A Simple Kinematic Source of Skewness in Atmospheric Flow Fields

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ABSTRACT

Geopotential height fields exhibit a well-known pattern of skewness, with distributions that are positively skewed on the poleward side of the midlatitude jets/storm tracks and negatively skewed on the equatorward side. This pattern has often been interpreted as a signature of nonlinear dynamical features, such as blocking highs and cutoff lows, and there is renewed interest in the higher moments of flow variables as indicators of the nature of the underlying dynamics. However, this paper suggests that skewness can arise as a simple kinematic consequence of the presence of jet streams and so may not be a reliable indicator of nonlinear dynamical behavior. In support of this, reanalysis data are analyzed to demonstrate a close link between the jet streams and the skewness patterns. Further evidence is provided by a simple stochastic kinematic model of a jet stream as a Gaussian wind profile. The parameters of this model are fitted to data from the reanalysis and also from an aquaplanet general circulation model. The skewness of the model’s geopotential height and zonal wind fields are then compared to those of the original data. This shows that a fluctuating jet stream can produce patterns of skewness that are qualitatively similar to those observed, although the magnitude of the skewness is significantly overestimated by the kinematic model. These results suggest that this simple kinematic effect does contribute to the observed patterns of skewness but that other processes (such as nonlinear dynamics) likely also play a role.

1. Introduction

A key issue in meteorology is the extent to which atmospheric variability is strongly nonlinear. While there are of course strong nonlinearities in the equations of motion, stochastically forced linear models show considerable skill in simulating features of the observed variability (e.g., Branstator 1995; Whitaker and Sardeshmukh 1998; Zhang and Held 1999). This has considerable practical implications for predictability and for the utility of reduced-complexity atmospheric models.

The validity of linear and nonlinear paradigms for atmospheric flow is often assessed by their ability to reproduce the observed statistics of variability. Deviations from Gaussian behavior have been interpreted as signatures of highly nonlinear regime behavior (e.g., Majda et al. 2006), cross-frequency coupling (Rennert and Wallace 2009), or as a consequence of multiplicative stochastic noise (Sura et al. 2005; Sardeshmukh and Sura 2009). Higher-order moments such as the skewness and kurtosis of flow variables are often used to characterize these non-Gaussian features.

The basic structure of these statistics has been described by White (1980), Trenberth and Mo (1985), Nakamura and Wallace (1991), and Holzer (1996). The clearest signature is for geopotential height to be positively skewed on the poleward side of jet streams/storm tracks and negatively skewed on the equatorward side. This means that the distribution of geopotential height at a grid point poleward of the jet will have an enhanced positive tail, which is often interpreted as a signature of blocking anticyclones (Nakamura and Wallace 1991). Similarly, the distribution at an equatorward grid point will have an enhanced negative tail, which is often attributed to cutoff low pressure systems.

In this paper, however, we suggest that this pattern of skewness can arise as a simple kinematic consequence of the presence of jet streams. That skewness should be expected to naturally accompany jet streams has already been demonstrated in an oceanographic context, where patterns of skewness and kurtosis can be used to identify the location of ocean jet streams using satellite altimetry data (Thompson and Demirov 2006; Hughes et al. 2010), but we are not aware of its discussion in the atmospheric literature. It is consistent, however, with the recent
derivation of atmospheric flow patterns of maximal skewness, since these patterns resemble jet stream shifts (Pasmanter and Selten 2010).

The argument is as follows. Consider for simplicity a Northern Hemisphere jet. This corresponds to a strong meridional gradient of geopotential height, which shifts north and south with the jet. If we consider a location just to the north of the mean jet position, the effect of jet stream shifts on this point is very asymmetric. When the jet stream shifts south there is little effect at this point, as the geopotential height gradient to the north of the jet is relatively weak. In contrast when the jet shifts north, the strong height gradient moves over the point and it experiences a strong increase in height. The height distribution at this point will then naturally feature an enhanced tail of large, but rare, positive values and so the distribution will be positively skewed. Similarly, a location to the south of the mean jet position will only occasionally encounter low height values on the northern side of the jet and so will have a negatively skewed distribution.

