Modeling Tropical Precipitation in a Single Column

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ABSTRACT

A modified formulation of the traditional single column model for representing a limited area near the equator is proposed. This formulation can also be considered a two-column model in the limit as the area represented by one of the columns becomes very large compared to the other. Only a single column is explicitly modeled, but its free tropospheric temperature, rather than its mean vertical velocity, is prescribed. This allows the precipitation and vertical velocity to be true prognostic variables, as in prior analytical theories of tropical precipitation. Two models developed by other authors are modified according to the proposed formulation. The first is the intermediate atmospheric model of J. D. Neelin and N. Zeng, but with the horizontal connections between columns broken, rendering it a set of disconnected column models. The second is the column model of N. O. Renno, K. A. Emanuel, and P. H. Stone. In the first model, the set of disconnected column models is run with a fixed temperature that is uniform in the Tropics, and insolation, SST, and surface wind speed taken from a control run of the original model. The column models produce a climatological precipitation field that is grossly similar to that of the control run, despite that the circulation implied by the column models is not required to conserve mass. The addition of horizontal moisture advection by the wind from the control run substantially improves the simulation in dry regions. In the second model the sensitivity of the modeled steady-state precipitation and relative humidity to varying SST and wind speed is examined. The transition from shallow to deep convection is simulated in a "Lagrangian" calculation in which the column model is subjected to an SST that increases in time. In this simulation, the onset of deep convection is delayed to a higher SST than in the steady-state case, due to the effect of horizontal moisture advection (viewed in a Lagrangian reference frame). In both of the models, the steady-state moisture convergence is a nearly unique function of the surface evaporation when horizontal moisture advection is neglected, a result that is explained in terms of the moisture and moist static energy budgets. The proposed formulation can also be applied to limited-area three-dimensional models, such as cloud-resolving models. Additionally, with further development, it may be possible to use the fixed-temperature constraint as the basis for a truncated atmospheric dynamics appropriate for the study of tropical climate.

1. Introduction

In the deep Tropics, horizontal temperature gradients are exceedingly weak in the free troposphere, due to the weakness of rotational constraints and the consequent nonlocal nature of dynamical adjustment (Charney 1963, 1969; Schneider 1977; Held and Hou 1980; Bretherton and Smolarkiewicz 1989). Precipitation, however, varies strongly as a function of both space and time. Even when averaged over climatic timescales, the precipitation field remains highly structured in space. We have some understanding of the factors controlling this structure. For example, it is well known that the regions of high sea surface temperature (SST) tend to have high precipitation rates, and the reasons for this are clear, at least qualitatively (Neelin and Held 1987). However, the importance of the issue warrants further attempts to formulate the simplest possible physical theory that can correctly explain the basic features of the observed precipitation field and associated circulations. We discuss a simple single-column model formulation for application to this problem, and present results from two models developed by other authors that have been modified according to this formulation.

Single-column models have been used by many investigators to study questions of climate (e.g., Manabe and Strickler 1964; Manabe and Wetherald 1967; Schneider and Dickinson 1974; Ramanathan and Coak-
ley 1978; Sarachik and Sellers 1980; Nakajima et al. 1992; Rennó et al. 1994a,b; Li et al. 1997). Accordingly, it seems natural that temperature should be a prognostic variable in them. This choice implies that the vertical velocity must be specified in such models. If the column represents an average over an entire planet, its mean vertical velocity must be zero, so the choice is easy. If it represents a limited area, the mean vertical velocity can be nonzero, and its specification is both nontrivial and consequential. If the lapse rate and radiative cooling rate of the equilibrium atmosphere cannot vary greatly (as is true for any realistic tropical simulation) the external specification of the vertical velocity strongly constrains the precipitation. This prevents these models from being very useful in understanding the observed climatological distribution of precipitation, though of course they may be useful for many other purposes.

We explore an alternative approach, in which the vertical motion within the column is always such as to force its free tropospheric temperature profile in the single-column model to remain equal to a specified equatorial mean profile. In this approach, vertical motions maintain the horizontal uniformity of tropical tropospheric temperature in the face of surface forcings that vary from place to place.

This approach is not truly new, but we believe it can be more broadly exploited than in past studies. It can be regarded as a limiting case of two-column models (Pierrehumbert 1995; Miller 1997; Nilsson and Emanuel 1999; Larson et al. 1999; Raymond and Zeng 2000; Clement and Seager 1999; Mapes 2000, manuscript submitted to Quart. J. Roy. Meteor. Soc.), in which the column of interest has a horizontal area much smaller than that of a second column representing the remainder of the tropical atmosphere. A similar approach has also been used in a number of studies that have developed analytical theories for tropical precipitation (Neelin and Held 1987; Zeng 1998; Zeng and Neelin 1999; Raymond 2000a). In these theories, precipitation is determined by local thermodynamic considerations in a single column. Horizontal temperature gradients are assumed negligible in these theories, but additional simplifying assumptions are made as well. No horizontal moisture advection is considered, the number of degrees of freedom is one or two at most, and very simple physical parameterizations are employed. However, the assumption that horizontal temperature gradients are negligible near the equator is more broadly valid than these other simplifying assumptions. In this work, we explore the consequences of the first assumption, but in a broader context, in which multiple degrees of freedom may be retained, more complex physical parameterizations may be used, and some representation of horizontal moisture advection may be included. Besides true column models, this formulation can also be applied to three-dimensional models representing limited areas in the Tropics, though this is not done here.

