A Spatial and Temporal Energetics Analysis of a Baroclinic Disturbance in the Mediterranean

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ABSTRACT

An analysis of the energetics of an extratropical baroclinic depression is performed. The approach is based on the computation of the components of an energy flow for an open atmospheric region in which this disturbance is the major synoptic-scale feature. The local fields of motion and mass are partitioned into zonal and eddy terms, so that the resulting four energy forms are the zonal and eddy terms, so that the resulting four energy forms are the zonal and eddy components of the available potential and kinetic energies. The energy transformations, which can be considered to express well-defined physical processes and which are stable from a computational point of view, the diabatic generation of available potential energy, and the frictional dissipation of kinetic energy all determine a composite energy flow over the region. Interactions with the surrounding atmospheric volume are also taken into account in the form of energy transfers. Energies and their transformations and transfers are computed using the isobaric height, temperature, and wind analyses of the European Centre for Medium-Range Weather Forecasts. The generation of available potential energy is estimated as the residual required to maintain the energy balance.

This case study refers to the evolution of a central Mediterranean depression formed during the period from 1200 UTC 7 November to 1200 UTC 9 November 1981. The cyclone formed on an incipient quasi-stationary front over the central Mediterranean. Although its life cycle lasted about 3 days, its rapid development was associated with severe weather conditions over the central and eastern Mediterranean region.

The temporal and spatial variations of various components of the energy flow are presented in a detailed analysis. The energy contents and their changes in different atmospheric layers are discussed in the course of the cyclone’s development. From the analysis of the contribution of different layers to the energy conversions, it is shown that not only the intensity of the conversions but also their direction may vary greatly both in the horizontal and the vertical. The role of different layers in importing or exporting energy is also discussed. Integrating over the entire volume of the computational area, the complete energy flow of the cyclone is obtained and discussed within the framework of the available information on cyclone energetics.

1. Introduction

The energetics of the atmosphere have been studied by many investigators during the last three decades. A great effort has been made to identify and quantify the various forms of atmospheric energy and their transformations on the global scale. The energetics of the general circulation have been studied extensively and have been reviewed and summarized by Oort (1964), Wiin-Nielsen (1968), and Saltzman (1970). This effort has been coupled by an outstanding endeavor to quantify the role of midlatitude synoptic disturbances in the energy budget of the atmosphere. In this respect, this great interest is characterized by the variety of aspects studied and the diversity of approaches used. The kinetic energy budget of midlatitude cyclonic systems has received much attention (e.g., Palmen 1958; Sechrist and Dutton 1970; Petterssen and Smeybe 1971; Smith 1973a,b; Vincent and Chang 1975; Kung and Baker 1975; Ward and Smith 1976; Chien and Smith 1977; Kung 1977; Fuebreg and Scoggins 1978; Robertson and Smith 1980; Michaelides 1983). The available potential energy budget of such systems has received considerably less attention (Danard 1966; Widger 1969; Bullock and Johnson 1971; Vincent et al. 1977; Michaelides and Angouridakis 1980; Fuebreg et al. 1985). Studies that are upgraded in complexity by simultaneously considering the combined budgets of two or more energy forms are more revealing of the contribution of the synoptic-scale disturbances to the global energy budget (see Robertson and Smith 1983; Smith and Dare 1986; Michaelides 1987; Prezerakos and Michaelides 1989). This latter approach determines the framework within which the energetics of a Mediterranean cyclone will be studied in this article.

Extending Margules' (1903) ideas, Lorenz (1955) introduced the concept of available potential energy (APE) in atmospheric science. This has been defined

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as the fraction of the total potential energy (i.e., potential plus internal energies) that can be energetically active by an adiabatic redistribution of the mass of the atmosphere. Kinetic energy (KE) is released by an adiabatic transformation process at the expense of APE. Contributions to the generation of APE come mainly from diabatic sources such as heating by radiation, sensible heat addition, and latent heat release. The physical process responsible for the dissipation of KE is friction. The diabatic generation of APE, the conversion of APE into KE by reversible adiabatic processes, and the dissipation of KE by frictional processes determine a basic energy flow in the atmosphere.

The original mathematical formulations of the components of the above energy flow have been approximated by Lorenz (1955, 1967). Different approximations have been developed by Dutton and Johnson (1967). In addition, the concept of APE has been redefined by other leading investigators of atmospheric energetics (Van Mieghem 1973; Pearce 1978). Also, Robertson and Smith (1983) presented an eddy equation based on the so-called "exact" formulation that they then used in a cyclone diagnosis. However, Lorenz's expressions seem to be the only complete set of relationships describing the energy flow in an analysis involving zonal and eddy energy partitioning.

The present study makes use of Lorenz's approach for the energetics analysis of a midlatitude baroclinic disturbance and is one of a series investigating various aspects of the energetics of Mediterranean cyclogenesis (see also Michaelides and Angouridakis 1980; Michaelides 1983, 1987; Prezerakos and Michaelides 1989).

