Fine Adjustment of Large Scale Air–Sea Energy Flux Parameterizations by Direct Estimates of Ocean Heat Transport

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ABSTRACT

An inverse technique is used to adjust uncertain coefficients and parameters in the bulk formulae of climatological air–sea energy fluxes in order to obtain an agreement of indirect estimates of meridional heat transport with direct estimates in the North Atlantic Ocean. Three oceanographic estimates of ocean heat transport at the equator, at 25°N, and 32°N are compatible with meteorological evidence provided that the uncertainties of both direct and indirect estimates are taken into account. The transport coefficient $C_E$ for estimation of the latent heat flux is the major contributor to the overall uncertainty in estimates of ocean heat transport. The constraint of 1 PW northward transport across 25°N leads to a set of parameterizations for which the parameter adjustments are only less than half as large as the estimated uncertainties. Based on this set of constrained parameterizations monthly climatological fields of the individual fluxes in the North Atlantic Ocean are computed which are consistent with direct transport estimates.

With a larger set of heat transport observations this method will provide a possibility to discriminate between various bulk formulations, and to obtain more accurate estimates of the air–sea energy flux.

1. Introduction

Large scale climatological air–sea interaction fields are needed as boundary values in atmosphere and ocean circulation models. At present, the only practicable method for computing such long term averaged monthly fields is to process historical ship observations from the Voluntary Observing Fleet (VOF) and to deduce air–sea energy fluxes from the basic meteorological observations by means of parameterizations.

The net energy exchange $Q$ at the ocean surface is commonly computed as the sum of net shortwave radiation $Q_S$, net longwave radiation $Q_L$, and the turbulent fluxes of latent and sensible heat, $Q_L$ and $Q_H$:

$$Q = Q_S - Q_L - Q_L - Q_H.$$  (1)

For each flux component of $Q$, independent empirical parameterization formulae have to be applied. Various regional and global compilations which have been published in the last decade (e.g., Bunker 1976; Hastenrath and Lamb 1978; Esbensen and Kushnir 1981; Talley 1984; Hsiung 1986) led to systematic differences in air–sea energy fluxes. These differences are caused mainly by using different parameterization formulae or coefficients. Hence, an additional overall consistency check for those parameterizations is useful. One possibility is to compare indirect and direct estimates of ocean heat transport. Integrating the annual long term average of $Q$ over a whole ocean basin or the World Ocean—and assuming zero heat storage—yields an indirect estimate of the annual mean meridional heat transport in the ocean (e.g., Hastenrath 1980, 1982; Talley 1984; Hsiung 1985). Direct estimates of ocean heat transport across a latitude circle have been computed from hydrographic data of transoceanic sections in different parts of the World Ocean (e.g., Bennett 1978; Hall and Bryden 1982; Toole and Raymer 1985; Rago and Rosby 1987). These results do not exactly accord with estimates obtained by the indirect method but they often agree at least within their error bounds (e.g., see a discussion by Wunsch 1980). For the purpose of large scale oceanographic modeling, those estimates of air–sea energy fluxes are preferable which agree with direct ocean heat transport estimates. If a direct estimate is more accurate than its indirect counterpart, it will be useful to take the direct estimate as an independent constraint to the area integrals of air–sea energy fluxes and, thus, to the parameterization formulae of the individual fluxes.

The concept for establishing parameterizations for large scale air–sea heat fluxes used in this study consists of two steps. First, parameterizations for each component of the net air–sea energy flux are chosen independently based on the air–sea interaction physics involved and considering the best available experimental evidence from the open ocean. This step includes an identification and quantification of the uncertainties which cannot be resolved by those consid-
erations. Second, we adjust uncertain coefficients or parameters by using independent oceanographic constraints (i.e., direct estimates of the meridional oceanic heat transport) to the large scale area average of the net air–sea energy fluxes. These constraints can be introduced to the parameterizations by means of an inverse technique. The purpose of this paper is to describe such a calculation and its results for the North Atlantic Ocean.

2. Energy flux parameterizations and North Atlantic Ocean data

This research is based on a revised version of the climatological Bunker data set for the period 1941–72, which originally was defined on irregularly spaced subdivisions of 10° Marsden Squares (Bunker 1976). From this valuable data set long term averaged monthly North Atlantic Ocean fields of basic meteorological data and derived air–sea interaction quantities have been reanalyzed and interpolated onto a regular 1° grid net (Ismer and Hasse 1985). Calculations described here are based on these 1° grid fields in the North Atlantic Ocean south of 65°N (excluding the Baltic and Mediterranean seas). The parameterizations used differ in some aspects from those applied by Bunker. The following parameterization formulae are used here:

\[ Q_S = Q_0 \text{Tr}(1 - \alpha)(1 - c_n n + 0.0019h) \]  
\[ Q_L = \varepsilon o T_A^4 (0.254 - 0.00495 e_n)(1 - c_n d) + 4o o T_A^3 (T_s - T_A) \]  
\[ Q_L = -C_E p L U (q_A - q_s) \]  
\[ Q_H = -C_H p c_p U (T_A - T_s). \]  