Strictly, it is horizontal gradients of atmospheric density, rather than temperature, to which the arguments below apply most directly. For precision, the arguments could be straightforwardly modified so that temperature is replaced by virtual temperature, or even more precisely, the density temperature defined by Emanuel (1994, 111–113). We leave the arguments in terms of temperature for economy and simplicity. This choice is also probably a consistent one, since large-scale density variations due to horizontal temperature gradients (which are neglected) are probably at least as large as those due to water vapor and condensate.

2. Formulation

Consider the primitive temperature and moisture equations in pressure coordinates:

\[
\frac{\partial T}{\partial t} + u_h \cdot \nabla T + \omega S = Q_c + Q_R + Q_{\text{diff}}^T
\]

\[
\frac{\partial q}{\partial t} + u_h \cdot \nabla q + \omega \frac{\partial q}{\partial p} = Q_s + Q_{\text{diff}}^q,
\]

with \( T \) temperature, \( q \) specific humidity, \( p \) pressure, \( u_h \) horizontal velocity, \( \omega \) vertical (pressure) velocity, and \( S = (T/\theta)(\partial \theta/\partial p) \) the static stability, where \( \theta \) is the potential temperature. Here \( Q_c \) is the convective heating, \( Q_R \) the radiative heating, \( Q_{\text{diff}}^T \) the convergence of turbulent heat fluxes, \( Q_s \) the convective moisture source, and \( Q_{\text{diff}}^q \) the convergence of turbulent moisture fluxes. A single column model consists of these two equations, together with parameterizations for determining the terms on the right-hand sides of (1) and (2).

Having two equations, this system can be solved for two unknowns. Typically, these are taken to be \( T \) and \( q \). Given that choice, since a single column model can have no explicit momentum or continuity equation, \( \omega \) must be specified. To a first approximation, under typical tropical conditions, this fixes the precipitation as well. Assuming horizontal temperature advection is negligible, in steady state (1) reduces to

\[
\omega S = Q_c + Q_R + Q_{\text{diff}}^T
\]

while conservation of energy requires

\[
\int -Q_s \, dp = \frac{c_v}{L_v} \int Q_c \, dp
\]

with \( c_v \) and \( L_v \) the heat capacity of air at constant pressure and latent heat of vaporization of water, respectively, and the integrals taken over the depth of the troposphere. If we assume that moisture storage is small, that under typical conditions \( Q_s \) varies much more than \( Q_c \) or \( Q_{\text{diff}}^T \) (at least in the vertical integral and some reasonable time average), and that \( S \) is constrained to remain close to a moist adiabat, it follows that precipitation is largely constrained by the imposed vertical velocity in this formulation, regardless of the details of
the parameterizations. As an example, the column models of Hu and Randall (1994), formulated in the traditional way, exhibit fluctuations about a state of mean radiative-convective equilibrium ($\omega = 0$), but the amplitude of the precipitation variations shown in that study is less than 1 mm day$^{-1}$, in most cases much less than that value. This is small compared to the difference between deep convective and trade cumulus regimes.

Consider the alternative formulation in which we assume that the temperature tendency vanishes above the boundary layer. The temperature profile in the free troposphere must then be externally imposed. We also assume that the horizontal temperature gradient (thus horizontal temperature advection) vanishes in the free troposphere. Then, the vertical velocity can be diagnosed from (3), so the precipitation becomes a truly prognostic quantity.

In the planetary boundary layer (PBL), we leave (1) as in the standard formulation, so that temperature is interactive there. This is justified because the dynamical adjustment process that homogenizes temperature in the free troposphere is counteracted in the PBL by strong vertical mixing, which ties the atmospheric temperature there to the surface temperature. Since $T$ is interactive in the PBL, $\omega$ must be specified there. We do this by interpolating $\omega$ linearly in pressure between its value at the lowermost free tropospheric point and a surface boundary condition, taken here to be $\omega = 0$ (though this need not be the case, since in general surface pressure can vary). We leave (2) in its standard formulation throughout the entire atmosphere.

In section 3, we modify the quasi-equilibrium tropical circulation model of Neelin and Zeng (2000) and Zeng et al. (2000) according to the above formulation. The original model has full horizontal variability and two vertical modes. By turning off the momentum and continuity equations, the model is broken into a set of disconnected column models, with surface fluxes constrained by a fixed SST and surface wind distribution, which are the same as in a control run of the original model. Tropospheric temperature between 15$^\circ$S and 15$^\circ$N is fixed at a uniform value equal to the area and time mean from that region in the control run. In an additional simulation, horizontal moisture advection by the prescribed wind is added. The precipitation distribution from the set of disconnected column models with and without horizontal moisture advection is compared to that of the control run. In section 4, we modify the single column model developed by Renno et al. (1994a,b) in the same way. We examine the sensitivity of the reformulated model to varying (prescribed) SST and surface wind speed, and the stability of the model’s radiative-convective equilibrium state. By subjecting the model to a time-varying SST, we perform a “Lagrangian” calculation in which the transition from shallow to deep convection is simulated. We discuss the results and some implications in section 5, and conclude in section 6.

