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ABSTRACT

In Part II of this study, a new formulation of the spectral energy budget of moist available potential energy (MAPE) and kinetic energy is derived. Compared to previous formulations, there are three main improvements: (i) the Lorenz available potential energy is extended into a general moist atmosphere, (ii) the water vapor and hydrometeors are taken into account, and (iii) it is formulated in a nonhydrostatic framework. Using this formulation, the mesoscale MAPE spectra of the idealized mei-yu front system simulated in Part I are further analyzed.

At the mature stage, the MAPE spectra in the upper troposphere and lower stratosphere also show a distinct spectral transition in the mesoscale: they develop an approximately $2/3$ spectral slope for wavelengths longer than 400 km and $-5/3$ spectral slope for shorter wavelengths. In the upper troposphere, mesoscale MAPE is mainly deposited through latent heating and subsequently converted to other forms of energy at the same wavenumber. At wavelengths longer than roughly 400 km, the conversion of MAPE to horizontal kinetic energy (HKE) dominates, while at shorter wavelengths, the mechanical work produced by convective systems primarily adds to the potential energy of moist species and only secondarily generates HKE. However, this secondary conversion is enough to maintain the mesoscale $-5/3$ HKE spectral slope. Another positive contribution comes from the divergence term and the vertical flux. In the lower stratosphere, the main source of mesoscale MAPE is the conversion of HKE, although the vertical flux and the spectral transfer also have notable contributions.

1. Introduction

This is the second part of an investigation into the dynamics of the mesoscale energy spectra of the mei-yu front system. In Peng et al. (2014, hereafter Part I), we investigated the different physical mechanisms responsible for the mesoscale horizontal kinetic energy (HKE) spectra of the mei-yu front system in the upper troposphere and lower stratosphere. That study employed a spectral HKE budget analysis of an idealized mei-yu front system simulated by the Weather Research and Forecasting model (WRF v3.2) (Skamarock et al. 2008). To derive the spectral HKE budget equation suitable for convective systems, such as the mei-yu front system, we extended the pseudoincompressible approximation, which was proposed by Durran (1989) as an improved representation of the anelastic approximation, to a general moist atmosphere (see appendix A of Part I in detail). Spectral HKE budget analysis demonstrated that the mesoscale HKE in the upper troposphere is deposited through the buoyancy flux and removed by the advective nonlinearity and vertical pressure flux divergence, and that the mesoscale HKE in the lower stratosphere is deposited through the advective nonlinearity and vertical pressure flux divergence and removed by the
buoyancy flux. Additionally, we found that the spectrum of the upper-tropospheric buoyancy flux during the mature phase of a mei-yu front system has a peak at scales of around 300 km and a plateau throughout the mesoscale. In fact, the positive buoyancy flux spectrum implies different conversion rates of available potential energy (APE) to kinetic energy at each wavelength. In Part II, we further investigate the APE spectra of the mei-yu front system, specifically addressing how moist processes affect the spectral budget of APE with a focus on the spectral conversion of APE to HKE.

Building on the work of Margules (1910), Lorenz (1955) formulated the concept of APE as the small, active portion of the potential energy available for conversion into kinetic energy. He further derived an approximate expression for the APE, showing it to be proportional to the horizontal variance of the temperature fluctuations. Since then the concept of APE has been widely used to investigate the large-scale dynamics and the atmospheric general circulation (e.g., Boer 1989; Siegmund 1994; Steinheimer et al. 2008; Boer and Lambert 2008; Marques and Castanheira 2012; Zuo et al. 2012). However, a key limitation arises from the fact that the original concept of APE (and the corresponding discussion of the maintenance of the atmosphere circulation) has been focused on dry circulations. An early attempt to extend the discussion of kinetic energy generation to moist atmospheres was made by Lorenz (1978), who proposed the concept of moist available potential energy (MAPE) and the atmospheric general circulation (e.g., Boer 1989; Siegmund 1994; Steinheimer et al. 2008; Boer and Lambert 2008; Marques and Castanheira 2012; Zuo et al. 2012). However, a key limitation arises from the fact that the original concept of APE (and the corresponding discussion of the maintenance of the atmosphere circulation) has been focused on dry circulations. An early attempt to extend the discussion of kinetic energy generation to moist atmospheres was made by Lorenz (1978), who proposed the concept of moist available potential energy (MAPE) and the atmospheric general circulation (e.g., Boer 1989; Siegmund 1994; Steinheimer et al. 2008; Boer and Lambert 2008; Marques and Castanheira 2012; Zuo et al. 2012). However, a key limitation arises from the fact that the original concept of APE (and the corresponding discussion of the maintenance of the atmosphere circulation) has been focused on dry circulations. An early attempt to extend the discussion of kinetic energy generation to moist atmospheres was made by Lorenz (1978), who proposed the concept of moist available potential energy (MAPE) and the atmospheric general circulation (e.g., Boer 1989; Siegmund 1994; Steinheimer et al. 2008; Boer and Lambert 2008; Marques and Castanheira 2012; Zuo et al. 2012). However, a key limitation arises from the fact that the original concept of APE (and the corresponding discussion of the maintenance of the atmosphere circulation) has been focused on dry circulations.

To further investigate the energetics of the mei-yu front system, typically a moist nonhydrostatic system, it is desirable to derive nonhydrostatic spectral energy budget equations taking into account the effects of water vapor and hydrometeors. A formulation meeting these requirements is derived in section 2. Section 3 presents a brief overview of the idealized mei-yu front system simulated in Part I. Section 4 presents the mesoscale moist available potential energy (MAPE) spectra of the mei-yu front system, and discusses the sensitivity of the mesoscale MAPE \(-5/3\) spectra slope to the latent heating. Section 5a provides a diagnostic analysis of spectral MAPE budget, while section 5b presents the spectral conversions among different forms of energy, with section 5c presenting a further analysis of the MAPE nonlinear term. Conclusions are given in section 6.