The net shortwave radiation formula (2) is taken from Reed (1977). Here \( Q_0 \) is the solar radiation at the top of the atmosphere and \( h \) is noon solar altitude (in degrees). Originally, (2) was calibrated for daily averages. As the cloud reduction term is linearly dependent on cloud cover \( n \) (in fractions of unity), we used (2) with monthly averages of \( n \) taken from the Bunker data set. Values of \( h \) and \( Q_0 \) were calculated for each day of the month and averaged afterwards to obtain monthly mean values; \( \alpha \) is the sea surface albedo (in fractions of unity). Monthly averages were taken from tables of Payne (1972) with some modifications north of 50°N (see Bunker 1976). \( c_n = 0.62 \) is a constant cloud cover coefficient. The attenuation of the cloud-free atmosphere is incorporated into the atmospheric transmission \( T = 0.7 \) which is held constant for all months and all regions. Bunker (1976) relied on a formula already used by Budyko (1963). We used Reed’s (1977) parameterization instead of Budyko’s because it is more suitable for calculating solar radiation at sea: (2) was calibrated in maritime atmospheres in subtropical latitudes whereas Budyko’s formula was derived mainly from continental measurements. Reed (1977, 1982) (for tropical and subtropical latitudes) and Smith and Dobson (1984) (for midlatitudes) demonstrated that the Budyko formula significantly underestimates net shortwave radiation at sea, especially under conditions of high cloud amount and high solar altitudes. Budyko’s formula underestimates climatological monthly averages of \( Q_S \) in the North Atlantic Ocean by more than 50 W m\(^{-2}\) in tropical regions and in midlatitudes at 50°N in summer (=25 and 33 percent, respectively, of Bunker’s results; see Ismer and Hasse 1987).

The parameterization for net longwave radiation (3), based on a formula published by Efimova (1961), is identical to that used by Bunker (1976) with the cloud cover exponent \( d = 1 \); \( \sigma \) is the Stefan–Boltzmann constant and \( \varepsilon = 0.96 \) is the emissivity of the ocean surface. The cloud cover coefficient \( c \) depends on latitude (see Budyko 1974). Terms \( T_A \), \( T_s \), \( e_n \) and \( n \) are air temperature (K), sea surface temperature (K), water vapour pressure (hPa) and total cloud cover, respectively, again taken from the Bunker data set.

For the latent and sensible heat fluxes the well-known bulk formulae (4) and (5) are used. Here \( \rho \) (kg m\(^{-3}\)) is the density of the air, \( U \) (m s\(^{-1}\)) is scalar wind speed at 10 m height, \( q_A \) and \( q_s \) are specific humidity of the surface layer air and saturation-specific humidity at sea surface temperature, \( c_p \) is the specific heat at constant pressure, and \( L \) (J kg\(^{-1}\)) the heat of vaporization (depending on temperature); \( C_E \) and \( C_H \) are the bulk transfer coefficients. In the parameterization of \( Q_L \) and \( Q_H \) we include three major differences to Bunker:

(i) The transport coefficients \( C_E \) and \( C_H \) are reduced by 13 and 17 percent, respectively, compared to Bunker’s. Bunker’s (1976) used transport coefficients as a function of wind speed and air–sea temperature difference. He carefully estimated values for these coefficients from the literature published up to that time. Since then, however, a number of open ocean measurements of turbulent fluxes have been carried out at various sites of the World Ocean, especially under long fetch conditions. A comparison of a collection of these results (e.g., Hasse et al. 1978; Smith 1980; Large and Pond 1982; see a more complete list in Ismer and Hasse 1987) indicated Bunker’s coefficients to be too high by the average amount mentioned above. For example, our reduced transport coefficient \( C_{EN} \) (×10\(^5\)) increases from 1.11 at a wind speed at 10 m height of \( U_{10} = 5 \) m s\(^{-1}\) to 1.49 at \( U_{10} = 17 \) m s\(^{-1}\) for neutral boundary conditions. For the same wind speed range \( C_{HN} \) (×10\(^3\)) varies from 1.06 to 1.41.

(ii) For decoding of ship’s wind observations the revised scientific Beaufort scale published by Kaufer (1981) is used instead of the official scale of the World Meteorological Organization (WMO). There is widespread agreement that the official WMO Beaufort scale,
called "code 1100," for decoding of ship's wind observations is biased (see WMO 1970). Nevertheless, code 1100 is used in all climatological studies so far known to the authors. Kaufeld (1981) made a careful comparison of ocean weather ship data and VOF observations in the North Atlantic Ocean and found that code 1100 produces too low wind speeds in the range of 1–8 Beaufort and too high ones for wind speeds above 8 Beaufort. We found that the net effect of this bias in individual observations produces too low climatological wind speeds in all climate regions in the North Atlantic Ocean: from about 1 m s⁻¹ in the west wind drift in summer to more than 2 m s⁻¹ in the tropics throughout the year (Isemmer and Hasse 1987). Concerning the calculation of turbulent fluxes that bias is unacceptably large. We used the revised wind statistics for our recalculation of North Atlantic Ocean heat fluxes.

(iii) The vertical density gradient determines the static stability in the surface layer over the sea. Hence, as is done in all experimental work, we consider the surface layer humidity lapse profile over sea for the calculation of the stability dependence of the transport coefficients \( C_E \) and \( C_H \) using the virtual temperature difference \( (T_{aV} - T_{sV}) \) instead of \( (T_a - T_s) \). At least in the tropics and subtropics the influence of water vapor difference between air and sea surface on the density gradient is noticeable compared to the air–sea temperature difference.

In the following we restrict attention to six parameters (see Table 1) which in our opinion contain the major portion of the uncertainties in the bulk formulae that cannot be resolved by experimental evidence. The uncertainties are estimated from the available literature cited below.

Uncertainties of the atmospheric transmission \( T_r \) and the cloud cover coefficient \( c_n \) are based on comparisons of the Reed formula (2) with open ocean measurements (e.g., Reed 1982). We take 5 percent of the atmospheric transmission \( T_r \) and 10 percent of the cloud reduction coefficient \( c_n \) as their respective uncertainties.

The major uncertainty in the net longwave radiation \( Q_l \) is the parameterization of cloud effects. We decided to include the cloud cover exponent \( d \) because of apparently considerable confusion in the available literature concerning the value of \( d \), ranging from \( d = 1.0 \) to \( d = 2.2 \) with identical cloud cover coefficients \( c \) (e.g., see Sellers 1969). Budyko (1974) argued \( d = 2.0 \) to be the appropriate value but used \( d = 1.0 \) for his large scale calculations. We take 0.5 as the uncertainty of \( d \).

