Comparison of the Scaling Characteristics of Rainfall Derived from Space-Based and Ground-Based Radar Observations

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ABSTRACT

In view of the importance of tropical rainfall and the ubiquitous need for its estimates in climate modeling, the authors assess the ability of the Tropical Rainfall Measuring Mission (TRMM) precipitation radar (PR) to characterize the scaling characteristics of rainfall by comparing the derived results with those obtained from the ground-based radar (GR) data. The analysis is based on 59 months of PR and GR rain rates at three TRMM ground validation (GV) sites: Houston, Texas; Melbourne, Florida; and Kwajalein Atoll, Republic of the Marshall Islands. The authors consider spatial scales ranging from about 4 to 64 km at a fixed temporal scale corresponding to the sensor “instantaneous” snapshots (~15 min). The focus is on the scaling of the marginal moments, which allows estimation of the scaling parameters from a single scene of data. The standard rainfall products of the PR and the GR are compared in terms of distributions of the scaling parameter estimates, the connection between the scaling parameters and the large-scale spatial average rain rate, and deviations from scale invariance. The five main results are as follows: 1) the PR yields values of the rain intermittence scaling parameter within 20% of the GR estimate; 2) both the PR and GR data show a one-to-one relationship between the intermittence scaling parameter and the large-scale spatial average rain rate that can be fit with the same functional form; 3) the PR underestimates the curvature of the scaling function from 20% to 50%, implying that high rain-rate extremes would be missed in a downscaling procedure; 4) the majority of the scenes (>85%) from both the PR and GR are scale invariant at the moment orders q = 0 and 2; and 5) the scale-invariance property tends to break down more likely over ocean than over land; the rainfall regimes that are not scale invariant are dominated by light storms covering large areas. Our results further show that for a sampling size of one year of data, the TRMM temporal sampling does not significantly affect the derived scaling characteristics. The authors conclude that the TRMM PR has the ability to characterize the basic scaling properties of rainfall, though the resulting parameters are subject to some degree of uncertainty.

1. Introduction

Since the pioneering work of Lovejoy and Mandelbrot (1985) and Lovejoy and Schertzer (1985), the scaling properties of rainfall fields have attracted attention of many researchers. One reason for this increased interest is the need to fill the gap between the large scales of meteorological model outputs and the smaller hydrological scales. Studies have documented the importance of small-scale rainfall variability on runoff simulation (Ogden and Julien 1993, 1994; Winchell et al. 1998), radiative transfer computations (Harris et al. 2003), and estimation of land–atmosphere fluxes (Nykanen et al. 2001). The impact of ignoring the small-scale rainfall variability and the propagation of this variability via the nonlinear equations of hydrological models can result in significant biases of the predicted variables. Rainfall downscaling models often require only two to three parameters to reproduce rainfall over a large
range of scales, and hence could serve as a possible bridge for the transfer of information from large scales to small scales. To make the scaling-based approach an efficient tool for downscaling rainfall, the following research questions need to be addressed: “Are rainfall fields scale invariant?” “How can deviations from scale invariance be accommodated in the transformation methods?” “Can the scaling parameters be predicted from large-scale observables like rain rate at the synoptic scale?” “How do the scaling parameters vary in different regions of the world?” Investigation of these issues requires analysis of data collected in different rainfall regimes. Thus far, the number of such investigations reported in the literature is rather low. Because of the apparent lack of appropriate ground-based datasets, recent investigations have primarily dealt with radar data at a couple of locations (e.g., Lovejoy and Schertzer 1990; Gupta and Waymire 1990, 1993; Over and Gupta 1994, 1996; Perica and Foufoula-Georgiou 1996; Menabe et al. 1997; Deidda 2000).

Since its launch in November 1997, the Tropical Rainfall Measuring Mission (TRMM) satellite has provided nearly homogeneous global tropical rain estimates. TRMM’s precipitation radar (PR) is the first radar designed specifically for rainfall monitoring from space. The PR data provide a unique opportunity for globally investigating the scaling properties of rainfall. However, to our knowledge, the scaling characteristics of the PR-derived rainfall estimates have not yet been studied. The significant advances in the field of rainfall scaling that can result from this crucial information motivates the present study. In the exploitation of the PR data, a major task is to assess the accuracy of the scaling characteristics of rainfall derived from the PR.

To evaluate the accuracy of TRMM observations, NASA has implemented a ground validation (GV) program (Simpson et al. 1996; Kummerow et al. 2000), which is composed of ongoing rainfall measurements using ground-based radars (GRs) at four primary sites: Melbourne, Florida; Houston, Texas; Kwajalein Atoll, Republic of the Marshall Islands; and Darwin, Australia. The differences between the PR and GR radar characteristics and viewing geometries lead to important differences in sensitivity, attenuation, and resolution. Compared with the PR, the GRs have a lower minimum detectable signal (−108 dBm1 instead of 17 dBZ); a better horizontal resolution (the gate spacing is 250 m instead of 4.3-km resolution at nadir); and a nonattenuating wavelength (~10 cm instead of 2.2 cm). Comparison of the scaling characteristics of rainfall derived from the PR and GR is therefore instrumental to identifying the potential advantages and limitations of the PR.

In light of the above, the overall objective of this paper is to investigate, compare, and contrast the scaling characteristics of rainfall derived from the PR and GR in terms of

1) the scaling parameter estimates;
2) the relationship between the scaling parameters and the large-scale spatial average rain rate; and
3) deviations from scale invariance.

In section 2, we describe the GV sites and the datasets, including the differences in sensor characteristics between the PR and GR, the algorithms used to derive the rainfall products, and the climatological characteristics of the sites. Section 3 provides the analysis framework and discusses methodological issues associated with the application of the framework to the rainfall data. Section 4 is devoted to the comparison of the statistical and scaling properties of the PR and GR rainfall products. This section also investigates the connection of the scaling parameters with the large-scale spatial average rain rate and the effect of the TRMM temporal sampling on the inferred scaling characteristics. Finally, in section 5, we draw the conclusions.

2. Datasets

Of the four primary TRMM GV sites, we excluded Darwin because it currently has only two available months of GR rainfall products. Therefore, the three TRMM GV sites used in this study are Houston, Melbourne, and Kwajalein. Figure 1 shows the location of the GV sites in the global context. Houston and Melbourne are located on the eastern Atlantic coast in the subtropics, whereas Kwajalein is located in the deep Tropics and is nearly 100% oceanic. The amplitude of the diurnal cycle of rainfall at Kwajalein is fairly weak, consistent with other studies of diurnal rainfall over the open oceans (Wolff et al. 2005). The area observed by the radars at Melbourne and Houston is approximately 50% ocean and 50% land. The diurnal cycle of rainfall at Melbourne is highly periodic, being dominated by the frequent occurrence of sea-breeze-induced convection in the mid- to late afternoon, whereas the diurnality is less pronounced at Houston, suggestive of a less active convective heating cycle as compared to Melbourne (Wolff et al. 2005).