3. Experiments with the Neelin–Zeng model

a. Model description

In this section we present results using the quasi-equilibrium tropical circulation model (QTCM) introduced by Neelin and Zeng (2000) and Zeng et al. (2000), straightforwardly modified following suggestions made or implied in those studies and Zeng and Neelin (1999). As originally formulated, the QTCM is not a column model, but rather an “intermediate” model containing both horizontal dimensions and two modes in the vertical. However, the model is easily modified to function essentially as a set of disconnected column models, each formulated as described above. We compare results from this modified model to those from the original QTCM. We focus on precipitation over the tropical ocean, land having been treated in this way by Zeng and Neelin (1999; see also Zeng 1998).

The model contains a single baroclinic mode and a barotropic mode in its momentum equations. Temperature and moisture are each represented by a single number ($T_i$, $q_i$) at each horizontal point, each representing the first coefficient in a Galerkin expansion in the vertical. The convective scheme is based on the Betts–Miller scheme (Betts 1986; Betts and Miller 1986). All other significant processes are represented by parameterizations that are physically based and related to, if simpler than, those used in general circulation models. The parameterizations, like the dynamics, are projected on the two-mode truncation. For details of this elegant model’s formulation and basic behavior, the reader is referred to the papers by its creators (Neelin and Zeng 2000; Zeng et al. 2000). Useful theoretical background can be found in a number of precedent studies (Neelin and Yu 1994; Yu and Neelin 1994; Neelin 1997; Emanuel et al. 1994, and references therein).

In the modified version of the model used here, $T_i$ is fixed and uniform between 15$^\circ$S and 15$^\circ$N, with its value computed from a monthly and horizontal average over all points between those latitudes in the control run. The QTCM does not have a boundary layer per se, so in this case the tropospheric temperature interacts directly with the surface. The momentum equation is deactivated, and the horizontal wind field is set to the monthly mean value of the vector wind determined from the control run. In our basic modification, it interacts with the thermodynamic fields only through the surface wind’s effect on the surface fluxes. In a second modified

1 Some two-column models also make this approximation (Pierrehumbert 1995; Miller 1997; Larson et al. 1999; Clement and Seager 1999; Mapes 2000, manuscript submitted to Quart. J. Roy. Meteor. Soc.) while others do not (Nilsson and Emanuel 1999; Raymond and Zeng 2000). The extent to which this may make a difference in the model dynamics is left as an open question for future work.
version, we allow horizontal advection of the moisture field by the wind, as well as horizontal diffusion of moisture, so that different columns are explicitly coupled.

In both modified versions, the vertical velocity which appears in the moisture equation is not related to the divergence of the prescribed horizontal wind, but is determined from (3) as described in section 2. Thus rather than satisfying a true continuity equation, it can be said to satisfy a continuity equation in which the actual divergence is replaced by the “diabatic divergence” defined by Mapes and Houze (1995). This vertical velocity is explicitly constrained only by local quantities at each grid point, and hence at any given level is not required to average to zero over the horizontal domain. Therefore, while the resulting vertical divergence (equivalently, divergence) “field” can be thought of as implying a global divergent circulation, that circulation is not required to conserve mass.

c. Results

Figure 1 shows the precipitation between 20°S and 20°N from the control run and three modified runs described above. The 1979–98 average January mean precipitation obtained from the data set of Xie and Arkin (1997) is also shown. The control run produces a precipitation field that is similar to the climatological January mean obtained by Zeng et al. (2000) from a 1982–98 time-dependent run driven by observed SST. Differences between the control run and observations are thus essentially the same as discussed there, particularly the weak ITCZs over parts of the eastern Pacific and Atlantic. The modified versions of the model obviously should not be expected to improve the agreement with observations; rather, the proper test of the proposed idea is to what extent the modified runs can reproduce the results of the control run. While exhibiting a number of differences in detail with the control run, the modified runs are similar to it at a coarse level. The degree of similarity is perhaps even remarkable when one considers that the modified precipitation fields are essentially determined by isolated single column models (though in the case with moisture advection there is some connection between them) whose collective implied circulation is required to conserve neither momentum nor mass.

Perhaps as a consequence of this nonconservation, the areally averaged precipitation is not the same in the modified as the control runs. In both modified runs without moisture advection, the precipitation averaged over all longitudes between 15°S and 15°N is approximately 40% greater than the control run, with virtually no difference between the runs with and without horizontal temperature variations. Moisture advection, on the other hand, makes a substantial difference, reducing the discrepancy to 11%.

d. Relationship between evaporation and precipitation

There is a close relationship between evaporation and moisture convergence in these simulations. Figure 2 consists of scatterplots showing evaporation minus precipitation, \( E - P \) (strictly, the moisture divergence), versus \( E \) for all points in a region bounded by 15°S, 15°N, 120°E, and 270°E (i.e., the tropical Pacific region) in the control run and both modified runs with uniform temperature. Also shown are analogous results from observational datasets, specifically the Xie and Arkin dataset for precipitation, and the climatological dataset of da Silva et al. (1994) for evaporation. As might be expected from the above discussion of the areally averaged precipitation, moisture advection generally improves the comparison to the control run. Compared to the modified run without moisture advection, with moisture advection more points fall on or relatively near the zero precipitation line \( E - P = E \), especially at higher \( E \). These represent regions where shallow, nonprecipitating convection would be occurring in the real atmosphere, though the QTCM has no representation of this process. Moisture advection also causes a few points to fall at unrealistically high rain rates, but this is an artifact re-

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3 Strictly, it is the vertical mass transport, rather than velocity, which must average to zero over the domain if mass is to be conserved.

