A Theoretical Study on the Spontaneous Radiation of Inertia–Gravity Waves Using the Renormalization Group Method. Part II: Verification of the Theoretical Equations by Numerical Simulation

YUKI YASUDA AND KAORU SATO
Department of Earth and Planetary Science, University of Tokyo, Tokyo, Japan

NORIHIKO SUGIMOTO
Department of Physics, Keio University, Kanagawa, Japan

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ABSTRACT

The renormalization group equations (RGEs) describing spontaneous inertia–gravity wave (GW) radiation from part of a balanced flow through a quasi resonance that were derived in a companion paper by Yasuda et al. are validated through numerical simulations of the vortex dipole using the Japan Meteorological Agency nonhydrostatic model (JMA-NHM). The RGEs are integrated for two vortical flow fields: the first is the initial condition that does not contain GWs used for the JMA-NHM simulations, and the second is the simulated thirtieth-day field by the JMA-NHM. The theoretically obtained GW distributions in both RGE integrations are consistent with the numerical simulations using the JMA-NHM. This result supports the validity of the RGE theory. GW radiation in the dipole is physically interpreted either as the mountain-wave-like mechanism proposed by McIntyre or as the velocity-variation mechanism proposed by Vüede. The shear of the large-scale flow likely determines which mechanism is dominant. In addition, the distribution of GW momentum fluxes is examined based on the JMA-NHM simulation data. The GWs propagating upward from the jet have negative momentum fluxes, while those propagating downward have positive ones. The magnitude of momentum fluxes is approximately proportional to the sixth power of the Rossby number between 0.15 and 0.4.

1. Introduction

In the present paper and a companion paper (Yasuda et al. 2015, hereafter Part I), we propose a new mechanism for spontaneous inertia–gravity wave (GW) radiation: GWs are radiated through a quasi resonance with components that are slaved to (linear) potential vorticity (PV). These components, called slaved components, are part of a balanced flow. Here, a balanced flow is the sum of slaved components and a vortical flow that composes a (linear) PV field. An equation system describing this mechanism was derived in Part I from the hydrostatic Boussinesq system on the f plane by using the renormalization group (RG) method (Chen et al. 1994, 1996), which is a singular perturbation method. More specifically, the quasi resonance was formulated as the interaction between the vortical flow that has a slow time scale and the GWs that also have a slow time scale due to a significant Doppler shift caused by the vortical flow. It is important that the derived time-evolution equations [RG equations (RGEs)] describe the GW radiation reaction on the large-scale flow as well as the GW radiation through the quasi resonance. In the new theoretical framework, this slow-time-scale-flow system is considered to be a coexistence system composed of GWs, slaved components, and vortical flow, which interact with each other (see Fig. 1 in Part I).

It was shown that the RGE was formally reduced to a balanced adjustment equation (e.g., Plougonven and Zhang 2007) if the GW radiation reactions on the GW sources appearing in the RGE were ignored. Note that linear equations that describe the radiation of Doppler-shifted GWs from the residuals of a balanced-model solution are referred to as balanced adjustment equations.
(BAEs) in the present paper and Part I. The RGE theory can be regarded as an extension of the BAE theory and has revealed two important facts on the physical nature that are inherent in the BAEs. First, the GW sources are mainly composed of the slaved components. Second, the GW radiation is attributable to the quasi resonance between the GWs (i.e., imbalanced components) and slaved components (i.e., part of the balanced flow). Thus, it is generally not appropriate to call all GW sources in the BAEs “imbalance,” because imbalances are generally defined as the difference between the real flow and the balanced one. In addition, our theory has proposed a new adjustment process (i.e., resonance adjustment), which is different from the balanced adjustment (Zhang 2004): GWs are continuously radiated because the GW sources (i.e., slaved components) are always present, and the balanced flow weakens because of the GW radiation. When the balanced flow becomes too weak to cause the time-scale matching required for quasi resonance, the GW radiation is ceased, and the weak large-scale balanced flow remains. On the other hand, the RGE system has a few disadvantages compared with the BAEs. The most critical one is that the RGEs may be hard to apply to observation and/or numerical model data, because the RGE integration generally requires a lot of computational resources.

There are two main objectives in Part II. First, the RGE is verified by comparing with numerical simulations using the Japan Meteorological Agency nonhydrostatic model (JMA-NHM). It will later be demonstrated that the characteristics of the spontaneously radiated GWs in the vortex dipole obtained by the RGE are consistent with those by the JMA-NHM. The second objective is to clarify the GW radiation in vortex dipoles described by the RGE.

Observations have shown that GWs having long wave periods (i.e., those that are highly Doppler shifted) are frequently associated with jet streams (e.g., Hirota and Niki 1986; Uccellini and Koch 1987; Sato 1989, 1994; Sato and Yoshiki 2008). Numerical simulations have revealed that, even when the initial states do not contain GWs, GWs are radiated around jet streams through the evolution of unstable baroclinic waves (e.g., O’Sullivan and Dunkerton 1995; Zhang 2004; Plougonven and Snyder 2007; Wei and Zhang 2014). However, it is not easy to physically understand the GW radiation in a jet–front system. A system composed of both a jet stream and fronts is complicated, because GWs can be radiated from both of them. Moreover, it is usually difficult to identify GW sources because of the time variation in jet streams. Thus, a better understanding of the essential physics may be obtained by examining a simplified system that only includes a jet stream.

Spontaneous radiation in a vortex dipole (i.e., a pair of cyclonic and anticyclonic vortices) has been examined in recent studies (Snyder et al. 2007, 2009; Viúdez 2007, 2008; Wang et al. 2009, 2010; Wang and Zhang 2010). The flow near the center of the dipole is quite strong because of confluence and is similar to a localized jet stream observed in the real atmosphere. In addition, GW radiation is quasi steady in the dipole system, unlike in the jet–front system, which makes analyses easier (Snyder et al. 2007; Viúdez 2007, 2008; Wang et al. 2009; Wang and Zhang 2010).

Dipoles are roughly classified into surface dipoles and dipoles existing in the interior of fluids (referred to as interior dipoles). A surface dipole consists of high and low potential temperature anomalies (e.g., Snyder et al. 2007), while an interior dipole consists of high and low PV anomalies (e.g., Viúdez 2007). For both types of dipoles, GWs are dominant in the jet exit region and have short wavelengths both in the vertical and in the streamwise direction to the local jet. The BAEs have successfully reproduced the GWs simulated by the numerical models in both the surface dipoles (Snyder et al. 2009) and the interior ones (Wang and Zhang 2010). The main GW sources in the dipole are considered to be the residuals of the balanced-model solution, which are called imbalances in the BAE theory (e.g., Plougonven and Zhang 2007).

Viúdez (2007) proposed another mechanism by analyzing the numerical model data obtained from the simulations of the interior dipole: GWs were radiated by the acceleration/deceleration of the jet when the time scale of velocity variations in fluid parcels was comparable to the inertial period. On the other hand, McIntyre (2009) suggested that the spontaneous radiation in dipoles was similar to the orographic GW generation on the basis of their quasi steadiness and similarity of phase structures. However, these two studies did not make quantitative discussion on the GW structure.

The present study examines a dipole system using the RGEs that were newly derived in Part I. This RGE theory provides a unified perspective on the three studies by Plougonven and Zhang (2007), Viúdez (2007), and McIntyre (2009). The RGE can be formally reduced to a BAE, as was explained in Part I. It will later be shown that the main mechanism described by the RGE, that GWs are radiated through quasi resonances with slaved components, is divided into two mechanisms: the first one is the velocity-variation mechanism proposed by Viúdez (2007) (Fig. 3 in Part I), and the second one is the mountain-wave-like mechanism proposed by McIntyre (2009) (Fig. 2 in Part I). The RGE system provides the theoretical backgrounds for these two mechanisms.
The RGE theory also clarifies that GW sources, as well as a large-scale vortical flow, affect GW structures. Previous studies have considered that the distribution of spontaneously radiated GWs is mainly determined by the structure of a large-scale flow, and it does not significantly depend on the source structure itself. Its theoretical background is the wave-capture theory (Bühler and McIntyre 2005). Plougonven and Snyder (2005) compared the GW orientations with the dilatation axes of the strain-rate tensor in the numerically simulated jet–front system and showed that the GW distribution was consistent with the wave-capture theory. Wang et al. (2010) examined the phase structure of the GWs radiated from the prescribed Gaussian-shape forcing in the vortex dipole by using the linear numerical model and showed that the GW distribution was mainly determined by the large-scale background flow.

The RGE system can describe the wave-capture process similarly to the BAEs, although the process is not explicitly described. Instead of this implicit description, the RGE explicitly shows that the GW source structure, as well as the structure of the large-scale background flow, is important for the GW structure. This fact will also be confirmed later by theoretically calculating the GWs in the dipole.

In section 2, the RGEs [(46) and (54) in Part I] are briefly reviewed. The time evolutions of idealized dipoles in a three-dimensional fluid are simulated by using the JMA-NHM and are presented in section 3. The GW characteristics that the RGE theory should reproduce are also clarified. A modon solution (Berestov 1979; Flierl 1987) is taken to be the initial state, which is an exact solution of the quasigeostrophic (QG) system on the β plane. The β plane is used because a modon solution in the b plane has not yet been derived in previous studies. This usage of the β plane is different from the previous studies (Snyder et al. 2007, 2009; Viúdez 2007, 2008; Wang et al. 2009; Wang and Zhang 2010), which used the f plane. In section 4, the GW distributions are theoretically calculated by using the RGE and compared with those simulated by the JMA-NHM to verify the RGE theory. It is shown that the quasi resonance, which is essential for spontaneous GW radiation, occurs at an intermediate spatial scale between the balanced flow and the GWs. In section 5, physical interpretation of the GW sources and radiation mechanisms is provided by examining the RGE. It is shown that the GW radiation in the dipoles is classified into two groups. In section 6, the GW momentum fluxes are examined using the simulation data obtained by the JMA-NHM, particularly in terms of the dependence on Ro. A summary and the concluding remarks are given in section 7.

2. Renormalization group equations

The system of RGEs has been formulated in Part I in the wavenumber space using linear potential vorticity q, horizontal divergence δ, and ageostrophic vorticity γ. They were defined in the nondimensionalized Boussinesq system as follows:

\[ q = \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + \frac{b}{g}, \]

\[ \delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \] and

\[ \gamma = \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} - \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) p, \]

where (u, v, w) are respective components of velocity vector \( \mathbf{u} \) in the Cartesian coordinates (x, y, z) with x, y, and z, respectively, directed eastward, northward, and upward; p is pressure normalized by the basic-state density; and b (=gθ′/θ0) is buoyancy, where g, θ0, and θ′ are the gravitational acceleration, the constant basic potential temperature, and the perturbation of potential temperature from θ0, respectively. Here, the vortical flow is defined as the flow composing the q field, the slaved components are part of δ and γ, and the balanced components are part of q, and the balanced flow is the sum of the vortical flow and the slaved components. All equations and formulas were nondimensionalized with nondimensional numbers characterizing the system—namely, Rossby (Ro) and Burger (Bu) numbers:

\[ Ro = \frac{U}{fL} \quad \text{and} \quad Bu = \left( \frac{fL}{NH} \right)^2, \]

where U, L, and H are the characteristic scales of the large-scale flow for velocity, horizontal length, and height, respectively; f is a constant Coriolis parameter; and N is a constant buoyancy frequency. The value of Bu is taken to be unity for simplicity, because Bu = O(1) for most large-scale motions in the atmosphere.