Fifty-nine months of the PR and GR data were available for this study. This dataset includes 18 months of

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1 The relationship between dBm and dBZ for a given range (in km) is as follows: reflectivity factor (in dBZ) = received power (in dBm) + 20 × log(r) + C, where C is the radar constant (in the order of 60 dB for the ground-based radars used in this study).

a. TRMM PR

We used the TRMM PR product designated 2A25. A description of the various TRMM products is available at http://daac.gsfc.nasa.gov/precipitation/TRMM_README. The information on the PR swath width, resolution, and attenuation provided hereafter is applicable to the characteristics of the PR prior to August 2001, after which time the TRMM satellite was moved to a higher orbit to extend the lifetime of the data collection period. Among the several variables in 2A25, we used only the near-surface rainfall rate. The 2A25 products represent “instantaneous” rainfall-rate maps with a horizontal resolution of 4.3 km at nadir and about 5 km at the scan edge. The 2A25 products that were used correspond to version 5 and were processed by the TRMM Science Data and Information System (TSDIS). The first step in creating 2A25 was to correct for the effects of attenuation and nonuniform beam filling effect (NUBF) in the original reflectivity values. The NUBF effect correction method is described in Kozu and Iguchi (1999). The attenuation correction method is a hybrid method between the traditional Hitschfeld–Bordan (Hitschfeld and Bordan 1954) path integrated attenuation correction method and the surface reference technique correction method (Iguchi et al. 2000; Meneghini et al. 2000). The rain rate is then estimated from the corrected reflectivity values using a reflectivity–rain rate power law in which the parameters are functions of the rain type, existence of bright band, freezing height, storm height, and absolute height. We point out that the reflectivity to rain-rate relationship used for the TRMM-PR rainfall estimation is different from those used for the ground-based radars.

b. GR

We used the GR products designated 2A53, which represent “instantaneous” rainfall maps with 2-km Cartesian rainfall fields at a constant altitude of 1.5 km. The 2A53 products were processed by the TSDIS and correspond to the latest available versions: version 5 for Melbourne and Kwajalein and version 4 for Houston. The version-5 algorithm uses the Window Probability Matching Method (Rosenfeld et al. 1994), which relies on matching unconditional probabilities of rain rates ($R$) and radar reflectivity ($Z$) using rain gauge and radar data, respectively. The central pixel of the $3 \times 3$ reflectivity array centered over a given rain gauge location and the 7-min averaged gauge-measured rain rate centered on the time of the radar scan are taken to construct the probability distribution functions. The $R$ and $Z$ having the same cumulative probability are then matched. The version-4 algorithm involves the following steps: (i) estimation of rain rate using the relation $Z = AR^b$, with $A = 300$ and $b = 1.4$; (ii) comparison of the convective and stratiform monthly accumulations to gauge accumulations; and (iii) adjustment of the coefficient $A$ (while keeping the exponent $b$ as 1.4), separately for convective and stratiform groups, to force agreement between radar and gauge data.

The GR deployed at Houston and Melbourne is the Weather Surveillance Radar-1988 Doppler (WSR-88D), while the GR deployed at Kwajalein is the WSR-93D (Marks et al. 2000). The characteristics of WSR-88D and WSR-93D are quite similar; the main difference between the two lies in the scanning strategy. A typical WSR-93D scan strategy has 22 elevation angles versus the WSR-88D scan strategies of 9 to 14. The difference in the scanning strategies translates to a vertical resolution that is higher in the WSR-93D ($\sim0.25$ km) than it is in the WSR-88D ($\sim1$ km).
c. PR versus GR

The PR’s wavelength, viewing, and other radar characteristics are significantly different from those of the GR, leading to important differences in sensitivity, resolution, and attenuation. The PR transmits peak power of 500 W, which is significantly smaller in comparison to the GR (500 KW). Because the power transmitted by the PR is so small, the minimum detectable signal is relatively high. The minimum detectable signal that can be observed by the PR above the noise level is about 16-18 dBZ in the absence of attenuation, which roughly translates to a rainfall rate of about 0.5 mm h⁻¹ (Kummerow et al. 1998). In addition, the GR records signals at much shorter ranges than does the PR, which operates at an altitude near 350 km. Thus, backscatter from light precipitation recorded by the GR could fall below the noise level of the PR. The PR operates at a frequency of ~18 GHz, which is in the strong attenuation region for microwave radiation. Thus, attenuation of the PR signal is significant in medium to heavy precipitation. The large horizontal resolution of the PR (~4.5 km at nadir) in comparison to the GR (0.25-km gate spacing) leads to a larger effect of NUBF in the PR.

d. Other data

We used the TRMM and other satellites/sources precipitation estimate (3B43 version 5), which is one of the operational products of TRMM, to look at the characteristics of the precipitation regimes over the GV sites. The 3B43 products are on a calendar-month temporal resolution at 1° latitude × 1° longitude. The 3B43 combines TRMM Microwave Imager (TMI) and PR estimates, adjusted Geostationary Operational Environmental Satellite (GOES) precipitation index rain estimates (AGPI), and rain gauge data (Huffman et al. 1997; Adler et al. 2000). In Fig. 2, we present the monthly time series of precipitation during 1998-2000 obtained from 3B43 by extracting the data over each GV site. The average rain rate over the three years is higher for Kwajalein, amounting to 5.37 mm day⁻¹, as compared to that for Houston (2.79 mm day⁻¹) and Melbourne (3.06 mm day⁻¹). The shaded regions in Fig. 2 indicate the periods for which the GR rainfall products are available for this study within the 3-yr period. It is apparent that the periods cover both dry and wet seasons for all of the sites.

3. Approach

a. Scale-invariant fields

The (multi-) scaling characteristics of a geophysical field can be parameterized in several ways leading to many different notations and formalisms. As presented by Over and Gupta (1994, 1996), we describe the spatial moment scaling in this section as if the radar scene were a realization of a random cascade, that is, assuming it is scale invariant. The spatial scaling is best described by starting with the largest scale \( L_0 \). Consider a two-dimensional \((d = 2)\) region with dimensions \( L_0 \times L_0 \), which is successively divided into \( b \) equal parts \((b = 2d)\) at each step, and the \( i \)th subregion after \( n \) levels of subdivision is denoted by \( \Delta_i^n \). At the first level, the region is subdivided into \( b = 4 \) subregions denoted by \( \Delta_i^1 \), \( i = 1, 2, \ldots, 4 \). At the second level, each of the above subregions is further subdivided into \( b = 4 \) subregions, which are denoted by \( \Delta_i^2 \), \( i = 1, 2, \ldots, 16 \), for a total of \( b^2 = 16 \) subregions. At the \( n \)th level, we have a total of \( b^n \) subregions. Denoting the side length at the \( n \)th level as \( L_n \), the scale factor at level \( n \) is given by

\[ \lambda_n = L_n/L_0 = b^{-n/d}. \]  

For the subregion \( \Delta_i^n \), denote the volume of water falling in this subregion as \( \mu(\Delta_i^n) \). So \( \mu(\Delta_i^n) \) is calculated as the average of the PR (GR) data at full resolution falling within \( \Delta_i^n \) multiplied by the area of \( \Delta_i^n \) (i.e., \( L_i^n \)).