2 Except very indirectly, through the effect of the prescribed wind on the surface fluxes.
resulting from the advection of moisture across the 15°S boundary between regions of uniform and nonuniform temperature, particularly over land monsoon regions, as evidenced by the sharp meridional gradients near 15°S in such regions in Fig. 1. The higher the latitude, the worse the approximation proposed here is likely to be, and if meridional moisture advection is allowed this can cause more severe errors at higher latitudes to affect lower latitudes as well.

The close relationship between evaporation and precipitation in runs without horizontal moisture advection can be understood using bulk thermodynamic considerations. Following the argument of Zeng and Neelin (1999), but retaining horizontal moisture advection for the moment, leads to

\[ E - P = \frac{M}{M} (\hat{Q}_s + E + H + \mathbf{u}_s \cdot \nabla q) \]  

where \( E \) is evaporation, \( P \) precipitation, \( H \) surface sensible heat flux, \( M \) the gross moist stability, and \( M_q \) the gross moisture stratification (Neelin and Yu 1999; Yu...
**Fig. 2.** Scatterplots of $E$ vs $E - P$ for (a) the DaSilva and Xie and Arkin datasets, (b) the control run of the Neelin–Zeng model, (c) the column version of the Neelin–Zeng model with uniform temperature and no moisture advection, (d) as in c but with moisture advection.

**Fig. 3.** Scatterplot of $\bar{Q} + H$ vs $E$, for the run without horizontal moisture advection.

and Neelin 1997; Neelin 1997). The caret represents a vertical integral over the troposphere. In deep convective regimes, the factor $M_\rho / M$ is substantially greater than unity. In the present simulations $M_\rho / M \approx 3$ over the tropical oceans, somewhat smaller than the values obtained from observations by Yu and Neelin (1998).

If we neglect horizontal moisture advection in (4), treat $M_\rho / M$ as constant, and assume that $\bar{Q} + H$ is either small, or weakly varying compared to $E$, then the moisture convergence, $P - E$, is predominantly determined by the evaporation. This “convergence feedback” (Webster 1981; Zebiak 1986) means that anomalies in $E$ drive collocated anomalies in $P$ that can be several times as large. While in reality $M_\rho / M$ varies significantly over the tropical oceans, in the runs above without moisture advection it varies relatively little. The QTQCM also includes cloud–radiative feedbacks, and largely because of these ($H$ being small) $\bar{Q} + H$ varies significantly over the tropical oceans. Figure 3 is a scatterplot of $\bar{Q} + H$ versus $E$ for all tropical Pacific points in the run with fixed tropical temperature and no moisture advection. Most of the variation of $\bar{Q} + H$ is correlated with variations in $E$, and acts to approximately double the effective convergence feedback of $E$ on $P$. 
The above results might lead one to view variations in $E$ as causing variations in both $\dot{Q}_n + H$ and $P$. This is consistent with the close correlation of SST and $P$ in the model, since $E$ and SST are also correlated in the model. However, the model may be unrealistic in this respect. Observations have shown $E$ to be anticorrelated with SST at high SST, due to the tendency for wind speed to be low at high SST (Zhang and McPhaden 1995). We ran the QTCM with its default minimum wind speed of 5 m $s^{-1}$ in the surface flux parameterization, probably too large to allow the observed low values of $E$ to occur at high SST. The above discussion should therefore be viewed simply as explaining the model behavior. In reality, $P$ may be uncorrelated or anticorrelated with $E$, particularly if $\dot{Q}_n + H$ is similarly un- or anticorrelated.

That said, the correlation between the other energy fluxes and $E$, together with (4), explains the close correlation between the moisture convergence and evaporation in this simulation. Moisture advection changes this picture both by appearing explicitly on the right-hand side of (4), and by increasing the range of variations in $M_f/M$.

4. Results using a single column model

In this section we use the single column model developed by Rennó et al. (1994a,b) and maintained by K. A. Emanuel. The model uses the convective scheme of Emanuel (1991) and the radiative scheme developed by Chou et al. (1991). The reader is referred to the original papers for a detailed description of the model’s formulation, its basic behavior, and the parameterizations. The model was run with a vertical resolution of 50 hPa. While the specific results shown below depend on the specifics of the model’s formulation, the procedure does not, and could be applied to any single column model.

To derive the basic temperature profile, we first ran the model to steady state in its standard mode under radiative-convective equilibrium conditions ($\omega = 0$). We chose the SST to be fixed at 27.5°C. The radiation was computed interactively, though effects of cloud were not included. The insolation was set to its annual and diurnal average value at a latitude of 10°. Surface albedo was set to 0.05, surface wind speed to 7 m $s^{-1}$ and atmospheric CO$_2$ to 330 ppm. These choices yielded a temperature profile that was found to be reasonably near a January tropical mean profile determined from the National Centers for Environmental Prediction–National Center for Atmospheric Research reanalyses (Kalnay et al. 1996). This profile was then used as input to the version of the model that had been modified as described above, with the boundary layer taken to extend up to 850 hPa. Below that level, the input temperature profile is simply the initial condition, and the temperature is interactive (as discussed in section 2), while above it the input temperature profile is fixed for all time. The model was run a number of times under conditions identical to the initial run, except that the SST was varied.

