Relative Dispersion in the Atmosphere from Reanalysis Winds

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ABSTRACT

The dispersion of pairs of synthetic particles, advected with ECMWF winds, is examined. The particles were deployed at three latitudes and on three potential temperature surfaces in both hemispheres. Separation statistics are calculated and evaluated in relation to 2D turbulence theory and to Eulerian structure functions calculated directly from the wind data.

At the smallest sampled scales (100–1000 km), the pair-separation velocities are correlated, and the dispersion is laterally isotropic, at least at the higher latitudes. At larger scales, the dispersion is zonally anisotropic, and the pair velocities are uncorrelated. In all cases, the dispersion grows exponentially in time, and the second-order Eulerian structure functions consistently increase as separation squared. This implies nonlocal dispersion, which obtains with energy spectra at least as steep as $K^{-3}$.

Regional variations are seen in the parameters however. The $e$-folding times and the maximum scales for exponential growth are significantly larger on the 430-K surface than on the 315-K surface, and the dispersion is anisotropic at low latitudes, even at the smallest scales. Therefore, 2D homogeneous turbulence theory is applicable at best at subdeformation scales at the higher latitudes.

1. Introduction

Isentropic transport is important for many processes in the atmosphere: the redistribution of water vapor (Kelly et al. 1991; Yang and Pierrehumbert 1994; Fueglistaler et al. 2005), ozone (Leovy et al. 1985; Holton et al. 1995), airborne particulates, such as volcanic ash (Heffter and Stunder 1993; Prata et al. 2007), and active tracers, such as heat and potential vorticity (Hoskins et al. 1985; Haynes and McIntyre 1987; Juckes and McIntyre 1987). Such transport can be studied using particle (Lagrangian) dispersion. This includes the study of single particles (absolute dispersion) and groups of particles (relative dispersion). Relative dispersion is also of interest, as its properties depend on the kinetic energy spectrum (Bennett 1984). As such, pair dispersion gives insight into the Eulerian velocity statistics.

Relative dispersion was studied on large scales in the 1970s using constant-level balloons in the lower stratosphere. Two such experiments, conducted in the Southern Hemisphere (SH), were the EOLE (Morel and Bandeen 1973) and Tropical Wind, Energy Conversion, and Reference Level Experiment (TWERLE; Jullian et al. 1977) campaigns. In both experiments, pair separations were found to grow exponentially in time, with an $e$-folding scale of roughly 1 day, up to separations of 1000–2000 km (Morel and Larcheveque 1974; Er-El and Peskin 1981). Such exponential growth implies steep (nonlocal) energy spectra (Bennett 1984). The behavior at larger scales was less clear; Morel and Larcheveque (1974) found diffusive dispersion, while Er-El and Peskin (1981) observed faster (superdiffusive) growth.

The EOLE dispersion was reexamined by Lacorata et al. (2004), who employed a different measure [the finite-scale Lyapunov exponent (FSLE; Aurell et al. 2000)].
While the authors also identified exponential growth at separations below 100 km and diffusive growth at large scales, they suggested that the intermediate-scale (100–1000 km) dispersion exhibited a power-law dependence. This implies a shallower energy spectrum, and, in their case, one that was consistent with spectra derived from aircraft measurements (Nastrom and Gage 1985).

As balloons are expensive, field experiments involve limited numbers of instruments (of order 500 balloons in the EOLE experiment), and the statistics are often noisy. Large numbers of measurements can be obtained, though, using synthetic particles advected numerically. Although the advecting wind fields do not resolve smaller scales, the increased statistical aspects are appealing: error bars can be reduced by increasing the numbers of pairs.

Huber et al. (2001) studied the relative dispersion of synthetic particles advected by European Centre for Medium-Range Weather Forecasts (ECMWF) winds. They found that dispersion varied qualitatively with latitude; while it grew exponentially in time in the tropics, it exhibited a power-law growth in the mid- and high latitudes (and not greatly different than linear in time). This implies a corresponding variation in the kinetic energy spectra with latitude.

LaCasce (2010) also studied the EOLE balloon data, using probability density functions (PDFs) of pair separations. The dispersion, the second moment of the separations, derives from the PDF, but the PDF indicates additional aspects, such as the frequency of large separations. The PDFs in the EOLE experiment were consistent with exponential growth at separations below 1000 km. Again, though, statistical significance was hampered by the relatively small number of pairs available.

Herein, we use PDFs to reexamine the relative dispersion of synthetic particles advected by ECMWF winds. The goal is to obtain a clearer picture of the dispersion at the smallest scales captured by the winds. We deployed thousands of particles along latitude lines: in the midlatitudes and the tropics, in the troposphere and stratosphere, and in summer and winter seasons. We also compare the results to the Eulerian statistics derived from the same winds.

2. Theoretical background

A necessary first step in relative dispersion is to establish when the pair velocities are correlated. When separations exceed the scales of the energy-containing eddies, pair statistics are equivalent to those of single particles, and absolute dispersion suffices. Correlation can be tested in various ways: for example, with the mean-square separation velocity (e.g., Babiano et al. 1990; LaCasce and Bower 2000; Koszalka et al. 2009).

