Impact of Model Relative Accuracy in Framework of Rescaling Observations in Hydrological Data Assimilation Studies

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ABSTRACT

Soil moisture datasets vary greatly with respect to their time series variability and signal-to-noise characteristics. Minimizing differences in signal variances is particularly important in data assimilation to optimize the accuracy of the analysis obtained after merging model and observation datasets. Strategies that reduce these differences are typically based on rescaling the observation time series to match the model. As a result, the impact of the relative accuracy of the model reference dataset is often neglected. In this study, the impacts of the relative accuracies of model- and observation-based soil moisture time series—for seasonal and subseasonal (anomaly) components, respectively—on optimal model–observation integration are investigated. Experiments are performed using both well-controlled synthetic and real data test beds. Investigated experiments are based on rescaling observations to a model using strategies with decreasing aggressiveness: 1) using the seasonality of the model directly while matching the variance of the observed anomaly component, 2) rescaling the seasonality and the anomaly components separately, and 3) rescaling the entire time series as one piece or for each monthly climatology. All experiments use a simple antecedent precipitation index model and assimilate observations via a Kalman filtering approach. Synthetic and real data assimilation results demonstrate that rescaling observations more aggressively to the model is favorable when the model is more skillful than observations; however, rescaling observations more aggressively to the model can degrade the Kalman filter analysis if observations are relatively more accurate.

1. Introduction

In the hydrological sciences, observations are commonly integrated into models to reduce the random errors of models using Kalman filtering–based methods. These methods work well when model and observations have consistent signals in some statistical sense. However, systematic differences in signal intensity can interfere with the data assimilation (DA) process and degrade the accuracy of the data assimilation analysis (Yilmaz and Crow 2013). Such systematic model–observation inconsistencies are known to be particularly acute in soil moisture (Reichle et al. 2004). As a remedy, rescaling strategies (Reichle and Koster 2004; Yilmaz and Crow 2013; Su and Ryu 2015) are commonly applied to reduce systematic signal differences before observations are merged into hydrologic model simulations. These rescaling procedures lie at the heart of land data assimilation systems that are increasingly being employed within a wide range of science applications, including terrestrial water balance analysis (Zaitchik et al. 2008), climate trend detection (Carton and Giese 2008), and land–atmosphere coupling experiments (Koster et al. 2006).

When systematic differences between model and observations exist independently at multiple time scales, for example, at seasonal and subseasonal scales...
separately (Su et al. 2014), the success of any rescaling method [e.g., regression (REG)-, variance (VAR)-, or triple collocation analysis (TCA)-based methods (Yilmaz and Crow 2013); detailed discussion about these methods are given below under section 2a] varies, in part, on the degree to which observations are rescaled to a particular model. For example, rescaling both the seasonality and the anomaly (higher frequency) components of a particular observation to those of a model is a more aggressive rescaling strategy than rescaling only its anomaly components (and allowing the observation time series to retain its own seasonality). Likewise, rescaling observations using nonstationary rescaling coefficients (e.g., a separate rescaling coefficient for each month of the year) is a more aggressive rescaling strategy than using a single coefficient for the entire time series. The optimality of various rescaling methods has been investigated by earlier studies (Yilmaz and Crow 2013). However, the impacts of these variations in rescaling aggressiveness have received little attention and have not been extensively examined in the land data assimilation literature.

For some hydrological variables, large systematic differences between models and observations (Reichle and Koster 2004) must be addressed via preprocessing before the assimilation of observations. However, rescaling more accurate [i.e., higher signal-to-noise ratio (SNR) of variance] datasets to less accurate [i.e., lower SNR of variance] datasets poses the risk of amplifying sampling or rescaling errors and/or diminishing the true signal via overfitting. Hence, the efficacy of more aggressive rescaling strategies is closely tied to the relative accuracy of the model reference dataset versus the rescaled observations. On the other hand, current studies only select a particular rescaling method (e.g., REG, VAR, or TCA) and often neglect to investigate the impact of model inaccuracy on the optimality of any particular rescaling method. Accordingly, studies neglecting this investigation carry the risk of producing suboptimal analyses with increased errors through selection of an inappropriate rescaling strategy.

Here, we investigate the impact of this neglected case in hydrological data assimilation studies by analyzing the impact of model accuracy (relative to observations) on the performance of various rescaling strategies. Our goal here is quantifying the impact of systematic and random errors in hydrological models on data assimilation performance and exploring whether or not alternative rescaling strategies can alleviate the adverse impact of model error on analysis accuracy. The analysis is based on both well-controlled synthetic simulations using the antecedent precipitation index (API) model and real data assimilation experiments assimilating land parameter retrieval model (LPRM) products into an API model. For this study, we select the VAR rescaling approach (Crow and Ryu 2009; Draper et al. 2009) and implement a number of different rescaling strategies that rescale observations to models with various degrees of aggressiveness.