Uncertainties of the bulk transfer coefficients are estimated from results of a collection of well-performed open ocean measurements from the last 15 years (see Isemmer and Hasse 1987 for a list of references). Despite strong efforts to improve measurement techniques, the scatter in the results for \( C_E \) and \( C_H \) is considerable. We find an uncertainty of at least 12 percent in \( C_E \) and 24 percent in \( C_H \). The percentage values (see Table 1) are based on the reduced (as compared to Bunker's) coefficients.

Random and systematic errors of ship-based measurements of \( T_A \), \( T_D \) and \( T_S \) have been discussed frequently in the literature (e.g., see Taylor 1985 for a list of references). Concerning the parameterizations of the turbulent fluxes the net effect of these uncertainties on the vertical gradients \( T_A - T_S \) and \( T_D - T_S \) is important. We assume an uncertainty of 0.2 K in climatological averages of both quantities (abbreviated DT in Table 1).

From (2) to (5) and (1) climatological values of \( Q_S \), \( Q_A \), \( Q_L \), \( Q_H \) and \( Q^* \) [henceforth a "*" denotes results from (2) to (5) with unconstrained parameterizations] are computed for each 1° grid point in the North Atlantic Ocean for all calendar months. These fields are averaged to obtain the annual mean (see Fig. 1). From the zonally averaged annual mean of \( Q^* \) the indirect estimate of the meridional oceanic heat transport \( H^*(\phi) \) as a continuous function of latitude \( \phi \) is obtained by integrating \( Q^* \) starting at 65°N and proceeding equatorward. At 65°N a northward transport value \( H(\phi_N) = 0.1 \text{ PW} \) (1 PW = 10¹⁵ W) according to Aagaard and Greisman's (1975) study is used for all calculations presented here. Zonally averaged values of \( Q^* \) (Fig. 4a) vary between 70 Wm⁻² at the equator and −60 Wm⁻² at 35°N. Typical values in the trade wind region are 25 Wm⁻², and between −20 Wm⁻² and −60 Wm⁻² in the west wind drift and subpolar regions. The indirect estimate \( H^*(\phi) \) based on the parameterizations (2) to (5) shows a negligible northward transport at the equator and reaches a maximum of 0.62 PW at 25°N (Fig. 4b).

In Figs. 4, 5 and 6 the rms error of zonally averaged values of \( Q^* \) and the corresponding rms error of \( H^*(\phi) \) resulting from the uncertainties of the six parameters mentioned above is marked by shading (we have not incorporated the uncertainty of the northern boundary value at 65°N). The error in zonal averages of \( Q^* \) varies from 16 Wm⁻² at 55°N to about 27 Wm⁻² at 15°N. The resulting error for \( H^*(\phi) \) reaches 0.51 PW at 25°N (Table 1). South of 20°N the error is larger than the estimate of \( H^*(\phi) \). At the equator the error of \( H^*(\phi) \) embraces the range from −0.8 southward to +0.9 PW northward transport. This indicates that the direction of the cross-equatorial oceanic heat transport cannot clearly be identified from air–sea heat flux fields, although the Atlantic Ocean probably has the strongest interhemispheric heat transport signal among all ocean basins. We have to conclude that, without additional information, air–sea fluxes based on bulk parameterizations are of limited value for calculation of the cross-equatorial heat transport in the ocean.

As a reference we include the annual heat flux estimates of Bunker (1976) for comparison (Fig. 2). His results of annual zonal averages of \( Q \) and \( H(\phi) \) (Fig. 2).
4) are within our error estimates (except between 45°N and 55°N) and, hence indicate no significant differences to our estimates based on (2) to (5). Nevertheless, our estimates $H^*(\phi)$ are systematically smaller than Bunker's by about 0.2 PW. The main reason are our higher results for solar radiation in summer in the west wind drift, which overcompensate the enhancement of latent heat fluxes due to our higher (as compared to Bunker's) windspeed estimates. In this region annual zonal differences are about 30 Wm$^{-2}$ while monthly
zona larly averaged differences of $Q$ are larger and attain nearly 50 Wm$^{-2}$ in summer (not shown here). Between 10° and 35°N our estimates $Q^*$ are smaller by 10 to 15 Wm$^{-2}$ in the annual zonal mean. In this region the effect of higher latent heat fluxes due to the revised wind statistics dominates over the increase of solar radiation. In summer, the effects of higher insolation and higher evaporation nearly cancel in tropical and subtropical latitudes. We emphasize that, using the parameterizations (2) to (5), the resulting North Atlantic Ocean fields yield significant differences compared to Bunker’s concerning horizontal gradients and amplitudes of annual cycles (see Isemer and Hasse 1987, for details).

To prevent confusion concerning the Bunker data set we add the following note: In addition to his results published in Bunker (1976) he calculated time series of monthly averages for 10° Marsden Squares for the period 1948–72 using a different parameterization than (3) for $Q_L$ (Bunker and Goldsmith 1979). Some results of this compilation unfortunately remained in the files until quite recently (Bunker 1988). Hence, there are two versions of the “Bunker data set.” We relied on (3) because a comparison of the two $Q_L$-parameterizations with results of model calculations performed by Fung et al. (1984) leads to the conclusion that (3) incorporates smaller systematic errors in maritime atmospheres than the parameterization used by Bunker (1988). The latter results in a generally stronger energy loss of the North Atlantic Ocean of order 8 Wm$^{-2}$ leading to a higher northward cross-equatorial heat transport of more than 0.3 PW in the Atlantic Ocean. Compared to these results our estimate $\bar{H}^*(0^\circ)$ is even smaller by an amount of about 0.5 PW.

