Tropical–Extratropical Interactions of Intraseasonal Oscillations

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ABSTRACT

Tropical–extratropical interactions of intraseasonal oscillations (ISOs), based on 30 years (1979–2009) of northern winter observations and theory, are compared. The phase relationships between the tropical signal of the leading theoretical ISO for a January 1979 basic state and the development of Pacific–North America (PNA)-like and North Atlantic Oscillation (NAO) teleconnection patterns are found to compare closely with those for the observed Madden–Julian oscillation (MJO). For both observations and theory positive NAO occurs 5–15 days after MJO convection [negative outgoing longwave radiation (OLR) and positive precipitation] and negative upper-troposphere velocity potential ISO anomalies are focused over the central Indian Ocean. The fluxes of wave activity, based on the upper-troposphere streamfunction of the leading theoretical mode, indicate strong tropical–extratropical interactions and have very similar structures to those obtained by H. Lin et al. based on observations of extratropical anomalies associated with MJO convection.

The second leading theoretical ISO mode for January 1979 has quite similar properties to the leading ISO mode but has a longer period of 44.5 days compared with 34.4 days and a more distinct quadrupole streamfunction structure straddling the equator. Theoretical ISO modes for other observed basic states, including January 1988 and the 30-yr average of January 1980–2009, again link the tropical ISO signal with Northern Hemisphere teleconnection patterns, particularly the NAO. The growth rates of ISO modes increase with stronger baroclinicity of the basic-state zonal winds in the main jet streams and, importantly, with increased tropical–extratropical interactions because of stronger meridional winds.

1. Introduction

The intraseasonal variability of the atmospheric circulation has been a topic of intense interest since the discovery by Madden and Julian (1971) of the tropical intraseasonal oscillation that now bears their name. The Madden–Julian oscillation (MJO) is the dominant mode of tropical variability in the 20–100-day range, and its effect is felt both in the tropics and the extratropics. Detailed reviews of the advances in understanding the intraseasonal variability of atmospheric circulations are presented by Zhang (2005) and Khouider et al. (2013).

Most theoretical proposals for the formation of the MJO have focused on the tropical signal of the oscillation. The early theories of the MJO proposed that it arose from Kelvin wave dynamics sustained through the wave-CISK (Lindzen 1974; Lau and Peng 1987; Chang and Lim 1988) or frictional wave-CISK (Wang and Rui 1990; Salby et al. 1994) mechanism. Emanuel (1987) and Neelin et al. (1987) proposed that the generation mechanism of the tropical MJO involves a feedback between the tropical zonal wind perturbation and evaporation. Majda and Stechmann (2009) sought an origin of the MJO in the equations for tropical Kelvin and Rossby wave dynamics; they employed a Matsuno–Gill-type model (Matsuno 1966; Gill 1980) coupled to prognostic equations for moisture and synoptic-scale wave activity.

In the studies of Frederiksen and Frederiksen (1993, hereafter FF93, 1997, hereafter FF97) and Frederiksen (2002, hereafter F2002) the genesis mechanism of intraseasonal oscillations was sought in coupled
tropical–extratropical dynamics. They used a linear two-level primitive equation model with observed basic state for January 1979 together with a Kuo-type cumulus convection parameterization (FF93; FF97) and, in F2002, as well an evaporation–wind feedback parameterization. They found growing modes that couple the extratropics with circa-30–60-day-period disturbances that have properties similar to the MJO. They noted that their coupled tropical–extratropical intraseasonal oscillations derive their first internal (baroclinic) mode structure in the tropics because of convective interaction with dynamics (CID; Neelin and Yu 1994). To obtain realistic tropical structures the tropical moist static stability needs to be relatively small compared with the dry static stability. The differences between moist intraseasonal oscillation (ISO) modes and dry ISO modes are detailed in FF93 (section 5d), FF97 (section 6; Fig. 4), and F2002 (section 5b). For moist ISO modes the moist static stability can be negative, as is characteristic of wave-CISK, but this is not a requirement since the generation mechanism is primarily moist baroclinic–barotropic instability. Indeed, the structures of the intraseasonal oscillations are arguably more realistic with a small positive moist static stability (FF93; F2002). A positive moist static stability avoids the ultraviolet catastrophe of wave-CISK theories for which the smallest resolved scale grows fastest.

Scale selection is instead governed by moist baroclinic–barotropic instability with a dominant zonal wavenumber \( m^* = 1 \) for the velocity potential and baroclinic, or vertical shear, streamfunction. Emanuel et al. (1994) and Yu et al. (1998) have argued on the basis of observational studies that the tropical atmosphere is stable to disturbances growing due to pure wave-CISK but that wind-induced surface heat exchange, also known as evaporation–wind feedback, provides another source of instability. Their mechanism depends on the assumption of a basic state with easterly flow, which is not valid everywhere in the tropics (Wang 1988).

Evaporation–wind feedback, with an observed basic state with both easterly and westerly flow (Figs. 1b and 1d), was included in the study of F2002 and the intraseasonal oscillation modes obtained with the three-dimensional basic state were arguably more realistic in their eastward propagation and structural properties. In F2002 the properties of the convectively coupled and equatorially trapped waves in the theoretical model were also analyzed and found to compare closely with the results of Takayabu (1994) and Wheeler et al. (2000) based on observations. In particular, the theoretical intraseasonal oscillation modes and the \( m^* = 1 \) Kelvin wave were distinctly different, as in the observational results.
In their original paper, Madden and Julian (1971) argued that their oscillation ‘‘cannot be an atmospheric Kelvin wave similar to those discussed in the recent literature.’’ The difficulties with ascribing the MJO to pure Kelvin wave dynamics have also been noted in the observational studies of Nishi (1989), Hsu et al. (1990), and Hsu (1996). Wheeler et al. (2000) found that the MJO is not a conventional trapped equatorial wave and that the MJO and Kelvin wave signals are distinct even at zonal wavenumber 1, with a spectral gap at a period of about 25 days.