2. Methodology

a. Rescaling strategies

Many different rescaling methods have been utilized to alleviate inconsistencies between the signal components of datasets. Among them, REG, VAR, or TCA can be implemented by considering the most general linear relation between a reference dataset \( x \) and the dataset to be rescaled \( y \) in the form

\[
y^* = (y - \mu_y) c_y + \mu_x, \tag{1}
\]

where \( y^* \) is the rescaled \( y \), \( \mu_x \) and \( \mu_y \) are time averages of \( x \) and \( y \), and \( c_y \) is a scalar representing multiplicative bias between \( x \) and \( y \). Here, \( c_y \) is found using REG, VAR, or TCA linear methods as

\[
c^R_y = \rho_{xy} \sigma_x / \sigma_y, \tag{2}
\]

\[
c^V_y = \sigma_x / \sigma_y, \quad \text{and} \tag{3}
\]

\[
c^T_y = \frac{\bar{x} \bar{z}}{\bar{y} \bar{z}}, \tag{4}
\]

where \( z \) is a third product similar to products \( x \) and \( y \), \( x' \), \( y' \), and \( z' \) are temporal anomalies (obtained via the subtraction of a fixed climatology) of \( x \), \( y \), and \( z \), respectively; the overbar refers to time average; \( c^R_y \), \( c^V_y \), and \( c^T_y \) are linear rescaling factors for REG, VAR, or TCA methods, respectively; \( \sigma_x \) and \( \sigma_y \) are standard deviations of \( x \) and \( y \), respectively; and \( \rho_{xy} \) is the correlation coefficient between \( x \) and \( y \). For more details, see studies of Reichle and Koster (2004) or Yilmaz and Crow (2013, 2014).

In an earlier study, Yilmaz and Crow (2013) found that the TCA-based rescaling method in Eq. (4) is an optimal preprocessing approach for a data assimilation system that conforms to underlying assumptions. However, they did not consider seasonality and the anomaly components separately; instead, the entire time series was rescaled as one piece. On the other hand, rescaling the entire time series without considering potential differences between the scaling coefficients of seasonality and anomaly components does not necessarily guarantee an optimal analysis, which necessitates investigation of separate rescaling coefficients for different frequency signal components (Su and Ryu 2015). Here we investigate
the impact of scaling coefficient differences between the seasonality and the anomaly components of the observations and the control model (i.e., the model open loop) in a data assimilation framework.

Despite not being completely optimal, VAR rescaling is widely applied in land DA studies and, unlike TCA rescaling, does not require the presence of two independent observations (in addition to the model). Therefore, in order to maximize the relevance and generality of our results, the VAR rescaling methodology is utilized in this study. This is implemented by estimating $c_V^i$ via Eq. (3) for all experiments whenever observations (i.e., $y$) are required to be rescaled to control model (i.e., $x$) prior to assimilation. Hereinafter, $c$ will be used instead of $c_V^i$ for brevity.

The rescaling methods introduced above are implemented using either the same rescaling coefficient for the entire time series (by assuming they are stationary in time) or by utilizing separate rescaling factors for lower (e.g., seasonality)- and higher (e.g., anomaly)-frequency components:

\[ x^S_j = \frac{1}{29n} \sum_{i=1}^{31} \sum_{j=1}^{14} x_{ij}, \quad \text{and} \]
\[ x^A_j = x_{ij} - x^S_j, \]

where $i$ refers to year; $j$ refers to day of year; and $x^S$ and $x^A$ are the seasonality and anomaly components, respectively, of any given daily product $x$ that is available for $n$ many years. Summing these components recovers the original time series (i.e., $x = x^S + x^A$). Equation (5) defines the fixed seasonality of a dataset (i.e., $x$) for a particular day as the average of a 29-day moving-average window centered on a day of interest over all years of the dataset (e.g., the seasonality on 15 March is calculated as the average of all data between 1 and 29 March, using 29 days $\times$ 15 = 435 days of data). Data anomalies (i.e., $x^A$) are obtained by subtracting this daily seasonality time series from the raw dataset. Here, Eqs. (5) and (6) are implemented for any dataset when the seasonality and the anomaly components of this dataset are required.

Various rescaling methods (e.g., REG, VAR, or TCA) can be implemented in different studies using different strategies. These strategies often differ based on the component (i.e., seasonality or anomaly) that is rescaled and/or the assumption that rescaling coefficients are stationary (i.e., using 12 different monthly coefficients vs one single rescaling coefficient across all months). In this study, six different rescaling strategies are used to assess the impact of scaling coefficient differences that may exist between seasonality and anomaly components of products using only VAR rescaling method:

1) **DSI and Anom monthly**: Direct seasonality insertion (DSI) is when observation seasonality components are directly replaced by the control model seasonality while observation anomalies are rescaled to match the control model for each month separately via 12 different rescaling coefficients estimated using all time series data for the particular month (e.g., for a 15-yr dataset, the rescaling coefficient for May is calculated using data with length of 31 days $\times$ 15 = 465 days). Such rescaling can be expressed as

\[ y^s_i = x^S + [(y^A - \mu^A_{ym})c^A_m + \mu^A_{xm}], \]

where $y^s_i$ are rescaled observation anomalies obtained via a “DSI and Anom monthly” strategy; $y^A$ is an observation anomaly calculated using Eq. (6); $x^S$ is the control model (i.e., reference dataset) seasonality components calculated via Eq. (5); $m$ corresponds to month of year; $\mu^A_{ym}$ and $\mu^A_{xm}$ are the monthly temporal means of anomaly components of the daily time series $x$ and $y$, respectively; and $c^A_m$ represents monthly rescaling factors for the anomaly components calculated via variance matching using Eq. (3). Here, and in subsequent equations, the subscripts $i$ and $j$ are dropped from $x$ and $y$ for brevity (both variables are daily time series).

2) **DSI and Anom one piece**: DSI, but anomaly components are rescaled in one piece (i.e., lumped for the entire analysis period using a single-scalar rescaling coefficient rather than 12 separate coefficients). Such rescaling can be expressed as

\[ y^s_i = x^S + [(y^A - \mu^A_n)c^A + \mu^A_n], \]

where $y^s_i$ are rescaled observation anomalies obtained via a “DSI and Anom one piece” strategy. The scalar rescaling coefficient $c^A$ is sampled via variance matching between the entire model and the observation anomaly time series using Eq. (3), and $\mu^A_n$ and $\mu^A_n$ are scalar means of $y^A$ and $x^A$.