First, the dependent variables in the Boussinesq system were transformed into q, δ, and γ, and second, all equations were represented in the wavenumber space. The RG method (Chen et al. 1994, 1996) was applied to the transformed Boussinesq system under several relevant assumptions (i)–(vi) in section 3a of Part I, taking Ro to be a small number.

The RGE system is described by five dependent variables having slow time scales: the linear potential vorticity \( q^{GW} \), the horizontal divergence \( \delta^{GW} \) and ageostrophic vorticity \( \gamma^{GW} \) components describing GWs, and the
diagnostic components of horizontal divergence $\delta_{\text{diag}}$ and ageostrophic vorticity $\gamma_{\text{diag}}$. An eigenmode expansion of $\delta$ and $\gamma$ against a given vortical flow field was carried out after the RG method was applied, in order to take Doppler shift into account. The eigenmodes with eigenvalues (i.e., ground-based frequencies) on the order of $\text{Ro}$ were obtained as the GW modes or GW components ($\delta^{\text{GW}}$ and $\gamma^{\text{GW}}$). The superpositions of $\delta^{\text{GW}}$ and $\gamma^{\text{GW}}$ are regarded as the spontaneously radiated GWs. On the other hand, $\delta$ and $\gamma$ generally have nonlinear terms varying slowly, such as slaved components, which are different from GWs. To include these nonlinear terms in the RGE theory, the diagnostic components ($\delta_{\text{diag}}$ and $\gamma_{\text{diag}}$) have been introduced separately from the GW components. These components areagnostically obtained from $\delta$, $\delta^{\text{GW}}$, and $\gamma^{\text{GW}}$, and they are regarded as the sums of the slaved components and the GW radiation reactions.

The system of RGEs consists of the time-evolution equations for $\gamma$, $\delta^{\text{GW}}$, and $\gamma^{\text{GW}}$, and formulas giving $\delta_{\text{diag}}$ and $\gamma_{\text{diag}}$. The RGE system describes the GW radiation through the quasi resonance with the slaved components, as well as the time evolution of the large-scale vortical flow. The theory also describes the GW radiation reactions on the vortical flow and GW sources. The first-order RGE for $q$ describes the time evolution of the vortical flow that contains the GW radiation reaction. The second-order linearized RGE for ($\delta^{\text{GW}}$, $\gamma^{\text{GW}}$) describes the spontaneous GW radiation. These two RGEs are the most essential in our RGE theory.

The first-order RGE for $q$ [(46) in Part I] is as follows:

$$ q_k(t) = \tilde{q}_k(t) + O(\text{Ro}^2), $$

$$ \frac{d\tilde{q}_k}{dt} = \text{Ro} \sum_m \left[ I_{\gamma \gamma}^{(0)} q_{k-m} \gamma_{\text{balance}(1)}_{k} + I_{\gamma \gamma}^{(0)} q_{m} \gamma_{\text{GW}}^{\text{GG}(1)}_{k-m} + I_{\gamma \gamma}^{(0)} q_{m} \gamma_{\text{GW}}^{\text{QG}(1)}_{k-m} \right]. $$

Here, subscript $k$ denotes a Fourier coefficient of wavenumber $k$ depending on time $t$, $d/dt$ is an ordinary time derivative, and $t$ is the fast time defined as $t = s/\text{Ro}$, where $s$ is the slow time nondimensionalized by the advection time scale ($L/U$). The coefficient $I_{\gamma \gamma}^{(0)}$ and the others on the right-hand side (RHS) are the interaction coefficients dependent only on wavenumbers. Their specific forms are shown in appendix B in Part I. The terms with superscripts “balance,” “$R_{\text{GG}}$,” and “$R_{\text{QG}}$,” respectively, represent the nonlinear terms consisting of $\tilde{q}_k$ only, those of $q_k$ and GW components, and those of GW components only. Note that the interaction coefficients in $I_{\gamma \gamma}^{(0)}$ denoted with superscripts (0) (i.e., $I_{\gamma \gamma}^{(0)}$ and $I_{\gamma \gamma}^{(0)}$) are the leading terms of the original coefficients with respect to Ro.

The form of $\tilde{q}_k^{\text{balance}(1)}$ is the same as that of the time-tendency term for $\tilde{q}_k$ in the first-order balanced model (see appendix E in Part I). The term $q_{\text{GG}(1)}^{(0)}$ includes part of the advection of $q$ by the GW components, while $\tilde{q}_{\text{QG}(1)}^{(0)}$ represents the formation of $q$ by the nonlinear interactions between the GW components. These $q_{\text{GG}(1)}^{(0)}$ and $\tilde{q}_{\text{QG}(1)}^{(0)}$ represent the GW radiation reaction on $\tilde{q}_k$. All terms on the RHS of (5) are proportional to Ro, meaning that $\tilde{q}$ varies in slow time $s$.

The linearized second-order RGE for ($\delta^{\text{GW}}$, $\gamma^{\text{GW}}$) [(54) in Part I] describes the GW radiation from the GW sources (i.e., slaved components) through the quasi resonance:

$$ \frac{d}{dt} \left( \begin{array}{c} \gamma_{\text{GW}}^{\text{GG}(1)} \\ \delta_{\text{GW}}^{\text{GG}(1)} \end{array} \right) = \text{Ro} \left( \begin{array}{c} \gamma_{\text{GW}}^{\text{GG}(1)} \\ \delta_{\text{GW}}^{\text{GG}(1)} \end{array} \right) + \text{Ro}^2 \left( \begin{array}{c} \alpha_k^{\delta_{\text{slave}(1)}} \delta_{\text{GW}}^{\text{GG}(1)} - \gamma_{\text{Slave}(1)} \\ \gamma_{\text{GW}}^{\text{GG}(1)} - \gamma_{\text{_slave}(1)} \end{array} \right) \right) + O(\text{Ro}^3). $$

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where $\mathbf{k}_H$ represents horizontal wavenumber vector. The matrix $\mathbf{L}$ on the left-hand side (LHS) is given as follows:

$$
\ddot{\mathbf{L}} = \frac{1}{\text{Ro}} \begin{pmatrix}
-\Delta \mathbf{L}_{\text{GW}} & \omega_k^2 \\
1 & -\Delta \mathbf{L}_{\text{GW}}
\end{pmatrix}
+ \begin{pmatrix}
-\Gamma_{\text{GW}}^{(0)} & -\Gamma_{\text{GW}}^{(0)} \\
0 & -\Gamma_{\text{GW}}^{(0)}
\end{pmatrix},
$$

where the first matrix proportional to $\text{Ro}^{-1}$ describes the GW propagation, including the advection of GWs by the vortical flow, whereas the second matrix represents the modification to the GW propagation by the shear of the vortical flow. Linearization with respect to the GW component in the GW sources was carried out to derive (6) from the original nonlinear second-order RGE [(D4) in Part I]. This manipulation is equivalent to considering only the slaved components to be the GW sources and to ignoring both the GW reaction radiations on the sources and the advection of GWs by the velocity associated with the diagnostic components. The specific forms of the slaved components on the RHS are shown in the appendix. The RGE (6) has a similar form to a BAE (e.g., Plougonven and Zhang 2007), as was explained in Part I.

The eddy diffusion effect is usually not important near the GW source region, because GWs should have relatively large scales comparable with those of the GW sources (i.e., slaved components), as was discussed in Part I. On the other hand, near the edge of a region where wave capture occurs, the radiated GWs generally break (Bühler and McIntyre 2005), which indicates that the eddy diffusion effect is important there. This fact means that the diffusion term can be added to the RGE without changing its form, when the GW source region is distinguished from the edge of the wave-capture region, as in the dipoles. This is because diffusion does not largely affect the GW radiation, which occurs near the GW source region. By choosing the same form for the diffusion term as that in the JMA-NHM (i.e., fourth-order horizontal diffusion) the RGE with the diffusion is given by (6):

$$
\frac{d}{dt} \left( \begin{array}{c}
\mathbf{g}^\text{GW} \\
\mathbf{\delta}_k^\text{GW}
\end{array} \right) + \text{Ro} \mathbf{L} \left( \begin{array}{c}
\mathbf{g}^\text{GW} \\
\mathbf{\delta}_k^\text{GW}
\end{array} \right) = \text{Ro} \left( \begin{array}{c}
\omega_k \mathbf{\delta}_k^\text{slave(1)} \\
-\mathbf{\gamma}_k^\text{slave(1)}
\end{array} \right) + \text{Ro}^2 \left( \begin{array}{c}
\omega_k(\mathbf{\delta}_k^\text{slave(2)} + \mathbf{\delta}_k^\text{slave(2)}) \\
-\mathbf{\gamma}_k(\mathbf{\gamma}_k^\text{slave(2)} + \mathbf{\gamma}_k^\text{slave(2)})
\end{array} \right)
+ \left( \begin{array}{c}
-\nu \mathbf{\delta}_k^H \\
0
\end{array} \right) \mathbf{\delta}_k^\text{GW} + O(\text{Ro}^3),
$$

where $\nu$ is the diffusion coefficient.