There have been numerous analyses of the tropical–extratropical interactions associated with the MJO (Knutson and Weickmann 1987; Weickmann and Khalsa 1990; Schubert and Park 1991; Kiladis et al. 1994; Hsu 1996; Meehl et al. 1996; Matthews and Kiladis 1999; Straus and Lindzen 2000). In particular, the observational study of Straus and Lindzen (2000) showed that there was strong coherence between the subtropics and the tropics and supported the theory that planetary-scale baroclinic instability and the tropical MJO are connected. Further support for this idea has also come from model studies by Lin et al. (2007), Pan and Li (2008), Ray et al. (2009), Ray and Zhang (2010), and Ray and Li (2013).

Recently, there has been further important progress in elucidating this interaction between the tropics and extratropics in observational studies such as those of Zhou and Miller (2005), L’Heureux and Higgins (2008), Pan and Li (2008), Mori and Watanabe (2008), Lin et al. (2009, hereafter LBD2009), Riddle et al. (2013), and Zhao et al. (2013). These studies have established robust phase relationships between tropical convection on the intraseasonal time scale and the development of Pacific–North America (PNA), Arctic Oscillation (AO), or North Atlantic Oscillation (NAO) teleconnection patterns. It seems of interest to examine whether these same phase relationships hold for the theoretical intraseasonal oscillation modes. As well, in observational studies such as LBD2009 wave-activity fluxes have been calculated to characterize the tropical–extratropical interactions on the intraseasonal time scale. Again, we seek here to explore whether wave-activity fluxes for the theoretical intraseasonal oscillation modes have similar characteristics to those based on observations.

The plan of this paper is as follows. In section 2, we summarize how the theoretical intraseasonal oscillation modes were constructed and the basic state for January 1979 on which they grow. In section 3, the evolution of tropical convection and circulation anomalies associated with the Madden–Julian intraseasonal oscillation are examined based on observations for 30 northern winters. The upper-tropospheric velocity potential based on composites from the observations is compared with that from the theoretical intraseasonal oscillation at eight phases that summarize the cycle of evolution. Section 4 contains an analysis and comparison of extratropical circulation anomalies associated with the tropical convection of intraseasonal oscillations based on observations and from theory. In section 5, the similarities between the tropical and extratropical signals of intraseasonal variability, based on observations and the theoretical intraseasonal oscillation, are quantified by projections of the theoretical mode fields onto EOFs of extratropical variability, like the NAO, and velocity potential patterns strongly related to the two leading EOFs of outgoing longwave radiation (OLR). Section 6 focuses on the patterns of wave-activity fluxes of the theoretical intraseasonal oscillation mode, their implications for tropical–extratropical interactions on the intraseasonal time scale, and comparisons with the corresponding wave-activity fluxes based on observations by LBD2009. In section 7, we examine some of the properties of the second leading theoretical ISO mode for January 1979 with a period of 44.5 days. In section 8 we study how theoretical ISOs depend on the basic-state flow field by analyzing their properties for other basic states focusing on January 1988 and the 30-yr average of January 1980–2009. Our conclusions are summarized in section 9. Appendix A summarizes the equations for the linearized primitive equation model that are used to derive the theoretical intraseasonal oscillation modes while appendix B contains a description of the basic states used.

2. Theoretical intraseasonal oscillations for January 1979

The theoretical modes for the intraseasonal oscillations for which we make our primary comparisons with observational studies are taken from the study of F2002. There, intraseasonal oscillation modes were studied for a basic state corresponding to the global circulation for the month of January 1979 [based on European Centre for Medium-Range Weather Forecasts (ECMWF) analyses]. Figure 1 shows some of the important fields describing the circulation. Figure 1a shows the 300-hPa zonal wind while Fig. 1b shows the corresponding 700-hPa wind. Figure 1c shows the moist static stability that is positive everywhere so that wave-CISK is not possible. Figure 1d shows the evaporation structure function $f_{L}$ for the zonal wind defined in Eq. (A.3a). It has similar structure to the 700-hPa zonal wind but is more confined to the tropics. The 700-hPa zonal wind has westerlies, associated with the Australian monsoon, as well as easterlies and this is reflected in the evaporation structure function. The corresponding evaporation structure function
for the meridional wind defined in Eq. (A.3b) is shown in Fig. 1d of F2002. As discussed in more detail in FF93, 1979 was a time of large-amplitude intraseasonal oscillation and very active Australian monsoon, seen from the 700-hPa westerlies (Fig. 1b) in the tropics across and north of northern Australia. It was also a time of large-scale blocking in the Northern Hemisphere (NH) and a strong negative phase of the NAO.

With the flow fields for January 1979, some of which are shown in Fig. 1, F2002 considered four three-dimensional basic states in studies of the generation mechanisms of intraseasonal oscillations. A summary of the specification of these states is given in appendix B. Briefly, they consist of a DRY basic state employing the dry static stability and no evaporation wind feedback, the MOISTA basic state with a moist static stability that is everywhere positive but near zero in the equatorial regions and with no evaporation–wind feedback, an EVAP basic state that is like the MOISTA basic state but including evaporation–wind feedback, and a DISS basic state that is the same as the EVAP basic state but with much stronger dissipation.

The methodology for calculating the instability modes is summarized in appendix A. The generation of the modes as internal fluctuations or as the response to anomalous forcing is described in section 7 of Frederiksen and Webster (1988) and in section 5 of F2002.

Throughout the first six sections of this paper we focus on the leading intraseasonal oscillation mode for the EVAP basic state, with a period of 34.4 days and an e-folding time of 8.2 days, that was found in F2002 to be the most realistic leading mode. We also briefly discuss some properties of the second leading intraseasonal mode for the EVAP January 1979 basic state in section 7 and in section 8 we consider leading theoretical ISOs for other months, including January 1988 and January 1980–2009.