3) **Decom (Season–Anom) monthly**: Observations (i.e., $y$) and control model (i.e., $x$) time series are decomposed into their seasonality and anomaly components and both components are rescaled to corresponding coefficients of the control model separately for each month. Such rescaling can be expressed as

\[ y^s_i = [(y^S - \mu^S_{ym})c^S_m + \mu^S_{xm}] + [(y^A - \mu^A_{ym})c^A_m + \mu^A_{xm}], \]

where $y^s_i$ are rescaled observation anomalies obtained using a “Decom (Season–Anom) monthly”
strategy, and $c_{m}^{s}$ and $c_{m}^{A}$ represent the monthly rescaling factors for the seasonality and anomaly components, respectively, calculated via variance matching method using Eq. (3). Variables $\mu_{ym}^{s}$ and $\mu_{ym}^{A}$ are means of the seasonality components of observation and control model for month $m$, and $\mu_{ym}^{s}$ and $\mu_{ym}^{A}$ are the means of the anomaly components of observation and model datasets for month $m$. Here, $y^{s}$ and $y^{A}$ have the lengths of 365 and 365$m$, respectively. These components are rescaled using rescaling coefficients calculated for each month separately ($c_{m}^{s}$ and $c_{m}^{A}$). Overall, 24 different rescaling coefficients are used in Eq. (9), 12 for the seasonality (i.e., $c_{m}^{s}$) and 12 for the anomaly (i.e., $c_{m}^{A}$) components. For example, for 15 years of daily data, the scalar rescaling factor for the anomaly component during May $c_{m}^{A}$ is calculated using $31 \times 15 = 465$ values of $x^{A}$ and $y^{A}$ while the seasonality component $c_{m}^{s}$ is calculated using 31 values of $x^{s}$ and $y^{s}$ via Eq. (3).

4) Decom (Season–Anom) one piece: The seasonality and anomaly components of observations are rescaled separately to the seasonality and anomaly components of the control model, respectively, as one piece. Such rescaling can be expressed as

$$y_{4}^{s} = [(y^{s} - \mu_{Ym}^{s})c_{m}^{s} + \mu_{Xm}^{s}] + [(y^{A} - \mu_{Ym}^{A})c_{m}^{A} + \mu_{Xm}^{A}],$$

(10)

where $y_{4}^{s}$ are rescaled observation anomalies obtained using a “Decom (Season–Anom) one piece” strategy. Here $c^{s}$ and $c^{A}$ are scalars used to rescale seasonality and anomaly components, respectively, and $\mu_{Ym}^{s}$ and $\mu_{Xm}^{A}$ are scalar time averages calculated using entire time series (i.e., using $15 \times 365 = 5475$ values for a 15-yr daily dataset). The scalars $\mu_{Ym}^{s}$ and $\mu_{Ym}^{A}$ are calculated using 365 values (i.e., the length of seasonality components is always 365).

5) Nondecom monthly: Twelve different monthly rescaling coefficients are applied to nondecomposed observations. Such rescaling can be expressed as

$$y_{5}^{s} = [(y - \mu_{Ym}^{s})c_{m}^{s} + \mu_{Xm}^{s}],$$

(11)

where $y_{5}^{s}$ are rescaled observation anomalies obtained using a “Nondecom monthly” strategy.

6) Nondecom one piece: A single rescaling factor is applied to the entire time series of the nondecomposed observations:

$$y_{6}^{s} = [(y - \mu_{Y})c + \mu_{X}],$$

(12)

where $y_{6}^{s}$ are rescaled observation anomalies obtained using a “Nondecom one piece” strategy. This equation is identical to Eq. (1), which is the most commonly implemented form of linear rescaling in studies utilizing soil moisture datasets.

All rescaling coefficients ($c_{m}^{s}, c_{m}^{A}, c^{s}, c^{A}, c_{m}$, and $c$) given in Eqs. (7)–(12) are calculated using the VAR rescaling method given in Eq. (3). The number of rescaling coefficients required for each strategy is given in Table 1. Here, the nondecomposed type of rescaling is identical to the use of the same rescaling coefficients for seasonality and anomaly components. Any difference in seasonality and the anomaly components can be taken into account by rescaling these components separately. Among these six rescaling strategies, the ones that directly apply the model seasonality to observations (i.e., the DSI approaches in the first and the second strategies) are considered to represent a more aggressive rescaling strategy than rescaling anomaly and seasonality components separately (i.e., the Decom approaches in the third and the fourth strategies). These approaches, in turn, are more aggressive than rescaling the entire observation time series without decomposing it into seasonality and anomaly components (i.e., the Nondecom approach in the fifth and

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**Table 1. Number of different rescaling coefficients used for each rescaling strategy.**

<table>
<thead>
<tr>
<th>Rescaling strategy</th>
<th>Rescaling coefficient $c$</th>
<th>Same $c$ for season and anomaly</th>
<th>Different $c$ for season and anomaly</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSI and Anom monthly</td>
<td>$c_{m}^{A}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>DSI and Anom one piece</td>
<td>$c_{m}^{A}$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Decom (Season–Anom) monthly</td>
<td>$c_{m}^{s}$ and $c_{m}^{A}$</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Decom (Season–Anom) one piece</td>
<td>$c_{m}^{s}$ and $c_{m}^{A}$</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Nondecom monthly</td>
<td>$c_{m}$</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Nondecom one piece</td>
<td>$c_{m}$</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>
the sixth strategies). Also, rescaling observations for each month separately (i.e., nonstationary rescaling coefficient assumption) is considered a more aggressive rescaling strategy than matching the entire observation time series in one piece (i.e., stationary rescaling coefficient assumption). Accordingly, these approaches are listed above in inverse order of their relative aggressiveness (from most to least) with regard to their rescaling of observations to match systematic elements of the model.