2. Formulation of the spectral available potential energy budget

2a. Governing equations in z coordinates

Consider atmospheric matter to consist of dry air, water vapor, cloud water, cloud ice, rainwater, etc. Let \(q_v\) denote the mixing ratio of water vapor and \(q_m = q_c, q_r, \ldots\) denote the mixing ratios of cloud water, cloud ice, rainwater, and any other hydrometeors, respectively. The total mixing ratio \(q_t\) is given by \(q_t = q_v + q_c + q_r + \ldots\). Here the mixing ratio is defined as the mass of moist species per unit mass of dry air.

Defining the Exner pressure by

\[
\Pi = \left(\frac{P}{P_0}\right)^{R_d/\rho}, \tag{1}
\]

the state equation for moist air can be expressed as

\[
\rho = \rho_d R_d T + \rho_v R_v T = \rho_d R_d T \left(1 + \frac{R_v}{R_d} q_v\right)
= \rho_d R_d \pi \theta \left(1 + \frac{R_v}{R_d} q_v\right), \tag{2}
\]
where \( p \) is the pressure, \( T \) is the temperature, \( \theta \) is the potential temperature, \( \rho_d \) is the density of the dry air, \( \rho_v \) is the density of the water vapor, \( R_d \) is the gas constant for dry air, \( R_h \) is the gas constant for water vapor, \( c_p \) is the specific heat of dry air at constant pressure, and \( \rho_0 \) is a reference surface pressure. Defining a modified potential temperature as in Part I (appendix A) by

\[
\theta_m = \theta \left( 1 + \frac{R_h}{R_d} q_v \right) \simeq \theta (1 + 1.61 q_v),
\]

the state equation for moist air can be further written as

\[
p = \rho_0 \left( \frac{R_h}{R_d} \right) c_v \left( \theta_m \right),
\]

where \( c_v \) is the specific heat of dry air at constant volume and \( \alpha_d = 1/\rho_d \) is the specific volume of dry air. From Eq. (4), it is clear that \( \theta_m \) is the potential temperature that dry air would have if its pressure was equal to that of the given sample of moist air while its specific volume remains unchanged. There are two unique advantages to this formulation that are worth noting: (i) the state of moist air is completely determined by two independent variables \( \alpha_d \) and \( \theta_m \), while the effect of water vapor is entirely contained in \( \theta_m \) and (ii) the mass of dry air is a conserved quantity. Consequently, it will be convenient to solve the prognostic equations with \( \alpha_d \) as a state variable.

The total thermodynamic fields can be divided into a time-invariant, hydrostatically balanced, dry reference state and a perturbation as follows:

\[
p = \bar{p}(z) + p' ; \quad \pi = \bar{\pi}(z) + \pi' ; \\
\rho_d = \bar{\rho}_d(z) + \rho'_d ; \quad \theta_m = \bar{\theta}(z) + \theta'_m.
\]

Taking the logarithm and the gradients of Eq. (1) yields

\[
\begin{align*}
\nabla \bar{p} &= c_p \rho'_d \bar{\theta} \nabla \bar{\pi} \\
\frac{\partial \bar{p}}{\partial z} &= c_p \rho'_d \bar{\theta} \frac{\partial \bar{\pi}}{\partial z}
\end{align*}
\]

Substituting the hydrostatically balanced dry reference state into Eq. (6), we obtain

\[
\frac{\partial \bar{\pi}}{\partial z} = \frac{g}{c_p \bar{\theta}}.
\]

Defining \( A = (1 + q_v + q_c + q_l + q_r \ldots)^{-1} \simeq 1 - (q_v + q_c + q_l + q_r + \cdots) = 1 - q_v \), we obtain

\[
\rho = \rho_d A^{-1}
\]

and

\[
\begin{align*}
\frac{1}{\rho} \frac{\partial p}{\partial z} - g &= -c_p A \frac{\partial \pi'}{\partial z} - c_p A \frac{\partial \bar{\pi}}{\partial z} - g \\
&= -c_p A \frac{\partial \pi'}{\partial z} + (1 - q_v) \frac{\partial \bar{\pi}}{\partial z} - g \\
&= -c_p A \frac{\partial \pi'}{\partial z} + \frac{\partial \rho}{\partial \bar{\theta}} g - g q_v.
\end{align*}
\]

Using the above relations, the governing equations for the mei-yu front system [Eqs. (1)–(7) in Part I] can then be recast as follows:

\[
\begin{align*}
\frac{\partial u}{\partial t} &= - (u \cdot \nabla u + w \frac{\partial u}{\partial z}) - c_p \frac{\partial \pi'}{\partial x} + f(v - V_g) + D_u, \\
\frac{\partial v}{\partial t} &= - (u \cdot \nabla v + w \frac{\partial v}{\partial z}) - c_p \frac{\partial \pi'}{\partial y} - fu + D_v, \\
\frac{\partial w}{\partial t} &= - (u \cdot \nabla w + w \frac{\partial w}{\partial z}) - c_p \frac{\partial \pi'}{\partial z} + \frac{\partial \rho}{\partial \bar{\theta}} g - g q_v + D_w, \\
\frac{\partial \theta'_m}{\partial t} &= - (u \cdot \nabla \theta'_m + w \frac{\partial \theta'_m}{\partial z}) - \frac{\partial \bar{\theta}}{\partial z} + H_m + G_m + D_m, \\
\frac{dq_m}{dt} &= S_{q_m} + D_{q_m},
\end{align*}
\]

and

\[
\begin{align*}
\frac{dp}{dt} + \frac{\rho_d}{dt} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) &= 0.
\end{align*}
\]

Here \( \mathbf{u} = (u, v) \), where \( u \) and \( v \) are horizontal velocities along the zonal and meridional directions, respectively; \( w \) is the vertical velocity; \( g \) is the gravitational acceleration; and \( V_g \) is the meridional geostrophic wind velocity.