3. Evolution of tropical circulation anomalies

Empirical orthogonal function (EOF) analysis is often a useful way of capturing the essentially low-dimensional dynamics of many atmospheric processes. Wheeler and Hendon (2004, hereafter WH2004) characterized the evolution of convection, measured by OLR, and dynamics represented by 850- and 200-hPa winds, in terms of a combined EOF analysis of these three fields (normalized by their standard deviation). They found that the two leading EOFs based on data averaged over latitudes 15°S–15°N, which they termed Real-Time Multivariate MJO series 1 and 2 (RMM1 and RMM2), captured much of the intraseasonal variability in the equatorial sector. Figure 8 of WH2004, for example, shows the evolution of northern winter [December–February (DJF)] composite OLR and 850-hPa winds on the intraseasonal time scale based on data from 1979 to 2001. They also divided the MJO cycle of evolution into eight phases. In the phase space of the joint EOFs, RMM1 and RMM2, typical MJO events undergo evolutions like those shown in Fig. 7 of WH2004 and Fig. 2 of LBD2009.

For comparison with the results of F2002 on the evolution of the theoretical intraseasonal oscillation modes, it is useful to also examine how the corresponding observed velocity potential changes with the eight phases of the WH2004 index. The evolution of the 200-hPa velocity potential is shown in Fig. 2; the results are based on 30 northern winters (DJF) between 1979/80 and 2008/09 and pentad data derived from the daily averaged National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis data. In this study, the observed intraseasonal anomaly at a grid point was obtained by first removing the seasonal cycle, which is the time mean of the 30-yr pentad climatology, and then subtracting the seasonal average, which represents the interannual variability, following the method of LBD2009. At phase 1 the velocity potential is negative between circa 80°W and 80°E and positive between around 100°E and 130°W while negative OLR (WH2004) and positive rainfall anomalies (LBD2009) occur between 0° and 80°E in the equatorial sector. By phase 3 the negative velocity potential anomaly has its minimum at the equator over the central Indian Ocean, at essentially the same location as the corresponding negative OLR anomaly in Fig. 8 of WH2004 and the positive rainfall anomaly in Fig. 3 of LBD2009. At subsequent phases the velocity potential anomalies continue the eastward propagation, with increasing phase, across the equatorial sector.

In the theoretical study of F2002, the velocity potential of the intraseasonal oscillation is one of the primary fields describing the disturbance while the OLR and precipitation are not directly calculated. Thus, the observational results for the velocity potential in Fig. 2 are more directly applicable for comparison with the theoretical model results. In Fig. 3, we show the corresponding phase evolution of the 300-hPa velocity potential (determined as described in section 5) for the leading intraseasonal mode (mode 66) for the EVAP basic state. The mode consists of a number of wavenumbers, all with the same frequency (F2002); it is the fastest growing mode with $m^* = 1$ for the velocity potential and baroclinic, or vertical shear, streamfunction. The mode number is taken to increase with decreasing growth rate. We have found that for three-dimensional moist basic states there are many small-scale disturbances, such as storm-track systems, that can contribute to the evolution of the velocity potential.
modes, that grow faster than the leading intraseasonal modes with $m^* = 1$. This parallels the situation for zonally averaged dry basic states (e.g., Frederiksen 1978) where there may be of the order of 100 modes with $m > 1$ that grow faster than the leading mode with $m = 1$. However, in nonlinear simulations the smaller-scale modes saturate at lower amplitudes than the slower-growing larger-scale modes (e.g., Frederiksen 1981).

Comparing Fig. 3 with Fig. 2, we see that, at corresponding phases the broad features of the theoretical mode follow a similar evolution to that of the observational composite. The theoretical mode, however, has smaller-scale features imbedded in the largely zonal wavenumber–1 structure compared with the 30-yr composite. This is perhaps not surprising since compositing will tend to smooth out the higher-wavenumber features.
that occur in a single realization of the intraseasonal oscillation. Here, and throughout this paper, the non-dimensional units of the leading theoretical mode have been scaled down by a factor of 10 compared with those in F2002 so that the units in Fig. 3 can be interpreted as being in kilometers squared per second. Thus, the velocity potential of the theoretical mode has an appropriate magnitude at 300 hPa compared with the slightly larger amplitude of the observations at 200 hPa that are also in kilometers squared per second.

4. Evolution of extratropical circulation anomalies

As detailed in the introduction, recent observational analyses have found that the tropical MJO convection is associated with NH extratropical circulation anomalies that at certain phases correspond to PNA-like and NAO/AO teleconnection patterns. In particular, Fig. 4 of LBD2009 shows the phase evolution of composite 500-hPa geopotential height anomalies corresponding to tropical convection associated with the MJO in
different phases. Here, we describe the phase evolution of composite 300-hPa streamfunction anomalies that are directly comparable with the 300-hPa streamfunction of the theoretical mode intraseasonal oscillation for the EVAP basic state. These results are again based on pentad data for the 30 northern winters (DJF) from 1979/80 to 2008/09. Figure 4 shows the extratropical NH circulation anomalies at 300 hPa and at three phases. Results are depicted for phase 3 when the convection is focused over the central Indian Ocean and for the subsequent phase 4 and phase 5 (lagged by 5 and 10 days, respectively). We note from Fig. 4 that, at phases 3 and 4, there are wave trains extending from the northern central Pacific across North America. The wave trains, particularly at phase 4, are very reminiscent of the negative phase of the PNA teleconnection pattern, although the centers in Fig. 4 evolve with phase or time on an intraseasonal time scale and are slightly shifted from those defining the conventional PNA pattern (e.g., Wallace and Gutzler 1981). We term this transient wave train the intraseasonal PNA, or just PNA, in the following discussions, regardless of the slight shift compared to the conventional PNA. The positive NAO teleconnection pattern is clearly seen at phase 5, with a large negative anomaly centered over Greenland and positive anomalies stretching from North America to eastern Europe in a band centered on 40°N.

