\). The error in the \(D_m\) values resulting from bimodality in some of the midlatitude PSDs is found from calculations to be insignificant.
Cumulative Fraction of IWC and Total Area

A: Size Distribution

B: Cumulative Fractions

Fig. 2. (a) Measured (5-s average) and gamma-fitted size distributions, and (b) corresponding cumulative distribution of IWC and total cross-sectional area ($A_c$) with diameter centered on 2217:08 UTC on 19 Aug 1999 from the TRMM observations. The temperature is $-5.4^\circ C$.

Five horizontal lines show selected fractions of the total IWC.

area-weighted distribution with size $D_a$ were also derived, as illustrated in Fig. 2b. The ice water content is found by integrating the concentration times the size-dependent masses from Part I over all sizes.

The $D_m$ for a gamma-type ice particle size distribution with a mass versus diameter distribution that can be represented by a single power law for all sizes can also be found analytically (Mitchell 1991) using the relationship

$$D_m = \frac{\varphi + \mu + 0.67}{\lambda},$$

where $D_m$ is the exponent in the mass versus diameter relationship and has average values of $2.25 \pm 0.15$ (mid-latitude) and $1.97 \pm 0.14$ (TRMM) from Part I, and $\lambda$ and $\mu$ are the slope and dispersion of the size spectra, respectively. In this calculation, the $D_m$ values are not dependent on the PSD intercept parameter ($N_0$) but do depend on $\mu$. The $D_m$ also depends on habit, from the dependence of the term $\varphi$ on habit.

The median area-weighted diameter can be derived analytically, analogously to the derivation of $D_m$, from

$$D_a = \frac{2 + b + \mu + 0.67}{\lambda},$$

where the parameter $b$ is related to particle area and has a value of $b = -0.27 \pm 0.16$ (TRMM) $b = -0.33 \pm 0.07$ (midlatitude) from Part I. Although Eqs. (2) and (3) are not used to derive $D_m$ here, they do provide insight into the factors that influence two variables examined in this study, $D_m$ and $D_a$.

3. Results

The observed properties of the PSD ($\lambda$, $N_0$, and $\mu$) are presented in this section, as are bulk properties, including the median mass diameter, the ice water content, and the median terminal velocity. Power-law curves that represent the relationship among the properties of the PSDs, and between the various bulk properties, are plotted in the figures. Polynomial curves of order three and four are fitted between the bulk properties and temperature (except between $\lambda$ and temperature) to accurately represent the nonlinear trends with temperature (Table 1). In the plots that follow, the midlatitude data points
are shaded in gray to permit them to be easily distinguished from the black points representing the tropical data.

a. Particle size distributions

Plots of \( N_0 \) versus \( \lambda \), \( \mu \) versus \( \lambda \), and the correlation coefficients of the curve fits to the PSDs are shown in Figs. 3a to 3c. Each data point represents a size distribution averaged over about 1 km of horizontal distance. As noted in section 2, data where the correlation coefficients are below 0.6, or about 12% of all spectra, are omitted from the analysis. For clarity in Figs. 3a–c and most of the other figures in this paper, only one of each four data points is plotted, except for the replicator (FIRE II) points because they are few in number. The curve fits are based on all data points.

A well-defined relationship is found between \( N_0 \) and \( \lambda \) (Fig. 3a), a feature that has already been reported for the TRMM dataset in the H02 study but is now extended to include midlatitude PSDs. The \( \lambda \) values for the midlatitude PSDs are generally higher than those for the tropical PSDs. Some \( \lambda \) values below 50 cm\(^{-1}\) are found for a few of the midlatitude PSDs but are found for many of the tropical PSDs, which may clearly be attributed to temperature differences between the midlatitude and tropical datasets. A well-defined relationship is also found between \( \mu \) and \( \lambda \) (Fig. 3b). The midlatitude points tended to have larger \( \mu \) values for given \( \lambda \), signifying narrower spectra.

Correlation coefficients \( (r^2) \) for the fits to the PSDs (Fig. 3c) are mostly above 0.8, signifying that the gamma distributions generally fit the PSDs accurately. The lowest \( r^2 \) values are found for the midlatitude data points where \( \lambda > 100 \text{ cm}^{-1} \). Bimodality observed in many of the midlatitude PSDs is responsible for these relatively low correlation coefficients. The \( r^2 \) values for the tropical PSD are generally above 0.90, where there was little bimodality.