Many of our expectations for relative dispersion on synoptic scales derive from homogeneous 2D turbulence theory (e.g., Morel and Larchevêque 1974; Er-El and Peskin 1981; Lacorata et al. 2004; Bennett 2006; LaCasce 2008). In turbulent flows, relative dispersion is determined by the Eulerian energy spectrum (Bennett 1984). One can distinguish two regimes. Assuming a power-law kinetic energy spectrum

\[ E(K) \propto K^{-\alpha}, \]

the dispersion is local if \( 1 \leq \alpha < 3 \) and nonlocal if \( \alpha \geq 3 \). Under local dispersion, pair separations exhibit a power-law dependence on time, while the growth is exponential under nonlocal dispersion.

Richardson (1926) proposed that pair separations should be governed by a Fokker–Planck (F–P) equation (see also Kraichnan 1966; Lundgren 1981; Bennett 2006). In 2D, this is

\[ \frac{\partial p}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \kappa_2 \frac{\partial p}{\partial r} \right), \]

where \( r \) is the separation, \( \kappa_2(r) \) is the relative diffusivity, and \( p(r, t) \) is the PDF of the separations. The Eulerian statistics determine how the diffusivity \( \kappa_2 \) varies with separation. Once \( \kappa_2 \) is known, the equation can be solved for \( p(r, t) \).

Solutions for the 2D turbulent inertial ranges have been derived previously (Lundgren 1981; Bennett 2006; LaCasce 2010) and are given in the appendix. With uncorrelated pair velocities, the relative diffusivity is constant, and the asymptotic solution is self-similar, proportional to the Rayleigh distribution. Under a turbulent energy cascade, the pair velocities are correlated, and \( \kappa_2 = \beta r^{4/3} \), where \( \beta \) is a constant proportional to the third root of the energy dissipation rate. The solution (herein called the “Richardson PDF”) also asymptotically approaches a self-similar distribution. Under a turbulent enstrophy cascade, \( \kappa_3 = r^2/T \), where \( T \) is a time scale inversely proportional to the third root of the enstrophy dissipation rate. The solution is proportional to a lognormal distribution (herein called the “Lundgren PDF”). In contrast to the Rayleigh and Richardson PDFs, the Lundgren PDF is never self-similar but becomes increasingly peaked with time.

With the PDF, one can derive the separation moments: \(^1\)

\(^1\) We use the raw moments, as separations are positive definite.
The two of greatest interest are the dispersion (the second moment) and the kurtosis (the fourth moment, normalized by the second moment squared).

For uncorrelated motion, the asymptotic dispersion grows linearly in time, as for a diffusive process (Taylor 1921), and the kurtosis, initially 1, asymptotically approaches 2. In an energy cascade, the asymptotic dispersion increases as $t^3$, now known as Richardson’s law. The kurtosis asymptotically approaches a value of 5.6. In an enstrophy cascade, both the dispersion and kurtosis increase exponentially in time and at the same rate. Such growth also pertains to spectra steeper than $K^{-3}$ (Bennett 1984).

There are other measures that have been used to study relative dispersion. The finite-time Lyapunov exponent (FTLE; e.g., Lapeyre 2002) is particularly suitable under exponential growth. FSLE is another measure, reflecting the time required for pair separations to grow by specified amounts. Both have been used with synthetic data to map regions of intensified mixing. The advantage with PDFs is that we have analytical solutions that dictate how they should evolve in time.

3. Model

For particle advection, we used the Lagrangian Flexible Particle dispersion model (FLEXPART; Stohl and Seibert 1998; Stohl and Thomson 1999; Stohl et al. 2005). This solves the Lagrangian velocity relation

$$\langle r^n \rangle = 2\pi \int_0^{\infty} r^{n+1} p \, dr.$$  (3)

$$\frac{d}{dt} x = u[x(t)],$$  (4)

where the velocity $u$ comprises winds from gridded forcing data and stochastic components representing turbulent mixing. The latter components are primarily important in the planetary boundary layer and in the near-equatorial region.

The wind data come from ECMWF and span the period 1991–2009. The data have 3-h resolution, $1^\circ \times 1^\circ$ spatial resolution, and 60 vertical hybrid levels. At 0000, 0600, 1200, and 1800 UTC, we used data from the ECMWF interim reanalysis (Dee et al. 2011). For the intermediate hours (0300, 0900, 1500, and 2100 UTC), we used ECMWF analysis data. The subsequent results are not sensitive to this choice; using only analysis or reanalysis winds every 6 h yields nearly the same results.

The particles were deployed on latitude circles in square clusters of four, separated by $1^\circ$ of longitude. This yields two pairs, oriented along the diagonals, with equal longitudinal and latitudinal initial separations. The initial separation for each pair was 100 km, roughly the same resolution as for the wind data. The particles were placed at 10°, 30°, and 60° in both hemispheres.

The particles were also deployed on three isentropic surfaces: 315, 350, and 430 K (Fig. 1). The latter two surfaces are relatively flat and lie near 12- and 18-km heights. The height of the 315-K surface varies, however, being lower in the tropics and higher in the mid- and high latitudes. In addition, we deployed particles on 1 January and 1 September to test seasonal variations. The isentropes are qualitatively similar in September, albeit somewhat higher in the SH and lower in the

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Northern Hemisphere (NH) (Fig. 1). The trajectories were calculated over 1 month in both seasons, and the process was repeated for the 19 Januaries and Septembers in the time period 1991–2009. The statistics were then averaged for all years for each season.