In (13) \( H_m = (1 + 1.61 q_v) \bar{\theta}_g + 1.61 \theta S_{q_v} \) is the diabatic influence, which includes the diabatic contributions to potential temperature and water vapor; \( G_m = (1 + 1.61 q_v) \bar{\theta}_g + 1.61 \theta \bar{q}_v \) with \( \bar{\theta}_g = -u(\theta \bar{\theta}_g)_{LS} \) and \( \bar{q}_v = -u(\theta \bar{q}_v)_{LS} \) represents the large-scale forcing of the geostrophic potential temperature \( \bar{\theta}_g \) and water vapor \( \bar{q}_v \); and \( D_m = (1 + 1.61 q_v) D_{\theta} + 1.61 \theta D_{q_v} \) represents the contributions of diffusion. Also, \( D_{q_v} \) denotes the dissipation of \( \varphi \) (with \( \varphi \) a placeholder for the fields \( u, \ v, \ etc. \) and is given as \( D_{\varphi} = [K_h (\varphi^2 + \varphi_y^2) + K_v \varphi_z^2] \varphi + 2^{-6}(2 \Delta t)^{-1} \beta (\varphi^2 + \varphi_y^2) \varphi \), where the eddy viscosities \( K_h \) and \( K_v \) are given by the turbulent kinetic energy closure method, \( \Delta t \) is the time step, and the
parameter $\beta = 0.12$ (Skamarock et al. 2008). The diffusion acts on all the three components of winds, on potential temperature, on all moisture variables, etc. Additionally, the eddy viscosities used for scalars ($\theta$, $q_v$, $q_m$) are divided by a turbulent Prandtl number $Pr = \frac{1}{\beta}$.

The terms $S_\theta$, $S_{q_v}$, and $S_{q_m}$ represent the diabatic contribution to $d\theta/dt$, $dq_v/dt$, and $dq_m/dt$, respectively.

b. Moist available potential energy and its spectral budget equation

The original concept of Lorenz APE can be easily extended to moist atmospheres by using the modified potential temperature defined as Eq. (3). Thus, moist available potential energy (MAPE) per unit mass can be defined as

$$E_A = \frac{g^2}{2N^2} C \frac{\partial^2 \theta^'}{\partial t^2},$$

where $N^2 = g\beta \ln \bar{\theta}/\partial z$ is the Brunt–Väisälä frequency.

Energy spectra are computed by using a two-dimensional discrete cosine transform (denoted by DCT, as in Part I) at each vertical level. Let $\phi(k)$ be the DCT of the field $\phi$, where $k = (k_x, k_y)$ is the horizontal wave vector. To facilitate the comparison between the simulated energy spectra and observations (e.g., Gage and Nastrom 1986; Lindborg 1999), we consider energy spectra per unit mass. The MAPE spectrum per unit mass can be expressed as

$$E_A(k) = \frac{1}{2} [\gamma(z) \hat{\theta}^'_m(k) \hat{\theta}^'_m(k)] - \frac{1}{2} \gamma(z) \hat{\theta}^'_m(k) \hat{\theta}^'_m(k),$$

with $\gamma(z) = g^2/(N^2z^2)$.

Taking the partial derivative of Eq. (17) with respect to time, we obtain

$$\frac{\partial}{\partial t} E_A(k) = \gamma(z) \hat{\theta}^'_m(k) \frac{\partial \hat{\theta}^'_m(k)}{\partial t}.$$  

Substituting Eq. (13) into Eq. (18) allows the spectral MAPE budget to be written as

$$\frac{\partial}{\partial t} E_A(k) = T_A(k) - C(k) + H_A(k) + G_A(k) + D_A(k),$$

where the nonlinear term, the conversion term of MAPE to other forms of energy, the latent heating term, the large-scale forcing term, and the MAPE diffusion term are

$$T_A(k) = -\gamma(z) \hat{\theta}^'_m(k) \text{DCT}(u \cdot \nabla \hat{\theta}^'_m + w \partial_z \hat{\theta}^'_m).$$

Combining Eqs. (21), (22), and (24) in Part I and with some modifications, the spectral HKE budget can be written as

$$\frac{\partial}{\partial t} E_h(k) = T_h(k) + F_p^\dagger(k) + C_{E_a \rightarrow E_h}(k) + H_h(k) + G_h(k) + D_h(k),$$

where $T_h(k) = -\hat{u} \cdot \text{DCT}(u \cdot \nabla \hat{\theta}^'_m + w \partial_z \hat{\theta}^'_m)$ is the horizontal kinetic energy transfer due to advection and

$$F_p^\dagger(k) = -\frac{c_p}{\rho_d} \frac{\partial p_\dagger}{\partial z} \partial \hat{w} \hat{\theta}^'_m$$

is the pressure correlation term, which corresponds to the flux of inertial gravity wave (IGW) energy; $C_{E_a \rightarrow E_h}(k) = c_p \partial \hat{w} \partial_z \hat{\theta}^'_m$ is the conversion of MAPE to HKE at wave vector $k$; $H_h(k) = c_p H_m \hat{\theta}^'_m$ is the direct effect associated with the latent heating; $G_h(k) = c_p G_m \hat{\theta}^'_m$ is the direct effect due to the large-scale forcing; and $D_h(k) = \hat{u} \hat{D} + \hat{v} \hat{D} v$ is the diffusion term.

The large-scale motions are dominated by the horizontal kinetic energy components, whereas the contribution of the vertical kinetic energy (VKE) component increases with decreasing wavelength (Bierdel et al. 2012). Especially in mesoscale convective systems (e.g., mei-yu front, tropical cyclones), the vertical velocity can reach the same order of magnitude as horizontal velocities (e.g., Fig. 5 in Part I). To reveal the interactions between the conversion terms, $C(k)$ in Eq. (19) and $C_{E_a \rightarrow E_h}(k)$ in Eq. (26), we introduce the VKE. The VKE per unit mass is defined as

$$C(k) = \frac{g \hat{\theta}^'_m}{\partial}.$$
The spectral VKE budget can be derived as follows:

$$\frac{\partial}{\partial t} E_z(k) = \dot{w}(k) \frac{\partial w(k)}{\partial t}.$$  

Substituting Eq. (12) into Eq. (28), we obtain

$$\frac{\partial}{\partial t} E_z(k) = T_z(k) + C_{E_a \rightarrow E_z}(k) + D_z(k),$$

where $T_z(k)$ is the vertical kinetic energy transfer due to the total advection,

$$T_z(k) = -\dot{w} \text{DCT}(\mathbf{u} \cdot \mathbf{v}_w + w \dot{\varphi} w),$$

$C_{E_a \rightarrow E_z}(k)$ is the conversion of MAPE to VKE,

$$C_{E_a \rightarrow E_z}(k) = g \dot{w} \cdot \frac{\partial \theta_m}{\partial \theta} - c_r \dot{\theta}_m \dot{\varphi} w \dot{\varphi}_z,$$

and $D_z(k) = w \dot{\varphi} D_n$ is the dissipation term.