It is anticipated that the zonal wind will also exhibit deviations from Gaussianity. At both the northern and southern points, the wind will most often be weak, but on the occasions when the jet stream shifts over the points, they will experience strong winds. The distribution of zonal wind is therefore expected to be positively skewed on both flanks of the jet. In contrast, a location on the time-mean jet axis will be dominated by strong wind but will experience weak values on the occasions when the jet is displaced farther away. The wind distribution there might therefore be expected to be negatively skewed.

In this paper we present some evidence in support of this argument. We begin in section 2 with an analysis of the observed skewness and in section 3 we present a simple stochastic, kinematic jet stream model to demonstrate the existence of this phenomenon. A key motivating question was whether this model can generate skewness values as large as those observed. By fitting the parameters of the jet model to data from both a reanalysis and a simplified aquaplanet general circulation model, we show that this kinematic effect can in fact easily generate skewness values larger than those observed.

2. Observations

The data used for this study come from the 40-yr European Centre for Medium-Range Weather Forecasts (ECMWF) Re-Analysis (ERA-40) data during 1958–2001. The 6-hourly data are averaged to daily means and reduced to a grid of 80° latitude × 160° longitude, which covers the whole globe. Data from two seasons have been used: December–February (DJF; no leap days) and June–August (JJA). We analyze the skewness of geopotential height and zonal wind, with no filtering applied.

We begin by showing in Fig. 1 maps of the skewness of unfiltered daily geopotential height at 250 hPa (Z_{250}). These generally show positive values of skewness at high latitudes and negative values at low latitudes, as in the literature described above. In each case the climatological jet axes have been identified by maxima in the zonal wind climatology, and these are marked by black dots. This reveals a clear tendency for the reversal of the sign of the skewness to coincide with the jet axis. Multiple bands of alternately signed skewness are evident over Eurasia in DJF and the South Pacific in JJA, consistent with the presence of separated subtropical and eddy-driven jet streams, each with their associated skewness dipole. The skewness of 250-hPa zonal wind U_{250} is shown in Fig. 2. While noisier than the \( Z_{250} \) versions, these show a tendency for the skewness of \( U_{250} \) to be positive on both the poleward and equatorward sides of the jets, as suggested in the introduction. The hypothesis also suggests negative skewness along the jet axes, and this is seen to some extent. All of the regions of strong negative skewness do indeed lie along jet axes, although in many other regions the skewness along the jet axes is close to zero. As shown next, this negative skewness is stronger at 850 hPa, suggesting that the presence of multiple jets and strong wave activity might be obscuring this signal at upper levels.

As further examples we also show maps of the skewness of geopotential height and zonal wind at the near-surface level of 850 hPa. The \( Z_{850} \) maps in Fig. 3 also reveal a tendency for more positive values on the poleward side of the jets and more negative values on the equatorward side, albeit superimposed on a background of negative skewness. While White (1980) observed generally negative skewness values in his unfiltered surface data, these are less apparent in the filtered analysis of Nakamura and Wallace (1991). This suggests that the tendency for negative skewness values in geopotential height at low levels is related to the high-frequency transients, perhaps reflecting cyclone–anticyclone asymmetry (Donohoe and Battisti 2009). The \( U_{850} \) skewness maps in Fig. 4 more clearly show negative skewness along the jet axes and positive values on both the poleward and equatorward sides of the jets, in line with the simple jet stream signature suggested here. Positive skewness is particularly clear in the sub tropics, which may, of course, also be influenced by tropical processes. However, jet stream shifts regularly exceed 10° of latitude (Woollings et al. 2010) so that westerly jet stream winds do contribute to the distribution of subtropical zonal winds. As an aside, we note that the jet stream signature in low-level wind skewness may account for some of the discrepancy between the observed distribution of sea...
surface winds and that expected to arise from boundary layer physics (see Monahan 2006).