The linearized second-order RGE (8) describes only the time evolution of the slowly varying eigenmodes (GW modes) [i.e., modes whose eigenvalues (i.e., ground-based frequencies) are $O(\text{Ro})$]. Thus, the eigendecomposition of $\mathbf{L}$ is carried out to obtain the GW modes. The eigenmode equation is written as $\mathbf{L}_p = \Omega_p \mathbf{\hat{p}}$, where $\mathbf{\hat{p}}$ is an eigenmode and $\Omega_p$ is an eigenvalue. The basis functions in (8) are transformed from the plane waves into the eigenmodes by using a matrix, $\mathbf{P} = (p_1, p_2, \ldots)$, and the following equation is obtained:

$$
\frac{d}{dt} \mathbf{\hat{g}} + \text{Ro} \mathbf{\Omega} \mathbf{\hat{g}} + \text{Ro} \mathbf{F}_p^{\text{slave(1)}} + \text{Ro}^2 \mathbf{F}_p^{\text{slave(2)}} = -\sum_k \left( \mathbf{P}^{-1} \frac{\partial \mathbf{P}}{\partial \mathbf{g}_k} \frac{d}{dt} \mathbf{g}_k + \mathbf{K}_p \mathbf{g}_k + O(\text{Ro}^3),
$$

where

$$
\Omega = \text{diag}(\Omega_1, \Omega_2, \ldots) = \mathbf{P}^{-1} \mathbf{L} \mathbf{P},
$$

$$
\mathbf{K}_p = \mathbf{P}^{-1} \mathbf{K} \mathbf{P}, \quad \text{and} \quad \mathbf{g} = \mathbf{P}^{-1} \left( \begin{array}{c}
\mathbf{g}^\text{GW} \\
\mathbf{\delta}_k^\text{GW}
\end{array} \right),
$$

and

$$
\mathbf{F}_p^{\text{slave(1)}} = \mathbf{P}^{-1} \left( \begin{array}{c}
\omega_k^2 \mathbf{\delta}_k^\text{slave(1)} \\
-\mathbf{\gamma}_k^\text{slave(1)}
\end{array} \right) \text{ and}
$$

$$
\mathbf{F}_p^{\text{slave(2)}} = \mathbf{P}^{-1} \left( \begin{array}{c}
\omega_k^2 \mathbf{\gamma}_k^\text{slave(2)} + \mathbf{\delta}_k^\text{slave(2)} \\
-\mathbf{\gamma}_k(\mathbf{\gamma}_k^\text{slave(2)} + \mathbf{\delta}_k^\text{slave(2)})
\end{array} \right).
$$

The fourth term on the RHS of (9) represents the variation of $\mathbf{g}$ due to the change in the basis functions (i.e., eigenmodes). This term is considered to be quite small for a system having a quasi-steady structure, such as a dipole; hence, it is ignored in the present paper.
3. Spontaneous radiation of GWs in dipole obtained by JMA-NHM

a. Model setting and initial values

The compressible nonhydrostatic equations on the $\beta$ plane are integrated numerically by using JMA-NHM [see Saito et al. (2006) for details]. Arakawa C grids are used horizontally in this model, and Lorenz grids are used vertically. The advection terms are discretized by fourth-order flux formulas. Integration is performed by using the split-explicit scheme; the terms related to acoustic waves are integrated explicitly horizontally and implicitly vertically. Fourth-order horizontal numerical diffusion is included in the time-evolution equations for the respective predictor variables, the coefficient of which is chosen to set the $\epsilon$-folding time for two-grid noises of 50 min.

The physical space of the model is taken to be 2980 km $\times$ 2980 km horizontally, and 35.8 km vertically. The grid spacings are 20 km horizontally and 200 m vertically. The boundary conditions for the $x$, $y$, and $z$ directions correspond to periodic, Orlanski radiation, and rigid-lid conditions, respectively. Sponge layers with thicknesses of 200 and 6 km are employed, with damping rates that linearly increase from 0 to $10^5$ s$^{-1}$ and from 0 to $1500$ s$^{-1}$ in the latitudinal ($y$) boundaries and the top boundary, respectively. Note that there is no sponge layer near the bottom. The model is run over 60 days with a time step of 30 s.

The initial values are taken to be geostrophic velocity, potential temperature, and pressure of the three-dimensional modon solution (Berestov 1979; Flierl 1987). This solution is an exact solution of the QG system on the $\beta$ plane. This initial state is well balanced and contains little GWs in the model. Thus, it is expected that the GW radiation through the initial adjustment will not be quite as strong as that with other initial states. It is considered that the RGE theory can be applied to the dipole on the $\beta$ plane, as the intrinsic frequency and spatial structure of GWs are not largely affected by the $\beta$ effect.

The modon solution consists of a pair of cyclonic and anticyclonic vortices and has a strong eastward flow near the center of the modon (Fig. 1). In the coordinate system for which the $z$ axis is extended by factor $N/f_0$, the modon has a steady spherical structure with radius $a$ and propagates eastward with phase speed $c$. Here, the Coriolis parameter is $f = f_0 + \beta(y - y_0)$ as a function of $y$, where $f_0$ is the Coriolis parameter at the center in the $y$ direction ($y = y_0$). In the present paper, values at $45^\circ$N are used for $f_0$ and $\beta$, and $N = 0.01$ s$^{-1}$. All quantities with the modon are determined by the two parameters $a$ and $c$. We set $a = 500$ km and $c = 1.915$ m s$^{-1}$. The characteristic scales of the large-scale flow are taken as $L = a$ and $H = af_0/N$ (i.e., $Bu = 1$), and $U$ is an average of $u$ between $-a/2 \leq x \leq a/2$ (see Fig. 1a). The corresponding Ro is equal to 0.25. The case of Ro = 0.10 is also examined for comparison by taking $a = 500$ km and $c = 0.540$ m s$^{-1}$. The location of the modon center was initially set at ($x$, $y$, $z$) = (0 km, 1500 km, 15 km). The modon solution (i.e., the initial state) is referred to as the initial modon, while the vortex dipole on the $n$th day is referred to as the $n$th-day dipole.

b. Time evolution of dipole

The time evolution of $q$ in the $x$-$y$ section for Ro = 0.25 obtained by the JMA-NHM is shown in Fig. 2. The
The dipole propagates nearly eastward while maintaining a quasi-steady structure. The dipole with $\text{Ro} = 0.10$ evolved similarly (not shown). It is worth noting that their trajectories are different from those of dipoles on the $f$ plane that propagate eastward and slightly northward (Snyder et al. 2007; Viúdez 2007, 2008; Wang et al. 2009; Wang and Zhang 2010). The differences in trajectories are likely attributable to the $\beta$ effect that prevents northward propagation. This nearly eastward trajectory made our analyses easier.

It can be found from Fig. 2 that the mean propagation speed of the dipole has reduced rapidly over the first 20 days, which indicates that an initial adjustment has taken place. During this period, the GWs with high frequencies are radiated from the dipole and propagate outside the dipole (not shown). The RGE system cannot describe such high-frequency GWs accompanied with the initial adjustment, as was discussed in section 4a in Part I. The main objectives of the present paper are to verify the RGE theory describing the radiation of GWs having low ground-based frequencies and to clarify the physical mechanism. The initial adjustment is beyond the scope of the present study [see Snyder et al. (2007, 2009) for a detailed discussion on the initial adjustment in dipoles].

It may be worthwhile to note the structure and magnitude of local $\text{Ro}$ is defined as the ratio of the vertical component of vorticity to $f$. The structure of local $\text{Ro}$ is quite similar to that of $q$ in Figs. 1 and 2. The minimum of local $\text{Ro}$ at the center of the anticyclone is slightly less than $-1$ even in the initial state. This suggests that the inertial instability hardly occurs, as is consistent with the fact that the dipole retains its structure, as seen in Fig. 2.

The dipole reaches a quasi-steady state around the thirtieth day, as confirmed by nearly constant maximum and minimum values of $u, v, w, \text{ and }$ and $\theta$ at respective heights and the spatial distribution of $w$. In the following sections, the initial modon and thirtieth-day dipole are analyzed by employing comoving frames with the modon and dipole, respectively. The comoving frame with the modon is obtained from the phase speed $c \approx 1.9 \text{ m s}^{-1}$ determined theoretically by the modon solution, while that with the thirtieth-day dipole is obtained from the instantaneous propagation speed ($\approx 1.3 \text{ m s}^{-1}$) determined by tracking the dipole center. The origin of the comoving frame is the point where $u$ is maximum [i.e., the dipole (or modon) center].

c. Characteristics of spontaneously radiated GWs, which theory should reproduce

In this subsection, we examine the GW structure in the thirtieth-day dipole. The horizontal divergence $\delta_{\text{total}}$ calculated from the JMA-NHM data is considered to contain both diagnostic and GW components. Thus, it is necessary to remove the diagnostic components to see the GW spatial structure. However, it is difficult to obtain all diagnostic components, because they are composed of both the components slaved to $q$ and the GW radiation reaction. Thus, we used the sum of the first- and second-order slaved components $\delta_{\text{slave}}$ as the diagnostic component to extract the GW components $\delta_{\text{GW}}$.

Figures 3a–f show the $x$–$y$ and $x$–$z$ sections of $\delta_{\text{total}}, \delta_{\text{slave}}, \text{ and } \delta_{\text{total}} - \delta_{\text{slave}} (=\delta_{\text{GW}})$, respectively, in the thirtieth-day dipole with $\text{Ro} = 0.25$ obtained by using the JMA-NHM. Note that the $\delta_{\text{slave}}$ distribution is roughly symmetric with respect to the $z = 0.0$-km plane in a confined physical space (horizontal size: $1200$ km; and vertical size: $12$ km). This $\delta_{\text{slave}}$ component will also be used in section 4b for the RGE integrations. It was confirmed from calculations using a larger physical space that the confined domain size did not greatly affect the structure or amplitude of $\delta_{\text{slave}}$. A wave structure can clearly be observed in the $\delta_{\text{total}} - \delta_{\text{slave}}$ field and approximately expresses spontaneously radiated GWs. On the other hand, such GWs could hardly be recognized in the dipole with $\text{Ro} = 0.10$ (not shown). The observed GWs have three main characteristics:

(i) The GWs have a fine phase structure that is roughly vertically symmetrical in the jet exit region.
(ii) The GWs are hardly recognized outside the dipole. The GW wavelength decreases as it approaches the eastern edge of the dipole.
(iii) The GW phase structure is slightly different between the cyclonic and anticyclonic sides; the GWs are more strongly wrapped in the anticyclone.

The GWs with these three characteristics were continuously observed from the thirtieth to sixtieth days, although their amplitude was reduced by about one-third. This slight reduction was likely due to the numerical diffusion. This fact means that the GW structure is quasi steady, indicating that the GWs are generated not because of the initial adjustment but because of the spontaneous radiation. Figures 4a and 4b also show the
4. Spontaneous radiation of GWs in dipole obtained by RGE

GWs are theoretically calculated using the linearized second-order RGE (9) for the initial modon (section 4a) and the thirtieth-day dipole (section 4b). For these calculations, the RGE (9) is transformed into the equation in the comoving frame, as was mentioned above. The RGE theory is verified by comparing with results of the numerical simulation by the JMA-NHM in section 3c. In addition, the GW radiation reactions on \( q \) (section 4c) and GW sources (section 4d) are discussed.

