Effect of Stratiform Heating on the Planetary-Scale Organization of Tropical Convection

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ABSTRACT

It is widely recognized that stratiform heating contributes significantly to tropical rainfall and to the dynamics of tropical convective systems by inducing a front-to-rear tilt in the heating profile. Precipitating stratiform anvils that form from deep convection play a central role in the dynamics of tropical mesoscale convective systems. The wide spreading of downdrafts that are induced by the evaporation of stratiform rain and originate from in the lower troposphere strengthens the recirculation of subsiding air in the neighborhood of the convection center and triggers cold pools and gravity currents in the boundary layer, leading to further lifting. Here, aquaplanet simulations with a warm pool–like surface forcing, based on a coarse-resolution GCM of approximately 170-km grid mesh, coupled with a stochastic multicloud parameterization, are used to demonstrate the importance of stratiform heating for the organization of convection on planetary and intraseasonal scales. When the model parameters, which control the heating fraction and decay time scale of the stratiform clouds, are set to produce higher stratiform heating, the model produces low-frequency and planetary-scale MJO-like wave disturbances, while parameters associated with lower-to-moderate stratiform heating yield mainly synoptic-scale convectively coupled Kelvin-like waves. Furthermore, it is shown that, when the effect of stratiform downdrafts is reduced in the model, the MJO-scale organization is weakened, and a transition to synoptic-scale organization appears despite the use of larger stratiform heating parameters. Rooted in the stratiform instability, it is conjectured here that the strength and extent of stratiform downdrafts are key contributors to the scale selection of convective organizations, perhaps with mechanisms that are, in essence, similar to those of mesoscale convective systems.

1. Introduction

Despite the continued progress in our understanding of precipitation and cloud processes in the tropics, their representation in coarse-resolution global climate models (GCMs) remains a challenge (Lin et al. 2006; Kim et al. 2009; Hung et al. 2013). The difficulty arises, because the underlying cumulus parameterization schemes used to represent unresolved convective processes do not take into account the multiscale character of organized tropical convection and the inherent interactions across time and spatial scales (Moncrieff and Klinker 1997; Majda 2007). At the heart of these complex interactions, tropical convection involves four main cloud types: 1) shallow cumulus clouds with tops below the temperature inversion just
above the planetary boundary layer (PBL) that evolve in the subsidence regions of high convective inhibition; 2) cumulus congestus clouds with tops below the freezing level that are abundant in regions of low midtropospheric humidity; 3) deep convective towers that nearly reach the tropopause, which take over when the midtroposphere is moist enough; and 4) stratiform clouds that typically develop in the upper troposphere, above the freezing level, in the wake of deep convection (Johnson et al. 1999; Lin et al. 2004; Mapes et al. 2006). We note that, in contrast to stratiform clouds, which expand horizontally over a few hundred kilometers, the former three types are narrow (0.1–10 km in horizontal extent) and expand rather vertically. Their cloud base is found just above the top of the PBL at the (first) lifting condensation level.

The occurrence of these cloud types at a given point in time and space is largely controlled by the environmental conditions, as described above. However, because of large uncertainties and inaccuracies in the GCM state variables, a variety of cloud-type populations could, in principle, be present at the same time within the same GCM grid box. Moreover, clouds have the ability to change considerably their environment, either directly through exchange of latent heat associated with phase change of water and turbulent mixing (i.e., detrainment) or indirectly by modification of the radiation budget (Emanuel 1994). In this way, clouds are able to interact with each other and constitute a non-negligible source of climate variability not directly captured by the GCM mesh size or time step. In terms of heating, the congestus, deep, and stratiform clouds are known to affect the environment in three different ways. Cumulus congestus warm the lower troposphere and cool the upper troposphere by radiative cooling and detrainment at cloud top, and deep convective towers warm the whole troposphere quasi uniformly (Lin and Johnson 1996; Johnson et al. 1999; Khouider and Majda 2006, 2008), while stratiform clouds warm the upper troposphere and cool the lower troposphere through the evaporation of stratiform rain (Lin et al. 2004; Mapes 2000). In addition to the warming and cooling effects, clouds have an impact on the distribution of moisture. Preexisting cloud types are thus able to create conditions that are favorable or unfavorable to new cloud types. For example, it is hypothesized that congestus clouds help moisten the lower troposphere prior to deep convection (Johnson et al. 1999; Waite and Khouider 2010; Hohenegger and Stevens 2013; Hagos et al. 2014; Bellenger et al. 2015), while stratiform cloud decks develop on the remains of deep convective towers (Mapes 2000; Straub and Kiladis 2002; Kiladis et al. 2005). In return, the evaporative cooling induced by stratiform rain drives downdrafts that cool and dry the PBL (Majda and Shefter 2001; Khouider and Majda 2006) with a twofold consequence. The cooling and drying of the PBL locally decreases the instability for moist convection [i.e., convective available potential energy (CAPE)], which causes convection to cease and at the same time triggers the propagation of cold pools and gravity currents or bores, which cause the deepening of the PBL in the neighborhood of the initial cloud (Mapes 1993; Houze 1997). The PBL deepening decreases convective inhibition and allows convection to develop in the neighborhood of an existing or recently ceased convective tower. Moreover, a realistic simulation of the stratiform heating is critical to reproduce the front-to-rear-tilted heating profile, which plays a crucial role in the Madden–Julian oscillation (MJO) dynamics and organized convective systems in general (Kiladis et al. 2005; Lin et al. 2004; Lappen and Schumacher 2014).

These complex processes just described are considered to be among the main mechanisms that allow convection to be “gregarious” (Mapes 1993) and lead to its organization into mesoscale cloud clusters and superclusters (Nakazawa 1988). Nonetheless, large-scale circulation patterns associated with synoptic- and planetary-scale convective systems, such as convectively coupled equatorial waves (CCEWs) and the MJO, are found to be favorable for convective organizations by providing, for instance, large-scale convergence of moisture at low levels (Tompkins 2001; Grabowski and Moncrieff 2004; Khouider and Majda 2008; Khouider et al. 2011). The chicken-and-egg question associated with the two-way interactions between convection and the large-scale flow is a long-standing problem. However, there appears to be some sort of consensus in the tropical meteorological community that, at least for MJO initiation, both bottom-up and top-down energy cascades can be encountered in nature, depending on whether we are in the presence of a primary or a successive MJO event (Matthews 2008; Zhang et al. 2013). Successive MJOs seem to be triggered by the remains of preceding MJO events, for example, in the form of circumnavigating dry Kelvin waves (Matthews et al. 1999; Ajayamohan et al. 2013), while primary MJOs are believed to be initiated in situ, perhaps because of the gregarious nature of tropical clouds.