Among these strategies, replacing the observation seasonality with the model seasonality (i.e., DSI rescaling) results in correlations between observation seasonality and truth seasonality that are identical to correlations between model seasonality and truth seasonality. Likewise, linearly rescaling the observation seasonality component against the model seasonality component in one piece does not alter the correlation between observations and the truth. However, when linear rescaling of observations is performed for each month separately, the correlation between observation seasonality and the truth seasonality is impacted as a result of variations in the rescaling coefficients between months. As in the case with seasonality components, the monthly rescaling of observation anomaly components also results in modified correlations between observation anomaly and true anomalies as a result of variations in rescaling coefficients for different months.

b. Synthetic twin experiments

Because of its simplicity and transparency, the API model is frequently used in synthetic twin experiments aimed at clarifying particular methodological issues in land data assimilation (Loew et al. 2009; Crow and Yilmaz 2014; Han et al. 2014). The output of this model is derived from cumulative precipitation and representative of soil moisture; hence, it can be directly compared against soil moisture observations and/or estimates. Here, we apply an API analysis at a daily time step to test the impact of the rescaling coefficient differences between seasonality and anomaly components of observations and models in a well-controlled experiment setup. The API model is given as

$$x_d = y_p x_{d-1} + p_d$$ and

$$y_d = a + b \cos(2\pi d/365),$$

where \(d\) is the time step (days); \(x_d\) is the API value at \(d\); \(p_d\) is the precipitation at \(d\); and \(a\) and \(b\) coefficients are selected as 0.85 and 0.10, respectively (Crow and Zhan 2007). Simulations are performed using Tropical Rainfall Measuring Mission (TRMM) 3B42, version 7 (v7), daily data (Huffman and Bolvin 2014) for the 15 years between 1998 and 2012 within a single 0.25° pixel (35.0°N, 98.0°W) over Little Washita, Oklahoma.

| Table 2: Rescaling coefficients used in synthetic simulations. Here, “better” is defined as a higher SNR with respect to synthetic truth. |
|---------------|------------------|------------------|------------------|
| Scaling coef  | Expt 1 (model better) | Expt 2 (equal) | Expt 3 (obs better) |
| \(\alpha_{a_s}\) | 0.4 | 0.65 | 1.0 |
| \(\alpha_{A}\) | 0.4 | 0.65 | 1.0 |
| \(\alpha_{a_A}\) | 1.1 | 0.65 | 0.2 |
| \(\alpha_{A}\) | 1.1 | 0.85 | 0.6 |

The truth API (api\(_{tru}\)) synthetic simulations are obtained by using unperturbed TRMM data to represent the true precipitation forcing \(p^T\) in Eq. (13). This true forcing is then split into low-frequency seasonality \(p^S\) and high-frequency anomaly \(p^A\) components (i.e., \(p^T = p^S + p^A\)) using Eqs. (5) and (6).

Individual seasonality and anomaly components of api\(_{tru}\) (api\(_{stra}\) and api\(_{stru}\)) are obtained by forcing the API model in Eqs. (13) and (14) using \(p^S\) and \(p^A\) precipitation components, respectively. Because of the linearity of the API model, summing the seasonality (i.e., api\(_{stra}\)) and anomaly (i.e., api\(_{stru}\)) components of api\(_{tru}\) generated via parallel runs forced by \(p^S\) and \(p^A\), respectively, recovers the exact api\(_{tru}\) time series forced by \(p^T\). Similarly, the decomposition of api\(_{tru}\) using the above-described moving window–based averaging method in Eqs. (5) and (6) gives the same seasonality and anomaly components (api\(_{stra}\) and api\(_{stru}\)) obtained by two parallel runs forced by \(p^S\) and \(p^A\), respectively. This property is particularly important for obtaining full control over model simulations in generating time series with specific seasonality and anomaly scaling coefficients.

Observations are created by both multiplicatively biasing the seasonality and the anomaly components of api\(_{tru}\) and introducing additive noise to api\(_{tru}\) via

$$\text{api}_{obs} = (\text{api}_{stra} \alpha_{a_s} + \text{api}_{stru} \alpha_{A}) + [\varepsilon_{o} \sigma_{\text{api}_{tru}} (0.7)],$$

where \(\alpha_{a_s}\) and \(\alpha_{a_A}\) are observation rescaling coefficients for seasonality and anomaly components, respectively; \(\varepsilon_{o}\) is white noise time series with the same unit as api\(_{tru}\) and normal distribution with 0 mean and 1 standard deviation [N(0, 1)]; \(\sigma_{\text{api}_{tru}}\) is the standard deviation of api\(_{tru}\) time series; and the arbitrarily selected unitless 0.7 factor ensures that the standard deviation of added noise is 70% of \(\sigma_{\text{api}_{tru}}\). Values for the scaling coefficients \(\alpha_{a_s}\) and \(\alpha_{a_A}\) are given in Table 2. These values are selected before assimilation experiments are performed with the intent of designing specific experimental cases reflecting: 1) relatively better (i.e., higher SNR) model estimates, 2) model and observation estimates with similar SNR characteristics, or 3) relatively better observation
estimates, where SNR refers to the ratio between the variance of the first bracket term (i.e., the signal) and the second bracket term (i.e., the white noise) in Eq. (15).