The anomalies in Fig. 4 have similar centers and structures to the 500-hPa geopotential height anomalies, based on 25 northern winters (DJF) from 1979/80 to 2003/04, shown in Fig. 4 of LBD2009 at phase 3 and lagged by multiples of 5 days, phase 4 (or phase 3, lag 1) and phase 5 (or phase 3, lag 2). LBD2009 also show the circulation anomalies for the essentially opposite phases—namely, phase 7, 8, and 1. At these respective phases the opposite signs of the teleconnection patterns are also seen if perhaps less clearly.

The 300-hPa streamfunction for the theoretical intraseasonal oscillation for the EVAP basic state is shown in Fig. 5 at phases 3–5. For this linear mode the fields at phases 7, 8, and 1, respectively, are just the negative of those in Fig. 5. Figure 5 shows very clear patterns corresponding to the negative phase of the PNA at phases 3 and 4 followed by a strong positive NAO teleconnection pattern at phase 5. With our current scaling, the units in Fig. 5, like those in Fig. 4, are kilometers per second and the amplitude of the theoretical circulation patterns are somewhat larger than for the composite in Fig. 4. We expect there are several reasons for the stronger extratropical streamfunctions for the theoretical mode. First, the January 1979 negative NAO circulation was particularly strong and that would be reflected in a strong instability in the extratropical circulation (Frederiksen

![Fig. 4. Extratropical NH 300-hPa streamfunction anomalies for reanalysis data, as in Fig. 2 at phases 3–5. Contour interval is 1.5 km² s⁻¹.](image)
and Bell 1990). Second, compositing of the observations would tend to mix strong and weak events that would destructively interfere at some locations because of phase shifts between ensemble members. This latter reason would tend to affect the smaller scales more than the larger scales and thus impact the broad scales of the velocity potential less than the more complex extratropical teleconnection patterns.

In this section and in section 3 we have analyzed the similarities between the leading theoretical intraseasonal oscillation for the EVAP basic state and composites of observed circulation anomalies on the intraseasonal time scale in the tropics and extratropics. We have noted the close correspondence between the broad structure and phase propagation of the tropical signal in the velocity potential in the theory and observations. Importantly, the extratropical circulation anomalies associated with particular phases of the tropical convection found in the observations are also seen to a large measure in the theoretical mode. In both theory and observations the negative phase of the PNA pattern occurs most clearly at phase 4 whereas the positive phase of the NAO/AO teleconnection pattern occurs at phase 5. This seems quite a remarkable result given that the observational analyses are based on composites over many northern winters and pertain mainly to the mature phase of the intraseasonal disturbances while the theoretical mode is linear, describes the genesis, and is for the single month of January 1979. In the following section, we provide a more quantitative analysis of these broad qualitative similarities in the structure and phase relationships of tropical–extratropical interaction on the intraseasonal time scale.

5. Circulation indices and empirical orthogonal functions

In this section, we examine how well the structures and evolution of the leading theoretical intraseasonal oscillation for the EVAP basic state compare with observational results based on EOF analysis. First, we have calculated the EOFs of OLR anomalies in the tropical region between 20°S and 20°N from the 30 northern winters (1979/80–2008/09) of pentad data. The structures of the two leading EOFs are shown in Fig. 6. The two patterns are in approximate quadrature, indicating that between them they represent a propagating mode. The leading EOF1 explains 11% of the variance and EOF2 explains 10%; these two EOFs are well separated from the higher-order EOFs according to the criterion of North et al. (1982) (EOF3 explains just 5% of the variance). The corresponding principal component PC1 has a correlation with RMM1 of 0.62 while PC2 has a

![Fig. 5](image_url)
correlation with RMM2 of 0.76. In fact, the eight phases based on the one-dimensional RMM1 and RMM2 (for latitudinal averages between 15°S and 15°N) are essentially the same as those based on the two-dimensional EOF1 and EOF2 in Fig. 6. The two leading EOFs represent the eastward-propagating MJO disturbance, in agreement with previous studies that performed similar EOF analysis of OLR data (e.g., Ferranti et al. 1990; Lin et al. 2010).

Next, from the leading EOFs of OLR anomalies and the corresponding PC time series we determine the 200-hPa velocity potential (\(\psi_{200}\)) patterns that correspond most closely to EOF1 and EOF2 of OLR through regression onto the PC1 and PC2 time series. The consequent velocity potential patterns, denoted EOF\(_x^1\) and EOF\(_x^2\), from regression onto PC1 and PC2 are shown in Fig. 7 for the domain 30°S–30°N. The patterns have been normalized to a magnitude corresponding to one standard deviation of the corresponding PC. Again, the velocity potential EOF\(_x^x\) patterns are essentially in quadrature. The velocity potential EOF\(_x^x\) patterns can then be used to quantify the similarity of the broad features of the velocity potential from observations on the intraseasonal time scale with the theoretical EVAP intraseasonal oscillation mode. The time evolution of the EOF\(_x^x\) patterns can be expressed by two time series, which are referred to as pattern time series (PTS1 and PTS2), and are calculated by projecting the velocity potential anomaly onto the EOF\(_x^x\) patterns.