The predominance of stratiform clouds in organized tropical convective systems and their importance for the latter’s propagation and maintenance is widely recognized (Houze 1997; Tokay et al. 1999; Schumacher and Houze 2003; Jakob and Schumacher 2008; Sharma et al. 2009; Zhang and Hagos 2009; Tao et al. 2010). For instance, the upper-tropospheric outflows and the implied subsidence associated with stratiform anvils are thought to play a pivotal role in the dynamics, organization, and
overall morphological structure of mesoscale systems that develop in the Atlantic and eastern Pacific ITCZ and many other parts of the globe (Dudhia and Moncrieff 1987; Parker and Johnson 2004; Khouider and Moncrieff 2015). Also, several observation and modeling studies have demonstrated that stratiform clouds directly associated with deep convection and the implied tilted heating structure play a crucial role in the dynamics and propagation of the MJO and CCEWs, as well as monsoon intraseasonal oscillations and low pressure systems (Lin et al. 2004; Lappen and Schumacher 2014; Choudhury and Krishnan 2011). Some important questions, however, remain. For example, how significantly and differently does stratiform heating and stratiform rain affect the MJO versus CCEWs? What distinguishes planetary-scale organization of tropical convection from its synoptic-scale counterpart? In this paper, we use a coarse-resolution GCM with a stochastic parameterization of convection, based on the multicloud paradigm discussed above, to address these questions.

The multicloud model, in its deterministic version, was introduced in Khouider and Majda (2006) and further modified in Khouider and Majda (2008) as a refinement of the Majda and Shelter (2001) model for stratiform instability, which itself was inspired by Maps (1993, 2000). While from the linear theory point of view, the stratiform instability yields a scale-selective growth of moisture-coupled gravity waves at synoptic scales, nicely mimicking convectively coupled tropical waves, nonlinear simulations require an additional mechanism—namely, wind-induced surface heat exchange (WISHE; Emanuel 1987; Neelin et al. 1987)—for the maintenance and propagation of these waves (Majda et al. 2004). Also, in a warm pool setting [i.e., a horizontal distribution of the imposed sea surface temperature (SST) mimicking the Indian Ocean and western Pacific warm pool], the wave activity occurs within the descending branch of the induced Walker cell where the surface wind is the strongest. Not only is this nonphysical, but the simulation has the peculiar feature of exhibiting eastward-moving waves in the region of near-surface easterly winds (the eastern side) and westward-moving waves in the region of near-surface westerlies, which is one of the main characteristics of WISHE waves.

WISHE is discarded as a viable mechanism for the MJO because it requires background easterlies to produce an MJO-like eastward-moving disturbance, but, overall, westerlies often prevail over the Indian Ocean—western Pacific region in winter, when the MJO is most active. As demonstrated in Khouider and Majda (2006, 2008, and subsequent papers), WISHE-free waves with the right physical features are obtained when a model based on the three cloud types cumulus congestus, deep, and stratiform is used instead of one based only on deep and stratiform. While the stratiform heating is able to destabilize the system at the right synoptic scale with the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1$</td>
<td>38.47 K</td>
<td>First baroclinic projection of the background moisture gradient in (2)</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>38.35 K</td>
<td>Second baroclinic projection of the background moisture gradient in (2)</td>
</tr>
<tr>
<td>$Q_{K_1}$</td>
<td>1 K day$^{-1}$</td>
<td>First baroclinic radiative cooling rate</td>
</tr>
<tr>
<td>$\overline{\theta}<em>{S,1} - \overline{\theta}</em>{m,1}$</td>
<td>11.00 K</td>
<td>Discrepancy between $\overline{\theta}<em>{S,1}$ and $\overline{\theta}</em>{m,1}$ at RCE in (3)</td>
</tr>
<tr>
<td>$\overline{\theta}<em>{S,2} - \overline{\theta}</em>{S,1}$</td>
<td>10.00 K</td>
<td>Discrepancy between saturation and actual $\overline{\theta}_{S,1}$ at RCE in (3)</td>
</tr>
<tr>
<td>$\alpha_1 (\alpha_2)$</td>
<td>0.1 (0.9)</td>
<td>Relative contribution of $\overline{\theta}_{S,1}$ and $q$ to deep convection in (1)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>0.5</td>
<td>Dry convective buoyancy frequency in deep and congestus heating in (1)</td>
</tr>
<tr>
<td>$\gamma_2 (\gamma_3)$</td>
<td>0.25, 0.6</td>
<td>Relative contribution of $\theta_3$ to deep (congestus) heating in (1) and to CAPE (CAPE$_0$) in Table 2</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.2</td>
<td>Relative contribution of stratiform and congestus to downdrafts in (3)</td>
</tr>
<tr>
<td>$\alpha_1 (\alpha_2)$</td>
<td>0.25 (0.5)</td>
<td>Congestus (stratiform) heating fraction in (1)</td>
</tr>
<tr>
<td>$\alpha_1 (\alpha_2)$</td>
<td>1 (3) h</td>
<td>Congestus (stratiform) adjustment time scale in (1)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>2 h</td>
<td>Convective time scale in (1)</td>
</tr>
<tr>
<td>$h$</td>
<td>500 m</td>
<td>Prescribed boundary layer height</td>
</tr>
<tr>
<td>$H$</td>
<td>16 km</td>
<td>Average height of the tropical troposphere</td>
</tr>
<tr>
<td>$n_m = \overline{\theta} = \overline{\theta}<em>{S,2} - \overline{\theta}</em>{S,1}$</td>
<td>0.007 34 m$^{-1}$</td>
<td>Scale of downdraft mass flux; value set by RCE solution</td>
</tr>
<tr>
<td>$\tau_{ev}$</td>
<td>14.8 h (in EXPI)</td>
<td>Evaporation time scale; value set by RCE solution</td>
</tr>
<tr>
<td>$\tilde{\alpha}$</td>
<td>0.1</td>
<td>Coefficient of second baroclinic velocity component in moisture equation</td>
</tr>
<tr>
<td>$R$</td>
<td>32.0 J kg$^{-1}$</td>
<td>CAPE constant in Table 2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.7</td>
<td>Contribution of $\theta_1$ to CAPE anomalies in Table 2</td>
</tr>
<tr>
<td>$T_0$</td>
<td>30 K</td>
<td>Scaling factor of dryness in Table 2</td>
</tr>
<tr>
<td>CAPE$_0$</td>
<td>400 J kg$^{-1}$</td>
<td>Scaling factor of CAPE in Table 2</td>
</tr>
<tr>
<td>$n \times n$</td>
<td>1600</td>
<td>Number of lattice sites within each CGM grid box</td>
</tr>
</tbody>
</table>
right eigenstructures of superclusters, it is not able to sustain it. Cumulus congestus cloud decks that are observed to prevail in the front of organized convective systems of all scales are the missing link for a successful model for the two-way interactions between tropical clouds of various types and the associated large-scale waves, including the MJO and CCEWs. Nonetheless, the stratiform instability remains an essential ingredient, though not the only one. As demonstrated in Majda et al. (2004), the stratiform instability is tied to the parameter $\mu$ which controls the relative contribution of stratiform heating in the downdraft closure formula. In the present study, we further demonstrate that the (relative) amount of stratiform heating is the key parameter that controls the horizontal length scale at which convection is organized in coarse-resolution GCM simulations, using the stochastic multicloud model as a cumulus parameterization (Deng et al. 2015, hereafter DKM15).