2. The energy budget of open atmospheric systems

Midlatitude baroclinic disturbances have long been treated as eddying motions evolving in a zonal current. The partition of energies into zonal and eddy components leads to a reformulation of the energy flow. The set of differential equations (with respect to time) describing the large-scale atmospheric energy balance of an open atmospheric system takes the form

\[
\begin{align*}
\dot{A}_Z &= C(K_Z \rightarrow A_Z) + C(A_E \rightarrow A_Z) + \text{BA}_Z + G_Z \\
\dot{K}_Z &= C(K_E \rightarrow K_Z) + C(A_Z \rightarrow K_Z) \\
&\quad + \text{BK}_Z + \text{B}\Phi_Z + D_Z \\
\dot{A}_E &= C(A_Z \rightarrow A_E) + C(K_E \rightarrow A_E) + \text{BA}_E + G_E \\
\dot{K}_E &= C(K_Z \rightarrow K_E) + C(A_E \rightarrow K_E) \\
&\quad + \text{BK}_E + \text{B}\Phi_E + D_E. \quad (1)
\end{align*}
\]

The energy flow is shown graphically in Fig. 1 and the approximate mathematical expressions of its components are given in the Appendix.

The zonal kinetic energy \(K_Z\) corresponds to the zonally averaged motion, and the zonal available potential energy \(A_Z\) corresponds to the zonally averaged mass field. The amounts of KE and APE left are termed eddy kinetic energy \(K_E\) and eddy available potential energy \(A_E\), respectively. The generation of APE is resolved into generation of \(A_Z(G_Z)\) through heating at warmer and cooling at colder latitudes and generation of \(A_E(G_E)\) through heating of warmer and cooling of colder regions at the same latitude. Also, the dissipation of KE by friction is resolved into dissipation of \(K_Z(D_Z)\) and dissipation of \(K_E(D_E)\).

A conversion between energies \(X\) and \(Y\) is denoted by \(C(X \rightarrow Y)\); the direction of conversion is denoted accordingly [e.g., \(C(X \rightarrow Y)\) is the conversion of \(X\) into \(Y\)]. The transformation between \(A_Z\) and \(K_Z\) is achieved through the thermally driven mean meridional circulations. The transformation between \(A_E\) and \(K_E\) is achieved through the thermally driven circulations that act in the longitudinal sense. The eddy transport of sensible heat is the process responsible for the transformation between APE forms. Finally, the transformation between KE forms is accomplished by the eddy transport of momentum.

The above four energy forms with their transformations, the two APE generation terms, and the two KE dissipation terms define an energy flow sufficient to describe the atmospheric energetics of a closed system, provided no lateral boundaries are considered and vertical velocities at the bottom and top of the system are zero. However, in open atmospheric systems, several transformation terms are mathematically possible (Lettau 1954). Nevertheless, Smith (1970) states that a sufficient condition for retaining a transformation is that it establishes a direct physical relationship between the two energy forms involved. The four energy-transformation terms described above meet this require-
ment. Also, in open systems, the specification of lateral boundaries implies that nonzero boundary transports of energy must be taken into account (Muench 1965). These transport processes for $A_Z$, $A_E$, $K_Z$, and $K_E$ are denoted by $BA_Z$, $BA_E$, $BK_Z$, and $BK_E$, respectively. Another process that may lead to changes in $K_Z$ and $K_E$ is that of mechanical production of KE by pressure work at the boundaries at the expense of the KE of the surrounding volume, denoted by $B\Phi_Z$ and $B\Phi_E$, respectively.

The expressions for APE contents and their conversions and boundary transfers involve the area (isobaric) average of a static stability parameter. The expression for the static stability developed and used in this context conforms with the original considerations by Lorenz (1955, 1967). The present approximate expressions make direct use of the observed meteorological fields and can overcome difficulties in determining APE in regions with a temperature lapse rate approaching its dry-adiabatic value. The assumption of an isobarically constant static stability parameter has been criticized by Dutton and Johnson (1967), who argue that the importance of a variable static stability should not be ignored in diagnostic studies. Indeed, contributions to APE contents from layers with a near dry-adiabatic environmental lapse rate can be considerably larger than those estimated using a uniform static stability parameter. Local temperature profiles reveal such neutrally stratified layers near the earth’s surface and in medium and upper-tropospheric layers under conditions of prolonged subsidence. However, such layers have a very limited depth and should be undetected with the currently used vertical resolution of the temperature field. Also, if heating occurs in less stable air (as it often does in the area of cyclones), the generation of eddy APE will be enhanced. This fact does not alter the present estimates of eddy generation of APE, because this is estimated as the residual to the respective energy-balance relationship. Bearing in mind the above, the use of an isobarically averaged static stability factor remains a practical alternative.

At this point, it is considered necessary to clarify a fundamental issue regarding the application of the concept of APE to limited-area energetics. As stated by Lorenz (1967), APE is not defined within a particular column of the atmosphere. Consequently, the amount of APE computed within an open atmospheric region does not strictly determine the APE of the region, but it constitutes its contribution to the APE of the global atmosphere. Therefore, the quantities of APE calculated in the present context do not define the APE of the system but simply represent the fraction of the global APE, as defined by Lorenz, contained in the respective atmospheric volume. Smith (1969) and Johnson (1970) give the theoretical basis for a meaningful extension of the theory of APE to open systems.

In the present study, the Eulerian framework used to define the boundaries has been chosen carefully so that the cyclonic system under consideration always remains the predominant synoptic feature.

3. Data and computational procedures

For the computations presented here, a uniform grid is used bounded by meridians 0° and 40°E and by latitude circles 25° and 55°N; the gridpoint distance is $\delta \varphi = \delta \lambda = 5^\circ$ in both the meridional and longitudinal directions, respectively. Computations in this study refer to a somewhat smaller area, because a centered finite-difference scheme has been adopted to approximate space derivatives on isobaric surfaces. This area is bounded by meridians 2.5° and 37.5°E and by latitude circles 27.5° and 52.5°N.