Control model simulations are created by using a rescaled and perturbed forcing dataset:

$$p^C = (p^5 \alpha_{m_5} + p^A \alpha_{m_A}) + [e_m \sigma_{ap}^{\text{true}} (0.7)],$$  \hspace{1cm} (16)

where $p^C$ is precipitation forcing used in control model simulations; $\alpha_{m_5}$ and $\alpha_{m_A}$ (Table 2) are unitless rescaling coefficients for model seasonality and anomaly components, respectively; and $e_m$ is a unitless white noise time series following $[N(0, 1)]$. Here, the variance of the terms in the square brackets in Eq. (16) is constant, while the terms in the first parentheses control the total precipitation variability (hence the signal variance and the SNR). Accordingly, as with the generation of synthetic observations via Eq. (15), the model simulations are based on introducing systematic multiplicative biases into both the anomaly and the seasonality components (in addition to the above-described white noise random variations).

Here, the noise variance is kept constant $[\sigma_{ap}^{\text{true}}(0.7)]$ for different experiments while variations in the dimensionless scaling coefficients ($\alpha_{a_5}$, $\alpha_{a_A}$, $\alpha_{m_5}$, or $\alpha_{m_A}$ given in Table 2) result in different SNR values for different experiments. Using different scaling factors results in different additive and/or multiplicative linear biases in APIobs; however, the rescaling of these observations to the reference model using VAR rescaling method effectively removes these differences. Since variances of the $p^{\text{true}}$, $p^{\text{true}}$, $p^A$, $e$, and $e_m$ time series and $\sigma_{ap}^{\text{true}}$ term in Eqs. (15) and (16) are constant, the SNR of observations or model is directly proportional to these scaling coefficients.

In this study, the impact of the accuracy difference between the model and the observations over the analysis is the primary focus. The selection of scaling coefficients are made arbitrarily while this selection ensures that necessary accuracy differences between the model and the observation for the particular experiment design are obtained (e.g., ensuring that the model has a higher SNR than the observations or vice versa). Different experiments have the goal of fixing the seasonality and anomaly components of the model and observations to have different accuracies. Accordingly, the standard deviation of the noise [i.e., $\sigma_{ap}^{\text{true}}(0.7)$ used in Eqs. (15) and (16)] can be set arbitrarily since scaling coefficients can always be defined to yield our (desired) cases of a superior model, superior observations, or equally accurate model and observations used in different experiments. As a result, selecting a higher scaling coefficient results in a higher SNR product than selecting a lower scaling coefficient for both the control model and observations. However, given that the variance ratios of anomaly and seasonality components of products differ, utilizing the same scaling coefficients for both the seasonality and the anomaly components does not always yield the same impact on the observation to model relative accuracy (i.e., increasing the observation seasonality or anomaly scaling coefficients by 0.10 does not change the observation SNR similarly). Therefore, the selection of different scaling coefficients for seasonality and anomaly components (Table 2) is necessary to obtain the desired model and observation accuracy differences.

Once observations and control model simulations are obtained, these two datasets are split into their seasonality and anomaly components. First, seasonality components are found using the 29-day moving window methodology using Eq. (5). Later, anomaly components are retrieved by subtracting this seasonality time series from the original dataset using Eq. (6). The first four rescaling strategies given in Eqs. (7)–(10) rescale either or both observation components to the respective component of the model using different rescaling coefficients, while the fifth and the sixth strategies given in Eqs. (11) and (12) rescale the entire observation time series into the control model using a scalar rescaling coefficient. All of these rescaling coefficients are obtained using VAR rescaling approach method.

Once observations are rescaled to the control model simulations, they are assimilated into the API model using a standard Kalman filter (Kalman 1960). The above six experiments are then repeated for each of the three relative accuracy cases described above (i.e., the model better, the model/observations equally accurate, and the observations better cases). Accuracy assessments are performed for all of the complete datasets (i.e., nondecomposed time series), seasonality components, and anomaly components by calculating correlation coefficients (Albergel et al. 2012) against the truth, where the correlation coefficients for seasonality and anomaly components of control model (i.e., $x$) and observations (i.e., $y$) are assessed against seasonality and anomaly components of api_{true}, respectively.

\textit{c. Experiments with real datasets}

Experiments utilizing real datasets are performed by assimilating Advanced Microwave Scanning Radiometer for Earth Observing System (AMSR-E) observation-based LPRM soil moisture datasets (Owe et al. 2001; Mladenova et al. 2014) into the API model and validating the subsequent soil moisture analysis using station-based ground data. API model simulations are performed using the unperturbed native TRMM
3B42 v7 daily precipitation product at 0.25° spatial resolution. LPRM datasets, resampled to the 0.25° spatial resolution grid, are acquired from the Vrije Universiteit Amsterdam (R. Parinussa 2013, personal communication). LPRM datasets were validated and intercompared in many studies (Mladenova et al. 2011; Brocca et al. 2011; Su et al. 2013; Parinussa et al. 2011). For more information about LPRM, see Owe et al. (2001) and Mladenova et al. (2014).

Simulations are performed between 2007 and 2011 over four U.S. Department of Agriculture (USDA) Agricultural Research Service (ARS) Watersheds (Little River, Little Washita, Walnut Gulch, and Reynolds Creek). These watersheds contain dense soil moisture networks (16–29 stations over 150–610 km²) over dominantly cropland, grassland, agriculture, semiarid, and forest land-cover types, respectively (Jackson et al. 2010). Soil moisture values representative of the watershed are obtained by taking the spatial average of measurements made at a 0–5 cm depth at 20–60-min intervals at different locations (Jackson et al. 2010). These datasets have been used previously in the validation of AMSR-E and Soil Moisture Ocean Salinity (SMOS) surface soil moisture products (Jackson et al. 2010, 2012) and verified via comparisons against gravimetric soil moisture observations (Cosh et al. 2006, 2008). Hereinafter, they will be referred to as watershed-average soil moisture (WASM) values and used as an independent benchmark target for Kalman filtering analysis results.