The stochastic multicloud model (SMCM) was first introduced in Khouider et al. (2010) in order to take into account the unresolved variability due to interactions between various cloud types in coarse-resolution GCMs in the context of the multicloud model. It is used in Frenkel et al. (2012, 2013) to simulate convectively coupled gravity waves in a simplified primitive equations model. In DKM15, the SMCM was implemented in the High-Order Methods Modeling Environment (HOMME) dynamical core as a cumulus parameterization for coarse-resolution GCM simulations, following Khouider et al. (2011), who previously used the deterministic multicloud model (MCM). Taken together, HOMME-MCM and HOMME-SMCM are very successful in simulating the MJO and CCEWs as well as monsoon-like intraseasonal oscillations (Khouider et al. 2011; Ajayamohan et al. 2013, 2014; DKM15). However, so far HOMME-SMCM is used only for the aquaplanet MJO simulations on a uniform SST background. Here, we further introduce a warm pool–like SST in HOMME-SMCM and study the sensitivity of the results to two key parameters that control the amount (i.e., the strength and temporal and spatial extent) of stratiform heating produced by the model and further

<table>
<thead>
<tr>
<th>Transition</th>
<th>Transition rate</th>
<th>Time scale (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formation of congestus</td>
<td>$R_{01} = \frac{1}{\tau_{01}} \Gamma(C) \Gamma(D)$</td>
<td>$\tau_{01} = 40\tau_{grid}$</td>
</tr>
<tr>
<td>Decay of congestus</td>
<td>$R_{10} = \frac{1}{\tau_{10}} \Gamma(D)$</td>
<td>$\tau_{10} = 1\tau_{grid}$</td>
</tr>
<tr>
<td>Conversion of congestus to deep</td>
<td>$R_{12} = \frac{1}{\tau_{12}} \Gamma(C) [1 - \Gamma(D)]$</td>
<td>$\tau_{12} = 1\tau_{grid}$</td>
</tr>
<tr>
<td>Formation of deep</td>
<td>$R_{02} = \frac{1}{\tau_{02}} \Gamma(C) [1 - \Gamma(D)]$</td>
<td>$\tau_{02} = 4\tau_{grid}$</td>
</tr>
<tr>
<td>Conversion of deep to stratiform</td>
<td>$R_{23} = \frac{1}{\tau_{23}} \Gamma(C) [1 - \Gamma(D)]$</td>
<td>$\tau_{23} = 3\tau_{grid}$</td>
</tr>
<tr>
<td>Decay of deep</td>
<td>$R_{20} = \frac{1}{\tau_{20}} (1 - \Gamma(C))$</td>
<td>$\tau_{20} = 3\tau_{grid}$</td>
</tr>
<tr>
<td>Decay of stratiform</td>
<td>$R_{30} = \frac{1}{\tau_{30}}$</td>
<td>$\tau_{30} = 1\tau_{grid}, 5\tau_{grid}$, or $10\tau_{grid}$</td>
</tr>
</tbody>
</table>

$\Gamma(x) = \begin{cases} 1 - \exp(-x), & \text{if } x > 0; \\ 0, & \text{otherwise.} \end{cases}$

$$CAPE_l = \text{CAPE} + R[\theta_d - \gamma(\theta_1 + \gamma_2\theta_2)],$$

$$\text{CAPE} = \text{CAPE} + R[\theta_d - \gamma(\theta_1 + \gamma_2\theta_2)].$$

$D = (\theta_{e0} - \theta_{e0})/T_0$

$C_l = \text{CAPE}/\text{CAPE}_0$

$C = \text{CAPE}/\text{CAPE}_0$

**Fig. 1.** The prescribed warm pool structure for all the simulations. Contour plot of $\theta_{e0}$ minus $\bar{\theta}_{e0}$ [the discrepancy between the saturation and radiative–convective equilibrium (RCE) values of the equivalent potential temperature in the boundary layer] in the zonal and meridional directions, roughly mimicking the distribution of the sea surface temperature in the vicinity of the Indian Ocean–western Pacific warm pool.
investigate whether the stratiform instability mechanism is responsible for this sensitivity.