Although the size of the domain attending the surface and upper wave is comparable between various studies, an important point of difference is the gridpoint distance employed. Gridpoint distance ranges from around 1.5° (Petterssen and Smeybe 1971; Robertson and Smith 1983; Smith and Dare 1986; Michaelides 1983; Prezerakos and Michaelides 1989) to the currently used 5° (see also Kung and Baker 1975; Kung 1977; Michaelides 1987). The latter gridpoint distance is comparable to that commonly used in global energetics studies. Since the gridpoint distance pertains to the definition of the eddy components, the gridpoint distance employed here is considered to be appropriate in order to place the discussion of cyclone energetics within the framework of global energetics carried out on the basis of the energy flow of Fig. 1.

Nine isobaric surfaces are considered, namely those of 1000, 850, 700, 500, 400, 300, 250, 200, and 150 hPa. These surfaces specify eight sublayers of nonuniform depth that are used for the numerical evaluation of volume and surface energy integrals. At each gridpoint and for each of the above surfaces, data have been retrieved from the European Centre for Medium-Range Weather Forecasts (ECMWF) by using a package from the Department of Meteorology of the University of Reading, United Kingdom. These data refer to all the 0000 and 1200 UTC synoptic times, starting with 1200 UTC 7 November 1981 and ending with 1200 UTC 9 November 1981. The variables required for the computation of the energy-flow components are the temperature, the eastward and northward wind components, and the geopotential heights.

The results obtained in the present analysis are the outcome of the computational manipulation of the specific set of atmospheric data employed here. Therefore, it should be expected that these results are influenced by the various characteristics of the dataset upon which the energetics calculations have been based. In a recent energetics analysis of the general circulation, Kung and Tanaka (1983) have shown that the origin
of the dataset influences the computations profoundly. In particular, they were able to establish that the energy cycles produced using data from different sources may present noticeable differences. Such differences are attributed to the fundamental differences between the analyses used to prepare the datasets. The extent to which data from global circulation models exercise their influence on energetics calculations must depend on two aspects: first, the original observed meteorological data, and second, the techniques employed in the assimilation of these observed data to reach their final form utilized in global modeling. It is therefore necessary to consider such inherent characteristics of the analyses used here.

Observed meteorological data originate from a variety of synoptic and nonsynoptic observing systems, as described by Bengtsson et al. (1982). Also, these authors give an account of the four-dimensional data-assimilation system used at ECMWF, which consists of a three-dimensional multivariate optimum interpolation and nonlinear normal-mode initialization. Retaining the meteorologically significant structures of the fields is a very important aim of the assimilation scheme used. Collected data are assimilated in 6-h time periods assuming a geostrophic-type balance. The subsequent application of the nonlinear normal-mode initialization scheme leads to a realistic presentation of initial divergences and pressure tendencies, at least at high latitudes.

Vertical velocities were computed from the divergence of the horizontal wind field and were further adjusted at each gridpoint by an error-sharing procedure, assuming no vertical motions at 1000 and 150 hPa (see O'Brien 1970). Vertical motions derived from divergences appear to be more consistent to use in budget studies than the initialized vertical velocities from the ECMWF analyses (Wash et al. 1988). No smoothing of the kinematically computed vertical-motion field was attempted because such smoothing is thought to lead to an underestimate of the intensity of the energy-transformation processes (Dutton and Johnson 1967).

The computation of the energy contents and transformations requires the numerical solution of the respective volume integrals. Also, the computation of the energy transports requires the numerical solution of the respective surface integrals. Details of the numerical procedures and the form of the numerical analogs used are given in the Appendix.

The energy contents, transformations, and transports have been calculated over the area studied and for each sublayer. Totals of the respective integrals within the layers 1000–700, 700–300, and 300–150 hPa have subsequently been used in the discussion of the energetics of different atmospheric layers. Totaled over the depth of the computational area, from 1000 to 150 hPa, these quantities are then used to discuss the cyclone energy balance as determined by Eq. (1). In this energy balance, tendencies of the energy contents have been approximated as time differences at consecutive times, and energy transformations and boundary fluxes have been estimated as averages of the respective quantities at consecutive times. The $A_z$ and $A_E$ generation terms have been estimated as residuals to the respective APE balance relationships.

The pressure work terms $B_{\Phi Z}$ and $B_{\Phi E}$ were found to obtain unrealistically large values. This behavior has also been encountered elsewhere and is attributed to the magnifying effect that small deviations in the geopotential field have on the mechanical production of KE (see Muench 1965; Brennan and Vincent 1980; Michaelides 1987). As mentioned previously, an energy transformation is retainable in an energy flow if it represents a direct physical relationship. Although $B_{\Phi Z}$ and $B_{\Phi E}$ satisfy this condition, they cannot be retained in a meaningful evaluation of the energy balance because they are computationally unstable. Therefore, they were combined with the dissipation terms, $D_z$ and $D_E$, into one zonal and one eddy residual term denoted by $R_z (= D_z + B_{\Phi Z})$ and $R_E (= D_E + B_{\Phi E})$, respectively. These residuals contain not only the frictional dissipation of KE and the pressure work terms but also any computational errors due to scale transfers.

Bearing the above in mind, a schematic representation of the balance of large-scale energy is shown in Fig. 1. In this energy balance, the only energy-transformation terms retained are those terms that express a well-defined physical process and are stable from a computational point of view.