Statistics of seasonality and anomaly components of various datasets are compared. For these intercomparisons only, Noah land surface model (LSM; Ek et al. 2003) simulations are also used along with LPRM and station-based data. Noah, version 2.7, datasets are obtained from Global Land Data Assimilation System, version 2, simulations. Three-hourly Noah soil moisture simulations representing the top 10 cm over four USDA ARS watersheds are retrieved at 0.25° spatial resolution. These 3-hourly datasets are later averaged into daily values that are used in seasonality- and anomaly-related statistics calculation. [These datasets are obtained from http://hydro1.sci.gsfc.nasa.gov/dods/; for more information, see Rodell et al. (2004).]

3. Results and discussion

In all experiments, observations are rescaled to the reference model using the VAR rescaling method [Eq. (3)] and subsequently assimilated into API model using a Kalman filter. Here, the correlation coefficient is used as the performance metric since the error standard deviation metric can become unreliable when datasets with different dynamic ranges are compared (Entekhabi et al. 2010; Gupta et al. 2009).

Correlations between the synthetic data assimilation analysis and the synthetically generated truth for the six rescaling strategies described above [Eqs. (7)–(12)] are shown in Fig. 1. In the figure legend, rescaling strategies are numbered in inverse order of their aggressiveness. That is, DSI and Anom monthly is the most aggressive rescaling approach while Nondecom one piece is the least aggressive strategy. Analysis seasonality and anomaly components are correlated against both the seasonality and anomaly components of the synthetic truth. Synthetic data results in Fig. 1 illustrate the link between the relative accuracy of observations compared to model and the performance of a particular rescaling strategy. When the model is less accurate than the assimilated observations (see the right-hand side of Fig. 1), less aggressive rescaling strategies are advantageous. This is particularly true for the seasonality components (Fig. 1c). For example, rescaling observation seasonality components to relatively less accurate control model seasonality components using a “monthly” rescaling strategy (i.e., sampling and applying 12 separate rescaling coefficients for each month) results in analysis seasonality components being less accurate than observations. In contrast, such degradation does not occur if less aggressive methods (e.g., “one piece” rescaling) are applied. A similar degradation in analysis correlation also occurs when observations are more accurate (i.e., higher SNR) than the assimilation model and very aggressive strategies (i.e., DSI, regardless from single or 12 separate coefficients are used to rescale anomaly components) are applied (Fig. 1c). In contrast to seasonal components (Fig. 1c), applying the DSI strategy did not result in the degradation of correlation within anomaly components (Fig. 1b). This might be because the seasonality component of the analysis is based on the (less accurate) model seasonality (i.e., due to the application of DSI rescaling) while the analysis anomaly is a product of both observation and model anomaly components. Nevertheless, noticeable skill gain is obtained when less aggressive one-piece rescaling is chosen over a monthly rescaling strategy.

Likewise, more aggressive rescaling strategies (e.g., monthly) produce more accurate results than less aggressive rescaling strategies (e.g., one piece) when control model simulations are more accurate than observations (i.e., see the left-hand side of Fig. 1). Here, no particular advantage is found between the DSI monthly and DSI one-piece strategies and the Decom monthly and Nondecom monthly strategies (Fig. 1). When observations and model have similar SNR (i.e., center column of Fig. 1), the differences between
various rescaling strategies become less apparent. Overall, even though more aggressive strategies (DSI > Decom > Nondecom) yield more accurate analysis when the model has a higher SNR than observations (left-hand side of Fig. 1) and vice versa (right-hand side of Fig. 1), the differences between these three strategies are less apparent for the monthly rescaling case than for the one-piece rescaling case (Fig. 1).

Comparable real data assimilation results corresponding to the assimilation of AMSR-E LPRM retrievals into the API model are shown in Figs. 2 and 3. In particular, Fig. 2 demonstrates the impact of applying different rescaling strategies to assimilated observations during the calendar year 2007 only, and Fig. 3 investigates the impact of all rescaling strategies on analysis accuracy in a way similar to the presentation of synthetic experiment results in Fig. 1. As in the synthetic studies, the choice of rescaling strategy (i.e., less or more aggressive) is consistent with the relative accuracy of LPRM observations (i.e., correlations with the ground-based WASM; Table 3) compared to the background API model accuracy at anomaly/seasonality component level (Figs. 3b,c). The anomaly components of API have higher correlations than LPRM (i.e., model better) over Little River and Little Washita; hence, the more aggressive monthly strategy results in a better analysis than less aggressive one-piece strategy (Fig. 3b). Similarly, the anomaly components of LPRM have higher correlations than API (i.e., observations better) over Walnut Gulch and Reynolds Creek; hence, the less aggressive one-piece strategy results in a better analysis than the more aggressive monthly strategy (Fig. 3b). Similar to the anomaly component level, the results are also consistent at the individual seasonality level. That is, the seasonality components of API have higher correlations than the control model run (no assimilation), and the observations. Scaling coefficients used for seasonality and anomaly components of model and observations are given in Table 2. Observation rescaling approaches become increasingly aggressive with decreasing rescaling method number (i.e., 1 is the most aggressive rescaling method while 6 is the least aggressive method).
LPRM (i.e., model better) over Walnut Gulch; hence, the more aggressive monthly strategy results in a (marginally) better analysis than a less aggressive one-piece strategy (Fig. 3c). Similarly, the seasonality components of LPRM have considerably higher correlations than API (i.e., observations better) over Little River, Little Washita, and Reynolds Creek, and thus the less aggressive one-piece strategy results in a considerably better analysis than the more aggressive monthly strategy (Fig. 3c). Overall, when observation seasonality or anomaly components have higher SNR than model seasonality or anomaly components, then it is desirable to preserve this higher-quality observation component via one-piece rescaling strategy rather than degrading them via rescaling to less accurate model component using monthly or DSI strategies.