4a. Spontaneously radiated GWs in initial modon obtained by RGE

In this subsection, we discuss the spatial structure of GWs in the initial modon obtained by the RGE without numerical diffusion. The radiated GWs are amplified in time, which is likely because the RGE does not include diffusion. The amplitude of the GWs obtained by the RGE with the diffusion will be discussed for the thirtieth-day dipole in the next subsection. As will be shown there, the spatial structure of the GWs in the thirtieth-day dipole...
reaches a quasi-steady state in about 1 day; hence, the integration time is taken to be 1 day in this subsection. An eigendecomposition of $L$ is required to obtain the time evolution of GWs by the RGE, as was described in section 2. However, $L$ is composed of a large number of elements. Thus, calculations over the whole wavenumber space require many computational resources and a long computational time. To save computational resources and time, a confined physical space, the lengths of which are 1200 km horizontally and 12 km vertically, is used for the RGE integrations. It was confirmed that the size of the confined space did not largely affect the obtained GW characteristics. In addition, in order to save computational resources and time, a confined physical space, the lengths of which are 1200 km horizontally and 12 km vertically, is used for the RGE integrations. It was confirmed that the size of the confined space did not largely affect the obtained GW characteristics. In addition, in order to save computational resources and time, wavenumbers are limited to $k_1 \leq 15$, $k_2 \leq 8$, and $k_3 \leq 15$, where $k_1$, $k_2$, and $k_3$ are respective components of wavenumber vector $k$ for the $x$, $y$, and $z$ directions. Corresponding minimum resolvable wavelengths are 80, 150, and 0.8 km in the $x$, $y$, and $z$ directions, respectively. The resolution in the $x$ direction is finer than that in the $y$ direction, because GWs are expected to be contracted along the $x$ axis because of the wave-capture mechanism. It should be emphasized here that the restrictions on domain and wavenumbers do not affect the definition of the GW components in the RGE theory.

The $x$–$y$ and $x$–$z$ sections of $\delta^{GW}$ obtained by using the RGE are respectively shown in Figs. 5a and 5b. The GW modes are taken to be eigenmodes within $|\Omega_i| \leq 3$, where $\Omega_i$ is the eigenvalue (i.e., ground-based frequency) of the $i$th eigenmode. The ground-based wave periods of the eigenmodes are longer than or equal to about 0.94 day in the dimensional units. The inertial period (i.e., the longest intrinsic period of GWs) is about 0.93 day at the lowest latitude in the JMA-NHM domain ($y = -1500$ km). Thus, all GW modes included in the RGE integration are Doppler shifted. As can be seen from Figs. 5a and 5b, the GWs are dominant in the jet exit region. The phase structure is nearly symmetric with respect to the $z = 0.0$-km plane, which is consistent with the GW characteristic (i). The GWs are hardly recognized outside the initial modon, and the wavelength decreases as the GW approaches the eastern edge of the modon, which is consistent with the GW characteristic (ii). In addition, the GWs are wrapped more strongly in the anticyclone, which is consistent with the GW characteristic (iii), although it is weak compared with that in the JMA-NHM simulation. The weaker wrapping is probably due to the limited resolution in the $y$ direction for the RGE integration. These facts indicate that the RGE satisfactorily explains the GW spatial structure obtained from the JMA-NHM simulation.

The frequency range of GW modes is ambiguous because the GW mode is defined simply by the order of its frequency [i.e., $|\Omega_i| = O(1)$]. To confirm the relevance of the frequency range taken in the calculation, GWs are obtained from the GW modes defined within a narrower frequency range. Figures 5c and 5d, respectively, show the $x$–$y$ and $x$–$z$ sections of $\delta^{GW}$ obtained by the RGE with the GW modes of $|\Omega_i| \leq 1$ (i.e., wave periods longer than or equal to about 2.83 day). In contrast to the case of $|\Omega_i| \leq 3$ (Figs. 5a and 5b), the GWs are distributed in both the jet exit and entrance regions. This fact means that phase cancellation is important in the jet entrance region by superposing the GW modes, and calculations
taking the narrow frequency range for the GW mode definition cannot describe this cancellation. This result also suggests that the GWs are composed of eigenmodes within a wide frequency range in which the highest frequency is close to the inertial frequency as shown in Figs. 5a and 5b.

We next examine the dependence of GWs on wave-number limitations. First, wavenumbers were limited to high-wavenumber ranges: $6 \leq |k_1| \leq 15$ and $6 \leq |k_3| \leq 15$ for the RGE integration. The range of $k_3$ was the same as before. Resolved wavelengths were from 200 to 80 km in the $x$ direction and from 2 to 0.8 km in the $z$ direction. In this case, GWs were radiated nearly symmetrically outside the modon (not shown), which is not consistent with the JMA-NHM simulation (Fig. 3). As will be discussed later in section 4d, this is likely because the high-wavenumber components of the GW sources are symmetrical and dominant near the edge of the modon. Thus, it is necessary to include low-wavenumber components for the RGE integration to obtain a GW structure that is consistent with that obtained from the JMA-NHM simulation. In other words, our theory indicates that GWs should inherently have low-wavenumber components.

![Fig. 5](image-url)

Figure 5. (a) The $x$–$y$ section at $z = 3.0$ km and (b) the $x$–$z$ section at $y = 0.0$ km of $\delta^{GW}$ (colors) for the initial modon with $Ro = 0.25$, which were obtained by the RGE (9) without diffusion. The calculation was carried out for the GW modes within $|\Omega| \leq 3$ and wavenumber ranges: $-15 \leq k_1 \leq 15$ and $-15 \leq k_3 \leq 15$. (c),(d) As in (a) and (b), respectively, but for the GW modes within $|\Omega| \leq 1$. (e),(f) As in (a) and (b), respectively, but for wavenumber ranges: $-10 \leq k_1 \leq 10$ and $-10 \leq k_3 \leq 10$. The contours in (a),(c), and (e), and in (b),(d), and (f) indicate $q$ with a CI of $0.1 \text{ s}^{-1}$ and $u$ with a CI of $2.5 \text{ m s}^{-1}$ in the comoving frame, respectively, where $q$ is multiplied by $10^4$ in all panels.
Second, wavenumbers were limited to low-wavenumber ranges: \(-10 \leq k_1 \leq 10\) and \(-10 \leq k_3 \leq 10\). The results obtained from the RGE integration are presented in Figs. 5e and 5f. In this case, the GW distribution is nearly the same as that in Figs. 5a and 5b for \(-15 \leq k_1 \leq 15\) and \(-15 \leq k_3 \leq 15\). This fact suggests that components with wavenumbers much higher than 10 are not important to reproduce the GW structure observed in the JMA-NHM simulation.

The low-wavenumber components correspond to relatively large-scale GWs having horizontal wavelengths of about 400 km and vertical ones of about 4 km and having large amplitudes near the jet center (Figs. 3f and 5b). The characteristic spatial scales for the vortical flow, which are determined by the energy density spectra, are about 800 km horizontally and about 8 km vertically, while those for GWs are about 200 km horizontally and about 2 km vertically. This fact means that the spatial scales of the low-wavenumber GWs are intermediate between the characteristic scales for the GWs and those for the vortical flow. Thus, it is suggested that GWs are radiated spontaneously through the quasi resonance with their sources occurring at these intermediate scales (about 400 km horizontally and about 4 km vertically). Note that this spatial-scale matching as well as the time scale one is necessary for the quasi resonance, as was discussed in section 3c of Part I. It is worth noting here that such intermediate-scale GWs are modified into smaller and smaller-scale GWs through the wave-capture mechanism (Bühler and McIntyre 2005).

The RGE was also integrated for the modon with \(\text{Ro} = 0.10\) under the same conditions. The spatial structures of the slaved components (i.e., GW sources) are the same as those in the modon with \(\text{Ro} = 0.25\); nevertheless, GWs were radiated almost negligibly. An important difference from the case of \(\text{Ro} = 0.25\) is that the ratio of the lower speed of the vortical flow. Thus, this little GW radiation can be explained by the insufficient Doppler shift by the vortical flow for a quasi resonance with the slaved components at \(\text{Ro} = 0.10\).

b. Spontaneously radiated GWs in thirtieth-day dipole obtained by RGE

In this subsection, the linearized second-order RGE (9) is integrated using the field at the thirtieth day obtained from the JMA-NHM data. First, the RGE without numerical diffusion was integrated for the thirtieth-day dipole under the same model configuration as for the initial modon. Figures 6a and 6b show the \(x-y\) and \(x-z\) sections of horizontal divergence from the RGE integration, which is the sum of the first- and second-order slaved components \(\delta_{\text{slaved}}\) and the GW components \(\delta_{\text{GW}}\). Note that \(\delta_{\text{slaved}}\) is the same as that in Figs. 3c and 3d. In contrast, Figs. 6c and 6d, respectively, show the \(x-y\) and \(x-z\) sections of \(\delta_{\text{GW}}\) only. It is found from Figs. 6c and 6d that the RGE successfully reproduced again the GWs having characteristics (i)–(iii) and spatial structure similar to that obtained by the JMA-NHM (Figs. 3e and 3f). The GW wrapping in Fig. 6c is weaker than that in Fig. 3e (in other words, the GW wavelengths are longer in the RGE simulation). This is probably due to the low resolution in the \(y\) direction. A notable difference from the JMA-NHM simulation is that the \(\delta_{\text{GW}}\) amplitude from the RGE integration is about 6 times larger than that from the JMA-NHM simulation. This is likely because this RGE integration does not include the diffusion that the JMA-NHM has.

We integrated the RGE (9) with the same diffusion coefficient as in the JMA-NHM to quantitatively compare the GW amplitudes. The wave-capture theory (Bühler and McIntyre 2005) indicates that GW wavelengths in the dipole become smaller and smaller along the \(x\) and \(z\) directions. On the other hand, previous studies on the dependence of GW amplitude on model resolutions (e.g., O’Sullivan and Dunkerton 1995; Plougonven et al. 2013) have indicated that GW amplitude increases as model resolution increases because of smaller-scale GWs. These two facts strongly suggest that the resolutions in the \(x\) and \(z\) directions for the RGE integration should be taken to be as high as those in the JMA-NHM. Thus, wavenumbers were limited to \(-29 \leq k_1 \leq 29\), \(-4 \leq k_2 \leq 4\), and \(-29 \leq k_3 \leq 29\) for the RGE integration. Note that the maximum wavenumber in each \((x, y, z)\) direction is 30 on the basis of resolution in the JMA-NHM. The minimum resolvable wavelengths for the RGE integration are 40, 300, and 0.4 km in the \(x, y, z\) directions, respectively.

Figure 7 shows the time series of \(\delta_{\text{GW}}\) in the \(x-z\) sections obtained by the RGE, including the diffusion [(9)]. Here, the eigendecomposition of \(\mathbf{L}\) was carried out for the dominant \(O(\text{Ro}^{-1})\) terms in \(\mathbf{L}\) by neglecting the smaller \(O(\text{Ro}^0)\) terms [i.e., the last matrix in (7)], and the GW modes were defined as eigenmodes within \(|\Omega| < 2\). Note that the same color scales in Fig. 3f for the JMA-NHM result are used in Fig. 7. It is found that the RGE well reproduced the GW amplitudes simulated by the JMA-NHM. It is also seen from Fig. 7 that the GW structure is formed in about 1 day.