With a suitable reformulation of the energy integrals, values of the energy transformations have been derived within atmospheric columns of $\delta \lambda \times \delta \phi$ horizontal cross-sectional area. Such values, totaled over the layers 700–300 and 300–150 hPa separately, have been assigned to the respective central grid point of each $\delta \lambda \times \delta \phi$ area and are used for the construction of the spatial distributions of energy transformations.

4. Synoptic summary

Wintertime surface cyclogenesis in the Mediterranean is very frequent. However, the occurrence of cyclogenesis appears to have preference for certain localities (see Petterssen 1956; HMSO 1962; Reiter 1975). The frequency of cyclogenesis over the central Mediterranean is very low, and a rather rare case of cyclone initiation over this area is studied here. The dynamical characteristics of a similar case of cyclogenesis over this region have been discussed by Dent and Mason (1972). Here a brief summary of the synoptic event is given. The evolution of the system can be followed in the chart sequence of Fig. 2, in which the 500- and 1000-hPa ECMWF analyses are depicted. The
FIG. 2. Chart sequence depicting the synoptic evolution: ECMWF analyses of the 500-hPa (continuous lines drawn for every 6 dam) and 1000-hPa (dashed lines drawn for every 2 dam) isobaric surfaces. The positions of the fronts and the cyclone center (filled circles) are derived from subjectively analyzed surface weather charts.
frontal positions have been taken from subjectively analyzed mean sea level weather charts.

The transfer of colder air over the central and eastern Mediterranean over a period of a few days results in the establishment of a weak quasi-stationary front seen in Fig. 2 for 1200 UTC 7 November 1981. This front separates the colder and drier air to the north and the warmer and moister Mediterranean air to the south. Some rain showers and very isolated thunderstorms are reported over the northern Aegean Sea and in the warm air mass. Within the cold air mass, scattered snow showers are reported. In the upper levels, a marked trough moved from the northeastern Atlantic over central Europe. The movement of the trough enhances the advection of colder air over the central and western Mediterranean. Also, an area of cyclonic vorticity is advected by the upper wind field and is superimposed on the preexisting surface frontal zone. The conditions become favorable for the baroclinic development of a cyclonic disturbance, as discussed by Petterssen et al. (1955).

At 0000 UTC 8 November, the formation of a surface cyclone is evident. The warm-sector showers and thunderstorms become widespread and extend as far south as the North African coast. In the middle and upper troposphere, the flow is dominated by a well-developed elliptical vortex with a northeast-southwest-oriented trough axis. Ahead of this trough, the flow supports the northeastern advection of warm air and the further extension of the thermal ridge, which on the surface analysis is manifested as a northward-propagating warm front.

At 1200 UTC 8 November, the surface synoptic situation is dominated by a well-developed frontal depression associated with extensive cloudiness, heavy showers, and widespread thunder activity. Most of this activity is confined near and around the center of the cyclone. In the upper levels, the cold trough extends well over the Mediterranean Sea and its axis maintains its original northeast-southwest orientation.

In the next 48 h, the cyclone followed a northeast track and deepened further. At 0000 UTC 9 November, the center of the low is found over southern Greece. The associated cold front advanced farther east with a simultaneous advancement of the warm front to the north. Precipitation is less common and consists of rain in the warm sector and snow in the colder air mass. Some showers are still reported near the center of the depression. In the upper levels, there is a substantial change in the contour pattern. The extension of the trough to the south ceases and, simultaneously, the tilting of its axis changes to north–south.

In the next 12 h, the depression moves rapidly to the northeast, so at 1200 UTC 9 November its center is found over the western Black Sea. Light or continuous rain is reported in the warm air, while in the cold air precipitation falls mainly as snow.

### 5. Results

The discussion in this section starts with the presentation of the vertical totals for the four energy forms and the energy budgets over the computational region. The discussion on energy conversions and transports is further elaborated by considering three separate atmospheric layers that were found to exhibit a discrete behavior. These layers are:

1. the lower 1000–700-hPa layer, which, due to the elevated terrain surrounding the Mediterranean basin, is considered to comprise the tropospheric boundary layer;
2. the intermediate 700–300-hPa layer, which comprises the rest of the troposphere and is the most energetically active layer; and
3. the upper 300–150-hPa thermally stratified layer, which proved to be energetically activated during the development of the cyclone.

#### a. Energy contents and energy balance

The vertical totals for the four forms of atmospheric energy are shown in Table 1. The APE budget of mid-latitude cyclonic systems has received less attention compared to the volume of investigations on KE (see Smith 1980). Using the limited literature available, the presently calculated APE contents are compatible with results from other midlatitude cyclone case studies (see Michaelides and Angouridakis 1980; Smith 1980; Smith and Dare 1986; Michaelides 1987). Also, the KE contents calculated here are typical of well-developed midlatitude depressions (see Petterssen and Smebye 1971; Smith 1973a; Ward and Smith 1976; Michaelides and Angouridakis 1980; Robertson and Smith 1980; Fuelberg and Scoggin 1978; Michaelides 1983; Prezerakos and Michaelides 1989).

The energy balance during the evolution of the cyclone is shown in Fig. 3. The values (W m⁻²) in this figure refer to the components of the energy-flow diagram in Fig. 1.