The paper is organized as follows. In section 2, we discuss briefly the model setup and present the results of an MJO simulation in a typical parameter regime as our control simulation. Sensitivity tests to key stratiform parameters are presented in section 3. In section 4, we demonstrate that the effect on the planetary-scale organization due to stratiform heating can be explained in large part by the mechanism of downdrafts induced by the evaporation of stratiform rain, which helps remoisten the midtroposphere and cool and dry the boundary layer. The latter makes cold pools that expand and strengthen with the extent and strength of the stratiform heating and which are believed to play a major role in reinitiating new convection in the neighboring grid cells, thus leading to propagating organized convective systems [cf. the stratiform instability (Majda and Shefter 2001; Mapes 2000)]. A summary and conclusions are given in section 5.

2. Model setup and control experiment

In this section, we briefly review the implementation of the global atmospheric model HOMME-SMCM and highlight the key parameters that control the strength and extent of the stratiform heating, the focus of the present paper. More details on the model’s framework can be found in DKM15 and the references therein. A control experiment in a typical parameter regime simulating the MJO evolution and variability within the warm pool is also presented.

a. Model setup

The SMCM-HOMME model uses the HOMME core of the National Center for Atmospheric Research (Taylor et al. 1997; Dennis et al. 2005; Nair et al. 2009) as a dry dynamical core coupled to the SMCM of Khouider et al. (2010) as a cumulus parameterization. Apart from boundary layer and upper-tropospheric damping, the model is free of any other physics, except for the cumulus heating provided by SMCM. HOMME is a highly scalable dynamical core based on spectral elements in the horizontal and finite differences in the vertical and uses a cubed sphere geometry. In our setting, each face of the cube carries 20 integration elements of 4 degrees of freedom. This is roughly equivalent to a horizontal grid size of 167 km. We use 26 vertical levels and a time step of 30 s. SMCM carries

![Hovmöller diagram](image-url)
equations for the vertically integrated moisture and boundary layer equivalent potential temperature, which are integrated in parallel with the dynamical core. The SMCM routine includes an imposed uniform cooling with a strength of roughly 1 K day$^{-1}$ and a baroclinic vertical profile.

In SMCM, the vertical profiles of convective heating and cooling associated with the congestus, deep, and stratiform cloud types are prescribed, while the heating rates ($H_c$, $H_d$ and $H_s$) obey the closure equations in (1):

$$H_c = \sigma_c \frac{\alpha_c}{\text{CAPE}} \sqrt{\frac{\text{CAPE}}{m}},$$

$$H_d = \sigma_d \left( \frac{1}{\sigma_d \theta_{\text{conv}}} \left[ a_1 \theta_{eb} + a_2 q - a_0 (\theta_1 + \gamma \theta_2) \right] \right)^+, \quad \text{and}$$

$$H_s = \sigma_s \left( \frac{1}{\sigma_s \theta_{\text{conv}}} \left[ a_1 \theta_{eb} + a_2 q - a_0 (\theta_1 + \gamma \theta_2) \right] \right)^+. \quad (1)$$

Here, $q$ is the vertically averaged tropospheric moisture, and $\theta_{eb}$ is the boundary layer equivalent potential temperature, while $\theta_1$ and $\theta_2$ are, respectively, the first and second baroclinic components associated with the vertical mode expansion (Khouider et al. 2011). Moreover, $\sigma_c$, $\sigma_d$, and $\sigma_s$ are the stochastic area fractions of, respectively, congestus, deep, and stratiform clouds. The stochastic model is discussed briefly below. The parameters $\alpha_c$ and $\alpha_s$ are used to tune the magnitude of, respectively, the congestus and stratiform heating rates. For the sake of streamlining, the rest of the model variables and parameters are listed in Tables 1 and 2. The notation $\{ \cdot \}^+$ refers to the positive part of the function value [i.e., max$(\cdot, 0)$]. The heating profiles are based on the first and second baroclinic vertical structure basis functions so that deep convection heats the entire troposphere (up to 200 hPa), while congestus (stratiform) clouds warm (cool) the lower troposphere and cool (warm) the upper troposphere. More details can be found in Khouider et al. (2011). The combined heating profile provides the cumulus heating and cooling tendency for the temperature equation and drives the HOMME dynamical core.

The boundary layer equivalent potential temperature and vertical average moisture that appear in the closure equations in (1) satisfy

![Figure 3](image-url)

**Fig. 3.** As in Fig. 2, but for (a) deep convective, (b) congestus, and (c) stratiform heating rates and (d) vertically averaged moisture anomaly. The black dashed line in (a) marks an MJO-like event moving eastward at roughly 5 m s$^{-1}$. 

Note: The text is a natural reading of the provided document, ensuring clarity and coherence. Any graphical elements or equations are accurately represented as described.
Fig. 4. Spectral power of the (10°S–10°N) meridionally averaged (a) 800- and (b) 200-hPa zonal wind, (c)–(e) convective heating rates, and (f) moisture anomalies corresponding to those in Figs. 2 and 3, with the data from 1000 to 2000 days.
Here, $E_s$ is the evaporation from the sea surface, and $D$ is the downdraft mass flux:

$$
\frac{\partial \theta_{eb}}{\partial t} + \mathbf{u}(x, y, p_1, t) \cdot \nabla \theta_{eb} = \frac{1}{\bar{h}} E_s - \frac{1}{\bar{h}} D \quad \text{and}
$$

$$
\frac{\partial q}{\partial t} + \nabla \cdot [q(\mathbf{u} + \mathbf{u}_1 + \mathbf{u}_2)] + \nabla \cdot \mathbf{u}_1 + \nabla \cdot \mathbf{u}_2 = -P + \frac{D}{H}.
$$

(2)

Here, $E_s$ is the evaporation from the sea surface, and $D$ is the downdraft mass flux:

$$
\frac{1}{\bar{h}} E_s = \frac{1}{\tau_c} (\theta_{eb}^s - \theta_{eb}),
$$

$$
D = \frac{m_0}{Q_{R,1}} [Q_{R,1}^0 + \mu (H_s - H_c)] \left( \theta_{eb} - \theta_{en} \right),
$$