We also aim to quantify the extent to which the extratropical streamfunction structure of the theoretical intraseasonal mode is similar to the NAO pattern. For this, we identify the NAO pattern as the second EOF in a rotated EOF analysis of monthly mean 500-hPa geopotential height anomalies following the method of LBD2009. This NAO pattern is shown in Fig. 8. It has a negative

![Fig. 6. Horizontal structures of (a) EOF1 and (b) EOF2 of OLR anomalies in the tropical region between 20°S and 20°N from 30 northern winters (1979/80–2008/09) of pentad data. The magnitude corresponds to one standard deviation of the PC. Contour interval is 3 W m\(^{-2}\).](image)

![Fig. 7. Horizontal structures of (a) EOF\(_x^1\) and (b) EOF\(_x^2\) of the 200-hPa velocity potential patterns in the tropical region between 30°S and 30°N from 30 northern winters (1979/80–2008/09) of pentad data. Contour intervals are 0.4 and 0.5 km\(^2\) s\(^{-1}\).](image)
anomaly centered over Greenland and positive anomalies in a band centered on 40°N from North America, across the Atlantic, to eastern Europe. The NAO has a high correlation with the AO of 0.73, based on NCEP data from 1950 to 2010, and differs from the AO primarily in the fact that the AO also has a positive anomaly centered on 40°N that stretches across the North Pacific Ocean and an extension of the Greenland low into the rest of the Arctic region [Fig. 6 of Zhou and Miller (2005); Fig. 5 of L’Heureux and Higgins (2008)].

We are now in a position to quantify the connection between the tropical and extratropical circulations of the theoretical intraseasonal mode and the velocity potential EOF\(_1\) and EOF\(_2\) in Fig. 7 and the NAO rotated EOF in Fig. 8. Figure 9 shows the evolution of the NAO index with respect to the phase of the velocity potential pattern time series (PTS1 and PTS2). By taking into account the lagged relationship between PTS1 and PTS2, and the corresponding location of tropical convection, eight MJO phases can be identified in a similar way as in WH2004, which are marked in Fig. 9. Here, we take the phase of PTS2 as a reference, which represents the time evolution of velocity potential EOF\(_2\) that has a dipole structure of convection with centers of opposite signs over the central Indian Ocean and the western Pacific. As discussed in Lin et al. (2010), the dipole diabatic heating structure of EOF2 is more effective in forcing the extratropical atmosphere than EOF1. We define

\[
\chi_{IO} = \text{EOF}_2 \times \sin \Phi
\]  

as a measure of the velocity potential, and implicitly a dipole convection centered over the central Indian Ocean and the western Pacific. The relationship between the eight phases and the angle \(\Phi\) in degrees is also shown in Fig. 9. When \(\Phi = -90^\circ\), between phases 2 and 3, \(\chi_{IO} = -\text{EOF}_2\), PTS2 reaches the minimum, and the convection is a maximum over the central Indian Ocean. We also note that the NAO index of the theoretical mode peaks at phase 5, corresponding to \(\Phi = 22.5^\circ\). This is in a good agreement with the observational result of section 4 and LBD2009 that a positive NAO occurs at MJO phase 5, about two pentads after PHASE 3. More generally the evolution of the leading theoretical mode as determined by the graphs in Fig. 9 indicates that it is able to capture some of the essentials of the complex tropical–extratropical interaction of intraseasonal variability that have been established in many recent observational studies discussed in the introduction.

6. Wave-activity flux and tropical–extratropical interaction

Next, we analyze the wave activity associated with the leading theoretical intraseasonal oscillation for the EVAP basic state. We define the flux vector of wave activity in the manner proposed by Takaya and Nakamura (2001), who discussed the advantages of their formulation including the lack of sensitivity to the particular phase of small-scale features. We consider the horizontal components of the wave-activity vector \(\mathbf{W}\) based on the 300-hPa basic-state vector winds \(\mathbf{U} = (U, V)\) and the
300-hPa streamfunction $\psi$ for the theoretical intraseasonal mode. These components are given by

$$W = \frac{1}{2 |U|} \begin{bmatrix} U(\psi^2_x - \psi\psi_{xx}) + V(\psi_x \psi_y - \psi\psi_{xy}) \\ U(\psi^2_y - \psi\psi_{yy}) + V(\psi_x \psi_y - \psi\psi_{xy}) \end{bmatrix},$$

(6.1)

where the subscripts denote partial derivatives with respect to $x = a \sin \phi = a \mu$ and $y = a \lambda$. Here, $a$ is Earth’s radius, $\lambda$ is longitude, and $\phi$ is latitude.

Figure 10 shows the wave-activity flow vectors overlaid on the contours of NH 300-hPa streamfunctions for the leading theoretical intraseasonal mode at phases 3–5. Note that for this linear mode the wave-activity flow vectors are the same at the respective phases 7, 8, and 1, for which the perturbation streamfunction have the opposite signs. With the current scaling, the wave-activity flux vectors may be taken to have units of meters squared per second squared and the streamfunction units of kilometers squared per second. We note that at phase 3 there are wave-activity flux vectors emerging from the tropics near 120°E that propagate into the area over the Gulf of Alaska and bifurcate with one branch heading poleward and then eastward and another heading southeastward near the western coast of North America. At phase 4 the propagation from the tropics near 120°E toward the region over the Gulf of Alaska is again evident with the wave activity then broadening and propagating eastward and southeastward over North America. The northern branch of the $W$ vector continues to the area over Greenland and from there strengthens as it propagates over northern Europe with a southeasterly branch reaching into the subtropics near 60°E.

At phase 5 (and phase 1) the branch of the wave-activity flux across the Pacific from the subtropics weakens while it strengthens across North America to Greenland with an intensified flux across western Europe into the subtropics near 60°E over the Indian Ocean. In these respects, there is good agreement with the observations as shown in Fig. 5 of LBD2009. At phase 6 (and phase 2) the wave-activity flux has broadly similar features to that for phase 5 including the vectors into the region of the Indian Ocean. Such southeastward fluxes of wave activity (at phases 1 and 2) may contribute to the genesis of the tropical signal of ISO over the Indian Ocean.