The small reduction in $A_Z$ calculated in the first period is followed by an energy increase with a maximum in the period immediately after the onset of cyclogenesis. On one hand, the generation term $G_Z$ appears to

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be the major input of energy to the system. The role of this generation becomes increasingly important as the system develops, indicating that the $A_Z$ reservoir is continuously replenished through an intense diabatic generation on the local scale. The extensive cloud formations and precipitation soon after the initiation of the cyclone imply that the latent heat release, taking place primarily in the warmer air to the south, could be a major diabatic factor leading to the generation of $A_Z$. On the other hand, there exists a continuous depletion of the $A_Z$ reservoir through a transfer of zonal APE outside the region. This energy transfer is quite intense during the first period and decreases afterwards. This implies that the cyclone acted as an $A_Z$-exporting synoptic eddy.

The eddy APE presents an increase in the first period and a decrease in the second and third periods. In the final stages, it increases sharply at a rate of 3.29 W m$^{-2}$. The respective generation term, namely $G_E$, determines an energy input at all times, but its role in the local generation of APE is considerably less important than that of $G_Z$. On the global scale, $G_E$ appears to operate either as an APE generation process (see Newell et al. 1970) or as an APE destruction process, especially during the winter (see Krueger et al. 1965; Tomatsu 1979). However, Dutton and Johnson (1967) state that in a disturbance, all of the diabatic components can serve to generate $A_E$ and, moreover, to reinforce each other. In the first stages of the presently studied depression, latent heat release occurs primarily in the warmer air leading to local generation of $A_E$. The advection of colder air over the warmer Mediterranean waters results in a rapid destabilization of the air leading to local APE generation. Tantawy and Owais (1972), in a study of another Mediterranean depression, estimated that the local generation of APE due to sensible heat transfer from the sea surface to the overlying air amounts to 3 W m$^{-2}$. Fuelberg et al. (1985) studied a frontal depression over North America and estimated that the generation of APE by sensible heat transfer from the land surface to the warm-sector air mass reaches 0.2 W m$^{-2}$ in the afternoon.

Calculations of APE generation in cyclonic disturbances have been presented by many investigators. Although differences exist between various studies regarding the magnitude of this generation, the predominant role of latent heat liberation in generating APE is widely acknowledged (see Danard 1966; Bullock and Johnson 1971; Vincent et al. 1977; Lin and Smith 1982). On the contrary, the role of longwave radiation emission in the local generation of eddy APE is not clear (see Widger 1969; Lin and Smith 1979). The two APE generation terms calculated in this study have been derived as the residuals required to balance the respective energy budgets. These indirect estimates of APE generation are much greater than the direct estimates obtained for other extratropical cyclones. In particular, the highest rates of APE generation due to latent heat release calculated by Danard (1966) or due to latent heat release and infrared radiation emission

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calculated by Bullock and Johnson (1971) and by Lin and Smith (1982) are all one order of magnitude smaller than the present estimates. The maximum rate of 21 W m\(^{-2}\), due to latent heat and infrared radiation of Vincent et al. (1977), is comparable to the presently derived estimates, but it merely amounts to one-third of the present maximum of 65 W m\(^{-2}\) in the last stages of development. Bearing in mind the aforementioned, it appears that the residual technique applied here overestimates the generation of APE.

The transfer term BA\(_E\) acts as a source of eddy APE in all periods but the last one. Starting with a rate of 3.02 W m\(^{-2}\) in the first period, the contribution of BA\(_E\) to the A\(_E\) budget decreases as the system develops and is even reversed in the last period with a small amount of A\(_E\) being exported.

During the first two periods, there is a conversion of A\(_E\) into A\(_Z\). The direction of energy conversion changes in the following two periods. These results indicate that in the first stages of the cyclone development, the eddy flux of heat is against the temperature gradient, thus enhancing the meridional temperature contrast and therefore the east–west-oriented frontal zone; as the system matures, however, the flux reverses its performance by transferring heat in the direction of the temperature gradient, thus reducing the meridional temperature contrast and enhancing the eddy component of APE.

The changes in K\(_Z\) are positive during all periods, thus implying a continuous increase in the zonal motion field. In the first stages of the development of the system, there is an import of K\(_Z\). The possibility of KE import in the cyclone area, especially in its initial stages, has been stressed by Palmen and Newton (1969). The well-formed cyclone is associated with a marked export of K\(_Z\). Also, the residual term R\(_Z\) operates as a significant sink of K\(_Z\) at all times. Its maximum value 39.20 W m\(^{-2}\) is recorded in the last period.

The coupling of sinking at colder latitudes and rising at warmer latitudes appears to be a very effective mechanism for production of K\(_Z\) at the expense of A\(_Z\) in the area of the cyclone. However, the uncertainty regarding the direction in which this process operates on the global scale (see Wiin-Nielsen 1965) appears to extend to the regional scale as well. Indeed, comparing the present results to those of another case study infers that this conversion may operate in either direction during the evolution of a midlatitude depression (see Michaelides 1987).

The formation of the surface cyclone is accompanied by a maximum in eddy KE increase of 6 W m\(^{-2}\). This is followed by a smaller increase in the second period. In the third period, however, the eddy KE exhibits a marked decrease at a rate of 4.1 W m\(^{-2}\). Large rates of eddy KE decrease, soon after the formation of the surface cyclone, have also been noted in a similar case study (Michaelides 1987). Such a decrease during the development of the system can be explained by a general weakening of the wind field or by a reduction of the wave amplitude. Bearing in mind that this decrease in eddy KE is more pronounced in the intermediate and upper layers (not shown), it would appear to be related to a subtle east–west broadening of the upper-air trough.