The spectral slope \( \lambda \) is inversely related to the temperature \( (T) \) (Fig. 3d), although particular details of the cloud and sampling location play a central role in this relationship and the scatter in the figure. For example, the group of larger points that are distinctly to the left of the main tropical group of points, which are labeled 0822, are from a spiral that was conducted in relatively close proximity to convection (22 August 1999, during KWAJEX). This dataset is different from the other datasets in many respects, as pointed out in H02 and in the sections that follow. Much of the scatter in the \( \lambda \) versus \( T \) relationship results from the aggregation process, which is more a function of vertical depth below cloud top than \( T \) (Field and Heymsfield 2003). In the replicator datasets, the largest values of \( \lambda \) occur at the lowest temperatures. Two exponential curves fitted to the \( \lambda \) versus temperature data, without the inclusion of the 0822 data, are plotted in the figure, with the coefficients in the curve fit also shown.
Fig. 3. Properties of the gamma PSD fitted to the midlatitude (gray symbols) and tropical (dark symbols) datasets as a function of the slope parameter $\lambda$ (abscissa). The data are subsampled; one of each four data points is plotted. Ordinates are (a) $N_0$, (b) $\mu$, (c) correlation coefficient, (d) temperature, (e) diameter of the largest sampled particle, and (f) median mass diameter. Curve fit equations are shown.
As shown in Fig. 3e, a marked dependence may be noted between the largest particle measured for the PSD, $D_{\text{max}}$, and $\lambda$, a tendency reported on in H02 for the TRMM PSDs. As noted in H02, the largest measured size depends on what probe is used, because this size is related to the probe sample volume. In Fig. 3e, the $D_{\text{max}}$ values for the midlatitude datasets are clearly about 25% smaller than those for the tropical set, because the high-volume precipitation spectrometer (HVPS) probe (tropical) has about an order of magnitude larger sampling volume than the 2D-P probe (midlatitude). The replicator sampling volume is even smaller, leading to larger underestimates in $D_{\text{max}}$.

The median mass diameter characterizes the mass-weighted size of the PSDs. As shown in Fig. 3f, an inverse relationship is found between $D_m$ and $\lambda$, although considerable scatter is noted. Referring back to the analytic relationship found between $D_m$ and $\lambda$ given by (2), much of the scatter results from a nonunique relationship between $\mu$ and $\lambda$. Unlike what has been found in past studies for raindrop exponential PSD where approximately $D_m \propto \lambda^{-1}$, the average relationship for ice PSD is $D_m \propto \lambda^{0.60}$. The exponent in this equation is less than unity because $\mu$ in Eq. (2) is a function of $\lambda$ to a power less than unity (Fig. 3b).

PSDs have also been fitted to exponential functions ($\mu = 0$) to permit comparison with earlier studies that used exponential PSDs. Past studies employing exponentials have noted a well-defined relationship between the $N_m$ and $\lambda$ parameters; in H02, many of these earlier midlatitude observations are reported. The $N_m$ versus $\lambda$ relationship for both midlatitude and tropical observations conforms to these earlier reported trends (Fig. 4a). What is most striking in the figure is that, for given values of $\lambda$, the midlatitude $N_m$ values are about an order of magnitude lower than the tropical $N_m$ values. This effect is due to differences in the total ice particle concentrations and IWC, as IWC for an exponential PSD is directly proportional to the value of $N_m$ (H02, and as discussed further in section 3b). Therefore, the $N_m$ differences result from IWC differences or vice versa. The relationships between $\lambda_{\text{ga}}$ (for the gamma) and $\lambda_{\text{exp}}$ for the exponential fits are shown in Fig. 4b, a feature also examined by Mitchell (1991). A direct relationship is found between the two sets of $\lambda$ values, with the two being equal at a $\lambda$ value of approximately 50 cm$^{-1}$. This crossover point, where $\mu \approx 0$ (see Fig. 4b), represents the only place where an exponential would do as good a job at representing the small sizes of the PSD as a gamma distribution. For this reason, gamma distributions are used throughout this study. They do a better job of representing the end of the PSD comprising smaller particles.