To illustrate the behavior, we plot four sets of trajectories in Fig. 2. Each trajectory represents the first 4 days of advection. At 60°N, the January trajectories on the 315-K surface (Fig. 2a) are zonally oriented and wavy, as might be expected for the wintertime storm track (e.g., Chang et al. 2002; Hoskins and Hodges 2002; Ulbrich et al. 2009). The September trajectories (Fig. 2b) are less zonal, consistent with a weaker subtropical jet and storm track in summer. The trajectories in the stratosphere on the 430-K surface (Fig. 2c) exhibit large, elliptical swirls, suggesting advection by the stratospheric polar vortex.

![Figure 2](image_url)

**Fig. 2.** Trajectories of 360 (of 1440) particles deployed in 1991 in (a) January at 315 K and 60°N, (b) September at 315 K and 60°N, (c) January at 430 K and 60°N, and (d) January at 315 K and 10°N. For clarity, we only show the positions for the first 4 days after deployment. Note that the minimum latitude is 30°N in (a)–(c) and 10°S in (d).
The 315-K particles at 10°N, on the other hand, are more zonal (Fig. 2d).

Figure 2 suggests predominantly zonal advection, with variations in the spatial and temporal scales with latitude and height. The statistics discussed herein support such differences, although the generic behavior at the early periods will be seen to be similar.

4. Results

a. Correlation and isotropy

To establish the periods and scales over which pair motion is correlated, we use the pair velocity correlation normalized by twice the single-particle velocity variance:

\[ C = \frac{\langle \mathbf{u}_i \cdot \mathbf{u}_j \rangle}{2 \sigma^2}, \]

(e.g., Koszalka et al. 2009). The angle brackets represent the average over available pairs.

Figure 3a shows the result for the particles launched in January at 60°N on the 350-K surface. The solid curve corresponds to the full velocities, while the dashed curves are the meridional and zonal correlations. All three decrease with time, but while the meridional correlation falls to zero after 20–25 days, the zonal
correlation is still roughly 0.4 after 30 days. As such, the full velocity correlation is also nonzero at 30 days.

As the meridional curve provides the clearest indication of decorrelation, we use that in the other panels (Figs. 3b–d). At all three latitudes, the correlations in the troposphere (315 K) decay to zero soon after 10 days. This is true in both winter and summer. However, the decay on the 430-K surface takes about twice as long. The decay on the 350-K surface is intermediate between the two.

The decorrelation times, of course, depend on the initial separation. Using smaller initial separations lengthens the period. An alternative is to plot the correlations against rms separation (not shown). Doing so reveals that the correlations fall to zero at scales exceeding several thousand kilometers.

The theoretical solutions also assume isotropic dispersion. We test this by plotting the ratio of the mean square zonal and meridional pair separations (Morel and Larcheveque 1974). The ratio for the NH releases at 60°N is shown in Fig. 4. The ratio is near 1 for the first 2 days (Fig. 4a). That it is 1 initially follows from the choice of initial condition, with the pairs aligned along the diagonals of a square. It is nearly 2 by day 5, and at day 10 it is roughly 3. In terms of spatial scales, the ratio is 1 for separations less than roughly 1000 km and reaches 2 by 2000 km (Fig. 4b). There is little variation with height or season, except that the dispersion is more zonal on the 430-K surface in winter and more isotropic in summer. At 30°N, the flow becomes anisotropic earlier and at smaller scales (Figs. 4c, d). The ratio is nevertheless mostly between 1 and 2 in the 100–1000-km range (Fig. 4d). Zonality is more pronounced on the 430-K surface, where the ratio reaches 3 at 500-km separation. The anisotropy is also greater at 10°N (Fig. 4e), being first evident at separations of only several hundred kilometers (Fig. 4f).

The response in the SH (not shown) is like that in the NH, albeit more zonally anisotropic. The results, moreover, are consistent with the EOLE balloon results of Morel and Larcheveque (1974), who found that zonal dispersion dominated at scales exceeding 1000 km. After 15 days, the EOLE zonal dispersion was roughly 4 times larger than the meridional.

b. Dispersion at 60°N

As the dispersion at 60°N is the most persistently isotropic, we focus on those deployments first. We begin with the set launched on the 350-K surface in January. Thereafter, we consider the dependence on height, latitude, and season.

The dispersion for this set is shown in Fig. 5. As the spreading is initially isotropic, we plot the total dispersion, the sum of zonal and meridional components. For comparison, we also plot the Lundgren (nonlocal) and Richardson (energy) dispersion curves [using (A22) and (A12), respectively].

Each of the theoretical curves has two unknowns: the initial separation $r_0$ and the parameter that determines the growth rate—$T$ for the Lundgren curve and $\beta$ for the Richardson. All the pairs have the same initial separation, so we take $r_0$ to be the same. That leaves a single unknown for each case.