During the life cycle of the cyclone, eddy KE is continuously transferred outside the region. More intense rates of this transfer occur in the second and third periods, implying that the deepening of the depression is associated with a significant supply of eddy KE to the region surrounding the synoptic system. The residual term R\(_E\) also implies a continuous sink of eddy KE, with more intense rates in the second and third periods.

Zonal KE is converted into eddy KE at all times. The highest rate of this conversion, 14.28 W m\(^{-2}\), is observed immediately after cyclogenesis. These results suggest that the eddy transport of momentum operates in the direction of the maintenance of the KE of the disturbance at the expense of the zonal KE. Comparing these results with those of another case study (Michaelides 1987), it appears that, in the area of a midlatitude cyclone, the feeding of eddy motion by the zonal flow could be a predominant feature.

Finally, a continuous production of K\(_E\) at the expense of A\(_E\) is observed to take place in the present study; a result that agrees with results from other case studies. Palmen (1958) estimates that in the area of an intense depression, C(A\(_E\) → K\(_E\)) reaches the value of 52 W m\(^{-2}\), but Reiter (1969b) considers that a rate of 20 W m\(^{-2}\) represents a realistic figure for such systems (see also Danard 1966; Palmen 1966). However, the values for this conversion as calculated here are one order of magnitude smaller than the above estimates.

b. Vertical distribution of energy conversions and transfers

The time course of the four energy conversions over the area is shown in Fig. 4. Within the boundary layer, A\(_Z\) is converted into K\(_Z\) at all times, the maximum of this conversion taking place soon after the formation of the cyclone. In the intermediate layer, the conversion between the zonal energies is the most pronounced feature. In this layer, K\(_Z\) is continuously enhanced at the expense of A\(_Z\). In the stratospheric layer, K\(_Z\) is converted into A\(_Z\) at a rate of about 7 W m\(^{-2}\) before the onset of the surface cyclonic circulation. However, after the formation of the cyclone, the respective conversion process operates in the opposite direction, thus enhancing the zonal KE. That the thermally driven mean meridional circulations in the stratosphere can act so that the conversion between A\(_Z\) and K\(_Z\) changes direction in a matter of a few days has long been considered by Reed et al. (1963) and Julian and Labitzke (1965).
Even on the global scale, the direction of this conversion is not widely accepted, particularly in the cold season (see Wiin-Nielsen 1965).

In the boundary layer, $K_E$ is converted into $A_E$, initially, but as the system develops, a reversal of the conversion occurs. In the intermediate layer, $C(A_E \rightarrow K_E)$ is always a sink of $A_E$. However, in the upper layer, this conversion feeds the eddy APE at the expense of eddy KE. The conversion of $A_E$ into $K_E$ is considered to be the major process feeding the eddy KE of the synoptic-scale disturbances (see Oort 1964; Krueger et al. 1965; Wiin-Nielsen 1965). However, from the energetics analyses of the stratosphere, it has been revealed that the conversion in the opposite sense is possible, even on a global scale (see Reed et al. 1963; Julian and Labitzke 1965).

In the 1000–700-hPa layer, the conversion between the two APE forms favors the eddy component at all times. This conversion is negligible initially, but it increases, gradually reaching a rate of 2 W m$^{-2}$ by the end of the period studied. In the intermediate layer, the direction of $C(A_E \rightarrow A_Z)$ changes so that it favors $A_Z$. The sign of this conversion changes again in the uppermost layer and a gradual increase of its importance as a source of $A_E$ is noted.

Regarding the last conversion term, $C(K_E \rightarrow K_Z)$, the conversion between the two KE forms proceeds at negligibly small rates in the boundary layer. In both the intermediate and upper layers, the conversion process is quite active and enhances the eddy motion at the expense of its zonal counterpart. The maximum of this conversion is noted soon after the formation of the surface cyclone.

The time course of the four boundary transfers of energy is shown in Fig. 5. In the near-to-surface layer, 1000–700 hPa, $A_Z$ is almost always imported into the cyclone region. In the intermediate layer, however, the period under study commences with a marked export of $A_Z$ at a rate of 19 W m$^{-2}$. A sharp decrease of this export is noted with the onset of cyclogenesis. This export of energy continues at decreasing rates and eventually $A_Z$ becomes a small source of $A_Z$. In the stratospheric layer, there exists a continuous export of $A_Z$ from the cyclone area to the surrounding region.

The boundary-layer transfer of $A_E$ is similar to that of $A_Z$. In the intermediate layer, however, a striking difference between the two APE transfer terms is noted. Indeed, $A_E$ is initially an important source of $A_E$. Its importance as a source of $A_E$ decreases with time, and eventually it acts as a sink in the final stages. The effect
of $B_{AE}$ as an energy transfer process is negligible in the stratospheric layer.