In summary, the RGE well reproduced the GWs in the dipole simulated by the JMA-NHM in terms of the spatial structure and amplitude of the GWs. This strongly suggests that the RGE theory is valid.

c. Reaction of GW radiation on linear potential vorticity

In this and the next subsections, the reactions of the GW radiation are discussed by analyzing the instantaneous JMA-NHM data with the RGE theory. The RGE theory does not provide any algorithms to diagnostically
extract the GW components from observation and/or numerical model data. Thus, the components of $\delta$ and $\gamma$ having horizontal and vertical wavenumbers larger than 15 are designated as the GW components for the JMA-NHM data on the thirtieth day in this subsection, although the GWs actually have weak low-wavenumber components.

The GW radiation reaction on the vortical flow is represented by $\dot{q}_{\text{RGE}}^{\text{QG}(1)}$ and $\dot{q}_{\text{RGE}}^{\text{GG}(1)}$ in the first-order RGE (5), as was discussed in section 3c in Part I. Figures 8a–d, respectively, show the $x$–$y$ and $y$–$z$ sections of $\dot{q}_{\text{RGE}}^{\text{QG}(1)}$ and $\dot{q}_{\text{RGE}}^{\text{GG}(1)}$ in the thirtieth-day dipole. Both $\dot{q}_{\text{RGE}}^{\text{QG}(1)}$ and $\dot{q}_{\text{RGE}}^{\text{GG}(1)}$ have fine structures similar to those of the GWs in the jet exit region. The magnitude of $\dot{q}_{\text{RGE}}^{\text{QG}(1)}$ is two orders larger than that of $\dot{q}_{\text{RGE}}^{\text{GG}(1)}$.

Figures 8e and 8f, respectively, show the $x$–$y$ and $x$–$z$ sections of $q$ components with higher wavenumbers than 15 in each direction on the thirtieth day obtained by the JMA-NHM. Fine structures similar to those of $\dot{q}_{\text{RGE}}^{\text{QG}(1)}$ and $\dot{q}_{\text{RGE}}^{\text{GG}(1)}$ are observed in the jet exit region. In contrast, $q$ in the initial modon does not have such fine structures. These results indicate that the fine structures in $q$ on the thirtieth day simulated by the JMA-NHM (Figs. 8e and 8f) are likely due to the GW radiation reaction described by $\dot{q}_{\text{RGE}}^{\text{QG}(1)}$. Note that $\dot{q}_{\text{RGE}}^{\text{QG}(1)}$ and $\dot{q}_{\text{RGE}}^{\text{GG}(1)}$ are the time tendencies of $q$ [see (5)]. It is inferred that the time integrations of $\dot{q}_{\text{RGE}}^{\text{QG}(1)}$ and $\dot{q}_{\text{RGE}}^{\text{GG}(1)}$ having fine structures will provide $q$ components having high wavenumbers.

Such a finescale spatial structure of potential vorticity is recognized slightly above the tropopause at midlatitudes.

![30-day dipole with RGE](image-url)
in the general circulation model in which GWs are explicitly resolved (Miyazaki et al. 2010). The results in the present paper suggest that a strong candidate for causing such a fine potential vorticity structure is the GW radiation reaction on the large-scale flow.

**d. GW sources and their modification by the GW radiation**

In this subsection, the characteristics of the GW sources are examined. The results presented in the previous subsection suggest that the GW sources may also be affected by the GW radiation reaction, because the GW sources in the RGE consist of the slaved components diagnosed by \( q \) (i.e., \( \delta_{\text{slave}}^{(1)} + \delta_{\text{slave}}^{(2)} + \delta_{\text{slave}}^{(3)} \) and \( \gamma_{\text{slave}}^{(1)} + \gamma_{\text{slave}}^{(2)} + \gamma_{\text{slave}}^{(3)} \)), and \( q \) is modified by the GW radiation reaction. Note that the rest of the second-order slaved components (i.e., \( \delta_{\text{slave}}^{(2)} \) and \( \gamma_{\text{slave}}^{(2)} \)) are not GW sources in our theory, because they are absorbed in the renormalization constants. The \( x-z \) and \( y-z \) sections of the GW sources in the initial modon are shown in Fig. 9. Convergence \((-\delta = \partial w / \partial z < 0)\) and divergence \((-\delta = \partial w / \partial z > 0)\) of the vertical flow are observed in the

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**Fig. 7.** The \( x-z \) sections at \( y = 0.0 \) km of \( \delta_{GW} \) (colors) obtained by the RGE (9) for the thirtieth-day dipole with \( Ro = 0.25 \) on the (a) first, (b) second, (c) third, and (d) tenth days, including the same diffusion used for the JMA-NHM. Here, the wavenumber ranges for the RGE integration were taken to be \(-29 \leq k_1 \leq 29, -4 \leq k_2 \leq 4, \) and \(-29 \leq k_3 \leq 29;\) and the GW modes were defined by \( |\Omega| < 2.\) Note that the color scales are as in Fig. 3 for the JMA-NHM simulation. The contours indicate \( u \) with a CI of 2.5 m s\(^{-1}\) in the comoving frame.
jet entrance and exit regions, respectively. The physical interpretation of this convergence–divergence couplet will be given in section 5c.

The GW sources for the thirtieth-day dipole have large-scale structures similar to those in the initial modon, while their magnitudes are slightly smaller. This smaller magnitude is likely due to the numerical diffusion. Figure 10 shows the high-wavenumber components of the GW sources in the initial modon and the thirtieth-day dipole. The GW sources with high wavenumbers in the thirtieth-day dipole have fine structures in the jet exit region (Figs. 10a and 10b). On the other hand, similar fine structures cannot be observed in the GW sources of the initial modon. It is worth noting that, although the GW sources in the initial modon have high-wavenumber components, the structures are symmetric with respect to the jet center, and the magnitudes are one order smaller than those in the thirtieth-day dipole (Figs. 10c and 10d). The difference in the GW sources (i.e., the slaved components) between the initial modon and the thirtieth-day dipole is likely attributed to the modification of \( \mathbf{q} \) by the GW radiation (Fig. 8). This reaction effect slightly increases the GW amplitude in the jet exit region, although this effect is minor because the low-wavenumber components are dominant in the GW sources.

5. Physical interpretation of spontaneous GW radiation in dipoles

The RGE theory describes the GW radiation through a quasi resonance with slaved components which are

![Image](https://via.placeholder.com/150)
part of a balanced flow. The main GW sources (i.e., slaved components) are identified in section 5a, and they are interpreted physically in sections 5b and 5c. It is shown that the main mechanisms are divided into two groups: the velocity-variation mechanism (Viúdez 2007) and the mountain-wave-like mechanism (McIntyre 2009). The shear effect of the vortical flow on the two mechanisms is discussed in section 5d. Moreover, in section 5e, it is shown that the structures of the GW sources, as well as that of the large-scale vortical flow, may affect the spatial structure of GWs in the dipole. In this section, the GWs radiated in the initial modon are examined by using the second-order linearized RGE (9) without diffusion.

### a. Main GW sources

It is possible to independently obtain the GWs radiated from each source using the linearized RGE (9), except for the GW radiation reactions on the sources and the nonlinear interactions between the GWs, which are neglected to derive (9). The $x$–$y$ sections of all six GW sources (i.e., slaved components) are shown in Fig. 11. Note that Figs. 9a and 9c, respectively, correspond to the sum of Figs. 11a–11c and the sum of Figs. 11d and 11f. The color scales in Figs. 11d and 11f are about 12 times as large as those in Figs. 11a–11c, and those in Fig. 11e are about 20 times as small as those in Figs. 11d and 11f. The sources $\gamma^{\text{slave}(1)}$ (Fig. 11d), $\gamma^{\text{slave}(2)}$ (Fig. 11f), and $\delta^{\text{slave}(2)}$...
slaved comp. of 30th-day dipole (16 ≤ |k_H|, |k_3|)

(a) $\delta_{\text{slave}(1)} + \delta_{\text{slave} q}\delta_{\text{2}} + \delta_{\text{slave qy}}(2)$

(b) $\gamma_{\text{slave}(1)} + \gamma_{\text{slave} q}\delta_{\text{2}} + \gamma_{\text{slave qy}}(2)$

slaved comp. of modon (16 ≤ |k_H|, |k_3|)

(c) $\delta_{\text{slave}(1)} + \delta_{\text{slave} q}\delta_{\text{2}} + \delta_{\text{slave qy}}(2)$

(d) $\gamma_{\text{slave}(1)} + \gamma_{\text{slave} q}\delta_{\text{2}} + \gamma_{\text{slave qy}}(2)$

FIG. 10. The $x$–$y$ sections at $z = 3.0$ km of the GW sources composed of the slaved components (color) of (a) $\delta$ and (b) $\gamma$ in the thirtieth-day dipole with $Ro = 0.25$, the wavenumbers of which are larger than 15, which were obtained by the JMA-NHM. (c),(d) As in (a) and (b), respectively, but for the initial modon. The contours in all the figures indicate $q$ with a CI of 0.1 s$^{-1}$, where $q$ is multiplied by 10$^3$. The color scales are different for (a) and (c) and for (b) and (d).

(Fig. 11c) have large amplitudes and relatively small scales compared with $L$ (characteristic length of the large-scale flow) around the jet center, which is in contrast to the other sources. Note that $\gamma_{\text{slave}(1)}$ and $\gamma_{\text{slave qy}}(2)$ are zero on the jet axis, while they have comparable magnitudes to $\delta_{\text{slave qy}}(2)$ near the jet center. In addition, $\gamma_{\text{slave}(1)}$ and $\delta_{\text{slave qy}}(2)$ are meridionally symmetric with respect to the jet axis, while $\gamma_{\text{slave qy}}(2)$ is antisymmetric.

The main sources are identified as $\gamma_{\text{slave}(1)}$, $\gamma_{\text{slave qy}}(2)$, and $\delta_{\text{slave qy}}(2)$ by examining the GWs radiated from each source that are obtained by the RGE. Note that cancellation between GWs radiated from any sources is negligible. Figures 12a–f show the $x$–$y$ and $x$–$z$ sections of $\delta_{\text{GW}}$ from the three main sources. The spatial structures of the GWs from respective sources are quite similar, as discussed by Wang et al. (2010). This fact is also physically explained by the RGE theory; the GW eigenmodes that are potentially radiated from the slaved components are completely determined by the spatial structure of the large-scale vortical flow, indicating that all the GWs obtained as the superposition of the eigenmodes may have quite similar structures. As discussed later in section 5e, not only the spatial structure of the vortical flow, but also those of the GW sources, affects the GW phase structure. On the other hand, the GW amplitudes are largest for $\gamma_{\text{slave}(1)}$, second largest for $\gamma_{\text{slave qy}}(2)$, and third largest for $\delta_{\text{slave qy}}(2)$. The sum of the GWs from these three sources corresponds well to the total GWs shown in Figs. 5a and 5b. Although diffusion has not been included in the present calculations, diffusion...
probably affects the GWs from respective sources similarly, because the GW structures are quite similar in Fig. 12. Thus, $\tilde{g}_{\text{slave}}(1)$, $\tilde{g}_{\text{slave} q}(2)$, and $\tilde{d}_{\text{slave} q}(2)$ are considered to be the main sources.