Thus, $C(k) = C_{E_a \rightarrow E_0}(k) + C_{E_a \rightarrow E_z}(k) + C_{E_a \rightarrow E_p}(k)$, where $C_{E_a \rightarrow E_0}(k) = g \dot{w} \dot{q}_p$ denotes the conversion of MAPE to the potential energy of total moist species ($E_0$).

Based on the above equations, the individual components of the available energetics in a general moist atmosphere are summarized in Fig. 1. The MAPE in convective systems is partly used to lift moist species to the level at which it precipitates—namely, to increase the potential energy of moist species, and only partly to generate kinetic energy.

d. Further decomposition of the nonlinear terms

Because of the presence of vertical flux and a divergent component in the flow, the nonlinear terms $T_A(k)$, $T_b(k)$, and $T_z(k)$ do not act solely to redistribute energy among scales. In the following, the vertical flux and divergence component terms are separated from the between-scale spectral transfer term. In the present paper, we focus on the nonlinear term $T_A(k) = \gamma \dot{\theta}_m \text{DCT}(\mathbf{v} \cdot \mathbf{v}_m + w \partial_z \mathbf{v}_m)$. The nonlinear term $T_A(k)$ can be decomposed as follows (for details, see appendix):

$$T_A(k) = t_A(k) + F_{A_1}(k) + d_A(k) + J(k),$$

where $t_A(k)$ is the spectral transfer term due to nonlinear interactions,

$$t_A(k) = -\gamma \dot{\theta}_m \text{DCT}(\mathbf{v} \cdot \mathbf{v}_m + \frac{1}{2} \dot{\varphi}_m \mathbf{v} \cdot \mathbf{v}_m)$$

$$+ \frac{1}{2} \gamma \left[ \partial_z \dot{\theta}_m \text{DCT}(w \theta'_m) - \dot{\varphi}_m \text{DCT}(w \dot{\varphi}_m) \right].$$

$F_{A_1}(k)$ is the vertical convergence of the MAPE vertical flux,

$$F_{A_1}(k) = -\frac{\partial}{\partial z} \left[ \gamma \dot{\theta}_m \text{DCT}(w \theta'_m)/2 \right],$$

$d_A(k)$ is the spectral tendency due to the divergence of the 3D flows (henceforth the divergence term),

$$d_A(k) = \frac{1}{2} \gamma \dot{\theta}_m \text{DCT} \left[ \frac{1}{\theta_r} (H_m + G_m) \theta_m \right]$$

$$- \frac{1}{2} \gamma \dot{\varphi}_m \text{DCT} \left[ \frac{1}{\theta_r} \frac{\partial \theta}{\partial z} \theta_m \right],$$

and $J(k)$ is the spectral tendency due to adiabatic processes that do not conserve the sum of the MAPE and the total kinetic energy (henceforth the nonconservation term),

$$J(k) = \frac{\partial}{\partial z} \left[ \gamma \dot{\theta}_m \text{DCT}(w \theta'_m)/2 \right].$$

It is straightforward to show that the sum of $t_A(k)$ over all wave vectors $k$ is equal to $-\gamma \int_{S} |\mathbf{v} \cdot (\mathbf{u} \theta'_m)|^2 dS = -\int_{S} (\gamma/2) |\theta'_m|^2 \mathbf{v} \cdot \mathbf{n} dL$, where $S$ represents the horizontal domain, $L$ represents the lateral boundaries of $S$, and $\mathbf{n}$ denotes the unit vector pointing along the outward normal to $L$. If the spectral analysis is conducted on either a global area or a limited area with doubly periodic lateral boundaries, $-\gamma \int_{S} |\mathbf{v} \cdot (\mathbf{u} \theta'_m)|^2 dS = 0$, meaning that this transfer term $t_A(k)$ is exactly conservative and only redistributes energy among the different scales at a specific level.

To sum up, in contrast to the previous formulations, here the Lorenz available potential energy has been...
extended into a general, moist, nonhydrostatic atmosphere and the water vapor and hydrometeors are fully taken into account.

3. Overview

Based on the WRF model, Part I simulated an idealized mei-yu front system and obtained good agreement with the observed structure and temporal evolution. As observed, the simulated mei-yu front system in the control simulation (CNTL) exhibited the three distinct stages: the early phase \( t = 15 \text{–} 25 \text{~h} \) characterized by very strong vertical convection, the mature phase \( t = 26 \text{–} 38 \text{~h} \) possessing relatively weak vertical convection, and the late phase \( t = 39 \text{–} 48 \text{~h} \) when the vertical convection strengthens again. The simulated mei-yu front system also exhibited an upper-level divergence flow pattern and quasi-stationary precipitation with “core gape” structure.

In Part II, we utilize the same model simulations (CNTL and sensitivity experiments) at the same 5-km resolution used in Part I. Because Part I showed many 2D structures of the simulated mei-yu front, here we will focus on the modified potential temperature perturbation and hydrometeors. Figure 2 presents vertical cross sections of zonal-mean modified potential temperature perturbation and total mixing ratio of hydrometeors \( \hat{\theta}_m, \Sigma q_m \) at \( t = 12 \text{ and } 26 \text{~h} \) for CNTL. Negative contours are dashed. Only a 600 km \( \times \) 15 km of the domain is present.