We determine these by matching the theoretical curves to the modeled dispersion at a chosen separation. This approach is straightforward and, in the case of the Lundgren dispersion, allows the parameter to be determined directly. Note that matching the dispersion at a chosen time is less desirable, because the growth rates vary with latitude and height. So we determine the time $t^*$ at which the dispersion is $a$ times larger than the initial value: $\langle r^2(t^*) \rangle = ar_0^2$. Then $T$ is given by

$$T = \frac{8r^*}{\ln(\langle r^2(t^*)/r_0^2 \rangle)} = \frac{8r^*}{\ln(a)}. \quad (5)$$

We take $a = 5$ herein, although the results are relatively insensitive to the exact value.

The full Richardson dispersion [(A12)] does not lend itself to such a simple determination of $\beta$, so we adjust the value empirically until the dispersion matches the observed at $t^*$. A first guess is obtained using the dispersion’s small time asymptotic limit [(A15)].

The results are shown in Fig. 5. The modeled dispersion (dashed black curve) increases rapidly during the first 2 days and more slowly thereafter. The dispersion is close to the Lundgren curve (red) during the first 2 days, suggesting exponential growth. The Richardson curve (blue) increases somewhat faster initially and slower afterward. By design, the three curves intersect at $5 \times 10^4$ km$^2$, about 1 day after launch.

The PDFs are shown in Fig. 6. At 1 day, the Lundgren PDF (red) is similar to the modeled PDF (dashed black), with a peak near 100 km and a long tail, while the Richardson PDF (blue) is somewhat broader. The result at 2 days is similar. At 4 days, though, the distinction is less clear, but the curves differ in the tails; the Lundgren PDF indicates higher probability than the model’s PDF, and the Richardson PDF indicates lower probability. The differences are amplified by day 7, but this is also when the dispersion is significantly anisotropic.

Differences in the wings of the distribution are reflected in the kurtosis, shown in Fig. 7. The modeled kurtosis increases rapidly during the first 4 days and decreases thereafter, exceeding 30 at its peak. The Lundgren kurtosis grows comparably fast during the first
FIG. 4. The isotropy, defined as the ratio of the zonal dispersion to the meridional dispersion, for the deployments at (a),(b) 60°; (c),(d) 30°; and (e),(f) 10°N. The ratio is plotted as a function of time in (a),(c),(e) and as a function of rms separation in (b),(d),(f).
1.5 days and continues to increase thereafter while the Richardson kurtosis never exceeds 5.6. As such, the large value of the model kurtosis cannot be explained under Richardson dispersion. The results, thus, favor exponential growth, implying a steep energy spectrum. To check this, we calculated the Eulerian structure functions from the wind data. In homogeneous, isotropic turbulence, the Lagrangian and Eulerian structure functions are equivalent (Bennett 1984). In the Richardson regime, the second-order structure functions (longitudinal and transverse) exhibit a power-law dependence on separation $r$:

$$S_2(r) \propto r^{2/3}, \quad (6)$$

which is Kolmogorov’s 2/3 law (Kolmogorov 1941; Frisch 1995; Lindborg 1999). The longitudinal function is essentially the inverse Fourier transform of the energy spectrum, and the $r^{2/3}$ dependence for the structure function corresponds to the Kolmogorov $K^{-5/3}$ spectrum. In the Lundgren regime, the structure functions have a steeper slope:

$$S_2(r) \propto r^2 \quad (7)$$

(Lundgren 1981; Lindborg 1999; LaCasce 2002). The $r^2$ dependence implies an energy spectrum proportional to $K^{-3}$ or steeper (e.g., Bennett 1984).

We used the ECMWF winds, interpolated to constant-height surfaces, to compute the structure functions. Note that the latter do not correspond exactly to the winds experienced by the particles (which change height). However, the 350- and 430-K surfaces are relatively flat (Fig. 1), so the differences are likely small. As noted, the 315-K surface varies more with latitude, but as we are focusing on the first few days, the differences are relatively small.

The structure functions at 60°N and 12-km height are shown in Fig. 8. Both the longitudinal and transverse structure functions exhibit an $r^2$ dependence for separations less than 1000 km. In addition, the structure functions level off at separations greater than roughly 2000 km, implying the latter is the scale of the energy-containing eddies.

Note too that the transverse functions are larger than the longitudinal ones. This is as expected in isotropic turbulence. For a 2D incompressible flow, the functions obey

$$S_{2t} = \frac{d}{dr}(rS_{2l}) \quad (8)$$

(Batchelor 1953; Lindborg 1999; LaCasce 2002). Thus, if $S_{2l}$ is proportional to $r^2$, $S_{2t}$ is 3 times greater. In Fig. 8, the line indicating $r^2$ is plotted 3 times higher than the January longitudinal function. As this coincides with the January transverse function, the velocities are approximately isotropic.

c. Changes in latitude, changes in altitude

The initial dispersion is qualitatively similar for the other deployments. The kurtosis always increases rapidly and reaches a large value. The maximum kurtoses are mostly in the range 10–20, although values up to nearly 60 are also seen. Likewise, the initial growth in dispersion is approximately exponential in all cases.

What differs is the growth rate. Using the technique described in section 4b, we estimate $T$ for each of the 36 releases. On the 315-K surface (Fig. 9), $T$ varies between 2 and 7 days, with smaller values at the higher latitudes and larger ones at low latitudes. The corresponding $e$-folding times (equal to $T/8$) range from 6 to 21 h. A similar range is seen on the 350-K surface (middle panel) and there is strikingly little variation with latitude. Larger values are found, though, on the 430-K surface (right panel), varying from 6 to 12 days in most cases (or $e$-folding times from 18 to 36 h).