Figure 5 shows a plot of the $\lambda$ versus temperature values for the exponential fits, along with two curves fitted over different temperature ranges and a curve from Ryan (2000) that represents averaged values from a large number of midlatitude observations for temperatures ranging from 0° to −26°C. Again, only one of each four data points is plotted (although all FIRE II replicator points are plotted). Overall, results for this study conform well to these earlier observations, suggesting that a commonality exists between midlatitude and tropical ice cloud properties and the parameterizations developed from them.

b. Ice water content

Observations in cirrus at temperatures below −50°C suggest that particles below the detection threshold of the two-dimensional (2D-C) imaging probe (about 50 μm in size) contribute substantially to the total number concentration, the ice water content, and related moments of the PSD. The contribution of small particles to the IWC at low temperatures is assessed by using the binned size distribution data from the three replicator ascents. The ratios of the calculated IWC in sizes above 50 μm to the total calculated IWCs (above about 10 μm) are plotted as a function of temperature in Fig. 6a. In the uppermost (300 m deep) layer of each replicator ascent, these ratios drop to or below 80%, with the largest drop occurring for the coldest case. Aside from the 26 November case (dotted line), in which a dip in the curve occurred at the interface of a second layer comprising many small particles, particles above 50 μm contribute by far the dominant fraction of the IWC within the body of the cloud layer. Furthermore, size spectra measured by a balloon-borne hydrometeor videosonde during ascents through 18 cirrus cloud layers with cloud-top temperatures between −50° and −68°C have also been examined to assess the role of sub-50-μm particles (N. Orikasa 2001, personal communication). The ratio of IWC for sizes above 50 μm to the total IWC (sizes above about 10 μm) is a mean of 0.86 ± 0.10. Applying these results to the aircraft data leads to the conclusion that for all but the very uppermost part of each Lagrangian spiral descent, almost all of the IWC is contained in sizes measured by the imaging probes.

The parameterized forms of the PSDs are used to derive the values of the IWCs shown in Fig. 6b, where in the figure each data point along the abscissa represents an IWC value from one size distribution (or about 1 km of aircraft flight distance). Figure 1 shows the corresponding temperatures. Not shown (for brevity) are the IWCs derived from the measured PSD, which are mostly within ±10% of those shown.

Two features may be noted from the IWCs plotted in Fig. 6b: the range of IWCs is extensive, covering three orders of magnitude, and the IWC values from the TRMM dataset are about an order of magnitude larger than for the midlatitude cases. The second feature is not surprising, given that the TRMM observations were usually at warmer temperatures and the clouds formed through advection of particles from thunderstorms. These conditions would have the combined effect of yielding larger particles and higher IWCs.
The remaining portion of this section examines the factors that might influence the IWC. For the midlatitude samples, there is a clear tendency for the IWC to increase with temperature (Fig. 7a). The tendency is less clear with the tropical dataset, with the 0822 convective case skewing the results to higher IWCs at the lower temperatures. Curves fitted to the midlatitude and tropical datasets (omitting the 0822 case) are plotted in the figure, with fit coefficients given in Table 1.

Can $\lambda$ be used to provide an accurate estimate of the IWC, or visa versa? This question is investigated in Fig. 7b, where it is shown that, although there might be an inverse relationship between the two, there is considerable scatter in that relationship.

The findings of a large degree of variability in the IWC–$T$ and IWC–$\lambda$ relationships, noted by the large $\pm 1\sigma$ bounds in Figs. 7a and 7b, can be explained from analytic considerations. The IWC can be expressed in terms of the size distribution parameters $N_0$ and $\lambda$ from a relationship presented in H02, but simplified here:

$$IWC = \frac{N_0}{\lambda^p},$$

where the proportionality constant and the power $p$ depend on $\mu$ and the properties of the ice particles (density, e.g.). The inverse relationship found between IWC and $\lambda$ stems from this relationship. The variability in the parameter $N_0$, for a given value of $\lambda$, a feature clearly shown in Figs. 3a and 4a, leads to the scatter shown in Figs. 7a and 7b.