Define the spatial moments as

\[ M_n(q) = \sum_{i=1}^{b^n} \mu^q(\Delta_i^n), \]  

where \( q \) is the moment order. The scaling analysis in space can be performed by investigating the behavior of spatial moments \((2)\) for different spatial scales \( \lambda_n \). The rainfall intensity is considered to exhibit spatial scale invariance at moment order \( q \) if the following relationship holds:

\[ M_n(q) \propto \lambda_n^{-\nu(q)} \lambda, \]  

in the limit as \( n \) approaches infinity. Therefore, for scale invariance to hold, the parameters \( \tau(q) \), called (multi-) scaling parameters, should not depend on the spatial scale \( \lambda_n \). For \( \tau(q) \) to be observed to be independent of spatial scale presupposes the existence of a finite scaling range between two scales referred to here as the smallest scale \( L_{\text{min}} \) and the largest scale \( L_{\text{max}} \). This approach enables us to estimate the scaling parameters from a single scene. The knowledge of \( \tau(q) \) should help us in developing and calibrating a downscaling procedure able to reproduce observed rainfall properties in different spatial scales.

b. Application to rainfall data

Each radar scene is analyzed separately. We consider a square region \((b = 4)\) within the scene. We had to first
select $L_{\text{min}}$ and $L_{\text{max}}$ for which the scaling law would be investigated. We used $L_{\text{min}} = 4 \text{ km} \ (4.3 \text{ km})$ for the GR (PR) data and $L_{\text{max}} = 64 \text{ km} \ (69 \text{ km})$ for the GR (PR) data. There are four regions per scan at the largest scale. Our choice of the largest scale was dictated by the fact that the largest square of contiguous data with $2^n$ pixels in the GR scan (excluding areas near the radar center and outside of the radar circle) is $2^9 = 64 \text{ km}$ (see Fig. 3) and the swath width of the PR is 215 km. Our choice of the smallest scale was dictated by the PR resolution, 4.3 km at nadir. There are three levels in between the smallest and largest scales, and the corresponding scale factors are 1, $\sqrt{2}$, $\sqrt{4}$, $\sqrt{8}$, and $\sqrt{16}$; $\lambda_0 = 1$ corresponds to $L = L_{\text{max}}$, and $\lambda_6 = \sqrt{64}$ corresponds to $L = L_{\text{min}}$.

For the PR rainfall product, it is not feasible to create a geometrically fixed largest box because the PR crosses the ground-radar coverage in different orientations with each overpass. Rather, we defined the PR’s largest box as the region with dimensions 128 km $\times$ 128 km that has the largest overlapping area with the GR’s largest box. Although the data are contiguous at the largest scale, we discarded the largest scale from the analysis to treat the data similar to the GR data.

We based our analysis on nearly contemporaneous and overlapping PR and GR scans and selected the GR scans that occurred at times closest to the TRMM overpasses. From this set of nearly coincidental PR and GR pairs, we exclusively selected the PR–GR pairs that satisfy the following two criteria: (i) the area overlap between the largest GR and PR boxes should be at least 1%, and (ii) the rain rate averaged over each larg-
est box should not be smaller than 0.001 mm h\(^{-1}\) (e.g., only one wet pixel within each largest box registering intensity greater than about 1 mm h\(^{-1}\) would qualify the scans). The latter criterion is necessary to perform scaling analysis on the individual scans. These criteria resulted in the following number of GR–PR scan pairs: 92 for Houston, 210 for Melbourne, and 84 for Kwajalein. The distribution of the number of these scans as a function of the overlapping area between the PR and GR largest boxes is presented in Fig. 4. About half of the cases have overlaps of less than 50%, and there are no cases where the overlap exceeds 80%. Houston has the largest overlapping area, with 20% of the cases having an overlap of 70%–80% compared to Melbourne and Kwajalein, which have negligible cases with this amount of overlap.

Our interest is in the scaling function \(\tau(q)\), particularly its parameters \(\tau(0)\), \(\tau(2)\), and \(\tau'(1)\). The intermittence scaling parameter \(\tau(0)\) is the fractal dimension of the support of \(\mu\) and measures the rate of growth of the fraction of the rainy areas with scale (Hentschel and Procaccia 1983). The second-order-moment scaling parameter \(\tau(2)\) measures the variability (in the second-order sense) of positive rain rate with scale within the rainy areas. From the scale-invariant behavior of the multiplicative cascade process, Holley and Waymire (1992) demonstrated that \(\tau(q)\) has to be a convex function of \(q\), that is, \(\tau'(q) \geq 0\). The parameter \(\tau'(1)\), which is the curvature of \(\tau(q)\) evaluated at \(q = 1\), indicates the rate of change in the variability of the positive rain rate within rainy areas. These parameters are often adequate to characterize \(\tau(q)\) across a large range of moment orders and hence may be the only parameters required to simulate scale-invariant fields. For example, the lognormal cascade model proposed by Over and Gupta (1996) requires an estimate of \(\tau'(1)\) to fully describe the scaling properties of positive rain rate, that is, \(\tau(q > 0)\). Modeling of the partition of the rainy and nonrainy areas at each scale requires only \(\tau(0)\). We have also limited our analysis to relatively low order moments because the uncertainty of estimation of \(\tau(q)\) increases quickly with moment order \(q\) (Troutman and Vecchia 1999). Using three different simulation models,
Troutman and Vecchia (1999) reported that the error in the estimate of $\tau(q)$, for $|q| \leq 2$, is within 10%.

Estimation begins with deriving rainfall maps from each scene at different spatial scales. The PR rainfall data are available at $L_{\text{min}}$ scale. We aggregated the pixels of the GR rainfall data to $4 \text{ km} \times 4 \text{ km}$ to acquire the data at $L_{\text{min}}$ and subsequently aggregated them to obtain the data at different spatial scales up until $L_{\text{max}}$. From each scene of data, we estimated $\tau(q)$ as a slope of the regression equation $\ln M_\text{r}(q)$ versus $\ln n$ obtained by log-transforming Eq. (3) and applying evenly weighted least squares regression. We estimated $\tau'(1)$ using numerical differentiation as $[\tau(1.5) + \tau(0.5)]/0.25$ and used a 95% confidence level to test the significance of difference to assess the comparison between the PR and GR results.