**Fig. 2.** Vertical cross sections of (a),(b) zonal-mean potential temperature perturbation \( \theta' \) (1.0-K contour interval) and water vapor mixing ratio \( q_v, 2 \text{~g} \text{~kg}^{-1} \) shaded), and (c),(d) zonal-mean modified potential temperature perturbation \( \hat{\theta}_m \) (1.0-K contour interval) and total mixing ratio of hydrometeors \( \Sigma q_m \) shaded) at (a),(c) \( t = 12 \text{ and } (b),(d) \) \( t = 26 \text{~h} \) for CNTL. Negative contours are dashed.
to the level where condensation occurs. The phase change of water is accompanied by latent heat release (Fig. 2b), which significantly increases the $\theta'$ in the upper atmosphere. By $t = 26$ h (Fig. 2d), the modified potential temperature perturbation in the upper troposphere peaks at about 7.5 km with a maximum value of 6.9 K. Moreover, the nonuniformity of the latent heating results in a modified potential temperature with typical mesoscale features, which implies an increasing of mesoscale MAPE. In what follows, we look at the MAPE spectra in the upper troposphere and lower stratosphere.

4. Mesoscale moist available potential energy spectra of the mei-yu front system

a. Moist available potential energy spectra

Spectra as a function of $\mathbf{k}$, where $k_h = |\mathbf{k}| = \sqrt{k_x^2 + k_y^2}$, are constructed as outlined in Eq. (19) of Part I; that is, $E_A(k_h) = \sum_{\Delta k = 2\pi/|\mathbf{k}| < \Delta k} E_A(\mathbf{k})/\Delta k$ with $\Delta k = \pi/(\Delta N)$, where $\Delta$ is the horizontal grid spacing and $N = \min(N_x, N_y)$ in which $N_x, N_y$ are the number of grid points in the $x$ and $y$ directions, respectively.

The vertical profiles of $N^2$ and the dimensional prefactor $\gamma(z)/2$ (Gage and Nastrom 1986) for the simulated mei-yu front system are plotted in Fig. 3.

As shown in Fig. 4b of Part I, positive latent heating mainly takes place under the height of 12 km and peaks at about 8 km. The horizontal wavenumber MAPE spectra $E_A(k_h)$, averaged over $z = 0$–5, 5–10, and 12–15 km (henceforth lower troposphere, upper troposphere, and lower stratosphere, respectively) from $t = 0$ to 48 h for the CNTL simulation are plotted in Fig. 4. Overall, the evolution of the MAPE spectra is similar to that of the HKE spectra (Fig. 8 in Part I), which closely follows the convection pattern. Due to the effects of spinup, the MAPE spectra also undergo a significant adjustment at all the levels during the first 2 h. A rapid enhancement of the MAPE spectra in the troposphere commences around $t = 12$ h, following the onset of convection. In both the lower and upper troposphere, the MAPE spectra first grow significantly for $k_h > 2\pi \times 10^{-5}$ rad m$^{-1}$, corresponding to wavelengths less than 100 km, with the spectral range of MAPE growth extending to longer
MAPE spectrum develops a distinct peak around wavelength 330 km, which is consistent with the HKE spectrum. In the upper troposphere, the MAPE spectrum approaches a $-3.6$ spectral slope over the large-scale end of the mesoscale ($400 \leq \lambda \leq 1000$ km) and transitions to a shallower $-1.8$ spectral slope at smaller scales ($40 \leq \lambda \leq 400$ km). (Spectral slopes are estimated here and elsewhere by a least squares power fit over a given wavenumber range.) In the lower stratosphere, the MAPE spectrum also shows an apparent transition in the mesoscale: it develops an approximately $-3.8$ spectral slope over wavelengths $400 \leq \lambda \leq 1000$ km, and transitions to a roughly $-1.9$ spectral slope at smaller scales $40 \leq \lambda \leq 400$ km.

c. Comparison with observations

Figure 6 presents further comparison between the simulated upper-tropospheric and lower-stratospheric energy spectra and the reference spectrum obtained from the Measurement of Ozone and Water Vapor by Airbus In-Service (MOZAIC) aircraft observations by Lindborg (1999). In the upper troposphere (Fig. 6a), the HKE spectrum is in remarkably good agreement with the Lindborg spectrum over wavelengths $100 \leq \lambda \leq 1000$ km, although the level of HKE around the transition scale 400 km is lower than observations. The shape of the MAPE spectrum resembles that of HKE spectrum at all but the dissipation scales, while the amplitude of the former is much lower than the latter. In the lower stratosphere (Fig. 6b), a similar result can be observed, but the HKE spectrum over wavelengths $40 \leq \lambda \leq 400$ km is slightly steeper than the reference spectrum and consequently has much lower amplitude. At both levels, the ratio of mesoscale HKE to MAPE spectrum over wavelengths $40 \leq \lambda \leq 1000$ km is approximately 4. This ratio is larger than the observation in the real Nastrom–Gage spectrum, where the ratio of kinetic to potential energy spectrum is approximately 2 (e.g., Gage and Nastrom 1986). However, we argue that this disagreement may be not due to the WRF version used in this study, but to the weak baroclinicity of the simulated mei-yu front system, as this observation in the real Nastrom–Gage spectrum has been reproduced well in the baroclinic waves simulated with the same model by Waite and Snyder (2009) (see their Fig. 10).

d. Sensitivity to the latent heating

In Part I, we pointed out the importance of moist processes on the maintenance of the $-5/3$ spectral slope of the mesoscale HKE. The latent heating released by phase changes of water vapor adjusts the modified potential temperature and feeds the mesoscale MAPE, which in turn can enhance the buoyant production of...
HKE. Thus, moist processes firstly influence the MAPE. To further highlight the direct effects of the latent heating, we investigated the evolution of MAPE spectra simulated in an experiment with the latent heating turned off at $t = 24$ h (HEAT24hoff), which in all other respects is configured the same as the CNTL simulation. Not unexpectedly, the MAPE for wavelengths shorter than the transition scales in the troposphere (Figs. 7a,b) and stratosphere (Fig. 7c) decreases rapidly relative to the CNTL simulation after the latent heating is turned off. After about 12 h, the mesoscale MAPE spectra no longer show the distinct spectral transition. In conclusion, the increased MAPE due to the latent heating is partly used to maintain the $-\frac{5}{3}$ spectral slope of the mesoscale MAPE, and partly converted to kinetic energy, which is used to maintain the $-\frac{5}{3}$ spectral slope of the mesoscale HKE.