From the relationship between $D_m$ and $\lambda$ given by Eq.
(2), it is not surprising that $D_m$ values generally increase with the IWC (Fig. 7c). The smallest values of $D_m$ (about 100 $\mu$m) correspond to the lowest IWCs and the highest values (about 4 mm) occur at the largest values of the IWCs. The scatter is again due to variability in the parameter $N_0$, that influences the IWC values but not $D_m$.

What is also most notable in Figs. 7b–c is that the tropical IWC values are almost an order of magnitude larger than the midlatitude IWC for the same $\lambda$ or $D_m$, once again showing the influence of $N_0$ on IWC but not on $D_m$ (or $\lambda$), which only depend on the variables in Eq. (2).

c. Median mass diameter and effective radius

Increasing median mass diameter is likely to be a function of temperature, a tendency noted for $\lambda$ (Fig. 3d). The dependence of $D_m$ on temperature is examined in Fig. 8a, where it is noted that the $D_m$ values generally increase with temperature, from about 100 $\mu$m near $-55^\circ$C to about 20 times that value near 0°C. Again, however, considerable variability is observed in this relationship, because $D_m$ is not uniquely a function of temperature. The leveling off of the average values of $D_m$ (large squares) between $-25^\circ$ and $-15^\circ$C partially results from higher than average $D_m$ values at temperatures below $-25^\circ$C for the 0822 case (points markedly above the $D_m$ values for the other tropical clouds in the figure and identified with larger symbols).

Using the Ryan (2000) midlatitude $\lambda$ versus temperature relationship, we can compare the distribution of $D_m$ values found in Fig. 8a with those implied for exponential PSD for midlatitude clouds. The Ryan $D_m$ curve falls below that found in our observations, possibly because the concentrations in small sizes are represented over this temperature range by an exponential, not a gamma distribution. The curve labeled Ivanova et al. (2001) relates $D_m$ to temperature using gamma PSDs fit to measurements from midlatitude cirrus PSDs. This curve produces somewhat lower values of $D_m$ than we observed here.

Because PSDs may cover sizes that range over 1 cm or more, a single diameter ($D_m$) cannot adequately provide information about the sizes that dominate the distribution of IWCs. Is the IWC distributed uniformly across this 1-cm range, or is it peaked within a narrow range of sizes? To address this question, the diameters at various fractions of the total IWCs ($D_{mf}$, where $m$ represents mass and $f$ represents the fraction) were found, as illustrated schematically for five fractional IWC values in Fig. 1. In Fig. 8b, mean values of $D_{mf}$ are shown at intervals of 5°C for five different fractional IWC values (to maintain clarity, individual data points are not shown). All datasets, except for the data from the 0822 case, are combined to yield the curves shown in Fig. 8b. Notice that the $D_m$ values for the tropical and midlatitude datasets do not vary widely. For all fractional IWC values, there is a monotonic increase in
Fig. 7. IWC as a function of (a) temperature, (b) $\lambda$, and (c) $D_m$. One of each four data points is plotted. Vertical bars show $\pm 1\sigma$ bounds. Coefficients in the curve fits ("fits") are shown in Table 1.
Fig. 8. (a) Median mass diameter plotted as a function of temperature. Mean values per interval of temperature shown with squares; vertical bars show ±1σ bounds. Curves fitted to the data are plotted in the figure; coefficients of the fit parameters are given in Table 1. Curve based on the parameterized form of λ for exponential PSD by Ryan (2000), and curve from Ivanova et al. (2001) are shown. (b) Diameters at various fractions of the total IWC as a function of temperature (dark lines) and at various fractions of the total particle cross-sectional area (light lines).
with temperature. Particles less than 50 to 100 \( \mu m \) in size (reliably sampled by the replicator but below the reliable sizing capability of the 2D-C probes) contribute appreciably to IWC at temperatures below \(-50^\circ C\) only. The replicator measurements do not indicate a significant contribution for particles smaller than 50 \( \mu m \) for temperatures above \(-50^\circ C\) except in rare instances. This result is reinforced by the distribution of IWC with size from the 2D-C measurements and by measurements over a wide range of temperatures from the balloon-borne hydrometeor videosonde (HYVIS; N. Orikasa 2001, personal communication).