While in this paper we focus on flow statistics on pressure surfaces, it is of interest to briefly present an isentropic analysis. Hughes et al. (2010) modeled a jet stream as a step function between two regions of constant potential vorticity (PV) of different values. In this framework they were able to derive a theory that predicted not only that the skewness of PV changes sign across the jet stream, but also that the climatological jet axis should coincide with a minimum of kurtosis. Atmospheric PV on an isentropic surface intersecting the tropopause can be well modeled as two regions of constant PV (low tropospheric PV values and high stratospheric PV values) with a discontinuity at the tropopause (Swanson 2001), which will also correspond to the jet stream location. Figure 5 shows the skewness and kurtosis of PV on the 315-K isentropic surface. Note that PV has a larger magnitude toward the poles, where the 315-K surface is in the stratosphere, and that PV in the Southern Hemisphere is negative. These features explain why the Northern Hemisphere skewness map has the opposite

**FIG. 1.** Skewness of $Z_{250}$ in (left) DJF and (right) JJA. Jet axes are indicated by black dots that mark maxima with respect to latitude in the 250-hPa zonal wind climatology, with the added subjective criterion that the wind speed is at least 15 m s$^{-1}$.
sign to both the Southern Hemisphere version and the Northern Hemisphere map of geopotential height skewness. This figure shows that the tropopause does indeed clearly correspond to both a change in sign of the skewness and a minimum in kurtosis, so the theory of Hughes et al. appears to hold well in this case.

3. Model

In this section we present the results from a simple kinematic jet stream model to demonstrate that the general nature of the observed patterns of skewness is consistent with the signature of a varying jet stream. The kinematic model simply generates artificial time series of the wind speed along a latitude section via stochastic variations of a Gaussian wind profile. The model is based on that of Monahan and Fyfe (2006), with a zonal wind profile \( u \) defined by

\[
u(\phi, t) = U(t) \exp \left\{ -\frac{[\phi - \Phi(t)]^2}{2\sigma^2(t)} \right\}, \tag{1}\]

FIG. 2. As in Fig. 1, but for \( U_{250} \).
where $U(t)$, $\Phi(t)$, and $\sigma(t)$ are the jet strength, position, and width, respectively, which all vary stochastically following

$$U(t) = U_0 + \xi(t),$$  
$$\Phi(t) = \phi_0 + \lambda(t),$$  
$$\sigma^{-1}(t) = \sigma_0^{-1}[1 + \eta(t)].$$

In the above, $U_0$ is the time mean jet peak velocity, $\phi_0$ is the time-mean jet central latitude, and $\sigma_0^{-1}$ is the inverse time-mean jet width. The fluctuations in the jet peak velocity and central latitude are $\xi(t)$ and $\lambda(t)$, respectively, and the scaled fluctuations in inverse jet width is $\eta(t)$. The fluctuations $\xi(t)$, $\lambda(t)$, and $\eta(t)$ are Gaussian time series with mean zero.

Following Monahan and Fyfe (2006) the observed correlation between strength and inverse width is preserved by setting $\xi(t) = \rho \eta(t)$. This is used in conjunction with Eqs. (3) and (4) to determine the strength, position, and width of the jet. Latitude and width fluctuations are calculated using an AR1 process as follows:

$$\lambda(t) = r\lambda(t-1) + \omega \eta_1 \sqrt{1 - r^2},$$

FIG. 3. As in Fig. 1, but for $Z_{850}$. The thresholds used to identify the jet axes are 8 m s$^{-1}$ in DJF and 6 m s$^{-1}$ in JJA.
\[ \eta(t) = r_w \eta(t - 1) + \nu n_2 \sqrt{1 - r_w^2}, \]

where \( n_1 \) and \( n_2 \) are two independent random numbers from a standard Gaussian distribution, \( \omega \) and \( \nu \) are the standard deviations of \( \lambda \) and \( \eta \), and \( r_l \) and \( r_w \) are the lag-1 autocorrelations of the jet latitude and width. This enables the zonal wind to be determined via Eq. (1), and the geopotential height is obtained from this by assuming geostrophic balance. To conserve mass, the height field at each time is adjusted by adding or subtracting a constant so that the integration of the height over the latitude range remains identical to its original value.