The main sources are divided into two groups on the basis of their formulas: the first group consists of $\tilde{g}_{\text{slave}}(1)$ and $\tilde{g}_{\text{slave} q}(2)$, which describes the velocity-variation mechanism (Fig. 3 in Part I), and the second one consists of $\tilde{d}_{\text{slave} q}(2)$, which describes the mountain-wave-like mechanism (Fig. 2 in Part I). The first and second groups are examined separately in the following.

b. Velocity-variation mechanism

The GW sources are transformed into derivative forms to clarify the discussion in this and the next subsections. We first examine the sources $\tilde{g}_{\text{slave}}(1)$ and $\tilde{g}_{\text{slave} q}(2)$, which describes the velocity-variation mechanism. This mechanism was proposed by Viúdez (2007). The RGE theory can provide the theoretical background for this mechanism.

The term $\tilde{g}_{\text{slave}}(1)$ is given by (A2) and can be transformed into the following:

$$\tilde{g}_{\text{slave}}(1) = \text{Ro} \left[ \frac{\partial (\tilde{u}_H' \cdot \tilde{V}_H \tilde{u}'_H)}{\partial x} + \frac{\partial (\tilde{u}_H' \cdot \tilde{V}_H \tilde{u}'_H)}{\partial y} \right] = \text{Ro} \left( \frac{\partial}{\partial x} \frac{D\tilde{u}'}{Dt} + \frac{\partial}{\partial y} \frac{D\tilde{u}'}{Dt} \right), \tag{12}$$

where $\tilde{u}'_H$ and $\tilde{v}'_H$ are the velocity components associated with $\tilde{q}$ (i.e., vortical flow) that are obtained by using (17), (20), and (21) in Part I; $\tilde{V}_H$ is the nabla operator in the $x$–$y$ plane; and $D\tilde{q}/Dt$ is the material derivative by the vortical flow. We used the fact that $\tilde{u}'_H$ is horizontally nondivergent [see (20) and (21) in Part I] for the transformation to the most RHS of (12). Equation (12) means that $\tilde{g}_{\text{slave}}(1)$ is composed of the horizontal divergence of the vortical flow acceleration. The vortical flow is accelerated in the jet entrance region and decelerated in the exit one, indicating that $\tilde{g}_{\text{slave}}(1)$ is negative near the jet center, while it is positive in the jet entrance and exit regions (Fig. 11d). When the space–time scales of the horizontal divergence of the acceleration are partially comparable to those of the GW eigenmodes, the GWs are radiated through the quasi resonance (see schematic in Fig. 3 in Part I). Clearly, this mechanism corresponds to the velocity-variation mechanism proposed by Viúdez (2007), in which GWs are radiated when the time scale of particle-velocity variation is close to the inertial period.

Figures 13a and 13b, respectively, show the $x$–$y$ and $x$–$z$ sections of the slaved pressure $\tilde{p}_{\text{slave}}$ obtained by $\tilde{p}_{\text{slave}} = \tilde{g}_{\text{slave}}(1) / |\tilde{q}|^2$. The pressure gradient forces accelerate fluid particles in the jet entrance region, while they decelerate particles in the jet exit region. Moreover,
Figs. 13c and 13d, respectively, show the \( x \)-\( y \) and \( x \)-\( z \) sections of the slaved buoyancy \( \beta^\text{slave}(1) \) obtained from \( \rho^\text{slave}(1) \) by using the hydrostatic relation. Note that \( \beta^\text{slave}(1) \) corresponds to one of the main GW sources, which were called thermal imbalance in Snyder et al. (1993, 2009). The intervals between potential temperature surfaces are narrower around the jet center than in the jet entrance and exit regions (Fig. 13d). This fact is understood by the Bernoulli effect; the stream tube is thinner and the stream is faster around the jet center.

The term \( \gamma^\text{slaveq}(2) \) is composed of the product of \( \gamma^\text{slave}(1) \) and \( \tilde{q}_k \) as in (A7). Its derivative form is as follows:

\[
\tilde{\gamma}^\text{slaveq}(2) = \frac{\partial (\tilde{u}_H^q \cdot \nabla \gamma^\text{slave}(1))}{\partial x} + \frac{\partial (\tilde{u}_H^q \cdot \nabla \gamma^\text{slave}(1))}{\partial y} + \frac{\partial (\tilde{u}_H^q \cdot \nabla \gamma^\text{slave}(1))}{\partial x} + \frac{\partial (\tilde{u}_H^q \cdot \nabla \gamma^\text{slave}(1))}{\partial y} + \frac{\partial (\tilde{u}_H^q \cdot \nabla \gamma^\text{slave}(1))}{\partial x} + \frac{\partial (\tilde{u}_H^q \cdot \nabla \gamma^\text{slave}(1))}{\partial y} \]

\[
= \text{Ro} \left[ \frac{\partial D q}{\partial x} \tilde{q}_k^\text{slave}(1) + \frac{\partial D q}{\partial y} \tilde{q}_k^\text{slave}(1) + \frac{\partial D q}{\partial x} \tilde{q}_k^\text{slave}(1) + \frac{\partial D q}{\partial y} \tilde{q}_k^\text{slave}(1) \right]
\]

(13)

where \( \tilde{u}_H^\text{slave}(1) \) and \( \tilde{q}_k^\text{slave}(1) \) are velocity components associated with \( \gamma^\text{slave}(1) \) given by (19)–(21) in Part I; \( \tilde{u}_H^\text{slave}(1) = (\tilde{u}_H^\text{slave}(1), \tilde{v}_H^\text{slave}(1), 0) \); and \( D \gamma^\text{slave}(1)/Dt \) is the material derivative by \( \tilde{u}_H^\text{slave}(1) \). We used the fact that \( \tilde{u}_H^\text{slave}(1) \) is horizontally nondivergent as for \( \tilde{u}_H^q \) for the transformation to the most RHS of (13). Similar to
\( \gamma_{\text{slave}}(1) \) and \( \gamma_{\text{slave}}(2) \) is composed of the horizontal divergences of accelerations. Thus, \( \gamma_{\text{slave}}(2) \) also describes the GW radiation caused by the velocity-variation mechanism. Note that \( \gamma_{\text{slave}}(2) \) is composed of the horizontal divergences of the acceleration of both \( \tilde{u}_x \) (i.e., vortical flow) and \( \tilde{u}_y \), while \( \gamma_{\text{slave}}(1) \) only includes \( \tilde{u}_y \).

There is another difference between \( \gamma_{\text{slave}}(1) \) and \( \gamma_{\text{slave}}(2) \). The spatial structure of \( \gamma_{\text{slave}}(2) \) is meridionally antisymmetric with respect to the jet axis (Fig. 11f), while \( \gamma_{\text{slave}}(1) \) is symmetric (Fig. 11d). This fact is physically understood in the following. Roughly speaking, centripetal force acting on \( \tilde{u}_y \) directs to the vortex center on both the anticyclonic and cyclonic vortex sides. Thus, \( \gamma_{\text{slave}}(1) \) obtained as the horizontal divergence of the acceleration of \( \tilde{u}_y \) (i.e., centripetal force) is meridionally symmetric with respect to the jet axis. On the other hand, the vortical flow itself \( \tilde{u}_y \) is antisymmetric, because it is anticyclonic on the anticyclonic side of the dipole, while it is cyclonic on the cyclonic side. Loosely speaking, this antisymmetry leads to the meridional antisymmetry of \( \gamma_{\text{slave}}(2) \). More precisely, \( \tilde{u}_y \) is antisymmetric, while \( \tilde{u}_y \) is symmetric, because it is induced by the symmetric \( \gamma_{\text{slave}}(1) \). Thus, \( \gamma_{\text{slave}}(2) \) composed of the products of the symmetric \( \tilde{u}_y \) and antisymmetric \( \tilde{u}_y \) has the meridionally antisymmetric structure with respect to the jet axis [i.e., negative on the anticyclonic side and positive on the cyclonic side (Fig. 11f)].
c. Mountain-wave-like mechanism

Next, we examine the GW source \( \delta^\text{slave}\gamma^{(2)} \), which describes the mountain-wave-like mechanism. This mechanism was proposed by McIntyre (2009). The RGE theory can provide the theoretical background for this mechanism. We examine characteristics of the slaved vertical flow \( w^\text{slave}\gamma^{(2)} \), as \( \delta^\text{slave}\gamma^{(2)} \) is related to \( w^\text{slave}\gamma^{(2)} \) by \( w_k^\text{slave}\gamma^{(2)} = \frac{\gamma_k^\text{slave}\gamma^{(2)}}{k_3} \).

Figures 13e and 13f, respectively, show the \( x-y \) and \( x-z \) sections of \( w^\text{slave}\gamma^{(2)} \). It is found that \( w^\text{slave}\gamma^{(2)} \) has the descent–ascent couplets on the jet axis. Corresponding to the couplets, there are convergence \( \left( \partial w / \partial z < 0 \right) \) in the jet entrance and divergence \( \left( \partial w / \partial z > 0 \right) \) in the jet exit regions (Fig. 9b). It is shown by examining the dominant term that the formula giving \( w^\text{slave}\gamma^{(2)} \) is approximately reduced to

\[
\nabla^2 w^\text{slave}\gamma^{(2)} \approx - \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \mathbf{u}_H \cdot \nabla_H \delta^\text{slave}(1). \quad (14)
\]

It was confirmed that cancellation between the other term included in \( w^\text{slave}\gamma^{(2)} \) did not occur. The above formula indicates that the couplets are produced because of the vortical flow over the modified potential temperature surfaces described by \( \delta^\text{slave}(1) \). From the distribution of \( \delta^\text{slave}(1) \) in Figs. 13c and 13d, the situation can be schematically illustrated as Fig. 2 in Part I. When the space–time scale of \( w^\text{slave}\gamma^{(2)} \) is partially comparable to those of the GW eigenmodes, the GWs are radiated through the quasi resonance. Clearly, the modified potential temperature surfaces act as “mountains,” like ones generating orographic GWs; hence, this mechanism can be understood as the mountain-wave-like mechanism (McIntyre 2009).

d. Importance of shear effect by vortical flow on the mechanisms of GW radiation

In this subsection, the shear effect on the velocity variation and mountain-wave-like mechanisms is discussed. The linearized second-order RGEs including only the main GW sources are written in the following:

\[
\frac{d\gamma_k^{\text{GW}}}{dt} + \left( \sum_m \gamma^{(-1)}_{m \gamma_k} \delta_m \gamma_{k-m} + \omega_k^2 \gamma^{\text{GW}} \right) \\
+ \text{Ro} \left( \sum_m \left( I_{m \gamma_k}^{(0)} \delta_m \gamma_{k-m} + I_{m \gamma_k}^{(0)} \delta_m \gamma_{k-m} \right) \\
\approx \omega_k^2 \text{Ro}^2 \delta^\text{slave}(1) \quad \text{and} \quad (15)
\]

\[
\frac{d\delta_k^{\text{GW}}}{dt} + \left( \sum_m \delta^{(-1)}_{m \delta_k} \delta_m \gamma_{k-m} - \delta_{k-m} \right) \\
+ \text{Ro} \left( \sum_m \left( I_{m \delta_k}^{(0)} \delta_m \gamma_{k-m} + I_{m \delta_k}^{(0)} \delta_m \gamma_{k-m} \right) \\
\approx - \text{Ro} \delta^\text{slave}(1) - \text{Ro}^2 \delta^\text{slave}(2). \quad (16)
\]

The terms in the first sets of parentheses on the LHSs represent the GW advection by the vortical flow, and the terms in the second sets of parentheses represent the modification to the GW propagation by the shear of the vortical flow.