What is surprising is that more than 50% of the total IWC, on average, is contained in sizes above about 1 mm at temperatures above \(-8^\circ C\) for tropical ice clouds and \(-6^\circ C\) for midlatitude ice clouds.

Figure 8b may also be used to ascertain the range of sizes that contributes most of the IWC. On average, 80% of the IWC (10%—90%) is contained in particle sizes between 0.01 and 0.06 cm at a temperature of \(-50^\circ C\), a range that increases progressively with temperature.

As with mass-weighted diameters, the sizes that contribute to various fractions of the total cross-sectional area (or extinction) of the particle populations \(D_{af}\) were derived to provide a means of assessing whether a significant portion of the radiatively active part of the particle population has appreciable sizes and sedimentation velocities (discussed in section 3d). The total cross-sectional area of the population of ice particles \(A_c\), a number that is approximately 50% of the value of the extinction coefficient, is found analytically in H02 and takes on the following simplified form:

\[
A_c \propto \frac{N_o}{\lambda^r}.
\]  

The diameters at given fractions of the total areas \(D_{af}\) are found from the integrated fraction of the total area with size, as illustrated in Fig. 1.

In Fig. 8b, the median area-weighted diameters (gray lines), shown for fractions of the total area of 25%, 50%, and 75%, increase with temperature, and are only slightly smaller than values for \(D_{af}\), a conclusion that can be reached by factoring in the appropriate values of the coefficients from Part I into Eqs. (2) and (3). At other fractions of the total area, a monotonic increase in \(D_{af}\) values with temperature may also be noted. Figure 8b also indicates that the mass-weighted diameters at various fractions of the total IWC do not differ much from the corresponding area-weighted diameters, especially at the lower temperatures where particle densities and \(\lambda\) values are relatively large. As the \(D_{af}\) values increase and the \(\lambda\) values decrease, the area-weighted diameters become increasingly small relative to the mass-weighted diameters.

Figure 8b also indicates that significant fractions of the total area occur in small sizes at temperatures below \(-50^\circ C\), where we have reasonably good data on small particle concentrations from the replicator. The replicator data supports the view that small particles contribute very little to the total area between \(-30^\circ C\) and \(-50^\circ C\), except possibly near cloud top. It is also surmised that our TRMM dataset comprises most of the total cross-sectional areas of the PSDs. Heymsfield and McFarquhar (1996), using small particle data together with imaging probe data in tropical anvil cirrus, showed that almost all of the cross-sectional area of the particle populations are usually in particle sizes measured by the imaging probes when the IWCs are above about 0.1 g cm\(^{-3}\), comprising most of the TRMM observations. Clearly, more information is needed in small sizes to confirm these findings.

GCM researchers are concerned with the relationship between IWC and effective radius (or diameter), as the effective radius is a fundamental determinant of the radiative properties of a cloud. The effective diameter \(D_{ef}\) can be calculated using the definition in Fu (1996):

\[
D_{eff} = \frac{2\sqrt{3}}{3\rho_i} \frac{\text{IWC}}{A_c} \propto \lambda^{-r},
\]  

with the analytic expression derived in terms of the particle size distribution characteristics given in H02.\(^3\)

Recently, Mitchell (2002) used the effective photon path principle to develop a universal definition of \(D_{eff}\) of the form

\[
D_{eff} = 3 \frac{\text{IWC}}{2\rho_i} A_c.
\]  

The effective diameters as a function of IWC and temperature using the Fu (1996) definition are shown in Figs. 9a and 9b, where mean values are shown. (Standard deviations are omitted to improve clarity.) Values of \(D_{ef}\) using the Mitchell (2002) definition are 30% larger than those shown. In general, the trends are similar to those observed for \(D_{af}\): the effective diameter increases with IWC and temperature; differences are noted between the tropical and midlatitude datasets, but to a lesser extent, and for the tropical dataset, data points for the 0822 case fall distinctly above those for the cases more removed from convection.