The various parameter settings in the model are determined from the ERA-40 data using the jet-fitting procedure of Monahan and Fyfe (2006). The ERA-40 zonal wind is averaged over 30° sectors of longitude to reduce noise. Sections in longitude are chosen in both hemispheres where the jet streams are strongest in each season, and the model is applied to each of these. For each day the largest positive wind speed is identified and the fitting procedure is applied to the smallest latitude band surrounding this value that contains only positive values. When applied to all days of a given season in ERA-40, this procedure generates a distribution of
values of jet latitude, width, and speed. The means of these distributions are used to define $U_0$, $\phi_0$, and $\sigma_0$, and the standard deviations are used to define the standard deviations of $\xi(t)$, $\lambda(t)$, and $\eta(t)$. Occasional days were neglected when the fitting procedure returned unphysical values of latitude or width, or speeds in excess of 80 m s$^{-1}$ at 850 hPa or 200 m s$^{-1}$ at 250 hPa. At most 6% of the days were neglected in this way.

As defined so far the model in fact generates unrealistically large values of skewness. This is because the only wind values are associated with the varying Gaussian jet profile. This means that latitudes far from the mean jet axis only experience significant wind values on a few rare occasions and so exhibit highly skewed statistics. To remedy this we add some additional Gaussian distributed noise to $u(\phi, t)$ to represent high-frequency synoptic variations. This noise has zero mean and a standard deviation determined from the distribution of 2–6-day bandpass filtered meridional wind [using the Lanczos filter of Duchon (1979)]. For this purpose the meridional wind was sampled over the years 1996–2000 using all latitudes and longitudes in each sector, resulting in standard deviations of 2–6 m s$^{-1}$ depending on region (see captions for details). Experimentation

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**Fig. 5.** Skewness and kurtosis of potential vorticity on the 315-K isentropic surface. The black line marks a subjectively chosen contour of the potential vorticity climatology: 3.5 PVU in the Northern Hemisphere and −2.75 PVU in the Southern Hemisphere (1 PVU = $10^{-6}$ K m$^2$ kg$^{-1}$ s$^{-1}$).
suggests that simulations of length 10,000 days are sufficient for the skewness statistics to be very stable.

Figure 6 compares the observed and modeled skewness of $Z_{250}$ for the chosen sectors. The model skewness changes sign at the mean jet latitude, with negative skewness to the equatorward side and positive skewness to the poleward side. This confirms that this general skewness pattern can be a simple kinematic consequence of the presence of jet streams. In the region within 20°–30° of the jet axes there is in most cases a qualitative agreement between the modeled and observed values, but the model tends to overestimate the magnitude of the skewness. Further from the jet axis the agreement breaks down, likely because the jets have a weak influence in these regions in the observations and because the model variability has a relatively stronger influence from the mass correction term. In the case of the Southern Hemisphere in JJA there is a double jet structure in the observations, which is a clear limitation of this model.

At 850 hPa (Fig. 7) the model again overestimates the magnitude of the skewness values, often by a factor of 2 or more. Despite this there is some qualitative agreement, especially if the background negative skewness in the observations is accounted for.

To summarize, the model generates a pattern of skewness that is only qualitatively similar to the observations, in that height is positively skewed on the poleward side of the jet and negatively skewed on the equatorward side. Quantitatively, the model results are often quite different from the observations. There are several possible reasons for this discrepancy, such as the lack of low-frequency variability in the model. In addition this model, in its conservation of mass and imposed correlation between jet strength and jet width, is more representative of zonal mean flows than the regional cases shown here. This would be consistent with the results of Kushner and Lee (2007), which suggest that variability on these regional scales might be dominated by processes other than jet stream shifts.