To get a sense of the scaling parameter values, consider the extreme cases of the single rainy pixel (i.e., only one rainy pixel at all scales) and the uniform measure (i.e., all pixels at all scales are rainy, and all pixels at a given scale receive exactly the same amount of rainfall). At the largest scale, $M_{\text{r}}(q = 0) = c$, where $c$ is the proportionality constant implicit in (3). If there is any rain, $c = 1$ since there is only one box at that scale. Consider the minimum and maximum values of $\tau(0)$. If $\tau(0) = 0$, then $M_{\text{r}}(0) = 1$ at all scales, and there is a single box with rain at each scale. If $\tau(0) = 2$, then $M_{\text{r}}(0) = \lambda_n^2$. Notice that $\lambda_n^2$ also represents the number of boxes at scale $\lambda_n$, so $\tau(0) = 2$ corresponds to rain everywhere. Therefore, $\tau(0)$ has the range $0 \leq \tau(0) \leq 2$, with increasing $\tau(0)$ indicating increasingly rainy areas. Consider now $\tau(2)$: $\tau(2) = 0$ implies the single rainy pixel case and $\tau(2) = -2$ implies the uniform rain field case. In general for $\tau(q)$, $q > 1$, $\tau(q)$ is bounded from above by zero, and this case represents the strongest possible intensities at each scale. The more negative $\tau(q)$ becomes (for $q > 1$), the less intense the rain gets at each smaller scale.

4. Results

a. Statistics of rain

We first examine the nonscaling statistical properties of the rain rate. Figure 5 illustrates the distribution of the rain rate $R$ at the smallest scale, conditional on $R = 0.1 \text{ mm h}^{-1}$. Light gray bars denote PR, and outlined bars (darker when not overlapping the light gray bars) denote GR. The bins are uniformly spaced in $\log R$ with spacing 0.5.

where the corresponding amount is 23% and 26%, respectively. Recall also that the Houston GV data has been processed differently than the other GV data. It may be that the Window Probability Matching Method (Melbourne, Kwajalein) generates significantly less weak rain rates than the power law $Z-R$ method (Houston).

Figure 5 further reveals that at all sites the large rain rates obtained from the PR are lower than those obtained from the GR. The underestimation is much more pronounced at Kwajalein than at the other sites. The underestimation of the large rain rates by the PR
may be attributed to the attenuation of the PR signals. Although there are attempts in the PR retrieval algorithm to correct for the effect of attenuation, the simultaneous occurrence of both attenuation and NUBF introduces large uncertainty in the correction algorithms (Iguchi et al. 2000).

Next we calculated the fractional coverage of the analyzed domain by rain as the fraction of the number of wet boxes at the smallest scale. As Fig. 6 shows, the PR underestimates the fractional coverage at Houston, while overestimating at Kwajalein. The underestimation at Houston is as expected and could be attributed to the presence of a significant quantity of light rain that was missed by the PR. Despite the fact that the PR misses the light rains at Melbourne and Kwajalein, the resulting fractional coverages are equal to or higher than those obtained from the GR. There must then be some other factors that not only compensate for the PR sensitivity but also result in an overestimation of the fractional coverage.

Table 1 provides values of the average rain rate and the average rain rate conditioned on positive rain rate for each site and sensor. As compared to the GR, the PR tends to give higher rain rates at Houston and Melbourne and lower rain rates at Kwajalein. In terms of the conditional rain rate, the PR tends to give higher values at Houston and lower values at the other sites, compared to the GR.

From the above discussion, the discrepancies between the PR and GR statistics exhibit geographical variability. This could be attributed to three different factors, and it is not presently clear which factor is responsible for this result. One factor could be the misalignment between the PR and GR largest boxes, which changes with site. Another factor could be the accuracy of the GR processing algorithm, as the algorithm applied at Houston is different from those applied at the other sites (see section 2b). The PR processing algorithm might also have errors that vary geographically.

### b. Scaling parameters

In Fig. 7, we show the distributions of the scaling parameters $\tau(0)$, $\tau(2)$, and $\tau'(1)$ derived from the PR and GR data for each site by the methods described in section 3 under the assumption of scale invariance. We test this assumption later on in section 4e. Table 2 provides results of a comparison of the distributions of these parameters using the Kolmogorov–Smirnov (K-S) goodness-of-fit test, in terms of the P value. We begin with the intermittence parameter $\tau(0)$. Application of the K-S test indicates that the distributions of $\tau(0)$ obtained from the PR and GR do not differ significantly from each other at a 95% confidence level at

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**Table 1.** Values of average rain rate ($\bar{R}$) and average rain rate conditioned on positive rain rate ($\bar{R}|R > 0$) for each site and sensor.

| Site     | Sensor | $\bar{R}$ (mm h$^{-1}$) | $\bar{R}|R > 0$ (mm h$^{-1}$) |
|----------|--------|--------------------------|-------------------------------|
| Houston  | PR     | 0.746                    | 5.579                         |
|          | GR     | 0.608                    | 4.044                         |
| Melbourne| PR     | 0.644                    | 4.760                         |
|          | GR     | 0.557                    | 5.267                         |
| Kwajalein| PR     | 0.276                    | 2.283                         |
|          | GR     | 0.360                    | 3.867                         |
Melbourne and Kwajalein, but they do differ significantly at Houston. From the median values (see Fig. 8), the most probable PR result typically underestimates $\tau(0)$ by 20% at Houston, while it gives almost exact values at the other sites. One main difference between Houston and the other sites is that the Houston GV radar recorded much of the light rain that went undetected by the PR, and this has an implication on the estimation of $\tau(0)$.