Also plotted in Fig. 9b are curves derived by Ivanova et al. (2001) for planar polycrystals and by Boudala et al. (2002) from measurements in high-latitude clouds over the temperature range from \(0^\circ C\) to \(-40^\circ C\). The effective diameters are much smaller than observed here, partially because of their inclusion of particles measured

\(^3\) The absence of contributions to the IWC and \(A\) by particles below 50 \( \mu m \) in size for all but the replicator datasets could have adversely affected the estimates of \(D_{af}\). Analysis of the replicator data indicates that the \(D_{af}\) values as derived from the 2D-C probe data (with a minimum size detection threshold of about 50 \( \mu m \)) might be overestimating the effective diameter (underestimating the \(A\) values more than the IWC values) by about 20% near cloud top and by lesser amounts deeper into the cloud layers. Clearly, more data are needed to investigate this issue.
Fig. 9. Effective diameter based on the definition of Fu (1996) as a function of the (a) IWC; (b) temperature, with fitted curve plotted in the figure and listed in Table 1 along with curves developed by earlier researchers; and (c) median mass-weighted diameter, with fitted curves plotted and listed. The Mitchell (2002) definition yields $D_{\text{eff}}$ that are 30% larger.
by the forward scattering spectrometer probe, and in the
case of the Ivanova et al. observations, partially because of
their lack of particles larger than 1 mm. When the
forward scattering spectrometer probe (FSSP) data are
not included in the estimates of $D_{\text{eff}}$, the Bondala et al.
observations still fall considerably below the values
found here. The differences can be attributed to the
clouds sampled: for temperatures warmer than $-10^\circ\text{C}$,
for example, the mean IWCs in the Boudala et al. ob-
servations are $0.10 + 0.007 \text{ g m}^{-3}$, whereas the mean
IWCs are $0.54 + 0.38$. Clearly, higher IWCs lead to
higher $D_{\text{eff}}$ because IWC prevails over $A_s$ as tempera-
tures warm ($\lambda$ values decrease).

The results presented in Figs. 9a and 9b are consistent
with the relationship given by the right-hand side of Eq.
(6). This equation shows that $D_{\text{eff}}$ is an inverse function
of $\lambda$. Most importantly, $N_d$ does not appear in the
equation, as $D_{\text{eff}}$ is given by the ratio of two parameters (IWC,
$A_s$) that are a function of $N_d$. The absence of $N_d$ in the
relationship for $D_{\text{eff}}$ leads to the relatively low scatter
found in the plots.

The nature of the relationship found between the ef-
fective diameter and the median mass diameter was ex-
amined. There should be a direct relationship between
$D_{\text{eff}}$ and $D_m$ because both parameters are inversely re-
lated to $\lambda$. The direct relationship between $D_{\text{eff}}$ and $D_m$
is found in Fig. 9c. Marked differences may be noted
between the midlatitude and tropical datasets because the
coefficient $\varphi$ in the mass versus diameter relations-
ships differ between the two sets. Nevertheless, direct
relationships are found.

d. Ensemble terminal velocities

Reliable estimates of the sedimentation velocities of
ice particle ensembles are needed for mesoscale models
and GCMs, for reasons cited in the introduction. The
median mass-weighted terminal velocities of the particle
ensembles ($V_m$) are derived in this section. The pro-
cedure used to calculate $V_m$ parallels the approach used
to calculate $D_m$ in Fig. 1. Conceptually, the cumulative
distribution of the ice mass flux (the product of the ice
water content per size bin and the terminal velocity of
that bin), normalized by the total IWC, is plotted as a
function of the fall velocity (rather than the diameter
that was used in Fig. 1). The fall velocity where the
normalized cumulative fraction is equal to 0.5 defines
$V_m$. The approach used to calculate the fall velocities is
given in Part I.

In general, there is a tendency for $V_m$ to increase with
temperature (Fig. 10). For the tropical points (Fig. 10a),
$V_m$ values increase by about a factor of 4 between $-40^\circ$
and $0^\circ\text{C}$. Note the considerable scatter in the tropical
data, that results largely from the outlying points for the
0822 KWAJEX anvil case. For the midlatitude cases (Fig.
10b), the $V_m$ values increase by about a factor of
3 from $-60^\circ$ to $-20^\circ\text{C}$, with less scatter. A curve is
fitted to those observations (Table 1). Combining the
tropical and midlatitude datasets (Fig. 10c) results in
values for $V_m$ that, in general, increase with temperature,
but with a great deal of scatter, because of habit dif-
ferences, proximity to convection, etc. A curve fitted to
those observations, excluding the data from the 0822
case, appears in Table 1. Also shown in the table are
the coefficients for the ensembles if they had been fall-
ing at the surface (1000 hPa). Adjustments in the $V_m$
for other pressures can be made through the use of the
relationships given in Part I (Fig. 7), together with the
$D_m$ versus $T$ relationship in Table 1.