Figure 4 shows the steady-state $P$ and $E$ for this series of calculations. The two are approximately equal at 27.5 K. This indicates that the temperature profile that resulted from an assumption of zero vertical velocity in the original model yields a predicted value of zero vertical velocity when input to the modified model under otherwise identical conditions; the model is stable to the modification.4 This is in contrast to the two-column model of Raymond and Zeng (2000) in which the radiative-convective equilibrium state was found to be unstable even when the two columns had equal SST. It is not presently clear whether this difference is due to the different physical parameterizations or the different formulations of the two models. This question will be investigated in future work.

Figure 5 shows relative humidity profiles for the same set of runs shown in 4. As expected, the atmosphere moistens as the SST increases. However, it is not as dry as might be expected at low SST (where $P$ essentially vanishes and the atmosphere is subsiding). On the other hand, the minimum value on the profile remains below 60% even under conditions rainier than any found on Earth in a monthly mean. Of course, the relative humidity is particularly sensitive to the details of the convective scheme (e.g., Emanuel and Zivkovic-Rothman 1999).

Figure 6 shows $E - P$ as a function of $E$ for the same

4 There is actually a very weak instability, resulting in an oscillation whose amplitude is about 0.1 mm day$^{-1}$ in precipitation and whose period is about a week. A small increase in the surface albedo, which otherwise has no significant effect on the results, is sufficient to eliminate the oscillation.
set of runs as in Fig. 4, analogously to Fig. 2. Also shown are the same quantities for other sets of runs in which the imposed surface wind speed, rather than the SST, is varied, while SST is held fixed. The figure shows that once the tropospheric temperature profile and other parameters are fixed, the moisture convergence and evaporation are almost uniquely coupled, consistently with the arguments in section 3d, regardless of whether SST or surface wind speed is used to cause \( E \) to vary. The data from the QTCM run with uniform air temperature and without horizontal moisture advection (as in Fig. 2c) is also shown for comparison. While \( E - P \) is a nearly unique function of \( E \) in both models (though again, this may be unrealistic, and in particular is sensitive to cloud–radiative feedbacks, which are not simulated in the Rennó et al. model), it is a different function in each model.

**Transition from shallow to deep convection**

In a series of studies with column models and limited-area large-eddy simulation models, the dynamics of the transition from stratocumulus to trade-cumulus-topped boundary layers has been explored (Bretherton and Wyant 1997; Wyant et al. 1997). In these studies, the column or column-like models were subjected to temporally varying surface conditions in order to simulate a Lagrangian air column moving over an ocean with horizontally varying SST. The change in cloud type and boundary layer characteristics was then simulated and the results interpreted in terms of the various physical processes in the models. Above both types of boundary layers, the free troposphere was descending and so the free-tropospheric vertical velocity could be held fixed. It would be desirable to extend this approach to study the transition from trade cumulus to deep convection. In this transition, the vertical velocity changes sign. The traditional single column model is therefore not useful in this context.

Single-column models of trade cumulus topped boundary layers have been formulated in which either temperature or equivalent potential temperature is specified above the boundary layer, and which can predict the value of the tropospheric vertical velocity as long as it is subsiding; these models can be used to predict at what SST the trade cumulus boundary layer will be-
come unstable to deep convection (Betts and Ridgway 1989; Betts 1997). However, deep convection has not been explicitly represented in such models, so that the transition from shallow to deep convection, and associated change in sign of the vertical velocity, has not been explicitly modeled.

The present formulation allows us to simulate the entire transition. The “trade cumulus” regime is not particularly well simulated here, due to limitations of the present model, but its basic qualitative features are evident. Most significantly, the calculation below is able to capture at least one essential aspect of the transition that is not present in the earlier studies (Betts and Ridgway 1989; Betts 1997) in which the transition was represented by the inability of a model for the steady-state trade cumulus boundary layer to find a solution. This aspect is the role played by horizontal moisture advection.

Figure 7 shows the results of a time-dependent Lagrangian calculation using the modified Rennó et al. model as follows. The calculation is initialized with an SST of 23.5°C, and a temperature profile that is the same as used above for the free troposphere (above 850 hPa). Below 850 hPa, the temperature profile is obtained from the steady-state solution obtained at 23.5°C SST. The relative humidity profile for that solution is essentially zero throughout much of the troposphere (see Fig. 5). While the descending regions of the tropical and subtropical atmospheres may approach such conditions at times (Spencer and Braswell 1997), in general we expect some amount of lateral mixing (Pierrehumbert 1998) to keep the average relative humidity above zero even in these regions. We do not wish to take a strong position on this issue, but for present purposes we judge that using an initial condition of zero relative humidity might exaggerate the typical effects of dry air advection into convective regions. Therefore, the initial relative humidity profile above 850 hPa is taken from the steady-state solution for an SST of 25.5°C. As the calculation is integrated forward in time, the SST is continually increased at a rate of 7 \times 10^{-6} K s^{-1}, or roughly 0.6 K day^{-1}. This is roughly comparable to the rate of SST increase that might be experienced by a surface air parcel moving from the coast.