(3)

where $\theta_{eb}^s$ is the boundary layer saturation equivalent potential temperature, $\theta_{en}$ is the middle-tropospheric equivalent potential temperature, and $P$ is the surface precipitation. In (2), $\mathbf{u}(x, y, p_1, t)$ is the horizontal velocity at the lowest model (pressure) level, while $\mathbf{u}$, $\mathbf{u}_1$, and $\mathbf{u}_2$ are, respectively, the barotropic and first and second baroclinic horizontal velocity components. Here, $(x, y)$ are the longitude and latitude coordinates, and $\nabla$ is the associated horizontal gradient vector. We note that the stratiform heating directly affects the downdraft mass flux [(3)] through the parameter $\mu$, which, in turn, acts simultaneously on the tropospheric moisture $q$ and the boundary layer $\theta_e$. The latter effect drives cold pools in the boundary layer, which are believed to help the initiation of new convection in the neighboring grid cells and lead to propagating organized convective systems via the stratiform instability (Majda and Shefter 2001). The default parameters for the multicloud parameterization equations in (1)–(3) are given in Table 1. They are the same as in DKM15. Unless otherwise specified, these parameter values are used throughout the present

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Stratiform heating coefficient</th>
<th>Stratiform decay time scale</th>
<th>Other changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP1</td>
<td>0.50</td>
<td>$5\tau_{grid}$</td>
<td>—</td>
</tr>
<tr>
<td>EXP2</td>
<td>0.25</td>
<td>$5\tau_{grid}$</td>
<td>—</td>
</tr>
<tr>
<td>EXP3</td>
<td>0.25</td>
<td>$10\tau_{grid}$</td>
<td>—</td>
</tr>
<tr>
<td>EXP4</td>
<td>0.50</td>
<td>$2\tau_{grid}$</td>
<td>—</td>
</tr>
<tr>
<td>EXP5</td>
<td>0.50</td>
<td>$5\tau_{grid}$</td>
<td>$\mu = 0.1$</td>
</tr>
<tr>
<td>EXP6</td>
<td>0.50</td>
<td>$5\tau_{grid}$</td>
<td>$\mu = 0.05$</td>
</tr>
<tr>
<td>EXP7</td>
<td>0.50</td>
<td>$5\tau_{grid}$</td>
<td>$\mu = 0.01$</td>
</tr>
<tr>
<td>EXP8</td>
<td>0.50</td>
<td>$5\tau_{grid}$</td>
<td>$\tau_{ol} = 1\tau_{grid}, \tau_{ul} = 3\tau_{grid}$</td>
</tr>
</tbody>
</table>
study. We note that, for consistency, moisture variables are expressed in temperature units.

As formulated in (1), the heating rates are proportional to the congestus, deep, and stratiform cloud area fractions \( \sigma_c, \sigma_d, \) and \( \sigma_s \), respectively. The cloud area fractions are simulated by a stochastic lattice model, and they are the source of stochasticity (i.e., subgrid variability) in SMCM. Each GCM horizontal grid box is overlaid by a rectangular \( n \times n \) lattice, and each lattice site is assumed to be either clear sky or occupied by a congestus, deep, or stratiform cloud. The cloud area fractions are defined as the area coverage of the microscopic lattice by the sites occupied by each one of the three cloud types. A judicious coarse graining then permits us to recover the exact dynamics, in the case of nonlocal interactions between lattice sites (Khouider et al. 2010; the transition rates depend only on the given site’s state and the large-scale state and not on the rest of the microscopic configuration), or approximate dynamics, in the case of nearest-neighbor interactions (Khouider 2014; the transition rates depend also on the microscopic states in the neighboring sites), for the mesoscopic cloud area fraction, in the form of a three-species birth–death process at each GCM grid box. The area fraction birth–death process is evolved in time using Gillespie’s exact algorithm (Gillespie 1975, 1977) in a straightforward fashion with very little computational overhead. Because of the extra uncertain parameters associated with local interactions, only SMCM with nonlocal interactions of Khouider et al. (2010) is considered here. The implementation of SMCM with local interactions (Khouider 2014) is left for future developments.

In SMCM, each individual cloud site makes random transitions from one state to another according to intuitive probability rules, depending on whether the environment is favorable to one cloud type or another. This leads to a Markov process with conditional transition rates. The latter are given in Table 2 for streamlining. A given rate \( R_{kl} \) goes up or down according to whether the environment is favorable to the associated transition. For instance, the transition rates from clear sky to deep convection and from congestus to deep convection both increase with both CAPE and midtropospheric moisture. This mainly inhibits deep convection when the troposphere is dry to allow a more physical, progressive transition to deep convection as observed in nature.

The transition rates in Table 2 are given in terms of transition time scales denoted by \( \tau_{kl} \). They form a set of seven parameters, the values of which are uncertain. However, some attempts to infer them from data do
exist (Peters et al. 2013; De La Chevrotiere et al. 2014). To take into account the dependence of these parameters on the GCM grid resolution (in a crude way), the extra parameter $t_{\text{grid}}$ is introduced in Frenkel et al. (2012). Following DKM15, here we use $t_{\text{grid}} = 2$. The number of lattice sites, $n \times n$, is another important parameter of SMCM. Here, we use the conservative value of $n = 40$. The sensitivity of the results to both $t_{\text{grid}}$ and $n$ is documented in DKM15.

To limit the effect of the cumulus heating to the tropics, we introduce a mask in the meridional direction. It is set to one for latitudes between $30^\circ S$ and $30^\circ N$ and rapidly and smoothly decreases to zero toward the poles (Khouider et al. 2011). Moreover, a nonuniform SST mimicking the Indian Ocean–western Pacific warm pool is imposed through the prescribed surface evaporation rate, $(1/\tau_c)(\theta_{eb} - \bar{\theta}_{eb})$, which is raised above its spatial mean by up to 5 K per evaporation time scale $\tau_c$ inside the warm pool region and lowered by the same amount outside, as illustrated in Fig. 1 (Ajayamohan et al. 2013). As shown in (1), the stratiform heating rate is controlled by two key factors: namely, the stratiform fraction $\alpha_s$ and the stratiform cloud area fraction $\sigma_s$. While $\alpha_s$ is a constant that can be adjusted directly beforehand, $\sigma_s$ is a random variable that evolves during the simulation. However, the relative strength and dynamics of the cloud area fractions are strongly modulated by the transition rates in Table 2. For instance, persistent large areas of stratiform cloud decks can be easily achieved by using large stratiform cloud decay time scale $\tau_{30}$ values. The same can be achieved through changes of other transition time-scale combinations, but here only changes in $\tau_{30}$ are considered for the sake of simplicity.