Ivanova et al. (2001) used PSDs from midlatitude
cirrus clouds to develop a relationship between $V_m$
and temperature. This curve, plotted in Fig. 10b, shows low-
er values of $V_m$ than are observed in this study—which
may be attributed to the lower values of $D_{\text{eff}}$ found by
Ivanova et al. than is found here for midlatitude cirrus.

The median mass-weighted terminal velocity generally
increases with the ice water content (Fig. 11). Much of
the variability found in Fig. 11 (see $\pm$ bars) results
from variability in the coefficient $N_d$, which influences
IWC but not $V_m$. There is a great deal of variability in
the $V_m$-IWC relationship for the tropical dataset (Fig.
11a), largely because of the inclusion of points for the
0822 case; it is believed here that this variability reflects
the wide range of conditions that may be sampled in
association with convectively induced ice cloud layers.
The scatter observed in the midlatitude datasets (Fig.
11b) is smaller. In the composite dataset (Fig. 11c), on
average, there is approximately a fivefold increase in
$V_m$, from about 30 to 150 cm s$^{-1}$, with three orders of
magnitude increase in IWC. Curves representing the
midlatitude and combined datasets are shown in Figs.
11b and 11c.

The dashed curves in Figs. 11b and 11c were devel-
oped by Heymsfield and Donner (1990), who related $V_m$
to IWC for predominately midlatitude clouds. The curve
does a reasonable job of capturing the dependence of
$V_m$ on IWC but underestimates the mean values by about
50%.

The variable $V_m$ gives the terminal velocity at the size
at which the IWC is 50% of the total value. But how
do the terminal velocities vary at other fractional IWCs
($V_{mf}$), especially between very low and high fractional
IWCs? Estimates of $V_{mf}$ as a function of IWC would per-
mit an assessment of whether it is necessary in a
GCM to assign a range of terminal velocities to a given
IWC, or whether a $V_m$ alone is sufficient. Values of $V_{mf}$
for five fractional IWCs are averaged for all datasets
combined in even intervals of (log$_{10}$) IWC in Fig. 12a
and of temperature in Fig. 12b. Values of $V_{mf}$ at each
fractional IWC generally increase with IWC and tem-
perature, and they parallel each other over the full range
of IWCs (spanning three orders of magnitude) and of
temperature ($-60^\circ$ to $0^\circ\text{C}$). Particles constituting 80% of
the total IWC (between the 10% and 90% lines in these
figures) fall within a relatively narrow range of velocities of 50 cm s$^{-1}$ or less. This result points to the
Fig. 10. Median mass-weighted terminal velocity shown as a function of temperature for (a) tropical clouds, (b) midlatitude clouds, and (c) both groups combined. One of each four data points are plotted. Mean and standard deviation of values in temperature intervals of 5°C are shown. Curve fits from Table 1 are plotted in (b) and (c).
Fig. 11. Same as Fig. 10, except abscissa is IWC. (b), (c) Curves fitted to the observations.
important finding that the increase in $V_{mf}$ with the fractional IWC is nonlinear and that almost all of the IWC is contained in a narrow range of velocities. From Fig. 12b it can be concluded that the largest particles are falling at velocities close to $V_m$, a result also relevant to ice particle treatment in GCMs.

Differences between the mass- and area-weighted ensemble terminal velocities may lead to differential settling and, as discussed in the introduction, to enhanced shortwave reflection if these differences are large. The differences in mass- and area-related velocities at various fractions of the total IWC are shown in Fig. 13. Differential fall velocities are quite modest in magnitude for most IWCs (Fig. 13a), generally representing 5 to 20 cm s$^{-1}$ or only about 15% of the total velocity. The differential velocities are also quite modest for most
temperatures, amounting to 5–20 cm s\(^{-1}\) or 10%–20% of the total velocity. However, for temperatures below \(-40^\circ\text{C}\), the differential velocities are a larger fraction of the total, amounting to as 20%–50% of the total velocity at the lowest temperatures. Therefore, significant particle area is "left behind" at low temperatures.