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Fig. 7. Time-dependent Lagrangian calculation using modified Rennó et al. model with a time-varying SST boundary condition (see text for details). SST-pressure cross sections of (a) relative humidity and (b) pressure vertical velocity, analogous to Fig. 5. (c) Rain rate as a function of SST. (d) Moisture divergence vs evaporation for both the time-dependent and steady-state calculations.
of southern Baja California southwestward toward the ITCZ. In using a single column model to represent such a hypothetical Lagrangian trajectory, we are neglecting effects of vertical shear.

The figures show SST-height cross sections of relative humidity and $\omega$, which can be directly compared to the analogous plots in Fig. 5. Also shown are the rain rate as a function of SST, and the moisture divergence $E - P$ versus the evaporation $E$; in these plots the analogous curves from the steady-state calculations are also shown.

The model used here is not really capable of simulating a trade cumulus regime properly, though some aspects are at least qualitatively reasonable. At the lower SSTs, an inversion does develop at 850 hPa, where the cool boundary layer influenced by the low SST meets the fixed temperature profile above 850 hPa. This inversion caps the convection simulated by the Emanuel scheme, so the scheme does produce shallow, nonprecipitating convection. However, the inversion is maintained in a somewhat artificial way, and entrainment across it is not simulated (except by explicit vertical advection, as at any other grid point in the boundary layer). The inversion height cannot vary. The radiative effects of cloud, important in trade cumulus boundary layers, are neglected.

Despite these significant flaws in the model’s representation of shallow convection, this simulation does reveal at least one aspect of the transition to deep convection that is probably robust. Compared to the steady-state calculations, the transition to deep convection is delayed to higher SST in the time-dependent case. This delay is reflected in all the fields shown in Fig. 7. The delay occurs because, at any given SST, the relative humidity in the free troposphere is lower in the time-dependent simulation than in the steady-state simulation at the same SST. This reduces the precipitation efficiency of convection in the time-dependent case relative to the steady-state case. A number of additional simulations of this type have been carried out, and the degree to which the time-dependent and steady-state simulations differ depends, unsurprisingly, on the initial relative humidity and the rate of SST increase. However, the sense of the difference is always the same.

While there are clearly ways in which the simulation shown here can be improved, it is adequate to make two points. First, it illustrates the significance of horizontal moisture advection. It does this from a Lagrangian perspective, rather than the Eulerian one of the QTCM simulations in the preceding section. However, the two perspectives (and two models) both lead to the conclusion that horizontal moisture advection can be an important process in certain regions, where SST would be high enough to support precipitation but for the horizontal advection of dry air. Second, these simulations can be seen as a “proof of concept,” illustrating the potential usefulness of the modified single column model, in which free tropospheric vertical velocity rather than temperature is modeled, for studying this transition in a more careful way.

5. Discussion

a. General discussion

The present work does not address the problem of formulating good parameterizations of physical processes. Results obtained from models using the proposed formulation (including those presented above) will still only be as good as their parameterizations. However, perfect parameterizations are not required in order to determine the usefulness of the formulation itself, since it is just a strong simplification of the large-scale dynamics, a process that is relatively well handled in current numerical models (though numerical advection algorithms continue to determine some aspects of even large-scale behavior for quantities such as water vapor). In the Neelin–Zeng QTCM, however, the large-scale dynamics are already heavily truncated in the vertical, placing another caveat on the results. In future work, we will perform a set of experiments similar to those above with a full general circulation model.

It has long been recognized that the zero-order effect of fluid dynamics near the equator is simply to flatten horizontal density gradients, and that this causes a circulation to be determined by spatially variable heating (Charney 1963, 1969; Schneider 1977; Held and Hou 1980; Bretherton and Smolarkiewicz 1989). This has the implication, in our view a remarkable one, that much of the interaction between a single column and the rest of the tropical free troposphere can be represented in a model that does not explicitly simulate anything but the single column. If the surface wind speed is prescribed, a large fraction of the remaining interaction can be approximately encapsulated simply by fixing the column’s temperature profile to that of a mean profile over the deep Tropics. This, together with a set of physical parameterizations, can be thought of as constituting a sort of mean field theory (Herring 1963) for the quasi-steady component of tropical precipitation. This view is implicit in earlier studies (Neelin and Held 1987; Zeng 1998; Zeng and Neelin 1999; Raymond 2000a), which have modeled tropical convergence and precipitation using only local considerations, and neglecting horizontal temperature gradients. The present work further explores this view, by using it as a framework for more sophisticated physical parameterizations, by considering horizontal moisture advection, and by allowing more vertical degrees of freedom than the one or two in those studies.

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1 The version of the Emanuel convective scheme used in this work is not the most recent. In the more recent versions, convective clouds overshoot their levels of neutral buoyancy, so some entrainment would occur.
The dynamical feedbacks in this system operate as follows. If the convection scheme is triggered by the initial sounding, the resulting heating induces large-scale upward motion in the free troposphere. This advects moisture upward (implying moisture convergence), which can then be rained out or redistributed within the atmosphere by the convection scheme. Even though convection does not change the free-tropospheric temperature, two processes stabilize the sounding and can regulate the convective mass flux. First, parameterized convective downdrafts cool and dry the boundary layer. Second, the vertical profile of moisture can influence the buoyancy of convective updrafts and downdrafts; if environmental air entrained into updrafts and downdrafts is moist, it will be less effective at inducing cooling through evaporation of the liquid water in the drafts. At the same time, the sounding is destabilized by surface evaporation and radiation, both of which are also affected by convection. For many convective parameterizations, including those discussed in this paper, this quickly leads to a nearly steady state.