b. Control experiment

As a first experiment, we consider the standard parameter values in Tables 1 and 2. These are essentially the same values used in DKM15 to successfully simulate the MJO on a uniform SST background, except for the congestus cloud formation time scale $\tau_{01}$, which is increased from $\tau_{01} = 1 t_{\text{grid}}$ to $\tau_{01} = 40 t_{\text{grid}}$ (Table 2), and the deep cloud formation time scale $\tau_{02}$, which in turn is increased from $\tau_{02} = 3 t_{\text{grid}}$ to $\tau_{02} = 4 t_{\text{grid}}$. This allows us to decrease the amount of congestus heating that otherwise leads to unrealistic results in the present warm pool setting. This is essentially because the warm pool forcing yields large CAPE values, which increases the potential for congestus clouds; the new transition time scale counterbalances this increase in CAPE. In particular, the stratiform parameters are set to $\alpha_s = 0.50$ and $\tau_{30} = 5 t_{\text{grid}}$.

Fig. 7. As in Fig. 3, but for EXP2 ($\alpha_s = 0.25$ and $\tau_{30} = 5 t_{\text{grid}}$). The black dashed line in (a) marks an eastward wave speed of about 10.3 m s$^{-1}$. 
Fig. 8. As in Fig. 4, but for EXP2 ($\alpha_t = 0.25$ and $\tau_{30} = 5 \tau_{grid}$).
The Hovmöller diagrams of the meridionally averaged (10°S–10°N) lower-level and upper-level zonal winds are shown in Fig. 2, followed by those of the convective heating rates $H_d$, $H_c$, and $H_s$ and vertical average moisture $q$ in Fig. 3. Successive well-organized propagating convective systems are clearly seen in all of these plots, starting at the western edge of the warm pool and slowly moving to the east at roughly 5 m s$^{-1}$. Moreover, the low-frequency and small-wavenumber peaks in the spectrum power plots in Fig. 4 confirm the intraseasonal/planetary-scale variability, which characterizes the simulation. These MJO-like events have, in addition, the typical quadrupole vortex and tilted vertical structures (not shown) that characterize the MJO, consistent with earlier multicloud results (e.g., Khouider et al. 2011; Ajayamohan et al. 2013; DKM15).

Figure 5 depicts the time series of the cloud area fractions and heating rates averaged over a few grid points over the warm pool. As expected, the heating rates and the corresponding cloud area fractions are oscillating intensively and synchronously during the active phases of the MJO events. In particular, the area fraction time series exhibit an intermittent and yet causal variation, as seen in observation [e.g., the radar data at Darwin, Australia, in Peters et al. (2013)].

3. Stratiform transition from CCEWs to MJO regimes

Three more experiments (Table 3) are conducted to understand the response of planetary-scale organization to changes in the strength and extent of stratiform heating. As mentioned earlier, two key parameters that directly affect the stratiform heating strength and extent are $a_s$ ([1]) and $\tau_{30}$ (Table 2). The set of all numerical experiments conducted in this study is summarized in Table 3.

In EXP2, we decrease $a_s$ from 0.5 to 0.25 and keep $\tau_{30} = 5\tau_{\text{grid}}$ to separate the effect of the two parameters.
See the $H_s$ equation in (1). The associated Hovmöller and spectrum power diagrams are shown in Figs. 6–8. From these figures, we see that the variability has moved to synoptic scales, and the MJO-like propagating streaks and associated low-frequency spectral power have both diminished. While a few streaks of slowly moving planetary-scale wavelike signals are still visible in the Hovmöller diagram of the 200-hPa wind, the variability of deep and stratiform heating rates and especially moisture are dominated by synoptic-scale waves that move eastward at speeds approaching 10 m s$^{-1}$. While this speed is smaller than what is typically observed for convectively coupled Kelvin waves, it can be readily seen that the spectrum power is aligned linearly, as it is following a dispersionless dispersion curve of Kelvin waves with a reduced equivalent height (Fig. 8). However, when the data are filtered following the wavenumber–frequency box shown in the spectrum power plot of deep convection in Fig. 8 [a technique initially used in Wheeler and Kiladis (1999)], the corresponding horizontal and vertical structures reported in Fig. 9 do not seem to resemble those of typical convectively coupled Kelvin waves. Unlike those reported in Khouider et al. (2011) for the case of a uniform SST background, the waves in Fig. 9 carry a nontrivial meridional velocity converging at lower levels toward the equator within the region of active convection (corresponding roughly to the region of zonal convergence). This is consistent with the structure of Kelvin waves evolving in a meridional shear background (Ferguson et al. 2009; Han and Khouider 2010). Indeed, this is unlike the MJO, which instead exhibits Rossby gyres on both sides of the convection center and zonal convergence along the equator; for the MJO, the meridional divergence at low level is positive within the convection center. Nonetheless, the backward vertical tilts are still prominent, as seen in Figs. 9e and 9f.

Next (EXP3), we keep the small value $\alpha_s = 0.25$ as in EXP2 but increase the stratiform transition time scale to $\tau_{30} = 10 \tau_{\text{grid}}$. Intuitively, this will have the effect of making stratiform clouds last longer and thus expand in both time and space. In Figs. 10–12, we show the Hovmöller and spectrum power diagrams of the meridionally averaged zonal winds and heating rates. Clearly, the planetary-scale organization of MJO-like waves is successfully recovered as in the case of EXP1.

To further demonstrate the tendency of stronger stratiform heating to yield better MJO simulation, we did another experiment with $\alpha_s = 0.75$ and $\tau_{30} = 5 \tau_{\text{grid}}$ (results not shown). It resulted in similar and slightly
stronger MJO-like organization than EXP1 and EXP3. However, if, on the other hand, we use the same large stratiform fraction $\alpha_s = 0.50$ as in EXP1 but combine it with a smaller transition time $\tau_{30} = 2\tau_{\text{grid}}$ (EXP4), similar to EXP2 ($\alpha_s = 0.25$, $\tau_{30} = 5\tau_{\text{grid}}$), the planetary-scale organization of MJO-like waves is replaced by convectively coupled Kelvin waves, which, as in EXP2, dominate the warm pool region (results not shown).