The leading ISO mode has a streamfunction with largest magnitudes in the winter hemisphere (Fig. 3a of F2002 shows the 500-hPa streamfunction at phase 5) in...
agreement with the observed oscillation (Knutson and Weickmann 1987; Hendon and Liebmann 1990; Hendon and Salby 1994) and this is reflected in the wave fluxes. However, it does have a train of Rossby waves emanating from the southern Indian Ocean that propagate across Australia with one branch heading southward and another toward the equator. The associated wave fluxes (not shown) follow the two branches of dispersion with the equatorward branch contributing to extratropical–tropical interaction. Ray and Zhang (2010) in a model study of MJO initiation in April and Zhao et al. (2013) in a multiyear observational study for November–April and model study have previously noted that wave fluxes associated with Southern Hemisphere Rossby waves may contribute to extratropical–tropical interaction, but in their cases the fluxes are into the area over the southern Indian Ocean and may contribute to MJO initiation there.

7. Second leading intraseasonal oscillation
for January 1979

In this section, we briefly discuss the properties of the second-fastest-growing theoretical ISO mode for the EVAP basic state for January 1979. This is mode 83, which is the second leading mode with \( m^* = 1 \) for the velocity potential and baroclinic or vertical shear streamfunction. It has a period of 44.5 days and an \( e \)-folding time for growth of 10.2 days compared with 34.4 and 8.2 days, respectively, for mode 66 in the previous sections. Figure 11 shows the 300-hPa streamfunction and velocity potential, as well as the baroclinic (vertical shear) streamfunction and baroclinic zonal wind at phase 3 when the convection and negative 300-hPa velocity potential have largest magnitude over the Indian Ocean. The nondimensional units in Fig. 11 (and Fig. 12 below) can be interpreted as being in kilometers squared per second for streamfunctions and velocity potential; the wind can be taken as in meters per second by dividing by 6.37 (Earth’s radius/1000). Again, the velocity potential (Fig. 11b) has a wavenumber-1 structure within which are embedded smaller-scale structures. The 300-hPa streamfunction (Fig. 11a) again has wave trains across much of the NH including the Pacific–North American, North Atlantic, and European–Eurasian regions. In the subtropics and tropics there is evidence of a quadrupole straddling the equator, within which is embedded smaller-scale features. The quadrupole is also seen in the baroclinic streamfunction (Fig. 11c) while the baroclinic zonal wind has large magnitudes in the tropics and subtropics with eastward (between 80°E and international date line) and westward (between the international date line and 80°E) components along the equator. With the opposite sign of the mode, corresponding to phase 7, there are some broad similarities
between the streamfunctions and the zonal wind, in the tropics and subtropics, and the multiyear composite in Fig. 2 of Kiladis et al. (2005). As expected, their multiyear composite exhibits fewer of the small-scale features of the single instability mode. The quadrupole structure in Fig. 11c is more distinct than that for mode 91 for the MOISTA basic state in Fig. 7b of FF93, or for mode 66 for the EVAP basic state (not shown), although it is somewhat enhanced with stronger dissipation, as for mode 13 for the DISS basic state (not shown).

The second leading ISO mode for the EVAP January 1979 basic state has a qualitatively similar evolution to the leading mode of the previous sections with the positive NAO/AO pattern prominent in the NH 300-hPa perturbation streamfunction between 10 and 25 days after phase 3, as shown in Fig. 12a after 20 days.

8. Intraseasonal oscillation modes for other basic states

Next, we examine the properties of theoretical ISOs growing on other basic states. A variety of northern winter basic states for the months December–February have been examined. For the sake of brevity, and for elucidating the causes of the differences in growth rates for different basic states, it is convenient to focus on results for January.

We consider first the leading ISO for January 1988, which, as noted by Wheeler and McBride (2005), was a time of strong MJO activity. Here, and for other basic states considered, we use parameters corresponding to those for the EVAP January 1979 basic state (appendixes A and B), so that the moisture destabilization [Eq. (A.2)] and evaporation structure functions [Eq. (A.3)], as well as the dissipation, are comparable. In particular, the moist static stability (not shown) is positive everywhere with local minima in the equatorial regions of between 2 and 3 K, as is the case for the EVAP January 1979 basic state (Fig. 1c). The jets are also located in similar positions to those in Figs. 1a and 1b.

The leading ISO theoretical mode for January 1988 is mode 83, with a period of 33.3 days and an $e$-folding time of 8.0 days, which compare well with 34.4 and 8.2 days, respectively, for the leading ISO for the EVAP January 1979 basic state. In other respects, their properties also are broadly similar, including being the respective fastest growing modes with $\gamma^* = 1$ for the velocity potential and baroclinic (vertical shear) streamfunction. Figure 12b shows the NH 300-hPa perturbation streamfunction for the leading January 1988 ISO 12 days after phase 3. Again, the NAO/AO pattern for this ISO mode is broadly similar to that shown in Fig. 5c.

Wheeler and McBride (2005) note that many of the properties of the MJO are best seen in case studies for
a given month in a particular year since intraseasonal activity may be quite variable. Nevertheless, insights into how the growth of ISO modes depends on the basic state may be obtained by considering multiyear basic states that have less prominent planetary wave structures. We have examined average basic states for the same 30-yr time period used for the reanalyzed observations in the earlier sections, focusing here on January 1980–2009. The leading ISO for this basic state is mode 112 with a period of 37.5 days and an e-folding time of 13.2 days. Again, it is the fastest growing mode with $m^* = 1$ for the velocity potential and baroclinic streamfunction. The NH 300-hPa perturbation streamfunction for this mode is shown in Fig. 12c at 12 days after phase 3. Again, a structure with broad similarities with the NAO/AO pattern is present at this time.