To quantitatively assess the effect of the PR sensitivity (its minimum detectable rain rate) on $\tau(0)$ estimates, we created a new rainfall dataset from the GR data by simply assigning zero to all of the pixels with rain rates below 0.5 mm h$^{-1}$ (hereafter we call this product GRTH). Figure 7a illustrates that thresholding (ignoring the light rains below 0.5 mm h$^{-1}$) results in underestimating $\tau(0)$, as expected [recall $\tau(0)$ for a fully rainy 2D field and 0 for a field with a single pixel of rain]. This thresholding operation makes the Houston GR product more like the PR; however, this does not happen at the other sites (although the distributions derived from the GRTH and PR are still not significantly different at a 95% confidence level). Thresholding the GR data at Melbourne and Kwajalein results in underestimating $\tau(0)$ by ~20% at Melbourne and Kwajalein, while giving almost exactly the same value at Houston (Fig. 8). This means that the PR rainfall fields are characterized by a faster decrease with decreasing scale in the area occupied by high rain rates at Melbourne and Kwajalein, which is consistent with the distribution of high rain rates at the smallest scale (see Fig. 5). The effect of thresholding on the $\tau(2)$ value is negligible at Melbourne and Kwajalein, while it tends to

### Table 2

<table>
<thead>
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<th>Parameter</th>
<th>Site</th>
<th>PR vs GR</th>
<th>PR vs GRTH</th>
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<tbody>
<tr>
<td>$\tau(0)$</td>
<td>Houston</td>
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<td>Melbourne</td>
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<td>0.52</td>
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<tr>
<td></td>
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<tr>
<td></td>
<td>Kwajalein</td>
<td>0.06</td>
<td>0.02</td>
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<tr>
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</tr>
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<tr>
<td></td>
<td>Kwajalein</td>
<td>0.01</td>
<td>0.27</td>
</tr>
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</table>

Fig. 7: Box plots for (a) $\tau(0)$, (b) $\tau(2)$, and (c) $\tau'(1)$, estimated for the sites of Houston, Melbourne, and Kwajalein GV sites. Light gray bars denote PR rainfall fields, and medium gray bars denote GR and GRTH rainfall fields. GRTH fields are obtained from GR scans by assigning no rain to all the pixels with rain rates below 0.5 mm h$^{-1}$. The boxes represent the interquartile range, from the 25th to the 75th percentile, and the line through this box represents the median. The “whiskers” extend from the box to the 5% and 95% percentiles.

Melbourne and Kwajalein, but they do differ significantly at Houston. From the median values (see Fig. 8), the most probable PR result typically underestimates $\tau(0)$ by 20% at Houston, while it gives almost exact values at the other sites. One main difference between Houston and the other sites is that the Houston GV radar recorded much of the light rain that went undetected by the PR, and this has an implication on the estimation of $\tau(0)$.

To quantitatively assess the effect of the PR sensitivity (its minimum detectable rain rate) on $\tau(0)$ estimates, we created a new rainfall dataset from the GR data by simply assigning zero to all of the pixels with rain rates below 0.5 mm h$^{-1}$ (hereafter we call this product GRTH). Figure 7a illustrates that thresholding (ignoring the light rains below 0.5 mm h$^{-1}$) results in underestimating $\tau(0)$, as expected [recall $\tau(0)$ for a fully rainy 2D field and 0 for a field with a single pixel of rain]. This thresholding operation makes the Houston GR product more like the PR; however, this does not happen at the other sites (although the distributions derived from the GRTH and PR are still not significantly different at a 95% confidence level). Thresholding the GR data at Melbourne and Kwajalein results in underestimating $\tau(0)$ as compared to the PR, suggesting that there are other causes of difference between the PR and GR data aside from the threshold affecting $\tau(0)$ (see the discussion above in section 4a).

As Fig. 7b and Table 2 reveal, there are no statistically significant differences between the $\tau(2)$ distributions derived from the PR and GR at any site, though at Melbourne and Kwajalein the differences are nearly so. The most probable PR result underestimates the GR-derived $\tau(2)$ by ~20% at Melbourne and Kwajalein, while giving almost exactly the same value at Houston (Fig. 8). This means that the PR rainfall fields are characterized by a faster decrease with decreasing scale in the area occupied by high rain rates at Melbourne and Kwajalein, which is consistent with the distribution of high rain rates at the smallest scale (see Fig. 5). The effect of thresholding on the $\tau(2)$ value is negligible at Melbourne and Kwajalein, while it tends to
result in overestimating $\tau(2)$ at Houston, although the resulting distribution is still not significantly different from the PR estimates.

The differences between the PR and GR $\tau'(1)$ distributions are significant, with the PR underestimating $\tau'(1)$ at all sites (see Fig. 7c and Table 2). The underestimation is quite large at Houston ($\sim 50\%$) compared to Melbourne ($\sim 20\%$) and Kwajalein ($\sim 35\%$). Recall that we calculated $\tau'(1)$ by numerical differentiation as $[\tau(1.5) + \tau(0.5)]/0.25$. Since the PR sensitivity and attenuation limitations tend to lower $\tau(0.5)$ and $\tau(1.5)$, respectively, they both work in the direction of underestimating $\tau'(1)$. Thresholding makes the GR data surprisingly more like the PR with respect to its $\tau'(1)$ distribution not only at Houston but also at the other sites. In fact, there are no statistically significant differences between the thresholded GR data and the PR data with respect to this parameter. It seems that the threshold applied at Melbourne and Kwajalein is somewhat "excessive" and tends to compensate for the underestimation of large rain rates in $\tau'(1)$ estimates.

c. Dependence of scaling parameters on large-scale average rain rate

In this section, we investigate the dependence of the scaling parameters on a large-scale observable, the spatially average rain rate ($\overline{R}$) at the largest scale $L_{\text{max}}$. The threshold method of rainfall estimation is based on the observation that $\overline{R}$ is strongly correlated with the fractional area above some threshold intensity (Doneaud et al. 1981, 1984; Kedem et al. 1990; Krajewski et al. 1992). Since $\tau(0)$ governs the rate at which the rainy area decreases with increasing resolution in a scaling construction, one expects a relationship between $\tau(0)$ and $\overline{R}$. Figure 9 shows scatterplots of estimated $\hat{\tau}(0)$ versus $\overline{R}$. Both the PR and GR data show that $\tau(0)$ depends strongly but simply on $\overline{R}$ in the same functional form. The functional relationship fitted in Fig. 9 has the form

$$\hat{\tau}(0) = s \ln \overline{R} + i. \quad (4)$$

To estimate the parameters $s$ and $i$, we applied the so-called geometric mean functional relationship (Draper and Smith 1998). This method is preferred to the standard least squares method when both the regressed and regressor variables are subject to errors, which is clearly the case here.

Also shown in Fig. 9 are the slope ($s$) and the intercept ($i$) parameter estimates as well as the correlation ($r$) between $\tau(0)$ and $\ln \overline{R}$. The PR matches the GR in terms of characterizing the intersite variability of the slope parameter and the correlation. We performed a bootstrapping experiment to assess the significance of the differences between the actual parameter estimates (Table 3) and found that the slope estimates derived from the PR and GR differ significantly from each other at a 95% confidence level, and the PR overestimates the slope consistently by 20% at all sites. In terms
of the intercept parameter estimates, the differences between the PR and GR are mostly significant and remain within 10%.