Let us now compare the results for $H^* (\phi)$ with some recent direct observations $\bar{H} (\phi)$ (see Fig. 4). The error margin of Bryden and Hall’s (1980) direct estimate at 25°N, calculated from a hydrographic section performed in 1957, embraces the range between 0.8 and 1.4 PW with a mean value of 1.1 PW northward transport. Hall and Bryden (1982) revised this value to 1.22 ± 0.3 PW. This estimate is believed to be of high reliability because it was confirmed by a later hydrographic section performed in 1981 along this latitude circle (Roemmich and Wunsch 1985). Roemmich (1980) applied an inverse technique to three hydrographic sections in the subtropical North Atlantic Ocean and obtained estimates for $\bar{H}(25^\circN)$ ranging between 1.0 and 1.4 PW. Recently, Rago and Rossby (1987) obtained a direct estimate $\bar{H}(32^\circN) = 1.38 \pm 0.19$ PW. Their approach is similar to that of Hall and Bryden (1982). Rago and Rossby concluded that, though their estimate appears to be high, the difference to other results is not significant. Following the results of North Atlantic Ocean inverse modeling performed by Wunsch (1984) the transequatorial heat transport should be between 0.2 and 1.0 PW.

We conclude that our estimates $H^* (\phi)$, based solely on meteorological evidence using (2) to (5), are smaller than some of the best available direct estimates $\bar{H} (\phi)$. Nevertheless, the ranges of uncertainties, at least at the equator and at 25°N, overlap. This indicates that there
three constraints at different latitudes. We introduce the constraints in two different ways: with and without consideration of their uncertainties. In the next section we outline the technique used for this purpose.

3. Principles of the inverse calculation

The technique used in this paper is based on linear, discrete inverse theory which is readily available in other publications. For a review see, e.g., Menke (1984). We intend to give a description of our special application to the North Atlantic Ocean data. We describe the technique for the special case of one constraint in detail and, finally, extend the result to the use of several constraints.

a. One constraint

The components of the long term annual average net energy exchange in (1) depend on a number of parameters and coefficients which occur in the bulk parameterizations. From these one may choose $m$ coefficients and parameters, $p_1 \ldots p_m$, to be adjusted in the inverse calculation. Hence, $Q$ can be written as

$$Q = g(\lambda, \phi, p_1, \ldots, p_m)$$

where $\lambda$ and $\phi$ are longitude and latitude, respectively. The indirect estimate of the oceanic meridional heat transport $H(\phi)$ across a latitude $\phi$ in an ocean basin is then given by

$$H(\phi) = \int_{\phi_N}^{\phi_S} \int Q d\lambda + H(\phi_N)$$

$$= f(\phi, p_1, \ldots, p_m)$$

where $\phi_N$ is a northern latitude where the heat transport $H(\phi_N)$ vanishes or is known from other sources. Based on independent meteorological considerations (see discussion in the previous chapter) we obtained a priori estimates $p_1^*, \ldots, p_m^*$ for the parameters leading to an indirect heat transport estimate $H^*(\phi) = f(\phi, p_1^*, \ldots, p_m^*)$.

On the other hand we know independent direct oceanographic estimates $\hat{H}(\phi)$ which generally differ from $H^*(\phi)$. Let us consider one specific constraint $\hat{H}(\phi)$ (e.g., the ocean heat transport across 25$^\circ$N). Hence, we seek for a combination of parameter values $\hat{p}_1, \ldots, \hat{p}_m$, which satisfies

$$\hat{H}(\phi) - f(\phi, \hat{p}_1, \ldots, \hat{p}_m) = 0.$$ (8)

We restrict the variation to the $m = 6$ meteorological parameters already discussed in section 2. The determination of the $\hat{p}_i$ constitutes an underdetermined inverse problem, which generally is nonlinear as the function in (8) is nonlinear in the $p_i$. For sufficiently small parameter changes, however, (8) can be linearized according to

$$f(\phi, \hat{p}_1, \ldots, \hat{p}_m) = H^*(\phi) + \sum A_i(\hat{p}_i - p_i^*)$$.


Here, the \( A_i = \partial f(\phi, p_1^*, \ldots, p_m^*) / \partial p_i \) are the sensitivities of the meridional heat flux across latitude \( \phi \) to changes in the corresponding parameters \( p_i \).

From the meteorological viewpoint the parameter values \( p_i^* \) represent the most plausible solutions based on recent experimental evidence from the open ocean. If changes forced by oceanographic evidence are necessary, they should be as small as possible. We define

\[
x_i = \hat{p}_i - p_i^*, \quad i = 1, \ldots, m
\]

(10)
as the necessary changes (or fine adjustments) of the values of the uncertain parameters \( p_i^* \) to satisfy the constraint (8). The \( x_i \) are elements of the solution vector of the inverse problem. An appropriate least-squares condition is

\[
\sum_i \left( \frac{x_i^2}{e_i^2} \right) = \text{minimum}
\]

(11)
with the linear constraint

\[
\sum_i A_{ij} x_i = \hat{H}_i - H_i^*.
\]

(12)
The \( e_i \) are the estimated uncertainties of the \( p_i^* \) which cannot be resolved by meteorological evidence. The normalization (or weighting) by the \( e_i \) in (11) implies that the adjustments of those parameters with higher uncertainty will be larger than the adjustments of those parameters which are known with a higher degree of accuracy. Formally (11) follows from a Maximum Likelihood principle by assuming that the estimates \( p_i^* \) are independent and Gaussian random variables with standard deviations \( e_i \). (11) and (12) constitute a constrained least-squares problem which can be solved using the method of Lagrange multipliers (e.g., Menke 1984). It has the solution

\[
x_i = \{ (\hat{H}(\phi) - H^*(\phi)) e_i^2 A_i \} / \sigma_i^2
\]

(13)
where \( \sigma_i^2 = \sum_j e_j^2 A_j / \sigma_i^2 \).