Using these values of $T$, we rescale time and compare the dispersion curves. The results for the 12 releases on the 350-K surface are shown in Fig. 10. In the period up to roughly $t = T/2$, the dispersion curves overlap. The composite curve is nearly exponential and is similar to the Lundgren prediction, $\exp(8t/T)$ (black dashed curve). The curves differ only at later times (when the dispersion is zonally anisotropic).
Many of the similarities and differences between the releases can be seen in Fig. 11, which shows the structure functions at 60°N and 6 km (Fig. 11a), 60°N and 18 km (Fig. 11c), and 10°N and 12 km (Fig. 11e). The dispersion for the corresponding pair releases at 60°N and 315 K (Fig. 11b), 60°N and 430 K (Fig. 11d), and 10°N and 350 K (Fig. 11f) are shown on the right. In all cases, the dispersion is nearly exponential during the first 1–3 days and the structure functions increase as $r^2$ at the smallest scales. However, there are noticeable differences as well.

As the longitudinal structure function $S_2(r)$ is related to the energy spectrum, having a larger $S_2(r)$ implies more energetic variability. Compared with that at 12 km (Fig. 8), $S_2(r)$ is greater at 6 km and lesser at 18 km (Figs. 11a,c), so the energy levels decrease with height. In other locations, the energy levels at 12 and 6 km are similar, but the 18-km energy is usually less.

The Lundgren time scale (see bottom-right corners of Figs. 11b,d,f) depends on the enstrophy dissipation rate and so should be sensitive to the energy level. At 60°N, this is the case. The time scale is shortest in the troposphere and increases going up ($T = 2.73$ days at 315 K, $T = 5.14$ days at 350 K, and $T = 7.86$ days at 430 K). However, despite the energy level at 10°N being comparable to that at 60°N at the same height, the 10°N particles have a longer time scale ($T = 5.59$ days). Therefore, $T$ is affected by other factors at low latitudes.
Differences are also evident in the degree of isotropy. As noted, the $r^2$ lines in the left panels of Fig. 11 are plotted where the January transverse functions would lie if the fluctuations were isotropic. At 60°N and 6-km height (Fig. 11a), the line overlies the transverse function (dashed blue curve), and the same is true for the January $S_t$ curve at 18 km (Fig. 11c). The line is higher at 10°N (Fig. 11e), indicating anisotropy. Indeed, the longitudinal and transverse structure functions in summer overlie one another at 10°N, implying anisotropy is present at the smallest sampled scales.

To gauge the variations in isotropy among the deployments, we plot the ratio of $S_t(r)$ to $S_l(r)$ at the smallest separation, $r = 100$ km (Fig. 12a). There is a clear variation with latitude, with the midlatitudes being the most isotropic and the low latitudes being the most anisotropic. Only the high latitudes are close to isotropy at small separations; deviations are noticeable even at 30°N and 30°S. Anisotropy also increases to some extent with height, being more pronounced at 18 km than at 6 km.

These differences are mirrored in the relative dispersion (Fig. 4). The pair displacements are initially isotropic, but anisotropy develops in all cases and at different rates. In Fig. 12b, we plot the ratio of the squared meridional separation to the squared zonal separation at 1000 km. The parameter shows the same variation as in Fig 12a, with more isotropic dispersion at midlatitudes and more zonal dispersion at low latitudes. The similarity between Figs 12a and 12b implies the small-scale anisotropy in the structure functions manifests as growing anisotropy in the particle separations.

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2 Note that this is the inverse of the measure shown in Fig. 4.
FIG. 10. Relative dispersion for all deployments at 350 K. The Lundgren dispersion is indicated by the black dashed curve. The dispersion has been normalized by $r_2^3$ and time has been normalized by the relevant $T$ value for each deployment case. Line colors and styles are as indicated in the key. Hemispheres are distinguished by line weight: NH lines are thick and the SH lines are thin.

The energy-containing eddy scale also varies between deployments. To gauge this, we determined the scale at which $S_2(r)$ flattens out [$S_2(r)$ often continues to increase, because of persistent zonal correlations]. This scale is about 2000 km in the troposphere at 60°N (Fig. 11a) but is greater in the wintertime stratosphere (Fig. 11b). In most of the other cases, the scale is near 2000 km at 6 km and slightly larger at 12 km. But the scale is considerably larger at 18 km, lying between 3000 and 4000 km. So the principal factor for the size of the energy-containing eddies is height; latitude and season are less important.

On the other hand, the seasonal variations are modest. In all three cases in Fig. 11, the energy levels in winter are greater than those in summer. The difference is most pronounced in the stratosphere but less so at 10°N and 12 km and less still at 60°N and 6 km. Despite this, the qualitative features do not change with season.

Likewise, the differences between the hemispheres are relatively minor. In the collective plots (Figs. 9, 12), the symbols are arranged with the SH cases on the left and NH cases on the right. Broadly speaking, there is little asymmetry between the hemispheres. While we expect the dispersion to be more zonal in the SH because of the greater zonality of the mean circulation (e.g., Hoskins and Hodges 2005; Graff and LaCasce 2014), this is not apparent in the structure functions at the small scales.