The GWs from \( \gamma^{\text{slave}(1)}_k \) and \( \gamma^{\text{slave}(2)}_k \) obtained by the RGEs (15) and (16) with the shear terms are dominant, compared with those from \( \delta^\text{slave}(2) \), as shown in Fig. 12. In other words, the mountain-wave-like mechanism is not as important as the velocity-variation one. In contrast, when the shear terms are removed from the RGEs, the GWs from \( \gamma^{\text{slave}(1)}_k \) and \( \delta^\text{slave}(2) \) are dominant, compared with those from \( \gamma^{\text{slave}(2)}_k \) (not shown). Note that the second terms on the LHSs of (15) and (16) correspond to the \( O(\text{Ro}^{-1}) \) terms in \( \mathbf{L} \) [see (7)], while the third terms correspond to the \( O(\text{Ro}^0) \) terms.

This dependence on the vortical flow shear can be understood as follows. When the shear effect is negligible, the antisymmetry of the GW modes between the cyclonic and anticyclonic sides becomes quite weak, because the interaction coefficients \( O(\text{Ro}^{-1}) \) in \( \mathbf{L} \) are proportional to the first power of the wavenumber in the \( y \) direction. All GW modes have the meridionally symmetric structures in the modon solution, where \( q \) is antisymmetric in the \( y \) direction and symmetric in the \( x \) and \( z \) directions. Thus, GWs are expected to be effectively radiated from meridionally symmetric sources (see Fig. 11). Conversely, when the shear effect is not negligible, some \( O(\text{Ro}^0) \) terms in \( \mathbf{L} \) proportional to the second power of the wavenumber in the \( y \) direction become large. Thus, some GW modes have strong antisymmetry. As a result, the GW radiation occurs effectively from the antisymmetric sources.

These theoretical investigations suggest that the dominant mechanism for GW radiation significantly depends on the configurations of large-scale flows. It should be noted again that the large-scale vortical flow determines the spatial structure of each eigenmode, meaning that the degree of a quasi resonance with respective sources is determined as well. Wang et al. (2010) discussed the shear effect as the cause for the antisymmetric GW distribution through the modification to the GW propagation. The present paper newly indicates that the shear affects not only the GW propagation but also the radiation mechanism.

e. Relation between phase structures of GWs and their sources

Finally, in order to clarify how the phase structure of the GW sources affects that of the GWs, the following equation is examined, in which the slaved components in the first row of (8) are artificially exchanged with those of the second row:
\[ \frac{d}{dt} \left( \frac{\delta^{GW}_{k}}{\delta_{k}} \right) + \text{Ro} \cdot \Delta \left( \frac{\delta^{GW}_{k}}{\delta_{k} - \delta_{m}} \right) = \text{Ro} \left[ \omega_{k}^{2} \gamma_{k}^{\text{slave}(1)} \right] + \text{Ro}^{2} \left[ \omega_{k}^{2} \left( \gamma_{k}^{\text{slave}(2)} + \gamma_{k}^{\text{slave}(2)_{q}} \right) \right] + O(\text{Ro}^{3}). \]  

This exchange corresponds to the change in the phase structure of the GW sources. Figures 14a and 14b, respectively, show the x–y and x–z sections of \( \delta^{GW} \) obtained by (17). The GW phase structure is different from the original one in Figs. 5a and 5b by about 90°. This phase difference is attributed to the phase difference in the GW sources. This arbitrary experiment indicates that not only the spatial structure of the background flow but also those of the GW sources affect the GW structure.

Wang et al. (2010) examined the GWs radiated from the prescribed source with the Gaussian shape and showed that the GW structure was mainly determined by the structure of the background flow. However, while the width and height of the source were changed in their study, its phase structure remained the same. The results in this subsection complement their results.

6. Momentum flux distribution and dependence on Rossby number

In this section, we examine momentum fluxes associated with the GWs in terms of the spatial distribution and the dependence on Ro. By using an average over an appropriate limited area, we define the disturbance (mainly composed of GWs) and the mean flow (mainly composed of balanced flows) for a given physical quantity \( A \), as follows:

\[ \overline{A} = \overline{\int_{x-\Delta x}^{x+\Delta x} \int_{z-\Delta z}^{z+\Delta z} A(t, x', y, z') \, dx' \, dz} \]  

and

\[ A' = A - \overline{A}. \]

Note that the disturbance component defined in this way is regarded as GWs only in this section. A term \( \overline{u'w'} \) obtained from the JMA-NHM data on the thirtieth day is examined as GW momentum fluxes.

The momentum fluxes are important for the momentum budget in the middle atmosphere. GWs radiated from an isolated jet stream may be captured and hardly propagate upward (e.g., Plougonvén and Snyder 2007). However, a part of such GWs may be able to propagate upward without being captured when another large-scale vortex like the stratospheric polar vortex is present (McIntyre 2009) or in a more realistic complicated flow pattern (Tateno and Sato 2008). Thus, it is important and useful to quantitatively examine the momentum fluxes of the GWs radiated from the jet stream to improve GW parameterization schemes used in most climate models.
Two cautions are given regarding the momentum fluxes by Bühler and McIntyre (2005) and McIntyre (2009). First, it is necessary to carefully choose the area where momentum fluxes are estimated because GWs generally break at edges of wave-capture regions. Second, in a complicated three-dimensional fluid, a local wave forcing, such as the vertical derivative of $u_0w_0$ (i.e., $\partial u_0w_0/\partial z$), generally causes a nonlocal response to a mean flow like the Bretherton return flow.

Figures 15a and 15b, respectively, show the $x$–$y$ and $x$–$z$ sections of $u_0w_0$ on the thirtieth day with $Ro = 0.25$ obtained by the JMA-NHM. In these calculations, the averaging was done for respective areas with $(2D_x, 2D_z) = (120 \text{ km}, 1.2 \text{ km})$. The term $u_0w_0$ has a nearly antisymmetric structure vertically with respect to $z = 0.0 \text{ km}$ and is dominant in the central region surrounded by a contour of $u = 7.5 \text{ m s}^{-1}$ in Fig. 15b. In this region, the slaved component $\delta_{\text{slave}}q^2$ is also large and has a spatial scale comparable to that of “true” GWs (see Figs. 5a, 5b, 11c, 13e, and 13f). Thus, the disturbance defined by (18) and (19) may contain the slaved components as well as the GWs there. This inference is consistent with the fact that there is a similar structure of momentum fluxes around the central region even for the JMA-NHM simulation with $Ro = 0.10$, in which GWs are hardly recognized (not shown). In addition to the central region, the momentum fluxes near the edge of the dipole should not be analyzed, because GWs break there and are largely affected by the artificial numerical diffusion. Thus, the momentum fluxes within the rectangular region $D$ in Fig. 15 are examined:

$$D = \{(x, y, z) | x \in [200 \text{ km}, 300 \text{ km}], y \in [-300 \text{ km}, 300 \text{ km}], z \in [-3 \text{ km}, 3 \text{ km}]\}. \quad (20)$$

It is clear that $u_0w_0 < 0$ for the GWs in $D$ propagating upward from the jet and $u_0w_0 > 0$ for those propagating downward.

Next, in order to examine the dependence of $u_0w_0$ on $Ro$, the JMA-NHM was additionally run for initial modons in the range of $Ro = 0.05–0.50$ under the same conditions as those described in section 3a. Note again that the properties of an initial modon are uniquely determined by radius $a$ and phase speed $c$. Here, $a$ was fixed at 500 km, and $c$ was changed to set the required $Ro$. The dipoles had almost the same structures for all cases, as shown in Figs. 2–4.

The obtained $|u_0w_0|$ was averaged over the same $D$ on the thirtieth day with each $Ro$, which is referred to as $\text{Ave}|u_0w_0|$. Because the dipole was damped in time, $U$ was recalculated using the instantaneous $u$ field on the thirtieth day. As the size of the dipole did not significantly
Fig. 16. The characteristic magnitude of momentum fluxes plotted against $U$ and $Ro$, which were obtained from the JMA-NHM simulations. The closed circles indicate that GWs were hardly recognized in the dipoles, while the open circles indicate that the GWs, such as those in Figs. 3e and 3f, could be recognized. See text for details. The line is a least squares fit to the open circles, whose inclination is 6.23.

The characteristic scale of momentum fluxes is approximately proportional to the sixth power of $Ro$ or $L$. This means that the rough estimate by using the RGE does not contradict the results from the JMA-NHM simulations in Fig. 16. The above theoretical estimate is rough, and the characteristic magnitude of momentum fluxes is proportional to the fourth to sixth powers of $Ro$, as follows. Herein, a quantity with an asterisk represents the characteristic scale for the corresponding quantity. The scale of GW amplitude $\delta_{GW^*}$ should be mainly determined by the characteristic magnitudes of the GW sources (i.e., the slaved components). The first-order slaved components consist of the products of $\delta q_k$, while the second-order slaved components consist of the products of $\delta q_k$ and the first-order slaved components. In addition, the characteristic magnitude of a quantity increasing over time due to a resonance should be proportional to time. The results obtained by the RGE with the diffusion in section 4b suggest that the characteristic scale for the slow time should be used to estimate $\delta_{GW^*}$. Thus, $\delta_{GW^*}$ is estimated by using the linearized second-order RGE (8):

$$\delta_{GW^*} = (\mu + \sigma Ro^2) \frac{U}{L} \left( \frac{U}{L} \right)^2 = (\mu + \sigma Ro^2) \frac{U^2}{L^2} \frac{U}{L}, \quad (21)$$

where $\mu$ and $\sigma$ represent the constants. Note that the characteristic scale $s^k$ for the slow time ($s = Ro t$) is given by the advection time scale of the large-scale flow (i.e., $L/U$). Using $\delta_{GW^*} = u^*/(Ro L)$ and (21), the characteristic scale for $u'$ is obtained:

$$u'' = (\mu + \sigma Ro^2) U = \mu Ro^2 + \sigma Ro^3 \frac{f L}{U}. \quad (22)$$

The characteristic scale $w^s_*$ can also be estimated on the basis of the continuity equation as $(\mu Ro^2 + \sigma Ro^3) H$. Thus, the characteristic scale of momentum fluxes is obtained:

$$w'' = (\mu Ro^2 + \sigma Ro^3) f^2 L H. \quad (23)$$

This includes a term proportional to the sixth power of $Ro$ (or $U$), as well as terms proportional to $Ro^4$ and $Ro^5$. This means that the rough estimate by using the RGE does not contradict the results from the JMA-NHM simulations in Fig. 16. The above theoretical estimate is rough, and the characteristic magnitude of momentum fluxes

consistent with those obtained by Snyder et al. (2007), Wang et al. (2009) also examined the dependence of the GW amplitude on Ro in the dipole. In their study, Ro was changed by making the cyclonic center approach the anticyclonic one. This means that the characteristic GW amplitude in dipoles is proportional to the power of Ro.