These considerations motivate the investigation of a simpler case, and here we have chosen an aquaplanet general circulation model. We use output from
a high-resolution simulation of the Met Office Hadley Centre Atmospheric Model, version 3 (HadAM3), which has been run with the full package of physical parameterizations but with a simplified lower boundary with no continents or orography and forced with zonally symmetric sea surface temperatures. This is the control simulation of Brayshaw et al. (2008) using the QOBS sea surface temperature profile, which results in a single, broad jet in the zonal mean located at the limit of the Hadley cell circulation. This simulation provides 4320 days of data on the assumption that the two hemispheres are independent and can be combined in order to double the effective simulation length. See Brayshaw et al. (2008) for more details of the model and the experiment.

We have applied the fitting procedure as above to the zonal-mean daily zonal wind field of the aquaplanet model in order to derive parameters for the kinematic jet model. In this case, however, we neglect the additional synoptic noise term, which is not appropriate in the zonal mean. The results of this analysis applied to the 850-hPa level are shown in Fig. 8. This is the simplest case, since at low levels there is only one (eddy-driven) jet stream. Climatological means of the zonal wind and geopotential height are given in Figs. 8a and 8c, showing that through the fitting procedure the kinematic model reproduces the jet stream flow well. The kinematic model, however, does not simulate the easterly winds at both high and low latitudes exhibited by the aquaplanet model.

Figure 8b shows the skewness of the zonal wind. In the immediate vicinity of the mean jet core, the aquaplanet zonal winds exhibit negative skewness, as expected, and these are well reproduced by the kinematic model. Away from the jet, the aquaplanet winds exhibit positive skewness, again in agreement with the expectations outlined in the introduction. The kinematic model, however, greatly overestimates this positive skewness. These very large skewness values arise because, as described above, the only source of wind in the kinematic model is the Gaussian wind profile. In the absence of any other variability, a few rare westerly events set the high skewness values away from the mean jet position. At
these latitudes it seems unlikely that the jet stream is important in setting the skewness. This can be quantified by considering the leading empirical orthogonal function (EOF) of the zonal mean wind in the aquaplanet. This leading EOF (not shown) largely represents jet stream shifts and features negligible wind anomalies at latitudes more than 30° away from the jet axis. This shows that the region poleward of 70°N is very rarely directly influenced by the jet and so other processes must be dominant in setting the skewness there.

The skewness of geopotential height is also plotted in Fig. 8. The aquaplanet model shows the characteristic pattern of positive skewness poleward of the time mean jet location and negative skewness to the south. The kinematic model again overestimates the magnitude of the skewness but reproduces the latitudinal distribution reasonably well, except in the tropics where easterly winds are present in the aquaplanet model. Locations at both very high and very low latitudes are also relatively strongly affected by the mass correction term in the kinematic model.

This analysis has been repeated at 250 hPa and the results are shown in Fig. 9. In this case the characteristic skewness pattern is less clear, though still evident, in the aquaplanet zonal winds. The kinematic model compares rather poorly with the aquaplanet here. Interestingly, the mean jet core in the kinematic model is located slightly poleward of its counterpart in the aquaplanet model (Fig. 9a). This must arise because of asymmetries in the jet variability in the aquaplanet model, which highlights the role that nonlinear dynamics can play in shaping flow statistics. The height skewness is again reasonably quantitatively captured by the kinematic model, especially when the poleward bias of the kinematic model jet is taken into account; however, this quantitative agreement holds only in the vicinity of the mean jet core.