To summarize, the initial dispersion is exponential in all cases, and the structure functions are proportional to $r^2$ at small scales. But only at the high latitudes is the dispersion isotropic. At lower latitudes, the structure functions indicate zonal anisotropy at the smallest resolved scales, and this causes the relative dispersion to be increasingly zonal. The dispersion is also affected by differences in the energy-containing scales, with the largest such scales occurring in the stratosphere.

5. Comparison with previous studies

The present results agree well with those of Morel and Larcheveque (1974) and Er-El and Peskin (1981), who observed exponential relative dispersion and isotropic spreading at separations below 1000–2000 km in the EOLE and TWERLE experiments. Their $e$-folding time scales (1.35 and 1.2 days) are larger than that found here for the 350-K surface (roughly 0.5 days) but are of similar magnitude. They too observed zonally anisotropic dispersion at large scales. The papers differ on the dispersion rate at the large scales, with Morel and Larcheveque (1974) finding diffusive growth and Er-El and Peskin (1981) obtaining a faster rate, perhaps as fast as $r^2$. But if the latter is true, it cannot reflect Richardson dispersion, as the pair velocities are uncorrelated.

Lacorata et al. (2004) observed exponential growth in the EOLE separations at scales below 100 km, with an $e$-folding time (0.4 days) like that found here. They suggested a transition to $r^3$ growth occurred afterward and persisted until 1000 km. This would be consistent with the aircraft-derived spectra of Nastrom and Gage (1985), which indicate a $K^{−5/3}$ slope at scales below a few hundred kilometers. The dispersion in Fig. 5 arguably exhibits a power-law increase during the period from 1.5 to 6 days, but the kurtosis (Fig. 7) greatly exceeds the Richardson limit of 5.6 and continues growing until day 4. The ECMWF winds do not capture the $K^{−5/3}$ spectrum at small scales, which could well explain the difference with their results, but LaCasce (2010) suggested the separation PDFs for the EOLE data were also consistent with the Lundgren distribution.

Huber et al. (2001) also observed exponential dispersion, using synthetic particles and ECMWF winds, but only at low latitudes. However, the authors chose to fit the dispersion curves over the first 10 days. As seen here, the pair velocities are generally decorrelated by that time, so using the first few days is preferable. Alternately, one can identify the period over which the dispersion increases by some factor, as we did here. Doing so and rescaling time by the resulting Lundgren time scale collapses the present dispersion curves onto a single, nearly exponential curve during the initial period.

LaCasce (2010) studied the EOLE data using PDFs. However, he obtained the Richardson parameter by fitting the asymptotic Richardson dispersion relation.
Fig. 11. (left) The longitudinal (solid lines) and transverse (dashed) second-order structure functions for the
January (blue) and September (red) deployments at (a) 60°N and 6 km, (c) 60°N and 18 km, and (e) 10°N and 12 km.
(right) The relative dispersion curves for the January deployments (black dashed lines) at (b) 60°N and 315 K,
(d) 60°N and 430 K, and (f) 10°N and 350 K. The Lundgren dispersion (red) is also plotted for reference, and \( T \) is given in the bottom-right corner.
Using the full dispersion relation \[(A12)\] instead, we find closer agreement between the Lundgren and Richardson PDFs at early times. We also found that the full Richardson dispersion does not reach the asymptotic limit before roughly 10 days (not shown), by which time the pair velocities are decorrelated. It is therefore preferable to use the full Richardson PDF.

6. Summary and discussion

We examined relative dispersion using pairs of synthetic particles advected with ECMWF winds in the FLEXPART model. The particles were deployed at 10\(^\circ\), 30\(^\circ\), and 60\(^\circ\) in both hemispheres on three potential temperature surfaces (315, 350, and 430 K) and in two seasons (January and September). The wind data had 1\(^\circ\) × 1\(^\circ\) resolution, so the smallest sampled scale was roughly 100 km. We calculated various separation statistics and compared the results to Eulerian structure functions based on expectations from homogeneous 2D turbulence.

The pair velocities are correlated over roughly the first decade of sampled scales (100–1000 km). The dispersion is also laterally isotropic over these scales at the higher latitudes. At scales exceeding several thousand kilometers, the dispersion is zonally anisotropic, and the pair motion is decorrelated. However, both the dispersion and the structure functions indicate zonal anisotropy down to the smallest sampled scales at low latitudes.

For all deployments, the initial dispersion was nonlocal. The dispersion curves accordingly collapse onto an exponential after rescaling time by the e-folding time. The pair-separation PDFs evolve consistently with the nonlocal solution of Lundgren (1981), and the separation kurtoses grow exponentially in time. The second-order longitudinal structure functions exhibit an \(r^2\) dependence at the smallest scales, which is also consistent with nonlocal dispersion.

However, there are also variations. The e-folding times increase with height, with those at 430 K being up to 3 times larger than at the lower levels. The energy-containing scales are larger at 430 K also, implying the exponential growth proceeds to larger scales. And the dispersion is more anisotropic at low latitudes than at high latitudes, as noted. Therefore, 2D turbulence theory may only be relevant at the higher latitudes.