Using (11) implies that the direct estimate \( \hat{H}(\phi) \) would be perfectly accurate. In this case we denote \( \hat{H}(\phi) \) as an exact constraint. If an error \( \delta \) of the direct estimate is taken into account, the appropriate least-squares condition instead of (11) is

\[
\sum_i \left( \frac{x_i^2}{e_i^2} \right) + ((\hat{H}(\phi) - f(\phi, \hat{p}_1, \ldots, \hat{p}_m))^2 / \hat{\sigma}^2 = \text{minimum}
\]

(11')
with the solution

\[
x_i = \{ (\hat{H}(\phi) - H^*(\phi)) e_i^2 A_i \} / (\sigma_i^2 + \hat{\sigma}^2).
\]

(13')
In this case we will denote the constraint as inexact. Comparison with (13) shows that both solutions coincide if the direct estimate is sufficiently more accurate than the indirect one.

Whether the solution (13), or (13'), is considered to be acceptable, (i.e., compatible with the meteoro-
logical evidence) or not, depends on the magnitude of the \( x_i \) relative to the uncertainties \( e_i \). If the estimates for the parameters \( p_i^* \) were normally distributed with variance \( e_i^2 \), a \( \chi^2 \)-test is appropriate to test the compatibility at a certain confidence level. As we have insufficient information about the distribution and our estimates for the \( e_i \) are rather approximate, however, a determination of confidence levels is problematic. As a more simple consistency check, we will accept a solution if

\[
|x_i| < e_i,
\]

(14)
for all \( i \).

The \( A_i \) have been calculated numerically from the North Atlantic Ocean energy flux fields using

\[
A_i = \{ f(\phi, p_i^* + e_i) - f(\phi, p_i^* - e_i) \} / 2 e_i.
\]

(15)
In the parameter range discussed here (see Table 1), \( f(\phi, p_i) \) was found to depend linearly on the \( p_i \), to a very good approximation. The \( e_i \) are estimated based on the experimental evidence concerning the energy flux parameterizations as described in the previous chapter.

b. Multiple constraints

The extension to \( n > 1 \) direct heat transport constraints with \( m \) parameters is straightforward (Menke 1984). Let \( \mathbf{x} \) be a vector of dimension \( m \) containing the unknowns \( x_i \), \( \mathbf{h} \) a vector of dimension \( n \) containing the constraints \( (\hat{H}_j - H_j^*) \), \( j = 1, \ldots, n \), and \( \mathbf{A} \) the "model" matrix of dimension \( n \times m \) containing the \( A_{ji} \) as defined in (9). Then (9) may be rewritten as

\[
A \mathbf{x} = \mathbf{h}.
\]

(9*)
The problem is again solved by minimization of a combination of the solution vector and the prediction error. Corresponding to (11') the minimum condition may be written

\[
\sum_i \left( \frac{x_i^2}{e_i^2} \right) + \sum_j ((\hat{H}(\phi_j) - f(\phi_j, \hat{p}_1, \ldots, \hat{p}_m))^2 / \hat{\sigma}_j^2) = \text{minimum}.
\]

(11*)
In all cases considered in this paper the problem is underdetermined \( (n < m) \). The solution is

\[
\mathbf{x} = \mathbf{W}_e^{-1} \mathbf{A}^T [ (\mathbf{A} \mathbf{W}_e^{-1} \mathbf{A}^T) + \mathbf{W}_e^{-1} ]^{-1} \mathbf{h}.
\]

(13*)
where \( \mathbf{T} \) denotes the transpose. (13*) is the equivalent matrix notation of (13); \( \mathbf{W}_e \) is a weighting square matrix of dimension \( m \times m \) containing the reciprocals of the uncertainties \( e_i \) squared on its diagonal; and \( \mathbf{W}_e \) is the weighting matrix of the prediction errors and contains the \( \hat{\sigma}_j^{-2} \) on its diagonal, where \( \hat{\sigma}_j \) are the uncertainties of the direct heat transport estimates. From (13*) the equivalent form of (13) is obtained simply by omitting the weighting matrix \( \mathbf{W}_e \).
TABLE 1. List of parameters $p$ (see section 3) and solutions $x$ of the inverse calculation for net air–sea heat fluxes in the North Atlantic Ocean between 25°N and 65°N, $Q(25)$. The ocean heat transport $H(25)$ across 25°N is constrained to 1.0 PW (1 PW = 10¹² W) northward transport. The $e$ are the uncertainties of the parameters $p$, and $e\cdot A$ denote the response of $Q(25)$ to changes of the parameters $p$ by an amount of $e$. The $dQ(25)$ are the changes of $Q(25)$ due to the solutions $x$ of the inverse calculation and is expressed in Wm⁻² as area averaged value between 25°N and 65°N. In PW as change in $H(25)$ and in percent as percentage contribution to the total constrained change in $Q(25)$ ($= -15.9$ Wm⁻²) or $H(25)$ ($= +0.37$ PW). The total rms errors based on the $e\cdot A$ values of $Q(25)$ and $H(25)$ are 21.5 Wm⁻² and 0.51 PW, respectively.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$e$</th>
<th>$e\cdot A$ (Wm⁻²)</th>
<th>$x$ (PW)</th>
<th>$dQ(25)$ (Wm⁻²)</th>
</tr>
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<tr>
<td>$Tr$</td>
<td>0.035</td>
<td>7.8</td>
<td>-0.18</td>
<td>-0.010</td>
</tr>
<tr>
<td>$c_e$</td>
<td>0.06</td>
<td>-7.8</td>
<td>-0.18</td>
<td>0.016</td>
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<tr>
<td>$d$</td>
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<td>-5.5</td>
<td>0.13</td>
<td>0.1</td>
</tr>
<tr>
<td>$C_E$</td>
<td>12%</td>
<td>-13.5</td>
<td>-0.32</td>
<td>5.6‰</td>
</tr>
<tr>
<td>$C_H$</td>
<td>24%</td>
<td>-5.3</td>
<td>0.13</td>
<td>4.4‰</td>
</tr>
<tr>
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<td>-0.23</td>
<td>-0.07°C</td>
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</table>