5. Spectral moist available potential energy budget and the spectral conversion

a. Spectral moist available potential energy budget

To gain more insight into the dynamics of the MAPE spectra, spectral analysis of the moist available potential energy budget is presented in this section. Following Eq. (19), Fig. 8 presents the horizontal wavenumber spectra of the nonlinear term $T_A(k_h)$, the conversion term of MAPE to other forms of energy $C(k_h)$, the latent heating term $H_A(k_h)$, the large-scale forcing term $G_A(k_h)$, and the residual term $\Delta(k_h) = H_A(k_h) - C(k_h)$, averaged in the vertical over the upper troposphere and lower stratosphere and in time over $t = 15-25$ h and $t = 26-38$ h. Here, $t = 15-25$ and $26-38$ h indicate the early phase and mature phase of the mei-yu front system, respectively.

For $t = 15-25$ h, the strong direct forcing from latent heating occurs throughout the troposphere but peaks at about 8 km (Fig. 4b in Part I). In the upper troposphere (Fig. 8a), the dominant balance is between $H_A(k_h)$ and $C(k_h)$. Mesoscale MAPE is mainly deposited by latent heating at each horizontal wavenumber $k_h$ directly converted to other forms of energy at that wavenumber. Another small positive contribution to the MAPE comes from the nonlinear term. Although the nonlinear contribution to the MAPE is much smaller than the positive contribution from the latent heating term, it is comparable in magnitude to $\Delta(k_h)$. At wavelengths $100 \leq \lambda \leq 1000$ km, there appears to be a secondary balance between the residual term (negative) and the nonlinear term (positive); namely, the MAPE added by the nonlinear term is largely converted to other forms of energy. In addition, the large-scale forcing terms only have a weak, positive contribution to MAPE at large scales. In the lower stratosphere (Fig. 8b), where no direct forcing from latent heating occurs, the effect of the latent heating term is so weak that it can be neglected. The effect of $C(k_h)$ is to add MAPE at all wavelengths. That is to say, the other forms of energy, which are seen in Fig. 9b to be due primarily to the HKE, are converted to MAPE at large scales. In the lower stratosphere, the vast majority of this conversion occurs at the large scales exceeding 1000 km. In the mesoscale (less than the transition scale of 400 km) the...
conversion is characterized by a peak around wavelength 250 km. However, the nonlinear spectrum $T_A(k_h)$ in this band has a somewhat intricate shape. It reaches a minimum around $\lambda \approx 660$ km, crosses zero three times between $\lambda \approx 400$ and $\lambda \approx 200$ km, and increases to reach a small positive plateau between $\lambda \approx 200$ and $\lambda \approx 40$ km. Note that the wavelengths less than 40 km belong to the dissipation scales.

The time period $t = 26–38$ h is characterized by weakening convection. In the upper troposphere of the mei-yu front system (Fig. 8c), the dominant balance is still between $H_A(k_h)$ and $C(k_h)$, but the amplitude of these two terms is much weaker than earlier. In the lower stratosphere (Fig. 8d), overall the features of $C(k_h)$ are quite similar to the results shown in Fig. 8b, except that it reaches a plateau between 400 and 200 km.

However, the nonlinear term in the mature phase (Fig. 8d) is quite different from that in the early mature phase (Fig. 8b), especially at the larger scales. A further investigation of the nonlinear term will be given in section 5c.

b. Spectral conversion among different forms of energy

In previous studies (e.g., Koshyk and Hamilton 2001; Augier and Lindborg 2013), the spectral energy budget equation was usually formulated in the dry framework and hydrostatic balance is assumed. As a result, the latent heat release had to be regarded as an external energy source; thus only the conversion between dry available potential energy and horizontal kinetic energy could be considered. From Fig. 1, one can clearly see that moist available potential energy in moist convection systems, such as the mei-yu front system, can be converted to horizontal kinetic energy, vertical kinetic energy, or potential energy of moist species. To quantify these three conversion processes in the simulated mei-yu front system, we further calculate the horizontal wavenumber spectra of the conversion terms $C_{EA\rightarrowEH}(k_h)$, $C_{EA\rightarrowE}(k_h)$, and $C_{EA\rightarrowE_z}(k_h)$. The corresponding results are shown in Fig. 9.

In the upper troposphere, at both the early (Fig. 9a) and mature phase (Fig. 9c) of the mei-yu front system, the conversion of MAPE to HKE is dominant for large wavelengths, while the conversion of MAPE to potential energy of moist species is dominant for the smaller scales. More specifically, at wavelengths less than around 400 km, the mechanical work produced by convective systems is primarily used to add to the potential energy of moist species and only secondarily to generate HKE. Similar findings were reported by Pauluis et al. (2000). However, this secondary conversion is enough to maintain the mesoscale $^{2/3}$ HKE spectra. In the lower stratosphere (Figs. 9b,d), at both times, $C(k_h)$ is negative for almost all wavelengths, which indicates the conversion of the other forms of energy to MAPE. The conversion term, $C_{EA\rightarrowEH}(k_h)$, is also negative and its magnitude is comparable to, or even larger than, $C(k_h)$. Thus, the primary source of MAPE in the lower stratosphere is due to the conversion of HKE.

Furthermore, both in the upper troposphere and in the lower stratosphere, the conversion term $C_{EA\rightarrowE_z}(k_h)$ is always positive. This indicates that moist species always consume the mechanical work produced by convective systems. Thus the role of moist processes is proved to be twofold: (i) adding to the MAPE through the latent heating release and (ii) converting the MAPE to the potential energy of moist species, most of which is dissipated during precipitation.
c. Further analysis of the nonlinear term

Owing to the presence of vertical flux and a divergent component in the flow, the nonlinear terms $T_A(k)$ cannot act solely to redistribute energy among scales. This is especially true in the upper troposphere where the nonlinear term adds MAPE to all wavenumbers at the early phase in the evolution of the mei-yu front system (Fig. 8a). Further insight into the vertical flux is gained by integrating the vertical convergence of the cumulative vertical flux of MAPE $P_d(\kappa h)$ and $P_c(\kappa h)$:

$$
\Pi_d(\kappa h) = \int_{\kappa h}^{k_{\text{max}}} d_A(\kappa h) \, d\kappa h, \quad (37)
$$

and the spectral flux of MAPE

$$
\Pi_c(\kappa h) = \int_{\kappa h}^{k_{\text{max}}} \tau_A(\kappa h) \, d\kappa h, \quad (38)
$$

where $k_{\text{max}}$ denotes the largest wavenumber.