The results do not indicate Richardson dispersion at small scales, as would be expected from the \(K^{-5/3}\) spectra observed with aircraft data (Nastrom and Gage 1985). This is because the ECMWF winds lack an \(r^{2/3}\) range at small scales. The mesoscale \(K^{-5/3}\) range has been observed in several high-resolution numerical simulations (Koshyk and Hamilton 2001; Hamilton et al. 2008; Brune and Becker 2013), and it should be possible in future to examine relative dispersion in such runs. However, it appears that the divergent component of the winds is important in this range (VanZandt 1982; Cho and Lindborg 2001; Brune and Becker 2013), so a different framework for interpreting the dispersion may be required.

Often, as here, there are multiple dispersion regimes. The nonlocal dispersion at small scales is succeeded by a different type of dispersion at larger scales. Thus, the kurtosis does not increase indefinitely, as in a pure enstrophy cascade, but begins to fall at later times. With additional regimes (i.e., a Richardson range at even smaller scales), a more nuanced deployment strategy
would be desirable, comparing, for example, initial separations of 10 and 100 km. We are currently examining this with other data.

The present study admittedly employs unrealistically large numbers of pairs. Indeed, these calculations involve 720 pairs per deployment; repeated over 19 years, this yields 13,680 pairs for each release—a number greatly in excess of any in situ experiment. We found that using groups of 200 pairs produced variations in the dispersion curves but did not change the qualitative behavior (not shown). The kurtosis, being a fourth-order moment, was more sensitive to having fewer pairs. In all cases, the kurtosis increased to large values, but the maximum value varied.

As noted, the FLEXPART routine employs stochastic terms to represent small-scale mixing unresolved in the advecting winds. Such terms could also be used to represent uncertainties in those winds. Removing them leads to even larger values of the maximum kurtosis, as the pairs are strained only by the large-scale eddies. Nonlocal advection produces filaments, and, in the absence of small-scale mixing, the filaments become progressively thinner. This is a significant issue, for example, with regard to the dispersion of volcanic ash.

Last, we did not explicitly examine the predominantly zonal dispersion occurring at large separations because this is an issue of absolute dispersion (as the pair velocities are uncorrelated). However, the low latitudes exhibit zonally anisotropic, nonlocal relative dispersion; the separations still grow exponentially in time despite the lack of isotropy. This is interesting and perhaps warrants further examination.

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APPENDIX

PDFs and Moments

Solutions to the Fokker–Planck equation [(2)] are given by LaCasce (2010) and included here for convenience. The equation is solved using the Laplace transform assuming a delta-function initial condition (implying all pairs have the same initial separation).

When pair motion is uncorrelated, the relative diffusivity is constant, and the solution to (2) is as follows:

$$p(r, t) = \frac{1}{4\pi \kappa^2 t} \exp \left( -\frac{r^2}{4\kappa^2 t} \right) I_0 \left( \frac{r_0}{2\kappa^2 t} \right),$$  \hspace{1cm} (A1)

where $r_0$ is the initial pair separation and $I_0$ is a modified Bessel function. At long times and large separations ($t \gg r_0/\kappa^2$, $r \gg r_0$), this asymptotically approaches

$$p(r, t) = \frac{1}{4\pi \kappa^2 t} \exp \left( -\frac{r^2}{4\kappa^2 t} \right),$$  \hspace{1cm} (A2)

which is proportional to the Rayleigh distribution.

The separation moments are defined in (3):

$$\langle r^n \rangle = 2\pi \int_0^\infty r^{n+1} p(r) dr.$$  \hspace{1cm} (A3)

With (A2), this is

$$\langle r^n \rangle = (4\kappa^2 t)^{n/2} \Gamma \left( \frac{n}{2} + 1 \right),$$  \hspace{1cm} (A4)

where $\Gamma$ is the gamma function. The relative dispersion is the second moment:

$$\langle r^2 \rangle = 2\pi \int_0^\infty r^2 p(r) dr = 4\kappa^2 t.$$  \hspace{1cm} (A5)

The linear dependence on time is as expected for a diffusive process (Taylor 1921). Furthermore, the kurtosis (the fourth moment normalized by the square of the second moment) is 2 in the asymptotic limit.

In a turbulent energy cascade, the pair velocities are correlated and the diffusivity has the form

$$\kappa = \beta r^{4/3},$$  \hspace{1cm} (A6)

where $\beta$ is a constant (proportional to the third root of the energy dissipation rate). This applies both to the forward energy cascade in 3D turbulence and the inverse cascade in 2D turbulence. The corresponding solution to (2) is

$$p(r, t) = \frac{3}{4\pi \beta t r_0^{2/3}} \exp \left( -\frac{9(r_0^{2/3} + r^{2/3})}{4\beta t} \right) I_2 \left( \frac{9r_0^{2/3} r^{1/3}}{2\beta t} \right).$$  \hspace{1cm} (A7)

The moments of the Richardson distribution given by LaCasce (2010) apply in the long time limit. However, one can calculate the full dispersion (i.e., from the delta-function initial condition) as follows. Inserting (A7) into (3),
\[ \langle r^n \rangle = \left( \frac{3}{2} \right)^2 \frac{1}{\beta r_0^{3/2}} \exp \left( -9r_0^{2/3} \right) \int_0^\infty u^{(3n+2)/2} \exp \left( -\frac{9u}{4\beta t} \right) I_2 \left( \frac{9u^{1/3} \sqrt{u}}{2\beta t} \right) \, du, \]  