4. Results and discussion

Results with the constraint $H(25°N) = 1$ PW are given in Table 1. The changes from (or adjustments to) the previously revised parameterizations do not exceed the margins of the estimated uncertainties; between 18 percent and 47 percent of the error margins are used for the adjustments. Therefore we accept the results of this inverse calculation because the resulting "constrained" parameterizations do not violate the meteorological evidence which have led us to use (2) to (5). Monthly fields of the individual fluxes over the North Atlantic Ocean have been computed using these constrained parameterizations. Shortwave radiation $Q_S$ is calculated using (2) with a cloud cover coefficient $c_n = 0.636$ and the atmospheric transmission $Tr$ reduced from 0.70 to 0.69 corresponding to a change of the incoming solar radiation for cloudless sky by 1.4 percent. $Q_I$ is parameterized by (3) with the cloud cover exponent $d = 1.1$. The transfer coefficients $C_E$ and $C_H$ are increased by 5.6 and 4.4 percent, compared to the coefficients based on recent open ocean measurements. The resulting transfer coefficients $C_E$ are tabulated as a function of wind speed and stability in Table 2. From this table the transfer coefficients for sensible heat $C_H$ may be obtained by $C_H = 0.94 \cdot C_E$. The high sensitivity of average air–sea heat fluxes to small variations especially in $C_E$ is remarkable. A 10 percent bias in $C_E$ leads to a systematic error of 0.27 PW in $H(25°N)$ and 0.55 PW in the cross-equatorial ocean heat transport [Note, that a small 10 Wm⁻² bias in $Q$ results in a systematic error of 0.24 PW in $H(25°N)$ and 0.42 PW in the cross-equatorial value of $H$]. The choice of the value of $C_E$ strongly influences the air–sea heat balance, $C_E$ is probably the most critical parameter varied in the inverse calculation.

The correction of mean ship-based temperature measurements enters into the computation of air–sea interaction in two ways. First, in the bulk aerodynamic equations (4) and (5), the correction of $-0.07°C$ is added to the air–sea temperature difference and the dewpoint–sea surface temperature difference, respec-

TABLE 2. The adjusted transport coefficient $C_E (\times 10^3)$ for the calculation of the latent heat flux as a function of wind speed, $U_{10}$, and virtual air–sea temperature difference, both at 10 m height. The adjustment is obtained by considering the exact constraint $H(25°N) = 1.0$ PW. The adjusted transport coefficient $C_H (\times 10^3)$ may be obtained from this table by $C_H = 0.94 \cdot C_E$.

<table>
<thead>
<tr>
<th>$U_{10}$ (m s⁻¹)</th>
<th>$T_{Sv}$</th>
<th>$T_{AV}$</th>
<th>$T_{AV} - T_{Sv}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&gt;5.0°C</td>
<td>4.9°C to 0.9°C</td>
<td>0.9°C to 0.2°C</td>
</tr>
<tr>
<td>&lt;3</td>
<td>0.06</td>
<td>0.26</td>
<td>0.63</td>
</tr>
<tr>
<td>3–6</td>
<td>0.19</td>
<td>0.58</td>
<td>0.97</td>
</tr>
<tr>
<td>6–9</td>
<td>0.60</td>
<td>1.02</td>
<td>1.18</td>
</tr>
<tr>
<td>9–12</td>
<td>0.92</td>
<td>1.18</td>
<td>1.29</td>
</tr>
<tr>
<td>12–15</td>
<td>1.21</td>
<td>1.37</td>
<td>1.40</td>
</tr>
<tr>
<td>15–20</td>
<td>1.38</td>
<td>1.46</td>
<td>1.52</td>
</tr>
<tr>
<td>20–25</td>
<td>1.51</td>
<td>1.56</td>
<td>1.59</td>
</tr>
<tr>
<td>25–30</td>
<td>1.57</td>
<td>1.60</td>
<td>1.61</td>
</tr>
<tr>
<td>&gt;30</td>
<td>1.62</td>
<td>1.62</td>
<td>1.62</td>
</tr>
</tbody>
</table>
tively, and thus directly increases the turbulent fluxes. Second, the correction of $-0.07^\circ$C to $T_a - T_s$ makes the density gradient slightly more unstable and, in effect, increase the bulk transfer coefficients, which are taken as a function of stability. Note, that a 0.2$^\circ$C systematic change in $dT$ causes a bias of nearly 10 Wm$^{-2}$ in the average air–sea heat fluxes north of 25$^\circ$N (Table 1).

The results of the inverse calculation depend critically on the estimated uncertainties. Assume an uncertainty in $C_E$ of 20 percent (which probably would be accepted by most meteorologists) and leave the other $e_i$ unchanged. In this case the contribution due to the adjustment of $C_E$ to the total constrained change of $Q$ increases to 64 percent. If one were to consider $C_E$ exclusively in the inverse calculation, a hypothetical correction of $C_E$ by 14 percent only is required to constrain a value of $H(25^\circ N) = 1$ PW. This again demonstrates that one of the main reasons for the large uncertainties and discrepancies in air–sea heat fluxes and deduced ocean heat transport is the uncertainty of the transport coefficient $C_E$. Due to the integration procedure indirect estimates of ocean heat transport, especially in the equatorial and tropical region, are very sensitive to even small systematic errors in the parameterization formulae of the air–sea energy exchange.