LPRM complete time series correlations against WASM are higher than API correlations (i.e., observation better) over Little River, Walnut Gulch, and Reynolds Creek; the consistently less aggressive one-piece strategy results in better analysis than the more aggressive monthly strategy (Fig. 3a). Little Washita is an exception to this general relationship between model accuracy and appropriate levels of rescaling aggressiveness. In particular, API model total correlations over the Little Washita are higher than corresponding LPRM observations, yet a more aggressive monthly strategy does not yield a better analysis than a less aggressive one-piece scaling strategy (Fig. 3). This exception is perhaps due to the poor seasonal performance of the API model degrading the Kalman filtering analysis over the Little Washita. Poor API performance at the seasonal scale is likely associated with its neglect of seasonal variations in available energy for evapotranspiration.

However, when one-piece scaling strategies are applied, the assimilation of observations using their own native seasonality (via the application of the less aggressive Decom or Nondecom strategies) results in a more accurate analysis than directly inserting the model-based seasonality into observations (via the more aggressive strategy DSI; Fig. 3). As was the case with the synthetic results, this difference was only marginal when monthly varying rescaling strategies are chosen (Fig. 3).

As expected, the highly aggressive DSI scaling strategy yields the most accurate Kalman filter analysis (i.e., over Walnut Gulch) only when seasonality correlations between API and WASM are higher than the seasonality correlations between LPRM and WASM while these improvements are modest. Conversely, DSI rescaling strategies do not yield the most accurate results when LPRM seasonality correlations are better than API seasonality correlations. These real data results are consistent with both the correlation results given in Table 3 and with earlier synthetic simulation results, suggesting that rescaling observations more aggressively to a high-quality control model yields more accurate results. Likewise, less aggressive strategies yield more accurate results when observations are more accurate.

As in the synthetic studies, no particular advantage is found between the “Decom” and “Nondecom” rescaling strategies within real data experiments. The adverse impact of aggressive scaling is particularly obvious from the analysis skill obtained from monthly (Fig. 3) and one-piece (Fig. 3) rescaling strategies. When observations have relatively better seasonality skill than the model (e.g., in the Little River, Little Washita, and Reynolds Creek watersheds), monthly rescaling strategy results in observation seasonality that is overfitted to the model and, as a result, an assimilation analysis with degraded seasonality and overall accuracy (Fig. 3). This is why the Decom and Nondecom strategies result in higher analysis skill for one-piece rescaling than monthly strategies over these three watersheds.

An obvious issue is the degree to which results presented here are impacted by the simplicity of the API model. Modern LSMs that include a more complete
range of hydrological processes (e.g., Noah) often have higher correlation-based accuracies than observations or simple hydrological models (e.g., API). However, a large portion of this advantage is based on Noah’s enhanced ability to capture soil moisture seasonality (Table 3). When comparisons are limited only to evaluating soil moisture anomalies (calculated relative to a fixed seasonal climatology), the advantages of more complex LSMs largely disappear and simple models (like the API) perform relatively better (Table 3). Complex LSMs also require extensive datasets (forcing, parameters, etc.) and computational power, while simple models are an order of magnitude easier to run and require less effort (i.e., dataset, computational and coding time, etc.). Such advantages are critical in data assimilation applications to solve for unknown parameters required for the optimal analysis [e.g., the Auto-Tuned Land Data Assimilation System (ATLAS); Crow and Yilmaz 2014]. In addition, the linearity of the API model allows for the perfect decomposition of the model into seasonality and anomaly components (see section 2b), which is critical in obtaining full control over various experiments. Accordingly, it is reasonable to investigate the benefits obtained from merging systems that involve simple hydrological models with relatively better anomaly information (like API) and satellite-based observations with relatively more accurate seasonality components. A priori, there is no reason to

![Image](https://example.com/image.png)

**FIG. 3.** Analysis correlations against WASM over four USDA ARS watersheds for the six rescaling strategies described in Eqs. (7)-(12), the control model run (no assimilation), and the observations (details given in section 2b), where LPRM satellite-based observations are assimilated into API model. Observation rescaling approaches become increasingly aggressive with decreasing rescaling method number (i.e., 1 is the most aggressive rescaling method while 6 is the least aggressive method).
suspect that the use of more complex LSMs would qualitatively affect key results (about the rescaling strategies) presented here. Nevertheless, key conclusions presented here should be verified via follow-on studies utilizing more complex LSMs and data assimilation approaches (e.g., the ensemble Kalman filter) more suitable to capturing moderately nonlinear background forecast dynamics.

4. Conclusions

The projection of observations into model space (i.e., rescaling) is a common and necessary preprocessing step in hydrological data assimilation studies. Here, we investigate a neglected aspect of hydrological data assimilation studies—the impact of hydrological model accuracy relative to observations in the framework of rescaling observations before they are assimilated into models. In particular, we focus on the impact of the accuracy of the seasonality and anomaly components of LSMs (relative to observations) on rescaling strategy selection and assimilation analysis accuracy. We consider different accuracy levels and rescaling strategies using a series of synthetic twin experiments and real data simulations.