Two statistical measures (one being the autocorrelation function of the precipitation and column-averaged moisture and the other being the frequency of precipitation events) were performed to assess the statistical properties of convection for the four experiments presented so far (EXP1–4). As shown in Fig. 13, for all the experiments, the autocorrelation of the precipitation is much shorter than the moisture, which is qualitatively consistent with observational studies (e.g., Holloway and Neelin 2009, 2010), as in DKM15. Also consistent with DKM15 in the two experiments with clear MJO-like events (EXP1 and EXP3), the moisture has much longer autocorrelation times. Moreover, the observed two-power-law structure of precipitation event distributions is captured in all of these four experiments (results not shown), consistent with the observations reported in Neelin et al. (2008) and Peters et al. (2010), confirming the results of DKM15.

4. The stratiform organization mechanism

In this section, we attempt to elucidate the physical mechanism through which the stratiform heating affects the development of planetary-scale organized MJO-like waves. The stratiform heating affects the coupled HOMME-SMCM in two distinct fashions. One is through the differential heating and cooling of the upper and lower troposphere, which results in a tilted heating profile acting directly on the free-tropospheric dynamics, and the other is through the moist thermodynamic variables. As already pointed out, the evaporation of stratiform rain in the lower troposphere acts as a source of midtropospheric remoistening and, at the same time, dries and cools the boundary layer via the induced downdrafts. The latter effects are taken into account in SMCM via the parameter $\mu$ present in the downdraft equation [(3)]. The tilted heating is undoubtedly important for successful simulation of the MJO and CCEWs in GCMs (e.g., Khouider et al. 2011; Lappen and Schumacher 2014), but, by SMCM design, both congestus and stratiform heating contribute to the
FIG. 12. As in Fig. 4, but for EXP3 ($\alpha_s = 0.25$ and $\tau_{30} = 10\tau_{grid}$).
As will be demonstrated below by the results of EXP8 (and other tests not reported here), a strong congestus heating does not lead necessarily to a better MJO simulation. It becomes worthwhile to investigate the effect of the contribution of the stratiform heating to the downdraft field.

As demonstrated in Majda et al. (2004), through the parameter $\mu$, the stratiform heating controls the scale-selective instability of superclusters in terms of convectively coupled gravity waves; thus, the phrase “stratiform instability.” As pointed out in Majda et al. (2004), the effect of the downdraft on the boundary layer mimics the dynamics of cold pools as they expand and spread in time and space following the stratiform heating: the stronger and more expanded the stratiform heating is, the more prominent the cold pool effect is.

Here, we test this mechanism in the context of GCM simulations and see whether it can explain in part why the simulation of MJO-like planetary-scale organization versus synoptic-scale CCEWs is tied to the amount of stratiform heating, as demonstrated above. To do so, we conduct a few more experiments where we keep $\alpha_s = 0.5$ and $\tau_{30} = 5 \tau_{\text{grd}}$ as in EXP1 but gradually decrease the value of $\mu$. In Fig. 14, we plot the Hovmöller diagram of deep convective and congestus heating rates for the three values $\mu = 0.1, 0.05, \text{and } 0.01$. As we can see, with $\mu = 0.1$, which is equivalent to reducing the effect of stratiform heating on downdraft by a factor of roughly 2, the change in $\mu$ does not seem to have a big impact on the MJO simulation. In fact, the MJO events seem to be more persistent and more organized than in EXP1 (Fig. 3), although the heating strength seems to be a bit weaker with $\mu = 0.1$. This is indeed unlike the effect of reducing $\alpha_s$ by a factor of 2 as seen in EXP2 (Fig. 7). This is, in fact, a clear evidence that the importance of stratiform heating, for planetary-scale organization of convection, is not limited to its contribution to the downdraft fluxes but has significant impact on the MJO and CCEW dynamics through its direct contribution to the tilting of the heating (Khouider et al. 2011; Lappen and Schumacher 2014). Nonetheless, as shown in Figs. 14e and 14f, when $\mu$ is reduced significantly, to $\mu = 0.01$, the planetary-scale MJO-like organization is lost, but the simulation yields instead synoptic-scale eastward CCEWs as in EXP2. At $\mu = 0.05$, the planetary-scale blobs of convection begin to fracture and split, though MJO-like events clearly remain visible. The
progressive transition from MJO-like to synoptic-scale organization as $\mu$ decreases is confirmed by spectral power plots (not shown), although the changes are not as dramatic and as clear-cut as in tuning $a_s$ and/or $t_03$. This suggests that MJO dynamics are very complex and cannot be explained by a single mechanism or a single variable, such as column-integrated moisture (although it is extremely important).

Another way to counter the effect of stratiform heating, both in terms of the tilted heating and its contribution to downdraft, is increasing the amount of congestus heating while the stratiform parameters are kept as in EXP1. The increase of lower-level heating and upper-level cooling from increased congestus heating will compensate for the upper-troposphere heating and lower-troposphere cooling from stratiform heating $H_s$. At the same time, the congestus heating rate appears in the downdraft equation [(3)] in front of $H_s$ with a negative sign. The former will reduce the tilt in the overall heating profile, while the latter will have the effect of counterbalancing the contribution of stratiform heating to downdraft. In physical terms, the reduction of stratiform-induced downdraft by congestus heating is the result of the associated compensating updrafts.

We thus repeat the simulation EXP1 but with a smaller congestus formation time scale, $t_{01} = 1t_{\text{grid}}$ (EXP8), to allow substantial growth of congestus clouds. As we can see from Fig. 15, not surprisingly, this leads to CCEW-type organization as in EXP2 and EXP4. This indeed confirms the importance of the two mentioned effects of stratiform heating but also the detrimental effect of overcompensation by congestus heating. Nonetheless, the effect of congestus heating cannot be neglected or left aside, as it is the main driver of moisture preconditioning, first by delaying deep convection and thus allowing the atmosphere to moisten through the detrainment of shallow (and congestus) clouds, and then by effectively driving low-level moisture convergence (Khouider and Majda 2006, 2008). The persistence of convectively coupled Rossby
and Kelvin waves in this congestus-dominated dry environment is consistent with the results of Khouider et al. (2011), who obtained such behavior when the background moisture is weak, since, by the model design, a dry environment promotes more congestus heating. The crucial roles of congestus heating and moisture convergence have been thoroughly documented by two of the authors (e.g., Khouider and Majda 2006, 2008).