We have mapped three different versions of annual net air–sea heat exchange (Figs. 1, 2 and 3) and the corresponding curves of zonal averages of $Q$ and of meridional heat transport (Figs. 4 and 6). All three maps of net air–sea heat flux $Q$ show the well-known east–west asymmetry of net heat loss and gain. The most obvious difference seen in the maps is the position of the zero heat flux line. A prominent and noteworthy feature of the constrained parameterizations (Fig. 3) is the wavy pattern of the annual zero heat flux line in the tropics and subtropics, suggesting a large region of small oceanic heat loss between 5$^\circ$ and 20$^\circ$N, and a region of small heat gain between 20$^\circ$ and 30$^\circ$N. It is interesting to note that many ocean circulation models (which are usually forced by a prescribed surface temperature) show a similar pattern which can be attributed to the transport of heat by wind-induced Ekman currents (e.g., Sarmiento 1986). Though physically plausible, however, this feature is not significant because in those areas heat loss and gain nearly balance in the annual mean within 25 Wm$^{-2}$, and the uncertainty of $Q$ usually exceeds the value of $Q$. Hence, the sign of the annual net air–sea heat flux line and, consequently, the position of the annual zero heat flux line are poorly determined in these regions.

The constrained parameterizations lead to a northward heat transport of 0.76 PW at the equator, to the constrained value of 1 PW at 25$^\circ$N and to 0.91 PW at 32$^\circ$N, respectively (Fig. 4b). In contrast to the results of the unconstrained parameterizations and Bunker’s version the indirectly estimated values of $H$ do not vary much with latitude south of about 35$^\circ$N. North of 35$^\circ$N the ocean heat transport resulting from the constrained parameterizations is nearly identical with Bunker’s results. We emphasize, however, that our resulting monthly North Atlantic Ocean fields of the individual fluxes yield significant differences compared to Bunker’s concerning horizontal gradients and annual cycles of heat fluxes (not shown here). Differences in the net heat fluxes between Bunker’s estimates and those from the unconstrained parameterizations are only slightly modified by this inverse calculation. The main features originate from the elimination of biases discussed in section 2.

The value of $H(25^\circ N)$ constrains the air–sea heat flux parameterizations north of 25$^\circ$N. Transport values in the tropics and at the equator calculated with this set of constrained parameterizations may be regarded as a somewhat independent check of these results. It is satisfactory to note that the calculated cross-equatorial heat transport, although not used as a constraint in this calculation, is in better agreement with, e.g., results of inverse modeling presented by Wunsch (1984).

Monthly maps of the individual fluxes have been published separately in an air–sea interaction atlas of the North Atlantic Ocean (Isener and Hasse 1987). Above, we have given a number of arguments which led us to use this particular set of parameterizations. The special constraint used should be accepted by most oceanographers, although it is not a specific direct result but represents a synthesis of different direct and model results.

In addition to the result presented we have carried out a number of different calculations using exact constraints for the heat transport at 0$^\circ$, 25$^\circ$ and 32$^\circ$N, respectively. Constraints of either $H(0^\circ) = 0.6$ PW and $H(25^\circ N) = 1.22$ PW lead to insignificant changes in the parameters $p_i$ compared to the result presented above, while using $H(32^\circ) = 1.38$ PW as a single constraint the necessary adjustment of $C_E$ would reach 122 percent of the error margin $e$. The corresponding set of constrained parameterizations lead to $H(25^\circ N)$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$x$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(%)</td>
<td>(Wm$^{-2}$)</td>
</tr>
<tr>
<td>$T_T$</td>
<td>-0.077</td>
<td>219</td>
</tr>
<tr>
<td>$C_R$</td>
<td>0.669</td>
<td>1115</td>
</tr>
<tr>
<td>$d$</td>
<td>-2.751</td>
<td>550</td>
</tr>
<tr>
<td>$C_E$</td>
<td>-97.9%</td>
<td>817</td>
</tr>
<tr>
<td>$C_H$</td>
<td>80.1%</td>
<td>334</td>
</tr>
<tr>
<td>$D_T$</td>
<td>-1.58$^\circ$C</td>
<td>791</td>
</tr>
</tbody>
</table>

Table 3. As Table 1, but showing results of a calculation considering three exact constraints: $H(0^\circ) = 0.6$ PW, $H(25^\circ N) = 1.22$ PW and $H(32^\circ N) = 1.38$ PW. This solution is physically unreasonable.
= 1.66 PW and $H(0^\circ) = 1.98$ PW. The annual map of $Q$ (not shown here) indicates significant net heat losses in nearly all regions of the tropical and subtropical North Atlantic Ocean. Although a $X^2$-test does not indicate significant changes of the $p_i$ at the 95% level, we note unsatisfactory discrepancies in the resulting heat transports at the equator and at 25°N compared to direct results at these latitudes. Hence, acceptance of this latter solution would be at least problematic.

Forcing the solution to satisfy all three constraints simultaneously (and exactly) results in highly significant parameter changes. Although the resulting heat transport curve (Fig. 5) might still be considered to be acceptable, the corresponding plot of zonal annual averages of $Q$ indicates that this calculation leads to physically unrealistic solutions for the $p_i$ (see Table 3). This solution should be rejected. Hence, this ensemble of three exact constraints is clearly incompatible with the meteorological bulk formulae as defined by (2) to (5). To the extent that these formulae are a correct representation for the true air–sea fluxes one might conclude that this solution with three exact constraints violates meteorological evidence. It should be born in mind, however, that the physical basis for the parameterizations (2) to (5) is somewhat crucial in several aspects. Specifically, the assumption that some of the parameters (e.g., atmospheric transmission or the cloud cover coefficient) and also the adjustments to the chosen parameters are independent of season and region is certainly a questionable limitation. A latitude and time dependent variation of some of the parameters would physically be reasonable. Hence, it is well possible that the incompatibility reflects physical shortcomings in the bulk formulae (2) to (5).