There is a direct relationship found between \(V_m\) and \(D_{\text{eff}}\), with differences noted between the midlatitude and tropical points resulting largely from habit differences (Fig. 14). There is relatively little scatter noted in the relationship, because both \(V_m\) and \(D_m\) are independent of the parameter \(N_i\), and each depends on the ratio of mass to area. The exceptions are the data points from the 0822 case that fall above the other TRMM points.

4. Summary and conclusions

This study has sought to improve knowledge of the area, mass and terminal velocity properties of ensembles of ice particles found in deep cirrus and stratiform ice cloud layers in midlatitude and tropical regions. Using in situ data obtained during Lagrangian spiral descents...
through ice cloud layers, the dependence of the ice water content, the median mass diameter, the effective diameter, and the median mass-weighted fall velocity on temperature and the properties of the particle size distributions were examined, and the variability was characterized. Empirical relationships were developed from the observations, thereby allowing the results to be used in cloud and climate models.

A primary goal of this study was to develop a better understanding of the factors that influence the bulk properties of particle ensembles observed in deep ice clouds. The single most important finding is that for the properties that are independent of the size spectrum parameter $N_0$ ($\mu$, $D_{\text{eff}}$, and $V_m$) there appears to be relatively little dependence on the geographic location and cloud formation mechanisms, although the data from one case does suggest that the proximity to convection may be a factor requiring more data to confirm. These relationships do depend strongly on the size distribution slope parameter $\lambda$, which is a strong function of temperature. These "normalized" properties are related to each other as well, with relatively little scatter in the relationship. Conversely, for the properties that do depend on $N_0$ (the IWC and extinction, although this parameter was not directly examined here), there is considerable variability. The parameter $N_0$ was a factor of 10 larger for the clouds sampled during the TRMM campaigns than for the midlatitude cases. Therefore, although the midlatitude clouds had optical depths (that are also related to $N_0$) estimated to be an order of magnitude smaller than for the tropical clouds, the values of normalized properties at the same temperature are comparable.

Although midlatitude convectively induced ice clouds were not studied for this paper, the tropical results may apply to convectively produced anvils in midlatitude locations as well, because the ice particle concentrations, maximum sizes, and IWCs measured in the tropical anvils are comparable to those measured in midlatitude anvils by Heymsfield (1986) and Heymsfield and Palmer (1986).

Several aspects found for the properties of the particle size distributions and sedimentation velocities of particle ensembles are noteworthy. Temperature can be used as a first approximation to estimate $\lambda$ for gamma and exponential PSDs for areas removed from convection, and for gamma distributions $\mu$ is obtained from $\lambda$. Given a prognosed value for the IWC, the intercept parameter $N_0$ can then be obtained for either type of PSD. Investigation into the distribution of terminal velocities within an ensemble yielded the result that particles composing up to 90% of the total IWC fall within a narrow range of velocities of about 50 cm s$^{-1}$. This result indicates that $V_m$ values provide good estimates of the fall velocities of ice cloud particle ensembles. Furthermore, particles that compose most of the total area of the particle ensembles have fall velocities only 10 cm s$^{-1}$, or less than 10%, lower than those of particles that compose the same fractions of the total IWC. Therefore, the radiatively important part of the particle population appears to have fall velocities that are comparable to those dominating the mass distribution. Exceptions were noted in the results for temperatures below $-50^\circ$C, where fall-velocity differences between the mass- and area-dominated parts of the population are
large. With the exception of the regimes of these lower temperatures, current GCM and cloud models should treat the fallout of the area- and mass-related mean velocities of the PSD consistently.

One of the most intriguing findings of this study was a direct relationship with relatively little scatter between the effective diameter, a measure of the radiative size of a cloud particle population, and $D_m$ or $V_m$, because they are all independent of $N$. Each of these dependencies may be exploited in future cloud and climate model studies, as well as for remote sensing applications.

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