One might enquire as to the cause of the lower growth rate of the leading ISO mode for the 30-yr-average basic state of January 1980–2009 compared with those for January 1979 and January 1988. Some of the most significant differences between the January 1988 and January 1980–2009 basic states are shown in Fig. 13, which depicts the baroclinic (vertical shear) zonal and meridional winds for the two cases. Comparing Figs. 13a and 13c, we see that the extrema in the baroclinicity of the zonal wind generally have larger magnitudes, between 30° and 150°W, for the January 1988 basic state. Even larger differences in the baroclinicity of the meridional wind are seen in Figs. 13b and 13d, particularly in the Western Hemisphere reflecting the stronger planetary wave structure of the January 1988 basic state including in the Pacific–North American, North Atlantic, and European–Eurasian regions. Again, the strength of the mean, or 500-hPa, planetary wave structure is larger for the 1988 basic state compared with the 1980–2009 state (not shown).

Our results indicate that the growth rates of ISOs depend on the baroclinicity of the basic-state zonal winds in the main jet streams and, perhaps as much or more, on the tropical–extratropical interaction, indicated by the strength of the baroclinic and barotropic meridional winds. These conclusions are supported by analysis of the January 1979 basic state for which the planetary wave strength is similar to that of the January 1988 state. Both of these months were times of strong MJO activity (Hayashi and Golder 1993, their Fig. 2; WH2004, their Fig. 5) while in some other months (in the 1980–2009 period), such as January 1982, it was considerably weaker. This is reflected in the longer e-folding time of 11.8 (12.4) days for the theoretical ISO mode 106 (110) for January with a period of 34.2 (44.8) days. Our results are generally consistent with those of Lin et al. (2007), Ray et al. (2009), Ray and Zhang (2010), Ray and Li (2013), and Zhao et al. (2013), who emphasize the
importance of tropical–extratropical interactions in the
initiation of the MJO.

9. Discussion and conclusions

In this article, we have examined the phase relationships between the tropical signal of intraseasonal oscillations and the development of extratropical wave-train structures analogous to the PNA and NAO/AO teleconnection patterns. These connections, between the tropics and extratropics, have been analyzed for MJO convection and anomalous circulations based on 30 years of northern winters (DJF) and for the leading theoretical intraseasonal oscillation modes that were obtained in the instability study of F2002 with a January 1979 basic state in a primitive equation model. We have followed WH2004 and divided the intraseasonal oscillation cycle of evolution into eight phases. At phase 1 the observations indicate negative OLR (WH2004) and positive rainfall anomalies (LBD2009) that occur between 0° and 80°E in the equatorial sector. By phase 3 the convection, characterized by negative OLR and positive precipitation anomalies, is focused over the central Indian Ocean. We have examined the corresponding evolution of the 200-hPa velocity potential which, at phase 1, is negative between circa 80°W and 80°E and positive between around 100°E and 130°W. By phase 3, the negative velocity potential anomaly has its minimum at the equator over the central Indian Ocean, at essentially the same location as the corresponding negative OLR anomaly and positive rainfall anomaly. The velocity potential subsequently propagates eastward across the equatorial sector as the phase increases. The theoretical intraseasonal oscillation of F2002, with the EVAP basic state, including both cumulus convection and evaporation–wind feedback, follows a similar evolution to the observational composite.

Again, the evolution of the Northern Hemisphere 300-hPa streamfunction for the theoretical intraseasonal oscillation and observed anomalies are qualitatively similar. At phase 4, both show wave trains extending from the northern central Pacific to across North America, corresponding to the negative phase of an intraseasonal PNA-like pattern. At phase 5 both theory and observational anomalies produce an NAO teleconnection pattern with a large negative anomaly focused over Greenland and positive anomalies stretching from North America to eastern Europe in a band centered on 40°N. The similarity between the theoretical intraseasonal mode and observed anomalies, in their tropical and extratropical fields, has been quantified. The theoretical mode has been projected onto the NAO pattern (specified as the second rotated EOF in a rotated EOF analysis of monthly mean 500-hPa geopotential height anomalies), and its velocity potential has been projected onto velocity potential patterns related to the leading EOFs of OLR. It has been shown that the evolution of the theoretical mode captures some of the essentials of the complex tropical–extratropical interactions of intraseasonal variability that have been established in many recent observational studies discussed in the introduction.

We have also calculated the flux vector of wave activity of Takaya and Nakamura (2001) based on the 300-hPa streamfunction at eight phases in the evolution of the theoretical intraseasonal oscillation mode. We have found that at phases 3 and 4 there are wave-activity flux vectors emerging from the tropics near 120°E that propagate into the area over the Gulf of Alaska. At phases 5 and 6 (and phases 1 and 2) the branch of the wave-activity flux across the Pacific from the subtropics weakens while it strengthens across North America to Greenland with an intensified flux across western Europe into the subtropics near 60°E over the Indian Ocean. These results are also indicative of tropical–extratropical interactions and are in good qualitative agreement with the wave-activity flux vectors, based on observations of extratropical anomalies associated with MJO convection, calculated by LBD2009 (Fig. 5). Our results suggest that the southeastward fluxes of wave activity (at phases 1 and 2) may contribute to the genesis of the tropical signal of the MJO over the Indian Ocean.

We have also analyzed the properties of the second-leading theoretical ISO for the January 1979 EVAP basic state. It has a longer period of 44.5 days and grows more slowly with an e-folding time of 10.2 days compared with the leading ISO. The second leading ISO has quite a distinct quadrupole structure straddling the equator in the 300-hPa and vertical shear streamfunctions. Its properties in other fields are broadly similar to those of the leading ISO mode including extratropical PNA-like and positive NAO/AO that evolve at about 10 days after phase 3.