A similar style of thinking underlies the “boundary layer quasi-equilibrium” closure for convective mass flux proposed by Raymond (1995) and Emanuel (1995) and elaborated further in a number of theoretical and modeling studies (Raymond 1997, 2000a,b; Raymond and Torres 1998). In particular, while we consider convection to be governed by stability properties (e.g., convective available potential energy (CAPE)) of a tropical sounding, we neglect stability variations resulting from free-tropospheric temperature fluctuations. Since it is largely through such temperature fluctuations that an imposed vertical velocity can affect the stability and hence the convection [in quasi-equilibrium theory as formulated, e.g., by Emanuel et al. (1994)], removing the temperature fluctuations changes the role of the vertical velocity in the dynamics, rendering it more an effect than a cause of convection. Accordingly, the use of a convective parameterization in which the heating is based on the moisture convergence will fundamentally change the nature of the dynamics, making it more complex (and, in our view, less well posed). In this case, (3) is no longer a simple diagnostic equation for $\omega$, since $Q_v$ depends, through the moisture convergence, on $\omega$ itself, so that $\omega$ appears on both sides of the equation.

b. Application to limited-area modeling

The proposed formulation can be applied not only to true column models, but to limited-area models, which, conceptually, can function as though they were column models. Rapidly improving computational capabilities have drawn increasing interest toward the idea of using cloud-resolving models (CRMs) to study the interaction of deep convection with large-scale flows and with atmospheric radiation (e.g., Yamasaki 1975; Soong and Tao 1980; Nakajima and Matsuno 1988; Islam et al. 1993; Held et al. 1993; Sui et al. 1994; Randall et al. 1993; Nakajima and Matsuno 1988; Islam et al. 1993; Held et al. 1993; Sui et al. 1994; Randall et al. 1994; Grabowski et al. 1996a,b; Robe and Emanuel 1996; Moncrieff et al. 1997; Tompkins and Craig 1998, 1999; Xu and Randall 1996; Tao et al. 1999; Xu 1993, and references therein). CRMs can in fact be thought of as single column models in which the parameterizations are to a large degree replaced by explicit physics and dynamics, and hence the two sorts of models play complementary roles in climate studies (e.g., Randall et al. 1996).

Since at present CRMs are limited to small areas (particularly when run in three dimensions), the “large-scale flow” has in these studies been represented by an imposed vertical motion profile or something essentially equivalent to it (i.e., imposed large-scale cooling and moistening tendencies). The formulation proposed here for column models can be straightforwardly extended to CRMs, providing an alternative approach. At each time step, compute the horizontally averaged heating resulting from the model dynamics and physics. From this and the mean static stability, compute a large-scale vertical motion profile as in (3). Apply this in a horizontally uniform way in both the temperature and moisture equations above the boundary layer (the height of the boundary layer top must be either fixed in advance, or internally determined in some way). This vertical velocity will keep the horizontally averaged free-tropospheric temperature profile fixed by construction, but will act on the moisture field just as an imposed large-scale vertical motion would. Horizontal temperature variations, presumably important to the cloud- and mesoscale dynamics, are left unconstrained.

c. Three-dimensional dynamics

While our discussion to this point has been phrased mostly in terms of single column models or their analogs (such as CRMs), the calculations above using the Nee-Lin–Zeng QTGM clearly illustrate that one can consider the fixed temperature constraint instead as defining a truncated three-dimensional atmospheric dynamics. It is worth considering how this view could be refined, and for what it might be useful.

1) Mass conservation

We must first confront the fact that the system as posed in this paper does not conserve mass, as mentioned in section 3a. In this study, we have viewed this as a feature worth retaining, for two reasons. First, we have focused on single column or limited-area models, which are inherently incapable of constraining the global mass budget. Second, we view it as interesting to see to what degree the lack of a mass balance-constraint leads to errors in the solution (section 3c).

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6 Even simulations of radiative-convective equilibrium have an imposed vertical velocity, whose value is zero.
However, mass conservation is clearly a desirable feature in a three-dimensional atmospheric dynamics. Enforcing mass conservation requires an additional constraint, and several such constraints are possible, but one seems to us the most appealing. Rather than taking the free tropospheric mean temperature profile to be fixed externally, we could compute it interactively in such a way as to enforce mass conservation. We would first need to assume that the temperature profile has a specified vertical structure, such as a particular type of moist adiabat (the Neelin–Zeng QTCS already incorporates such an assumption). Since the temperature has no horizontal gradient by assumption, we then need only to determine a single parameter (e.g., the temperature at a specified level) to determine the temperature field throughout the troposphere. Given a fixed surface temperature distribution, changes in this parameter would vary the stability everywhere in the same sense. For example, decreasing the tropospheric temperature would increase CAPE, hence convection and hence heating, everywhere. By an iterative procedure, we could find the tropospheric temperature value that would cause \( \omega \) on a fixed pressure surface to integrate to zero (or if the integral is just carried out over the Tropics, to the net mass transport to mid-latitudes) on a single grid and pressure surface. One could enforce this constraint on multiple pressure surfaces by allowing the temperature profile to be determined by multiple parameters [e.g., retaining two vertical modes rather than one, as in Mapes (2000)].