The RGE theory explains that the characteristic magnitude of momentum fluxes is proportional to the fourth to sixth powers of $Ro$, as follows. Herein, a quantity with an asterisk represents the characteristic scale for the corresponding quantity. The scale of GW amplitude $\delta_{GW^*}$ should be mainly determined by the characteristic magnitudes of the GW sources (i.e., the slaved components). The first-order slaved components consist of the products of $\delta q_k$, while the second-order slaved components consist of the products of $\delta q_k$ and the first-order slaved components. In addition, the characteristic magnitude of a quantity increasing over time due to a resonance should be proportional to time. The results obtained by the RGE with the diffusion in section 4b suggest that the characteristic scale for the slow time should be used to estimate $\delta_{GW^*}$. Thus, $\delta_{GW^*}$ is estimated by using the linearized second-order RGE (8):

$$\delta_{GW^*} = (\mu + \sigma Ro) \left( \frac{U}{L} \right)^2 = (\mu + \sigma Ro) \frac{U^2}{L^2} \frac{U}{L}, \quad (21)$$

where $\mu$ and $\sigma$ represent the constants. Note that the characteristic scale $s^k$ for the slow time ($s = Ro t$) is given by the advection time scale of the large-scale flow (i.e., $L/U$). Using $\delta_{GW^*} = u^*/(Ro L)$ and (21), the characteristic scale for $u'$ is obtained:

$$u'' = (\mu Ro + \sigma Ro^2) U = (\mu Ro^2 + \sigma Ro^3) f L. \quad (22)$$

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$$\delta_{GW^*} = (\mu + \sigma Ro) \left( \frac{U}{L} \right)^2 = (\mu + \sigma Ro) \frac{U^2}{L^2} \frac{U}{L}, \quad (21)$$

where $\mu$ and $\sigma$ represent the constants. Note that the characteristic scale $s^k$ for the slow time ($s = Ro t$) is given by the advection time scale of the large-scale flow (i.e., $L/U$). Using $\delta_{GW^*} = u^*/(Ro L)$ and (21), the characteristic scale for $u'$ is obtained:

$$u'' = (\mu Ro + \sigma Ro^2) U = (\mu Ro^2 + \sigma Ro^3) f L. \quad (22)$$

The characteristic scale $w^s_*$ can also be estimated on the basis of the continuity equation as $(\mu Ro^2 + \sigma Ro^3) H$. Thus, the characteristic scale of momentum fluxes is obtained:

$$w'' = (\mu Ro^2 + \sigma Ro^3) f^2 L H. \quad (23)$$

This includes a term proportional to the sixth power of $Ro$ (or $U$), as well as terms proportional to $Ro^4$ and $Ro^5$. This means that the rough estimate by using the RGE does not contradict the results from the JMA-NHM simulations in Fig. 16. The above theoretical estimate is rough, and the characteristic magnitude of momentum fluxes
fluctuations is also considered to depend on the form of diffusion. However, this is beyond the scope of the present paper.

7. Summary and concluding remarks

The system of the renormalization group equations (RGEs) describing the spontaneous GW radiation that was derived in Part I has been validated by using numerical simulations with the Japan Meteorological Agency nonhydrostatic model (JMA-NHM). By taking the modon solution on the $\beta$ plane (Berestov 1979; Flierl 1987) to be the initial state, the quasi-steady GW radiation in the vortex dipole was simulated by the JMA-NHM. The spatial structures of the GWs in the dipole obtained by the JMA-NHM were quite similar to those reported by previous studies (Snyder et al. 2007; Viúdez 2007, 2008; Wang et al. 2009; Wang and Zhang 2010). It was shown that the RGE can reproduce the GWs in the dipole obtained by the JMA-NHM reasonably well, indicating the validity of the RGE theory. This fact strongly supports the validity of the mechanism on spontaneous GW radiation that was newly proposed in Part I and this paper: Doppler-shifted GWs are radiated through a quasi resonance with slaved components of the balanced flow, where the quasi resonance occurs as a result of time-scale matching between the GWs and slaved components.

It was shown that there are three main GW sources that are divided into two groups: the first consists of $\tilde{\gamma}_{\text{slave}}^{(1)}$ and $\tilde{\gamma}_{\text{slave}}^{(2)}$, and the second consists of $\delta_{\text{slave}}^{(2)}$. This fact means that the above GW radiation mechanism is further divided into two groups depending on the mechanisms of GW source production. The first-group sources (i.e., $\tilde{\gamma}_{\text{slave}}^{(1)}$ and $\tilde{\gamma}_{\text{slave}}^{(2)}$) are composed of the horizontal divergences of the large-scale flow acceleration [as in (12) and (13)]. Thus, the first-group sources describe the GW radiation with the velocity-variation mechanism (Viúdez 2007; see Fig. 3 in Part I). The second-group source (i.e., $\delta_{\text{slave}}^{(3)}$) is approximately produced by the vortical flow over the deformed potential temperature surfaces near the strong jet [as in (14)]. As the potential temperature surfaces act as mountains, like ones generating orographic GWs, $\delta_{\text{slave}}^{(2)}$ describes the GW radiation with the mountain-wave-like mechanism (McIntyre 2009; see Fig. 2 in Part I). In addition, it was suggested that the shear effect of the vortical flow determines which mechanism is dominant for the GW radiation in the dipole.

Last but not least, GW momentum fluxes (i.e., $\overline{u'w'}$) were estimated from the JMA-NHM simulation data. The GWs propagating upward from the jet had negative $\overline{u'w'}$, while those propagating downward had a positive flux. The characteristic magnitude of $\overline{u'w'}$ was approximately proportional to the sixth power of the Rossby number ($R_o$) between 0.15 and 0.4. This result did not conflict with the theoretical estimate obtained by the second-order RGE.

There are some important issues for future works. The first is the mountain-wave interpretation for the GW radiation from $\delta_{\text{slave}}^{(2)}$. In this interpretation, deformed potential temperature surfaces act as mountains. As was discussed by McIntyre (2009), these mountains vary over time. A large-scale flow should be weaker as GWs are radiated, indicating that the deformation of potential temperature surfaces by the Bernoulli effect becomes smaller. In other words, the mountains are expected to become smaller (or “lower”), which will probably be identified as the resonance adjustment (see section 4b of Part I). Confirming variations in potential temperature surfaces is one of the important issues regarding the reaction of GW radiation or the adjustment process.

It is important to examine whether the mechanisms proposed in this study can explain spontaneous GW radiation in more complicated systems, such as jet–front systems. The RGE system may be difficult to apply to such complicated systems, because it requires a lot of computational resources. In addition, GW radiation from a front demonstrates different characteristics than that from a jet (Plougonven and Snyder 2007). Large-scale flows vary on a relatively fast time scale during the evolution of the front (Snyder et al. 1993). In such situations, GWs having high frequencies may also be radiated; however, these high-frequency GWs cannot be described by the RGE theory. Thus, it is necessary to develop the RGE theory into a theory that is easier to apply and can treat GWs having both slow and fast time scales.

Wei and Zhang (2014) showed that the GW amplitude was larger in the moist jet–front system than in the dry one. The RGE theory gives a few implications for their study. Not only the latent heat release, but also potential temperature anomalies with vertical flows due to the latnet heat release, may become new GW sources. These sources may have short time scales and/or spatial scales. This expectation indicates that a quasi resonance would more easily occur in moist systems than in dry systems. Thus, the RGE theory indicates that larger GW amplitudes, as in Wei and Zhang (2014), can be attributed to the ease of the quasi resonance as well as the appearance of new GW sources. This is an interesting issue for future studies.

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APPENDIX

Specific Forms of Slaved Components

The specific forms of the slaved components are shown as follows. Note that the specific forms of the interaction coefficients are shown in appendix B in Part I. First, the nondimensionalized first-order slaved components are

$$
\delta_k^{\text{slave(1)}} = \frac{1}{\bar{\omega}_k} \sum_m I_{\alpha_m} \hat{g}_m \hat{q}_{k-m} \quad \text{and} \quad (A1)
$$

$$
\gamma_k^{\text{slave(1)}} = -\sum_m \hat{\delta}_m \hat{q}_{k-m} \hat{g}_m \hat{q}_{k-m}. \quad (A2)
$$

Here, \( \omega_k = |k|/|k_3| \), which can be regarded as the intrinsic frequency of the plane GW in the motionless atmosphere, the wavenumber vector of which is \( k \).

Second, the nondimensionalized second-order slaved components are

$$
\delta_k^{\text{slave(2)}} = \frac{1}{\bar{\omega}_k} \sum_m I_{\alpha_m} \hat{g}_m \hat{q}_{k-m}^{\text{balance(1)}} + \hat{q}_m^{\text{balance(1)}} \hat{g}_{k-m}. \quad (A3)
$$

$$
\gamma_k^{\text{slave(2)}} = \frac{1}{\bar{\omega}_k} \sum_m I_{\alpha_m} \hat{g}_m \hat{q}_{k-m}^{\text{slave(1)}} \hat{q}_{k-m} \quad \text{and} \quad (A4)
$$

$$
\delta_k^{\text{slave(2)}} = \frac{1}{\bar{\omega}_k} \sum_m I_{\alpha_m} \hat{g}_m \hat{q}_{k-m}^{\text{slave(1)}} \hat{q}_{k-m} \quad \text{and} \quad (A5)
$$

$$
\gamma_k^{\text{slave(2)}} = \frac{1}{\bar{\omega}_k} \sum_m I_{\alpha_m} \hat{g}_m \hat{q}_{k-m}^{\text{balance(1)}} + \hat{q}_m^{\text{balance(1)}} \hat{g}_{k-m}. \quad (A6)
$$

$$
\gamma_k^{\text{slave(2)}} = -\sum_m \hat{\delta}_m \hat{q}_{k-m} \hat{g}_m \hat{q}_{k-m} \hat{q}_{k-m} \quad \text{and} \quad (A7)
$$

$$
\gamma_k^{\text{slave(2)}} = -\sum_m \hat{\delta}_m \hat{q}_{k-m} \hat{g}_m \hat{q}_{k-m} \hat{q}_{k-m} \quad \text{and} \quad (A8)
$$

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