Zonal KE is transferred into the lower layer at small rates throughout the period. The same behavior is observed in the intermediate layer, but the energy import is accomplished at considerably higher rates. In the upper layer, however, the direction of the transfer process changes and $K_z$ is exported at high rates. A maximum of this export of $22 \text{ W m}^{-2}$ is calculated at 0000 UTC 9 November.
Finally, the behavior of $BK_E$ is similar to that of $BK_Z$: $K_E$ is imported in the cyclone area in both the lower and intermediate layers and is exported to the surrounding region in the stratospheric layer.

c. **Horizontal distribution of energy conversions**

The spatial distributions of the four energy-conversion terms within the most energetically active layers,
namely 700–300 and 300–150 hPa, and for the 1200 UTC synoptic times are shown in Figs. 6–9. Establishing a spatial relationship between specific contour patterns of the energy conversions, on one hand, and of the surface or upper-air analyses, on the other hand, is an extremely difficult task. The purpose of this section is to present the spatial characteristics of the energy conversions and to identify regions in which the associated physical processes take place. The discussion is made with reference to the various synoptic-scale features of the baroclinic disturbance.

The horizontal distributions of the conversion $C(A_Z \rightarrow A_E)$ are depicted in Figs. 6a and 6b. In the 700–300-hPa layer, the precyclogenetic period is characterized by a strong conversion of $A_Z$ to $A_E$ to the northwest of the upper trough, with local rates exceeding 125 W m$^{-2}$. This conversion is associated with the southward advection of colder air by the strong northern upper-tropospheric winds. However, this situation is rapidly modified as the tropospheric tilting of the trough axis decreases or, equivalently, as the phase difference between the thermal and contour fields diminishes (see Holton 1979). By the end of the period, the upper-tropospheric eddy transfer of heat weakens considerably, as does its associated conversion between $A_Z$ and $A_E$.

In the upper 300–150-hPa layer, a weak local conversion of $A_Z$ into $A_E$ and vice versa is noted during 7 and 8 November. By 1200 UTC 9 November, two areas of strong conversion of $A_E$ into $A_Z$ are observed; this is accomplished through a countergradient eddy transfer of heat. On the global scale, such behavior has been found to be closely related to periods of sudden stratospheric warming, in which case the stratospheric poleward eddy flux of heat continues despite the reversal of the temperature gradient (see Reed et al. 1963; Julian and Labitzke 1965).

The spatial distributions of $C(A_E \rightarrow K_E)$ are shown in Figs. 7a and 7b. In the precyclogenetic period, the predominant feature is an intense tropospheric conversion of $A_E$ into $K_E$ to the northwest of the upper trough. In the following, the intensity of this conversion weakens gradually, and several centers of conversion are formed by the end of the period.

In the stratospheric layer, the distribution of $C(A_E \rightarrow K_E)$ differs markedly from the respective distribution in the tropospheric layer. The stratospheric pattern at 1200 UTC 7 November, reveals a region of strong conversion of $K_E$ into $A_E$ superimposed on the region of strong tropospheric conversion in the opposite sense. At 1200 UTC 9 November, two distinct regions of conversion of $K_E$ into $A_E$ are present: one ahead and the other behind the cold front. From this pattern it is inferred that colder ascending stratospheric air is drawn into the occluding cyclone and upper-level descending motion occurs immediately to the east of the front.

The tendency of the negative stratospheric regions to superimpose on the positive tropospheric regions is maintained throughout the life cycle of the depression. Palmen (1958), in an early computational study of cyclone energetics, has shown that intense cyclonic disturbances constitute substantial source regions of eddy KE and sink regions of APE. He has also inferred that there should exist other regions in the atmosphere where eddy KE is converted back to eddy APE. The results presented here indicate that such marked source and sink regions can coexist in a developing cyclone: strong local generation of $K_E$ at the expense of $A_E$ takes place within the troposphere, whereas strong local conversion in the opposite sense taking place in the stratosphere offsets to a large extent the tropospheric gain of eddy KE.

The conversion between $K_E$ and $Z$ as distributed in the 700–300- and 300–150-hPa layers is shown in Figs. 8a and 8b. The examination of the distributional characteristics of this conversion reveals that both the tropospheric and stratospheric layers contribute mainly towards the maintenance of the KE of the disturbance, drawing energy from the zonal flow. Also, strong conversion is accomplished within concentrated and short lasting zones. In the 700–300-hPa layer, any specific patterns are initially absent. However, soon after the formation of the cyclone, a distinct region of strong conversion of $K_Z$ into $K_E$ appears to the east of the upper trough with rates exceeding 50 W m$^{-2}$. This distinct pattern disappears by the end of the period. A similar situation is observed in the stratospheric layer, where the distinct zones of strong conversion that formed soon after the cyclogenesis disappear shortly afterwards.

The horizontal distributions of the last conversion term $C(K_Z \rightarrow A_Z)$ are shown in Figs. 9a and 9b. In the 700–300-hPa layer, the meridional circulations produce the most intense local energy transformations. Sinking of cold air and rising of warm air in the meridional plane appears to be the prevailing process over most of the area, thus converting $A_Z$ into $K_Z$ at extremely high rates, especially to the north of the Mediterranean. Over the central Mediterranean, a region of conversion in the opposite sense is progressively established.

Initially, the meridionally driven stratospheric circulations act in favor of $A_Z$ and at the expense of $K_Z$ over most of the area. This situation is reversed 24 h later, so that the formation of the cyclone is accompanied by the establishment of a discrete region of strong conversion of $K_Z$ into $A_Z$.