FIG. 8. Statistics of the 850-hPa zonal mean flow in the aquaplanet simulations (solid lines) and the fitted kinematic jet model (dashed lines). Parameters used for the kinematic model were 18 m s\(^{-1}\), 7.7°, and 40° for the mean values of the jet strength, width, and latitude, and 3.8 m s\(^{-1}\), 3.9°, and 4.1° for their standard deviations, respectively.
4. Discussion

This paper has shown that spatial patterns of skewness in atmospheric flow fields have clear relationships with the jet streams. Skewness of geopotential height is positive on the poleward side of jets and negative on the equatorward side, while zonal wind tends to be negatively skewed in the jet core and positively skewed on either side of the jet. Similarly, the skewness and kurtosis of PV on an isentropic surface show clear signatures where the isentropic surface crosses the tropopause (Fig. 5).

The simple kinematic jet model shows that these patterns of skewness in wind and height fields can occur as natural consequences of the presence of jet streams, as outlined in the introduction. The quantitative skill of the kinematic model is generally poor, however, when applied to both observational and aquaplanet model data. The model performs best in the simplest case of zonal-mean low-level flow on the aquaplanet, when the assumption of one isolated jet stream is most appropriate. Even in this case, however, the kinematic model significantly overestimates the magnitude of the skewness. This discrepancy could arise from the simplicity of the jet stream representation in the model or from other processes not included. These include the effects of eddy–mean flow interaction on jet variability and the presence of medium- and low-frequency patterns of flow variability. The kinematic model performs worst at latitudes most removed from the mean jet position, where the flow experiences little direct effect of jet stream variations.

It is clear from these results that processes other than the simple kinematic jet stream effect must play a role in shaping the distributions of skewness. However, it also clear that whenever the flow features a fluctuating jet stream there will be some asymmetry of flow fields arising from the kinematic effect, and that these asymmetries can be large. This is true in particular at latitudes close to the mean jet locations. This effect should, therefore, be taken into account as an important source of skewness in flow fields, and so skewness is not necessarily a signature of
noting, however, that in some jet indices the distribution of jet latitude can exhibit marked deviations from Gaussianity (Woollings et al. 2010).

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**FIG. 10.** Skewness of the 850-hPa zonal wind from the aquaplanet model, viewed both in the usual frame of reference and a frame of reference that moves in latitude with the jet axis (defined as the latitude of maximum westerly wind). At a given latitude, results for this latter frame of reference are only plotted when fewer than 10% of days suffer from missing data. The original version has been shifted so that the time-mean jet location lies at a relative latitude of zero.

This shows that it does not seem possible for both Eulerian statistics and more Lagrangian flow quantities such as the jet latitude to exhibit Gaussian statistics, and it is not clear which of these is most relevant for assessing the nonlinearity of atmospheric variability. It is worthwhile nonlinear dynamics, cross-frequency coupling, or multiplicative noise.

It is interesting that even when the distribution of jet latitude is Gaussian, as in the model used here, the Eulerian flow fields will exhibit non-Gaussian statistics. This is essentially because the mappings from jet latitude to wind and height variations at fixed locations are themselves nonlinear. In this case the non-Gaussian statistics arise from our choice in performing an Eulerian analysis, in which we fix a location on the globe and then calculate the statistics of the flow at that point. As an alternative we could define a frame of reference that shifts in latitude with the jet stream, so that the y-coordinate is the distance in latitude from the jet axis. This has been tested by defining such a frame of reference for the zonal-mean zonal wind data from the aquaplanet model, and the skewness values are indeed very different when viewed in this frame of reference. As an example, Fig. 10 shows the skewness of zonal wind in this frame of reference, as compared to the original (Eulerian) frame. The skewness values near the mean jet location are reduced in this framework, as might be expected, but the skewness values at low latitudes are much increased. This seems to arise from the redistribution of the tropical easterly winds in this coordinate transformation.

This shows that it does not seem possible for both Eulerian statistics and more Lagrangian flow quantities such as the jet latitude to exhibit Gaussian statistics, and it is not clear which of these is most relevant for assessing the nonlinearity of atmospheric variability. It is worthwhile noting, however, that in some jet indices the distribution of jet latitude can exhibit marked deviations from Gaussianity (Woollings et al. 2010).

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