Besides the desirability of enforcing mass conservation for its own sake, this extension would allow the free tropospheric temperature to be a part of the solution, allowing us to investigate its dependence on other aspects of the model or boundary conditions.

2) WINDS AND SURFACE FLUXES

Besides the free tropospheric temperature, the other important quantity that has been specified in this study is the horizontal wind. Even from a purely thermodynamic point of view, this is important since the wind is needed to compute the surface fluxes as well as the horizontal moisture advection. As discussed previously, the heating computed from the fixed-temperature system (with some externally specified wind field as a first guess) implies a divergent flow field via the diagnosed vertical velocity. Once we have required this divergent flow to conserve mass as discussed immediately above, we could use this divergent flow to determine the rotational component of the flow by a number of approximate methods. Via thermal wind or some other balance assumption, this would also necessarily imply a computation of a temperature perturbation on top of the horizontally uniform temperature field. In fact, one could think of this procedure as an expansion to first order in a small parameter measuring in some appropriate way the size of the temperature fluctuations (the results in the present paper going only to zeroth order). We have not done this yet, so it would be inappropriate to discuss in any detail how one might best go about it, but we expect it to be tractable. Having both divergent and rotational wind components, one would then have a closed system, though a fair amount of iteration would presumably be necessary in order to assure that the surface fluxes, tropospheric temperature, moisture advection, etc., were part of a single consistent solution.

3) THE NATURE OF THE DYNAMICS

Having a hypothetical closed dynamical system, it is worth speculating briefly on what it could and could not be expected to do. A reasonable inference about this can be made from the results of the present study.

By fixing the temperature (at least at lowest order), we eliminate a large class of phenomena that depend on interactions between the mass and momentum fields. Gravity waves, notably, are eliminated, for precisely the same reason for which they are eliminated in balanced dynamics such as quasigeostrophy: they are assumed infinitely fast. Tropical phenomena such as the Madden–Julian oscillation most likely involve gravity-wave-like dynamics at some fundamental level, and hence will probably not be captured by the system described above. The same may well be true of a wide range of transient tropical phenomena, although purely barotropic transients could certainly exist in this system, and could interact with the thermodynamics through surface fluxes and horizontal moisture advection.

This system is most likely to be useful for describing in a simple way the dynamics of quasi-steady, large-scale tropical circulations. The Walker circulation in particular should be captured quite naturally. The Hadley circulation may seem less likely to be captured cleanly because of the importance of rotational constraints (e.g., Schneider 1977; Schneider and Lindzen 1977; Held and Hou 1980). However, it is possible that the thermodynamics of the Hadley circulation can be captured under our proposed system. That is, rotation can perhaps be thought of as setting the meridional scale of the region around the equator over which essentially nonrotating dynamics holds, and within that region the temperature could be assumed fixed and the vertical velocity diagnosed under our system. Rotational effects such as the transport of angular momentum, important for determining the horizontal winds, could perhaps be computed afterward from the implied divergent flow. Not having done this calculation, we cannot be sure that it would work well, but nothing obviously precludes it to our knowledge.

Given the predisposition of this system toward large-scale, quasi-steady circulations, it seems naturally suited to investigations of tropical climate dynamics. In particular, if surface fluxes can be computed as part of the solution as described above, the system could be cou-
pled to a dynamical ocean in order to simulate the evolution of the SST, rather than fixing the latter as above.

6. Conclusions

A formulation has been proposed for examining the roles of physical processes in determining the climatological distribution of precipitation using a single column model. This formulation, implicitly suggested by earlier investigators (Neelin and Held 1987; Zeng 1998; Zeng and Neelin 1999; Zeng et al. 2000; Raymond 2000a) can be considered a variant of the traditional single column model, or a limiting case of the two-column models used recently by several investigators. In this formulation, only a single column is explicitly modeled, but the free tropospheric temperature is prescribed so that the precipitation and vertical velocity can be true prognostic variables. This formulation has been applied to the Neelin–Zeng QTCM and the Rennó et al. column model, and the implications discussed. The primary conclusions are:

- The two models tested suffered no pathologies when run in this mode, but produced physically plausible results.
- When the QTCM was broken into a disconnected set of column models with a uniform tropospheric temperature, it produced a precipitation distribution broadly similar to that of the original model. Moisture advection by the mean horizontal wind from the control run improved the simulation significantly in dry regions.
- The modified Rennó et al. model was able to simulate the transition from shallow to deep convection with some success in a Lagrangian experiment with time-varying SST. Similar to the QTCM simulations, this simulation illustrated the effect of horizontal dry-air advection in suppressing deep convection in certain regions.
- In both models, the predicted steady-state moisture convergence was found to be a nearly unique function of the surface evaporation when horizontal moisture advection was neglected. This can be explained in terms of the moist static energy and water budgets, but is not necessarily a general or realistic result. It will be true if horizontal variations in the net radiative energy flux into a column are either small or correlated with the evaporation.
- The formulation presented here may also be usefully applied to limited-area models, such as cloud-resolving models.
- Further theoretical development may be possible, leading to a truncated atmospheric dynamics appropriate for the study of tropical climate.

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