Analogously, we can also define the cumulative divergent term

$$
\Pi_d(\kappa h) = \int_{\kappa h}^{k_{\text{max}}} F_A(\kappa h) \, d\kappa h, \quad (39)
$$

and the cumulative nonlinear term

$$
\Pi_T(\kappa h) = \int_{\kappa h}^{k_{\text{max}}} T_A(\kappa h) \, d\kappa h. \quad (40)
$$

FIG. 8. Horizontal wavenumber spectra of the nonlinear term $T_A(\kappa h)$ (black), large-scale forcing term $G_A(\kappa h)$ (blue), the conversion term $-C(\kappa h)$ (red), the latent heating term $H_A(\kappa h)$ (green), and the residual term $\Delta(\kappa h) = H_A(\kappa h) - C(\kappa h)$ (dotted line) for MAPE, averaged in the vertical over the (a),(c) upper troposphere and (b),(d) lower stratosphere and in time over (a),(b) $t = 15–25$ h and (c),(d) $t = 26–38$ h. The asterisk represents any one of the five terms $T_A(\kappa h), G_A(\kappa h), -C(\kappa h), H_A(\kappa h)$, and $\Delta(\kappa h)$. The plotted spectra (*) are multiplied by $k_h$ to preserve the area in log-linear coordinates.
Figure 10 presents the cumulative nonlinear term $P_T(k_h)$, spectral flux of MAPE $P_t(k_h)$, vertical convergence of the cumulative vertical flux of MAPE $P_F(k_h)$, and cumulative divergent term $P_d(k_h)$ over the whole $k_h$ range.

In the upper troposphere, during both time periods (Figs. 10a,c) the positive contribution of the nonlinear term to MAPE is dominated by the divergent term and the MAPE vertical flux. However, in the lower stratosphere, at both times (Figs. 10b,d), the nonlinear term is dominated by the MAPE vertical flux and the spectral transfer term, especially for the smaller scales $40 \leq \lambda \leq 400$ km.

In our idealized simulations conducted in limited-area domains, the lateral boundaries are periodic in $x$ and open in $y$, so the net MAPE flux integrated over the whole domain is not equal to zero. Since we cannot estimate the effect of the lateral boundaries on energy transfer, we hesitate to draw any strong conclusions about the existence of a mesoscale inverse MAPE cascade in our simulations, although our results indicate that it does occur to some degree in the upper troposphere (i.e., $\Pi_t < 0$ in Figs. 10a,c).

6. Summary

In Part II of this study, we extend the concept of Lorenz APE to include moisture effects by defining the moist available potential energy, and derive a new formulation of the spectral energy budget of MAPE and kinetic energy (horizontal and vertical). Compared to previous formulations, there are three main improvements: (i) the Lorenz available potential energy is extended into a general moist atmosphere, (ii) the water vapor and hydrometeors are taken into account, and (iii) it is formulated in a nonhydrostatic framework. This new formulation shows that moist available potential energy in a general moist atmosphere, such as in a mei-yu front system, can be converted to horizontal kinetic...
energy, vertical kinetic energy, or potential energy of moist species (Fig. 1). Using these spectral equations, the mesoscale MAPE spectra of the idealized mei-yu front system simulated in Part I are further analyzed.

During the mature phase of the mei-yu front system, the MAPE spectra in the upper troposphere and lower stratosphere also show a distinct spectral transition at the mesoscale: they develop an approximately $2\frac{5}{3}$ spectral slope for wavenumbers $k_0$, where $k_0 = \pi/2 \times 10^{-5}$ rad m$^{-1}$ corresponds to a wavelength of roughly 400 km. Specifically, the upper-tropospheric MAPE spectrum develops a $-3.6$ spectral slope over the large-scale end of the mesoscale ($400 \leq \lambda \leq 1000$ km) and transitions to a shallower $-1.8$ spectral slope at smaller scales ($40 \leq \lambda \leq 400$ km). Concurrently, the lower-stratospheric MAPE spectrum develops a roughly $-3.8$ spectral slope over the wavelengths $400 \leq \lambda \leq 1000$ km and roughly a $-1.9$ spectral slope at smaller wavelengths $40 \leq \lambda \leq 400$ km. Thus, the MAPE spectra of the mei-yu front system exhibit nearly the same spectral slopes and spectral break as the HKE spectra. Similar to the behavior of the HKE spectra, about 12 h after the latent heating is artificially turned off, the mesoscale MAPE spectra no longer show a distinct spectral transition. This suggests that, in the mei-yu front system, the MAPE injected through the latent heating is partly used to maintain the mesoscale MAPE $2\frac{5}{3}$ spectral slope, and is partly converted to kinetic energy, which acts to maintain the mesoscale HKE $2\frac{5}{3}$ spectral slope.

To further investigate the dynamics of the mesoscale MAPE spectra of the mei-yu front system, spectral MAPE budgets for various height ranges were analyzed and compared. In the upper troposphere, the dominant balance is between $H_A(k_h)$ and $C(k_h)$. This means that mesoscale MAPE is mainly deposited through the latent
heating term and subsequently converted to other forms of energy at the same wavenumber. Another positive, although weaker, contribution to the MAPE comes from the nonlinear term, which is dominated by the divergence term and the MAPE vertical flux. Although the nonlinear contribution to the MAPE is much smaller than the positive contribution from the latent heating term, it is comparable in magnitude to the residual term, \( \Delta(k_h) \). While in the lower stratosphere, the primary source of mesoscale MAPE is due to the conversion of HKE. The nonlinear term is dominated by the MAPE vertical flux and the spectral transfer term, especially for the smaller scales of \( 40 \leq \lambda \leq 400 \) km.