5. Summary and conclusions

Numerical simulations using an atmospheric GCM with an idealized water-only Earth surface with no land or topography (an aquaplanet) are presented and analyzed in terms of the ability of the model to simulate MJO-like wave disturbances. The model is based on the spectral elements HOMME dynamical core with coarse resolution using the stochastic multicloud model (SMCM) of Khouider et al. (2010) as a cumulus parameterization, following DKM15. Unlike in DKM15, however, here the surface forcing takes the more realistic shape of the Indian Ocean–western Pacific warm pool. The current study focuses on the role of stratiform heating in the model’s capability to reproduce organized tropical convection on multiple scales.

Many previous (modeling and observation) studies have identified stratiform heating as a major component of organized tropical convection and the MJO in particular (Moncrieff 1981; Dudhia and Moncrieff 1987; Houze 1997; Schumacher and Houze 2003; Lin et al. 2004; Parker and Johnson 2004; Mapes et al. 2006; Jakob and Schumacher 2008; Khouider et al. 2011). It is believed to be important mainly for providing the required tilted heating that characterizes tropical convective systems of all scales (Kiladis et al. 2009; Lin et al. 2004; Lappen and Schumacher 2014) and its ability to drive horizontal vorticity at mesoscales (Moncrieff 1981, 2010). In particular, it has been identified as a source of instability for superclusters (Mapes 2000; Majda and Shetter 2001). While many studies identified the importance of the tilted
heating structure in, for example, triggering gravity waves that precondition the environment downstream to new convection (Mapes 1993; Stechmann and Majda 2009). Majda and Shefter (2001) pinpointed that the so-called stratiform instability is mainly a result of the acceleration of downdrafts through the evaporation of stratiform rain, which in turn drives cold pools in the boundary layer.

Here, the crucial role of stratiform heating for the simulation of MJO-like convective organization in the coupled HOMME-SMCM is tested by changing a few key model parameters. In a first step, the amount of stratiform heating produced by the model was tested through two separate parameters: namely, the strength coefficient of stratiform heating $\alpha_s$ and the decay time scale of stratiform cloud area fraction $\tau_{30}$. It is found that these two parameters play a similar role in our model simulations. Large values of either $\alpha_s$ or $\tau_{30}$ lead to planetary-scale intraseasonal MJO-like organized convection, while smaller values yield synoptic-scale convectively coupled Kelvin waves. The fact that these two parameters lead to the same behavior is not very surprising, since $\tau_{30}$ strongly modulates the area fraction of stratiform heating $\sigma_s$, and both $\alpha_s$ and $\sigma_s$ appear as prefactors in the stratiform heating equation $[(1)]$. However, the reason why the MJO simulation is so sensitive to the stratiform heating remains to be elucidated.

To test the role of the stratiform instability as defined in Majda and Shefter (2001) through the contribution of stratiform heating to downdrafts by the evaporation of stratiform rain mechanism, we followed up with a series of simulations using decreasing values of the parameter $\mu$ which controls the contribution of stratiform rain evaporation to downdrafts. It is found that, for sufficiently small $\mu$ values, the MJO-like organization disappears and is replaced by convectively coupled Kelvin waves, as in the cases with small $\alpha_s$ or $\tau_{30}$. However, the amount by which $\mu$ needs to be reduced in order to shut off the MJO is not proportional to the amount by which $\alpha_s$ needs to be reduced in order to achieve similar results. While this clearly demonstrates the importance of stratiform-induced downdrafts for MJO simulation, as is the case for the stratiform instability, it also suggests that this is not the only mechanism that controls the scale selection of convective organization, and other factors, such as tilted heating, may play a role. The dynamics of the MJO and organized convection, in general, is very complex and may not be tied to one single physical mechanism (i.e., one single model parameter), such as the tilted heating (Lappen and Schumacher 2014) or the spreading of cold pools in the boundary layer (Savarin et al. 2012; Feng et al. 2015).

This extreme sensitivity of the model simulations to stratiform heating and especially evaporation of stratiform rain raise questions about the universality of such parameters. In nature, the strength and importance of such processes is often dictated by the large-scale conditions. While the amount of stratiform clearly depends on the strength and abundance of deep convection, the dependence is not a simple linear relationship. It highly depends on cloud microphysics, turbulent fluctuations in temperature and aerosol concentrations, and possibly many other factors. Already, the SMCM framework takes this notion of nonlinear dependence into account through the stochastic area fraction. However, this dependence is not strongly tied to all the physical processes that presumably control the amount of stratiform heating and especially the actual evaporation rate of stratiform rain in the lower troposphere. In state-of-the-art GCMs, stratiform rain is not parameterized but directly represented through grid-scale condensation. While this solves the issues of parameter tuning, it lacks the observed causality associated with deep convection and the effect of subgrid variability of the stratiform rain formation and evaporation.

It is worth noting that, in this work, with the simplified downdraft formula $[(3)]$, we focused on the role of stratiform clouds on the planetary-scale organization of convection, but downdrafts can also vary with cumulus convection (Cheng and Arakawa 1997). However, without the tilt and time delay inherent to stratiform heating, it is not clear whether convective downdrafts are able to produce cold pools that can expand in time and space and propagate in one direction.

The authors are currently working on more realistic ways of parameterizing stratiform heating fractions using the stochastic multicloud model framework applied to comprehensive bulk mass flux column-physical cumulus parameterizations (Tiedtke 1993; Zhang and McFarlane 1995). While this study provides an understanding of how stratiform heating affects the organization of convection on the planetary scale, the new framework will allow one to study the effects of interactive physics, such as radiation and cloud microphysics, on convective organization, especially through the modification of the effective stratiform heating.

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REFERENCES