We have examined the properties of leading ISO theoretical modes for a wide variety of northern winter basic states. We have focused on January 1988 and on the 30-yr average of January 1980–2009 for which the leading ISOS have periods of 33.3 and 37.5 days, respectively. Both modes have positive NAO patterns that peak 12 days after phase 3. Their growth rates, however, differ considerably with the mode for January 1988 having an e-folding time of 8.0 days while that for January 1980–2009 has one of 13.2 days. The cause of the lower growth rates for January 1980–2009 has been found to be the reduced baroclinicity of the basic-state zonal winds between 30° and 150°W and, importantly, the reduction in the tropical–extratropical interaction
due to the reduction in the strength of the meridional winds. These results add to those of FF93 and F2002 in which the dependence of ISO growth on the moist static stability and evaporation–wind feedback were considered.

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**APPENDIX A**

**Model Details**

The theoretical intraseasonal oscillation modes analyzed in this paper have been obtained using the two-level linearized primitive equation model described in F2002 and in more detail by Frederiksen and Frederiksen (2007). It includes a generalized Kuo-type heating parameterization (Kuo 1974) that incorporates closures for cumulus convection and evaporation–wind feedback. The linearized primitive equations are described in terms of a mean perturbation streamfunction $\psi$, which is the average between the values at the upper (level 1) and lower (level 3) levels; a vertical shear perturbation streamfunction $\chi$, which is half the difference between the upper- and lower-level values; the lower-level perturbation velocity potential $\theta$, which is equal to minus the upper-level velocity potential; and the mean and vertical shear perturbation potential temperatures $\sigma$ and $\sigma$, respectively.

The heating profile for the generalized Kuo-type heating is assumed to project completely onto the internal mode dynamics (F2002). The heating due to cumulus convection and surface evaporation then only enters the linearized mean potential temperature equation and is given by

$$\tilde{Q}_\theta = -\tilde{\nabla}^2 \chi + C_F (u^{(3)} f_u + \nu^{(3)} f_v),$$

(A.1)

where the basic-state moisture destabilization parameter is

$$\tilde{\nu} = Q_F \tilde{\nu},$$

(A.2)

and the evaporation structure functions are

$$f_u = (\tilde{\nu}_s - \tilde{\nu}) u^{(3)}/[(\tilde{\nu}^{(3)})^2 + (\tilde{\nu}^{(3)})^2]^{1/2},$$

(A.3a)

$$f_v = (\tilde{\nu}_s - \tilde{\nu}) \nu^{(3)}/[(\tilde{\nu}^{(3)})^2 + (\tilde{\nu}^{(3)})^2]^{1/2}.$$  

(A.3b)

Here, the parameter $C_F$ is an effective evaporation parameter and $Q_F$ is an effective cumulus heating parameter (F2002). Also, $u^{(3)}(\tilde{\nu}^{(3)})$ and $\nu^{(3)}(\tilde{\nu}^{(3)})$ are the lower-level zonal and meridional perturbation (basic state) velocities and $\tilde{\nu}$ is the basic-state specific humidity at the lower level.

The first term in Eq. (A.1) is the cumulus convection parameterization and the second term is the evaporation–wind feedback mechanism. With these closures included, the linearized mean perturbation potential temperature equation becomes

$$\frac{\partial \theta}{\partial t} = -J(\tilde{\nu}, \theta) - J(\psi, \tilde{\nu}) - J(\sigma, \sigma) - J(\tau, \sigma) + \nabla \cdot \sigma \nabla \chi + \nabla \cdot \sigma \nabla \chi - \frac{1}{C_F} \frac{1}{\sigma - \tilde{\nu}} (u^{(3)} f_u + \nu^{(3)} f_v),$$

(A.4)

where the overbar refers to the basic state and the notation is as in section 2 of Frederiksen and Frederiksen (1992). The equations for the other perturbation fields $\psi$, $\tau$, $\chi$, and $\sigma$ are unchanged and the complete set of nondimensionalized equations is given in F2002. For a slowly varying basic-state static stability, we can combine the moisture destabilization parameter with the dry static stability term in Eq. (A.4) and define a moist static stability by

$$\sigma_m = \sigma - \tilde{\nu}.$$  

(A.5)

The modes are obtained by first expanding each of the perturbation fields and basic-state fields in terms of spherical harmonics with the perturbations also having a time dependence $\exp(-i \omega t)$. Here, $t$ is the time and $\omega = \omega_r + i \omega_i$ is the complex angular frequency with $\omega_r$ being the frequency and $\omega_i$ being the growth rate. Then, a system of eigenvalue–eigenvector equations is obtained from which the modal structures and their frequencies and growth rates are calculated in the manner described in Frederiksen and Frederiksen (1992). The resolution of the calculations is rhomboidal 15.

**APPENDIX B**

**Basic States for January 1979**

As discussed in section 2, the flow fields for the basic states used are the monthly averaged global fields for January 1979, taken from ECMWF analyses for the First Global Atmospheric Research Program (GARP) Global Experiment (FGGE) Special Observing Period 1. The 300- and 700-mb fields are taken as representative of the upper and lower levels, respectively. The study of F2002...
considered four three-dimensional basic states denoted DRY, MOISTA, EVAP, and DISS (and corresponding zonally averaged basic states). The DRY case has $Q_F = 0 = C_F$ and the MOISTA case has $Q_F = 1000 K$ and $C_F = 0$; here, we focus on the EVAP basic state for which $Q_F = 1000 K$ and $C_F = 5 \times 10^{-4} \text{K m}^{-1}$ in dimensional units. These cases also have drag and biharmonic dissipation with dimensional values of $K = 8.39 \times 10^{-7} \text{s}^{-1}$ and $K' = 2.338 \times 10^{16} \text{m}^4 \text{s}^{-1}$. The DISS case is the same as the EVAP case, but with 8 times the biharmonic diffusion.

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