Previously, Yilmaz and Crow (2013) investigated the optimality of rescaling methodologies (REG, VAR, or TCA) in a data assimilation framework by focusing solely on one of the rescaling strategies considered here (i.e., Nondecom one piece) and examining various methods for estimating the single parameter required by this particular approach. This study extends rescaling to more general cases where systematic differences exist at multiple time scales and evaluates the impact of various structural forms for rescaling, each reflecting a different level of aggressiveness in modifying the observation time series to match systematic aspects of the model.

Many existing hydrological land data assimilation (particularly soil moisture based) systems are based on the concept of removing systematic differences between observations and the assimilation model via rescaling to match systematic aspects of the assimilation model. Results presented here show that rescaling observations more aggressively to the assimilation model (i.e., observation seasonality is replaced by model seasonality or rescaling observations to model for each month separately) is advantageous when the model has a higher SNR than observations. However, in contrast to prevailing land data assimilation practice, simpler/less aggressive rescaling strategies (i.e., rescaling entire time series without distinguishing for anomaly and seasonality components) are advantageous for cases in which observations have higher SNR than control model (i.e., the open loop). Minor skill gain is found in rescaling seasonality and anomaly components of observations separately in synthetic simulations but not in real data assimilation experiments. Therefore, the magnitude of aggressiveness of the rescaling strategy choice should ideally be made consistent with the relative accuracy of the seasonality and the anomaly components of observations and models.

<table>
<thead>
<tr>
<th>Product</th>
<th>Complete (seasonality + anomaly)</th>
<th>High frequency (anomaly)</th>
<th>Low frequency (seasonality)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watershed average</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LPRM</td>
<td>0.65</td>
<td>0.56</td>
<td>0.77</td>
</tr>
<tr>
<td>API</td>
<td>0.58</td>
<td>0.56</td>
<td>0.46</td>
</tr>
<tr>
<td>Noah</td>
<td>0.70</td>
<td>0.40</td>
<td>0.85</td>
</tr>
<tr>
<td>Little River</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0.47</td>
<td>0.95</td>
</tr>
<tr>
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<td>0.74</td>
<td>0.26</td>
</tr>
<tr>
<td>Noah</td>
<td>0.67</td>
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<td>0.92</td>
</tr>
<tr>
<td>Little Washita</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.6</td>
<td>0.61</td>
</tr>
<tr>
<td>API</td>
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<td>0.72</td>
<td>0.13</td>
</tr>
<tr>
<td>Noah</td>
<td>0.65</td>
<td>0.43</td>
<td>0.71</td>
</tr>
<tr>
<td>Walnut Gulch</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.76</td>
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<tr>
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<tr>
<td>Reynolds Creek</td>
<td></td>
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<td>0.58</td>
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<tr>
<td>Noah</td>
<td>0.77</td>
<td>0.35</td>
<td>0.94</td>
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</table>
To be of practical benefit, this advice must be accompanied by reliable information concerning the relative accuracy of the assimilation model versus the assimilated observations. Model and observation relative accuracies can be acquired prior to assimilation experiments via triple-collocation analysis (if multiple-platform-based datasets are available) or using correlations against a validation dataset or cross correlations between different products (higher cross correlations between products imply higher SNR). All of these methods can be used to robustly infer the relative accuracies of datasets. Higher observation accuracy relative to model would suggest the use of relatively simple rescaling strategies (see section 2a), while relatively lower observation accuracy would imply that more aggressive rescaling of observations would further increase the final analysis accuracy.

The overall accuracy of a soil moisture dataset is driven by the component (seasonality or anomaly) that provides the majority of its temporal variability. For example, the variability of Noah soil moisture anomaly and seasonality components are both more consistent with the station data variability than those of API and LPRM (Table 4). The majority of the Noah land surface variability is due to its anomaly component (Table 4); hence, it is the anomaly component that drives overall accuracy of Noah rather than the seasonality component. On the other hand, the Noah soil moisture anomaly component is much less accurate compared to LPRM anomaly (Table 3), which clearly presents an example of the “observation better” scenario tested in this study. Given that Noah’s soil moisture seasonality component has very high correlation (higher than LPRM observations) with independent ground-based observations (Table 3), LPRM seasonality component may be replaced by the Noah seasonality (e.g., DSI) or may be aggressively rescaled to Noah seasonality (e.g., 12 monthly) while anomaly components of LPRM soil moisture products may be rescaled to Noah anomalies using a less aggressive rescaling strategy (e.g., one piece). Similarly, given that the accuracy of seasonality of Noah is superior to API, while the anomaly component of API is superior to Noah, it is possible that seasonality and anomaly components of any given independent product [e.g., Advanced Scatterometer (ASCAT), SMOS, and Soil Moisture Active Passive (SMAP)] could be rescaled to those components of Noah and API separately to obtain a more accurate rescaled product. Despite this potential, such rescaling experiments have not been implemented here and will require future study.

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** REFERENCES **


** TABLE 4.** The ratio of low (seasonality)- and high (anomaly)-frequency soil moisture variance to total time series variance. Ratios are calculated over each USDA watershed separately and a single watershed average is sampled (across all watersheds) for each product.

<table>
<thead>
<tr>
<th>Variance/total variance</th>
<th>High frequency (anomaly)</th>
<th>Low frequency (seasonality)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WASM</td>
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<td>API</td>
<td>0.72</td>
<td>0.25</td>
</tr>
<tr>
<td>LPRM</td>
<td>0.48</td>
<td>0.49</td>
</tr>
<tr>
<td>Noah</td>
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