Further analysis of spectral conversion among different forms of energy demonstrates that in the upper troposphere, during both the early mature and mature phase of the evolution of the mei-yu front system, the conversion of MAPE to HKE is dominant for large wavelengths (larger than around 400 km), while at wavelengths less than around 400 km, the mechanical work produced by convective systems is primarily used to add to potential energy of moist species, with lesser amounts going to generate HKE. These results are similar to the findings reported by Pauluis et al. (2000). However, this secondary conversion is strong enough to maintain the \(-5/3\) spectral slope of the mesoscale HKE spectra. Thus, the effects of the moist processes are twofold: 1) they add to the MAPE through the latent heating release and 2) they convert MAPE to the potential energy of moist species, most of which is dissipated during precipitation.

Further work should consider the possible effects of the higher vertical and horizontal resolutions, the convection schemes, the diffusion schemes, etc. A further comparison between the spectral energy budget of the mei-yu front system and that of moist baroclinic waves should be also made.

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**APPENDIX**

**The Decomposition of the Nonlinear Terms**

The nonlinear term \( T_A(k) \) in Eq. (19) can be decomposed as follows:

\[
T_A(k) = -\gamma \hat{\theta}_m^r \text{DCT}(u \cdot \nabla \theta'_m + w \partial_z \theta'_m) \\
= -\gamma \hat{\theta}_m^r \text{DCT}(u \cdot \nabla \theta'_m + \theta'_m \nabla \cdot u/2) - \gamma \hat{\theta}_m^r \text{DCT}[w \partial_z \theta'_m + \partial_z (w \theta'_m) - \theta'_m \partial_z w - \theta'_m \nabla \cdot u]/2 \\
= -\gamma \hat{\theta}_m^r \text{DCT}(u \cdot \nabla \theta'_m + \theta'_m \nabla \cdot u/2) + \gamma [\partial_z \hat{\theta}_m^r \text{DCT}(w \theta'_m) - \hat{\theta}_m^r \cdot \text{DCT}(w \theta'_m)]/2 \\
+ \frac{\partial \ln g}{\partial z} [\gamma \hat{\theta}_m^r \text{DCT}(w \theta'_m)/2] \\
+ \gamma \hat{\theta}_m^r \text{DCT}[(\partial_z w + \nabla \cdot u)/2] \\
+ \gamma \hat{\theta}_m^r \text{DCT}(\theta'_m \partial_z w + \nabla \cdot u)/2 \\
\]

Using the pseudoincompressible equation [e.g., Eq. (A10) in Part I],

\[
w \partial_r (\vec{p} \hat{\theta}) - \partial_z (\vec{v} \cdot u + \partial_w) = \frac{1}{\theta} (H_m + G_m), \quad (A2)
\]

the divergence term \( d_A(k) \) can be further expressed as

\[
d_A(k) = \frac{1}{2} \gamma \hat{\theta}_m^r \text{DCT} \left[ \frac{1}{\theta} (H_m + G_m) \theta'_m \right] \\
- \frac{1}{2} \gamma \hat{\theta}_m^r \text{DCT} \left[ \frac{1}{\theta} \partial_r \vec{p} \hat{\theta} - w \theta'_m \right]. \quad (A3)
\]

Analogously, the nonlinear terms \( T_h(k) \) in Eq. (26) and \( T_c(k) \) in Eq. (29) also can be decomposed as

\[
T_h(k) = t_h(k) - \frac{\partial [\text{DCT}(w \theta)/2]}{\partial z} + d_h(k) \quad (A4)
\]

and

\[
T_c(k) = t_c(k) + F_{z \perp}(k) + d_c(k), \quad (A5)
\]

where the spectral transfer terms of HKE and VKE, the divergent terms of HKE and VKE, and the vertical convergence of the VKE vertical flux are
\[ t_h(k) = -\hat{u}DCT\left( \mathbf{u} \cdot \mathbf{v}u + \frac{1}{2} \mathbf{u} \mathbf{v} \cdot \mathbf{u} \right) \]
\[ + \frac{1}{2} \left[ \hat{\partial}_z \hat{DCT}(\mathbf{w}u) - \hat{u}DCT(\mathbf{w} \hat{\partial}_z \mathbf{u}) \right], \quad (A6) \]
\[ t_z(k) = -\hat{w}DCT\left( \mathbf{u} \cdot \mathbf{v}w + \frac{1}{2} \mathbf{w} \mathbf{v} \cdot \mathbf{u} \right) \]
\[ + \frac{1}{2} \left[ \hat{\partial}_z \hat{DCT}(\mathbf{w}w) - \hat{w}DCT(\mathbf{w} \hat{\partial}_z \mathbf{w}) \right], \quad (A7) \]
\[ d_h(k) = \frac{1}{2} \hat{u}DCT\left[ \frac{1}{\theta}(H_m + G_m)\mathbf{u} \right] \]
\[ - \frac{1}{2} \hat{u}DCT\left[ \frac{1}{\theta} \frac{\partial \theta}{\partial z} \mathbf{w} \right], \quad (A8) \]
\[ d_z(k) = \frac{1}{2} \hat{w}DCT\left[ \frac{1}{\theta}(H_m + G_m)\mathbf{w} \right] \]
\[ - \frac{1}{2} \hat{w}DCT\left[ \frac{1}{\theta} \frac{\partial \theta}{\partial z} \mathbf{w} \right], \quad (A9) \]

and
\[ F_{z1}(k) = -\hat{DCT}(\mathbf{w}w)/2 \frac{\partial}{\partial z}. \quad (A10) \]

Specifically, the vertical convergence of the total HKE vertical flux \( F_{z1}(k) \) is the sum of the pressure correlation term \( F_{p1}(k) \) and the turbulent HKE flux
\[ F_{h1}(k) = F_{p1}(k) = \frac{\partial (\mathbf{u} \cdot \mathbf{v} DCT(\mathbf{w}u))/2}{\partial z}. \quad (A11) \]

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