Over (1995, Fig. 4.26; see also Over and Gupta 1996) found a similar functional relationship between $\tau(0)$ and $\overline{R}$ using radar-derived rainfall from the Global Atmospheric Research Program (GARP) Atlantic Tropical Experiment (GATE). In terms of the parameters

Table 3. $\tau(0)$ vs $\overline{R}$ regression fits comparing the PR scans, the overpass GR scans (i.e., GR scans corresponding to TRMM overpasses), and the entire GR scans (GR ensemble). Listed are the best estimate and its confidence interval [2.5% quantile, 97.5% quantile].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Site</th>
<th>PR</th>
<th>GR overpass</th>
<th>GR ensemble</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>Houston</td>
<td>0.2185 [0.203, 0.240]</td>
<td>0.1816 [0.160, 0.197]</td>
<td>0.1776 [0.177, 0.179]</td>
</tr>
<tr>
<td></td>
<td>Melbourne</td>
<td>0.2275 [0.217, 0.241]</td>
<td>0.1925 [0.183, 0.205]</td>
<td>0.2055 [0.205, 0.206]</td>
</tr>
<tr>
<td></td>
<td>Kwajalein</td>
<td>0.2634 [0.249, 0.278]</td>
<td>0.2252 [0.197, 0.246]</td>
<td>0.1960 [0.195, 0.197]</td>
</tr>
</tbody>
</table>
defined in (4), he reported slope estimates of 0.2040 and 0.2064 for GATE phase I (28 June to 16 July 1974) and phase II (28 July to 15 August 1974), respectively. His intercept estimates were 1.5356 and 1.5104 for GATE phase I and phase II, respectively. The slopes and intercepts given by Over (1995) for the oceanic GATE region are therefore very much in line with the values we obtained for the oceanic Kwajalein site. Perica and Foufoula-Georgiou (1996) also found a strong correlation of a similar scaling parameter with the convective available potential energy (CAPE) in radar rainfall data from a squall-line storm in Oklahoma–Kansas. Perica and Foufoula-Georgiou’s explanation of the dependence on CAPE assumes positive correlation between $\overline{R}$ and CAPE.

Over (1995) also related the slope parameter to the number of levels $N$ between the largest scale at which scale invariance holds and the scale at which the probability distribution of rain rate is independent of $\overline{R}$. This relation follows analytically (see Over 1995 for details) from two observations: (a) the particular relation between $\tau(0)$ and $\overline{R}$ expressed in Eq. (4) and shown in Fig. 9, and (b) independence of the scaling of the positive rain rates, which is partially verified in Fig. 10. At larger scales, rain rates increase with $\overline{R}$, and at smaller scales, rain rates decrease with $\overline{R}$. Since the latter seems physically unlikely, this scale was interpreted by Over (1995) as the minimum scale at which scaling invariance can hold. Following Over’s approach, we found

![Fig. 10. Dependence of the estimated scaling parameter $\tau(1)$ on the spatial average rain rate $\overline{R}$ estimated from (left) PR and (right) GR scans, for (top) Houston, (middle) Melbourne, and (bottom) Kwajalein sites.](image-url)
For $s = 0.2$, (5) gives $N = 7$. Note that this is not a prediction of either the smallest or largest scales alone, but only the number of levels between them. To get the scales themselves, it is necessary to obtain one of them by independent means. If we assume the smallest scale ($L_{\text{min}}$) as 2 km (the typical size of a convective cell; Goldhirsh and Musiani 1986), the largest scale would be predicted to be about $L_{\text{min}} b^N = 2^N = 256$ km. The smallest and largest scales that we used in this study ($L_{\text{min}} \approx 4$ km, $L_{\text{max}} \approx 64$ km) therefore appear reasonable choices from a scale-invariance point of view.

How can the intercept $i$ parameter be interpreted? Notice that the fitted functional relationship assumes (and the data do not suggest otherwise) that there is a maximum spatial average rain rate $R_{\text{max}}$, which occurs where the fitted line intercepts $r(0) = 2$, and the scene is full of rain. Algebraic manipulation of Eq. (4) gives

$$i = 2 - s \ln R_{\text{max}}.$$  

The intercept parameter is thus related to the slope parameter and the maximum rain rate. For a given slope, the higher the maximum rain rate, the smaller the intercept becomes. It can be seen in Fig. 9 that the $R_{\text{max}}$ values observed here are approximately 10 mm h$^{-1}$, as also found by Over (1995) for GATE.

In Fig. 10, we show the dependence of $r'(1)$ on $\bar{R}$. Both the PR and GR data reveal a weak dependence of $r'(1)$ on $\bar{R}$. Over (1995) and Over and Gupta (1996) have also reported similar results for the GATE region.

d. Overpass sampling of scaling parameter distribution

Thus far, we have focused on the comparison of the PR and GR data taken at TRMM overpass times. In this section, we investigate the question: “Does the TRMM temporal sampling affect the distribution of the scaling parameter values and the dependence of $r(0)$ on $\bar{R}$?” In Fig. 11, we compare the $r(0)$ and $r'(1)$ distributions derived from the GR data corresponding to the TRMM overpass times (GR overpass) and the entire
GR data (GR ensemble). Aside from the sampling variability observed in the GR overpass results, the shapes of the two distributions are similar. In fact, application of K-S reveals that there are no significant differences between the distributions derived from the GR overpasses and the entire GR data at a 95% confidence level. Furthermore, Table 3 reveals that both datasets produce similar values of $\tau(0) - \bar{\tau}$ fit parameters. As evidenced by the [2.5%, 97.5%] quantiles of the parameter estimates obtained by a bootstrapping experiment, there are no significant differences between the fit parameters derived from the GR overpass and GR ensemble at a 95% confidence level. For a sample size of about one year, the sampling pattern of the TRMM PR can therefore adequately capture the scaling properties of rainfall. Our results show that the random samples used in this study are sampled from the same population.

e. Testing the scale-invariance hypothesis

In the previous sections, we have analyzed the data under the assumption of scale invariance. In this section, we test this hypothesis using the PR and GR data. Scale invariance implies that the relationship $\ln M_n(q)$ versus $\ln \lambda_n$ is linear in the limit as $n$ goes to infinity and approximately so for finite $n$. This means that the slope between adjacent (in scale) moment values is constant across scales. Deviations from scale invariance generally occur in a convex or concave pattern (Over 1995). Therefore, we constructed the following equation to test the null hypothesis that the slope between adjacent moment values is constant across scales by adding a term that allows the slope to increase or decrease with scale, thus having a convex or concave shape:

$$
\ln \left( \frac{M_{n+1}(q)}{M_n(q)} \right) = a(q) + c(q) \ln \lambda_n, \tag{7}
$$

where $a(q)$ and $c(q)$ are constants that depend on $q$ only. When $c(q)$ in (7) becomes zero, it means that the slope of $\ln M_n(q)$ versus $\ln \lambda_n$ is constant regardless of the cascade level $n$; hence, the process shows evidence of scale invariance. When $c(q) \neq 0$ it means that $\ln M_n(q)$ versus $\ln \lambda_n$ is nonlinear because its slope increases or decreases with $n$, and hence, the process shows evidence of the failure of scale invariance.