In all calculations presented up to now we assumed the oceanographic estimate to be exact, so that there is an exact agreement between $\bar{H}(\phi)$ and the heat transport value $H(\phi)$ obtained from the constrained parameterizations. Constraints of this kind are somewhat unrealistic because direct estimates are by no means known exactly. Hence, we now solve the inverse problem taking into account the uncertainties of the direct estimates. We consider the inexact constraints at $0^\circ$ ($0.6 \pm 0.4$ PW), 25°N ($1.22 \pm 0.3$ PW) and at 32°N ($1.38 \pm 0.19$ PW) simultaneously. The solution (Table 4) is considered acceptable as the necessary adjustments of each single parameter is smaller than the estimated uncertainty. They are not significantly different from the solutions with one constraint $\bar{H}(25^\circ\mathrm{N}) = 1.0$ PW. As multiple constraints are considered the ranking of the parameters due to both the magnitude of the adjustment and their contribution to the total change, respectively, is changed (compare Tables 1 and 4). For example, the adjustment of the sensible heat flux coefficient is more than three times larger, whereas the solution for atmospheric transmission is about half as large. The necessary change of $C_F$ is nearly identical. The total change in $Q$, however, is at the lower limit of the rms uncertainty range (see Fig. 6). Consequently,
the resulting heat transport curve is slightly outside the uncertainty range, leading to values of 1.05 PW, 1.17 PW and 1.04 PW at the equator, at 25° and 32°N, respectively. Hence, from an oceanographic viewpoint, this solution is only marginally acceptable, because the differences of ocean heat transport at 0° and 32°N are considerable. (The \( \chi^2 \)-test for these solutions indicates significant differences at the 89 percent level.)

A number of additional calculations have been performed with different combinations of the three constraints mentioned, both exact and inexact. Forcing the solutions to exactly fulfill one exact constraint \( \bar{H}(25^\circ\text{N}) = 1.22 \text{ PW} \) (the result of Hall and Bryden 1982) leads to nearly the same estimates of zonally averaged heat exchange and ocean heat transport as the solution with three inexact constraints (Fig. 6). Both solutions are not significantly different from the a priori estimates (the unconstrained parameterizations), although the single parameter adjustments are distinct due to the different constraints. (For example, considering \( \bar{H}(25^\circ\text{N}) = 1.22 \text{ PW} \) the adjustment of atmospheric transmission is twice as large whereas the change of the sensible heat flux coefficient is only half as large as in the case with three inexact constraints.)

In all cases considering \( \bar{H}(32^\circ\text{N}) \) both as exact and inexact constraint, we note a higher level of incompatibility between oceanographic and meteorological estimates of ocean heat transport. Having in mind the limits of the bulk formulae mentioned above, however, we cannot consider this estimate as clearly unacceptable. But we state that this direct estimate is at least at the upper limit of the compatibility range.

5. Summary and conclusions

Estimates of ocean heat transport based on bulk parameterization formulae for the large scale air–sea energy fluxes are very sensitive to uncertainties in the parameterizations and are generally less reliable than direct oceanographic estimates. Among six parameters which contribute mainly to the significance of air–sea energy fluxes the transport coefficient \( C_E \) of latent heat flux is especially crucial: In spite of a large amount of experimental data its uncertainty is unacceptably high for calculations of climatological heat transports. It is discouraging that even in the Atlantic Ocean the direction of the cross-equatorial heat transport cannot be determined from air–sea heat flux fields alone.

In this study we have shown that small but systematic changes can be applied to the air–sea energy flux parameters so that the resulting transports agree with oceanographic estimates without violating meteorological evidence. Using a standard inverse method, we have calculated adjusted parameter values which satisfy various direct heat transport estimates in the North Atlantic Ocean. The intention of this study is not to fix one specific result of meridional ocean heat transport from air–sea heat flux fields but to offer a method for testing meteorological bulk formulae and independent heat transport estimates upon their compatibility. We emphasize that the inverse method recommended offers a tool for a controlled adjustment of parameters.

In order to satisfy the constraint \( \bar{H}(25^\circ\text{N}) = 1.0 \text{ PW} \) exactly, an adjustment of all six parameters within less than half of the uncertainty range is sufficient. This particular parameter set predicts a clearly northward heat transport of 0.8 PW across the equator (and 0.9 PW across 32°N) into the North Atlantic Ocean. The fact that the transports in the tropical and equatorial North Atlantic Ocean, though not constrained directly, are consistent with oceanographic results is satisfactory. Maps for the individual fluxes calculated with this set of parameterizations have been published elsewhere (Isem and Hasse 1987). As different climate regimes are considered in this regional study one can hope to obtain a consistent description also for other parts of the World Ocean. These fields are potentially suitable as boundary conditions for circulation models as they
induce a climatological heat transport in accordance with observations.

We have demonstrated that three widely used direct oceanographic estimates of ocean heat transport [Wunsch 1984 (at the equator); Hall and Bryden 1982 (at 25°N); Rago and Rosby 1987 (at 32°N)] are compatible with meteorological evidence provided that the uncertainties of direct and indirect estimates are properly taken into account. The result of Rago and Rosby for 32°N is, however, at the upper limit of the compatibility range. The constrained solutions of this inverse calculation lead to ocean heat transport values of 1.05 PW, 1.17 PW and 1.04 PW at the equator, at 25° and at 32°N, respectively. If one would ignore the error bars of direct estimates and insist that the adjusted solution matches all three estimates exactly, a physically unrealistic set of parameters is obtained. This result shows the potential of the inverse method employed here: with a larger set of direct estimates at hand it would force either a revision of oceanic heat transport estimates, or of the parameterizations (2) to (5), or both.

In order to identify significant discrepancies between meteorological and oceanographic evidence more direct estimates of ocean heat transport are needed. The World Ocean Circulation Experiment (WOCE) is designed to yield a number of these direct observations in the World Ocean. Direct estimates are especially needed in tropical and subtropical latitudes and should include a careful analysis of their uncertainties. With more reliable constraints at hand one should extend such calculations to other parts of the World Ocean. Time- and latitude-dependent variations of the coefficients may need to be included, leading to a stronger physical basis of meteorological bulk parameterizations.

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