(A8)

where \( u = r^{2/3} \). We evaluate the integral in (A8), exploiting the following identity:

\[ \int_0^\infty x^{\mu - 1/2} \exp(-\alpha x)I_{2\nu}(2\gamma \sqrt{x}) \, dx = \frac{\Gamma(\mu + \nu + 1/2)}{\Gamma(2\nu + 1)} \gamma^{-1} \exp \left( \frac{\gamma^2}{2\alpha} \right) x^{-\mu} M_{-\mu,\nu} \left( \frac{\gamma^2}{\alpha} \right) \]  

(A9)

\[ \langle r^n \rangle = \left( \frac{3}{2} \right)^2 \frac{1}{\beta r_0^{3/2}} \left( \frac{4\beta t}{9} \right)^{(3n+3)/2} \frac{\Gamma(3n/2 + 3)}{\Gamma(3)} \exp \left( \frac{9r_0^{2/3}}{8\beta t} \right) M_{-(3n+3)/2,1} \left( \frac{9r_0^{2/3}}{4\beta t} \right) \]  

\[ = \left( \frac{3n + 6}{2} \right)^{3n/2} \frac{4\beta t}{9} \exp \left( -\frac{9r_0^{2/3}}{4\beta t} \right) M \left( 6 + 3n, \frac{9r_0^{2/3}}{4\beta t} \right). \]  

(A11)

So the relative dispersion is

\[ \langle r^2 \rangle = \frac{5t}{2} \left( \frac{4\beta t}{9} \right)^3 \exp \left( -\frac{9r_0^{2/3}}{4\beta t} \right) M \left( 6, 3, \frac{9r_0^{2/3}}{4\beta t} \right). \]  

(A12)

The initial dispersion, the small time limit of (A12), can be estimated using the large argument limit of the Kummer function (Abramowitz and Stegun 1964):

\[ \lim_{t \to 0} M(a, b, x) = \frac{\Gamma(b)}{\Gamma(a)} x^{a-b} \left[ 1 + \frac{\Gamma(2-a)\Gamma(b-a+1)}{\Gamma(1-a)\Gamma(b-a)} \right]. \]  

(A13)

One can show that

\[ \lim_{t \to 0} \langle r^2 \rangle = r_0^2 \left( 1 + \frac{20\beta t}{3r_0^{3/2}} \right). \]  

(A14)

As such, the first guess for \( \beta \), referred to in section 4b, is

\[ \beta = \frac{27r_0^{2/3}}{20r^{3/2}}. \]  

(A15)

At large separations and long times, the Richardson dispersion in (A7) asymptotically approaches a self-similar form:

\[ p(r, t) = \left( \frac{3}{2} \right)^5 \frac{1}{4\pi(\beta t)^3} \exp \left( -\frac{9r_0^{2/3}}{4\beta t} \right). \]  

(A16)

(Gradsteyn and Ryzhik 2007). Here, \( M_{-\mu,\nu} \) is the Whittaker \( M \) function. An alternate form is

\[ M_{-\mu,\nu}(z) = \exp \left( -\frac{1}{2}z \right) z^{\nu + 1/2} M \left( \frac{1}{2} + \nu + \mu, 1 + 2\nu, z \right), \]  

(A10)

where \( M \) here is Kummer’s function (Abramowitz and Stegun 1964). Thus,

\[ \langle r^n \rangle = \left( \frac{3n + 6}{2} \right)^{3n/2} \frac{4\beta t}{9} \exp \left( -\frac{9r_0^{2/3}}{4\beta t} \right) M \left( 6 + 3n, \frac{9r_0^{2/3}}{4\beta t} \right). \]  

The corresponding raw moments are

\[ \langle r^n \rangle = \left( \frac{3n + 6}{2} \right)^{3n/2} \frac{4\beta t}{9} \exp \left( -\frac{9r_0^{2/3}}{4\beta t} \right) M \left( 6 + 3n, \frac{9r_0^{2/3}}{4\beta t} \right). \]  

(A17)

with a relative dispersion

\[ \langle r^2 \rangle = 5.2675\beta^3 t^3. \]  

(A18)

The kurtosis is constant and equal to 5.6 in the asymptotic limit, and it is less than 5.6 at all times for the full dispersion.

The other inertial range in 2D turbulence is the enstrophy cascade range with an energy spectrum proportional to \( K^{-3} \). Here, the relative diffusivity is given by

\[ \kappa_2 = \frac{r_0^2}{T}, \]  

(A19)

where \( T \) is a time scale inversely proportional to the third root of the enstrophy dissipation rate. The corresponding solution to the F–P equation is

\[ p(r, t) = \frac{1}{4\pi^{3/2} t^{3/2} r_0^2} \exp \left\{ -\frac{\left[ \ln(r/r_0) + 2t/T \right]^2}{4t/T} \right\}. \]  

(A20)

(Lundgren 1981; Bennett 2006; LaCasce 2010). This has separation moments of

\[ \langle r^n \rangle = r_0^n \exp \left[ \frac{nn + 2n}{T} \right]. \]  

(A21)
So the dispersion
\[ \langle r^2 \rangle = r_0^2 \exp \left( \frac{8t}{T} \right) \] (A22)
grows exponentially in time. The kurtosis grows at the same rate.

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