We tested the scale-invariance hypothesis at $q = 0, 2$ by fitting Eq. (7) using ordinary least squares regression. Figure 12 shows that the $c(0)$ and $c(2)$ values derived from the PR and GR datasets are well within $\pm 0.5$. Application of a K-S test reveals that there are no significant differences between the $c(0)$ and $c(2)$ distributions derived from the PR and GR data, except for the $c(0)$ distribution at Houston. Thresholding the GR data at Houston makes the resulting $c(0)$ distributions not significantly different from the PR results (not shown here). The deviation of $c(q)$ from zero could be attributed to two possible hypotheses. The first is that the deviation is statistical and arises as a result of the small number of points (cascade levels) used to estimate $c(q)$. In this case, the process could be scale invariant. The second hypothesis is that the process is not really scale invariant. Hypothesis testing for the significance of $c(q) \neq 0$ requires determining whether or not this is the result of the second hypothesis. To perform this test, the uncertainty bound due to the sampling error in $c(q)$ is desired.

We obtained sampling uncertainty bounds for $c(q)$ using simulated data from the Beta-Lognormal cascade model. This model, proposed by Over and Gupta (1996), consists of a cascade generator with a mixed distribution consisting of a continuous lognormal distribution and an atom at zero. This model, like all random cascades with scale-invariant generators, generates scale-invariant fields and has been used in the modeling of spatial rainfall rates (Gupta et al. 1996; Johtiyangkoon et al. 2000).

We generated two-dimensional cascade realizations down to level $n = 4$. To obtain more accurate simulations, we generated independent “mini-cascades” for each box at level 4, down to an additional three levels, and then aggregated the results back to level 4 [see Over and Gupta (1996) for further discussion on the simulation technique]. The Beta-Lognormal model requires two parameters: $\beta$ and $\sigma^2$. The $\beta$ parameter is related to $\tau(0)$ by $\beta = 1 - \tau(0)/2$. The $\sigma^2$ parameter is related to $\tau(q)$ by $\sigma^2 = \tau(q)/\ln b$. The $\tau(q)$ function of the Beta-Lognormal model is quadratic in $q$ (Over and Gupta 1996); hence, $\tau(q)$ is a constant and does not depend on $q$. We used $\tau(1)$ to estimate $\sigma^2$.

We calculated the set of $\beta - \sigma^2$ parameters from each group of scans appearing in Table 4 and made 100 independent cascade simulations for each pair. For each group we obtained the [2.5%, 97.5%] quantiles for $c(0)$ and $c(2)$. As Table 4 reveals, the confidence intervals are not symmetric around zero.

Using these confidence intervals, we present in Table 4 the fraction of scenes (%) for which the $c(0)$ and $c(2)$ values estimated from data are significantly less than zero, more than zero, and not significantly different from zero at a 95% confidence level. In both the PR and GR data, the majority (85%-95%) of the scenes are scale invariant at $q = 0, 2$. It can further be seen in the table by comparing the GR overpass (i.e., corresponding to TRMM overpass times) and the GR en-
semble results that the TRMM temporal sampling effectively captures the relative proportion of scale-invariant scenes.

Although the vast majority of the rainfall fields are scale invariant, there are also some fields that are non-scale invariant, particularly for the oceanic Kwajalein site. This suggests that the scale-invariance property is more likely to break down over ocean than over land.

**Table 4.** Fraction of scenes (%) for which \( c(0) \) and \( c(2) \) are not significantly different from zero [\( "c(0) = 0" \) and \( "c(2) = 0" \)], significantly less than zero [\( "c(0) < 0" \) and \( "c(2) < 0" \)], and greater than zero [\( "c(0) > 0" \) and \( "c(2) > 0" \)] at a 95% confidence level, estimated from different collections of the PR and GR scans. Also shown is the confidence interval for each group.

<table>
<thead>
<tr>
<th>Site</th>
<th>Snapshots</th>
<th>Confidence interval [2.5%, 97.5%]</th>
<th>( c(0) )</th>
<th>( c(2) )</th>
<th>( = 0 &lt; 0 &gt; 0 )</th>
<th>( = 0 &lt; 0 &gt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Houston</td>
<td>PR</td>
<td>(-0.45, 0.29) (-0.28, 0.43)</td>
<td>(87)</td>
<td>(3)</td>
<td>(10)</td>
<td>(92)</td>
</tr>
<tr>
<td></td>
<td>GR</td>
<td>(-0.48, 0.29) (-0.30, 0.40)</td>
<td>(97)</td>
<td>(2)</td>
<td>(1)</td>
<td>(96)</td>
</tr>
<tr>
<td></td>
<td>GRTTH</td>
<td>(-0.48, 0.31) (-0.29, 0.41)</td>
<td>(94)</td>
<td>(0)</td>
<td>(6)</td>
<td>(95)</td>
</tr>
<tr>
<td></td>
<td>GR ensemble</td>
<td>(-0.46, 0.28) (-0.28, 0.40)</td>
<td>(96)</td>
<td>(2)</td>
<td>(2)</td>
<td>(91)</td>
</tr>
<tr>
<td>Melbourne</td>
<td>PR</td>
<td>(-0.46, 0.29) (-0.26, 0.42)</td>
<td>(96)</td>
<td>(0)</td>
<td>(4)</td>
<td>(92)</td>
</tr>
<tr>
<td></td>
<td>GR</td>
<td>(-0.48, 0.30) (-0.29, 0.40)</td>
<td>(95)</td>
<td>(1)</td>
<td>(4)</td>
<td>(93)</td>
</tr>
<tr>
<td></td>
<td>GR ensemble</td>
<td>(-0.44, 0.35) (-0.31, 0.40)</td>
<td>(95)</td>
<td>(3)</td>
<td>(2)</td>
<td>(91)</td>
</tr>
<tr>
<td>Kwajalein</td>
<td>PR</td>
<td>(-0.47, 0.32) (-0.27, 0.40)</td>
<td>(95)</td>
<td>(1)</td>
<td>(4)</td>
<td>(95)</td>
</tr>
<tr>
<td></td>
<td>GR</td>
<td>(-0.48, 0.30) (-0.29, 0.41)</td>
<td>(86)</td>
<td>(1)</td>
<td>(13)</td>
<td>(93)</td>
</tr>
<tr>
<td></td>
<td>GR ensemble</td>
<td>(-0.48, 0.32) (-0.29, 0.43)</td>
<td>(92)</td>
<td>(0)</td>
<td>(8)</td>
<td>(94)</td>
</tr>
</tbody>
</table>

Fig. 12. Probability distribution functions of (left) \( c(0) \) and (right) \( c(2) \) estimates obtained from (solid line) GR and (dotted line) PR scans for (top) Houston, (middle) Melbourne, and (bottom) Kwajalein sites.
From the Kwajalein GR datasets, the mean large-scale rain rate and the mean fractional coverage of the analyzed domain by rain for the scale-invariant cases are 0.39 mm h\(^{-1}\) and 4.1%, respectively, while they are 0.077 mm h\(^{-1}\) and 10.3% for the non-scale-invariant cases. So the rainfall regimes that are not scale invariant are dominated by light storms that cover large areas. At \(q = 0\), scenes that are not scale invariant have predominantly positive values of \(c(0)\) regardless of the site. At \(q = 2\), scenes that are not scale invariant may have positive or negative values of \(c(2)\) depending on the site: Houston and Melbourne are characterized by \(c(2) > 0\), whereas Kwajalein is characterized by \(c(2) < 0\).

To understand these results, notice from (7) that \(c(q) > 0\) indicates that growth in the \(q\)th moment with decreasing scale \(\lambda_n\) is slower at small scales than at large, while \(c(q) < 0\) indicates the reverse. Therefore, \(c(0) > 0\) indicates that rainfall tends to occur more intermittently at small scales than it does at large. Such behavior would be modeled as an “unsmoothed” or “roughening” cascade at \(q = 0\) (i.e., the small-scale field is more intermittent than scale invariance would predict). The case \(c(2) > 0\) indicates that the higher rain rates at large scales tend to be concentrated in a few regions, while they are more uniformly distributed at smaller scales, which represents a smoothed cascade for the positive rain rates. Likewise, the case \(c(2) < 0\) represents an unsmoothed cascade for the positive rain rates.

Let us now consider the implications of the behavior of deviation from scale invariance observed in our data for modeling using random cascades. For the cases of \(c(0) > 0\) and \(c(2) > 0\), the situation could be modeled as a random cascade with the zero generator whose variance increases as scale decreases and the positive rain generator whose variance decreases as scale decreases. This underscores the importance of treating the rain/no-rain and the positive rain regions separately. Bounded models proposed in a series of papers (Menabde et al. 1997; Menabde 1998; Menabde and Sivapalan 2001) can be used to generate smooth cascades at \(q > 0\). The Universal Multifractal Model of Schertzer and Lovejoy (see Tessier et al. 1993, and references therein) can be used to generate either smoothed or roughening cascades at \(q > 0\). But in neither case are models with zero rain rates presented. Our results demonstrate the need to develop cascade models that generate smooth/roughening cascades at \(q = 0\).

5. Conclusions

In this study, we have compared the statistical and scaling characteristics of rainfall derived from both the PR and GR standard rainfall products at three TRMM GV sites: Houston (Texas), Melbourne (Florida), and Kwajalein Atoll islands. Houston and Melbourne are located on the eastern Atlantic coast in the subtropics, whereas Kwajalein is located in the deep Tropics and is nearly 100% oceanic. The PR rainfall products correspond to version-5 2A25 near-surface rainfall rate, while the GR rainfall products correspond to version 5 for Melbourne and Kwajalein and version 4 for Houston. For the GR products, the version-5 algorithm uses the Window Probability Matching Method, while version 4 uses Z–R power-law relations. We have considered spatial scales ranging from about 4 to 64 km at a fixed temporal scale corresponding to the sensor “instantaneous” snapshots. Our focus has been on the scaling of the spatial moments, described by the function \(\tau(q)\), computed from each scene of data separately. The main findings of our study may be summarized as follows.

There is a consistent pattern at all sites in which the PR misses the majority of light rains, which may be explained by the detection threshold of the PR. The effect is more pronounced at Houston, where 47% of the wet pixels in the GR data receive rain rates below 0.5 mm h\(^{-1}\), as compared to Melbourne and Kwajalein, where the corresponding amount is 23% and 26%, respectively. It may be that the window probability matching method (Melbourne, Kwajalein) generates significantly less weak rain rates than the power law Z–R method (Houston). The PR underestimates the rainfall intermittency scaling parameter \(\tau(0)\) by 20% at Houston, while it gives almost exact values at the other sites.

The PR underestimates the second-order rain-rate variability scaling measure \(\tau(2)\) by ~20% at Melbourne and Kwajalein, while giving almost exactly the same value at Houston. This means that the PR rainfall fields are characterized by a faster decrease with decreasing scale in the area occupied by high rain rates at Melbourne and Kwajalein.

The PR underestimates the curvature of the \(\tau(q)\) function, \(\tau'(1)\), at all sites. The underestimation ranges from 20% to 50%. Both the PR sensitivity and attenuation limitations work in the direction of underestimating \(\tau'(1)\). The implications are that high rain-rate extremes would be missed in a downscaling procedure (and apparently were missed in the PR observations).

Both the PR and GR data show that there is a one-to-one relationship between \(\tau(0)\) and the large-scale spatial average rain rate \(\bar{R}: \tau(0) = s \ln \bar{R} + t\). The PR matches the GR in terms of characterizing the intersite variability of the slope parameter. The PR overestimates the \(s\) parameter by 20% at all sites, while it gives
estimates of the $i$ parameter to within ±10% accuracy. The slopes and intercepts given by Over (1995) for the oceanic GATE region are very much in line with the values we obtained for the oceanic Kwajalein site. Both the PR and GR data reveal a weak dependence of $\tau(1)$ on $\bar{R}$.

For a sample size of about one year of TRMM data, the TRMM temporal sampling does not significantly affect the ability of the TRMM PR to capture the overall distribution of the scaling characteristics of rainfall.

The majority of the scenes (85%–95%) are scale invariant at the moment order $q = 0, 2$. The scale-invariance property tends to break down more likely over ocean than over coast. The rainfall regimes that are not scale invariant are dominated by light storms that cover large areas. At $q = 0$, the deviation from scale invariance is mainly due to the increasing intermittency of rainfall at small scales as opposed to at large ones, at all sites. At $q = 2$, the deviation from scale invariance over the coastal sites is such that the higher rain rates at large scales tend to be concentrated in a few regions while they are more uniformly distributed at smaller scales, which represents a smoothed cascade for the positive rain rates. Whereas for the oceanic site, the deviation from scale invariance at $q = 2$ represents an unsmoothed cascade for the positive rain rates.

In general, using the TRMM GV sites as the “ground truth,” our results demonstrate that the TRMM PR has the ability to characterize the scaling properties of rainfall, although the resulting parameters will differ to some degree. Further investigation is needed to determine if this difference is due to PR errors or perhaps also due to GR errors. This paper is a starting point toward characterizing the scaling characteristics of rainfall across the Tropics based on the TRMM PR data. We are currently working on this, and our results will be published in a subsequent article.

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