6. Summary and conclusions

The spatial and temporal characteristics of the energetics of a baroclinic depression in the Mediterranean were presented. The partition of APE and KE into zonal and eddy components was adopted, and the fate
Fig. 6. (a) The spatial distribution of $C(A_z \rightarrow A_e)$ in the 700-300-hPa layer. Positive values imply conversion in the direction of the arrow, whereas negative values imply conversion in the opposite direction. The filled circle marks the position of the surface cyclone center. Isopleths are drawn for every 25 W m$^{-2}$. (b) The spatial distribution of $C(A_z \rightarrow A_e)$ in the 300-150-hPa layer. Positive values imply conversion in the direction of the arrow, whereas negative values imply conversion in the opposite direction. The filled circle marks the position of the surface cyclone center. Isopleths are drawn for every 10 W m$^{-2}$.
Fig. 7. (a) Same as Fig. 6a, except for $C(A_e \rightarrow K_e)$. (b) Same as Fig. 6b, except for $C(A_e \rightarrow K_e)$. 

700 - 300 hPa

300 - 150 hPa
Fig. 8. (a) Same as Fig. 6a, except for \( C(K_E \rightarrow K_Z) \). (b) Same as Fig. 6b, except for \( C(K_E \rightarrow K_Z) \).
Fig. 9. (a) Same as Fig. 6a, except for $C(K_2 \rightarrow A_2)$. (b) Same as Fig. 6b, except for $C(K_2 \rightarrow A_2)$. 

700–300 hPa

300–150 hPa
of atmospheric energy was studied within the framework of the resulting energy flow.

The conversion associated with APE and the conversion associated with KE are often thought of as the baroclinic and barotropic conversion, respectively. On the global scale, the baroclinic conversion is characterized by a downscale cascade from the zonal field to synoptic-scale disturbances; the barotropic conversion is characterized by an upscale cascade from the synoptic scale to the zonal motion (see Tanaka and Shaojian 1990). As it should be expected from the baroclinic instability theory, the energy balances in Fig. 3 confirm the baroclinic nature of the frontal depression, but, in addition, they stress the important contribution of the barotropic component.

In contrast to the expected behavior of the baroclinic component, the eddy APE was found initially to feed the zonal APE. With the development of the cyclone, however, the baroclinic component becomes increasingly important in maintaining the eddy APE.

As stated above, it is generally accepted that, on the average, the global $K_z$ is maintained partly through conversion from $K_E$. Also, several spectral studies agree that, on the average, the medium-scale waves (i.e., cyclones) export energy to longer and shorter waves through nonlinear KE exchange processes (Saltzman and Fleisher 1960; Tomatsu 1979). In the present study, the barotropic conversion was found to feed the eddy motion at the expense of the zonal field. The ability of the atmosphere to convert $K_z$ into $K_E$ and vice versa has been discussed by Wiin-Nielsen (1965). Also, Wiin-Nielsen et al. (1964) revealed that, on the global scale, a conversion between KE components may operate in favor of $K_E$ and at the expense of $K_z$, even for a period of a few days.

The magnitude of the cyclone energy conversions found in the present study is greater (in absolute terms) than their global counterparts. In general circulation studies, local centers of intense conversion (e.g., in the vicinity of cyclonic systems) cannot be detected because of the time averaging (usually of the order of a few months) and the global-scale space averaging employed.

The four transfer terms appearing in the energy balance (Fig. 3) indicate that the processes described by $BA_x$, $BA_y$, $BK_z$, and $BK_x$ are mostly depleting energy from the open system and transferring it to the surrounding region. Especially in the last period, all four terms act as energy sinks, indicating that the cyclone is not only effective in converting energy, but it progressively acts as an energy exporting system.

A comparison of the results of another case study (Michaëliades 1987) and the ones presented here reveals that the energetics of midlatitude cyclones may differ in some respects. For example, in the former case, an intense dependence of the cyclonic system on energy input was found, whereas in the present case, the cyclonic system acted mostly as an energy-exporting eddy. Although such differences are mainly attributed to the inherent variability of the thermal and motion fields, another reason could be the difference between the actual dimensions of the synoptic system studied.

The presentation of the spatial characteristics of the energy conversions has shown that various parts of the cyclone can make discretely different contributions to the energy balance of the overall cyclonic environment. This finding agrees with results from other case studies (see Lin and Smith 1979; Fuelberg et al. 1985).

The KE budget of individual midlatitude cyclonic disturbances has been the favorite subject of numerous investigators and the literature on this matter is extensive. Although the budget of APE on the global scale has been elaborated in several energetics studies, little is known about the APE budget of individual midlatitude cyclones. However, one particular component of this budget, namely the APE generation taking place during the evolution of extratropical cyclones, has received some attention. Much less effort has been made in investigating the fate of atmospheric energy associated with midlatitude disturbances in composite energetics analyses. This more general aspect of limited-area energetics is currently receiving more attention (e.g., Robertson and Smith 1983; Smith and Dare 1986; Michaëliades 1987; Prezerakos and Michaëliades 1989). Such composite energetics analyses, in which the budgets of two or more forms of atmospheric energy are studied together within a common framework, appear to be more elucidatory of the role that midlatitude cyclonic systems play in the global atmospheric energy balance. The analysis presented here is a further endeavor in this direction.

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APPENDIX

Numerical Solution of Energy Integrals

Reiter (1969a) proposed a notation for presenting mean and eddy atmospheric motions that was subsequently adopted for the presentation of energy integrals (see Reiter 1969b). The numerical analogs of the energy integrals as used in the present study are given below, using a notation that is considered suitable for a comprehensive presentation of numerical integration schemes (volume and surface) involving mean and eddy components.
The position of a grid point in the three-dimensional grid used here is given by three indices \(i, j, \) and \(k\) \((i = 1, 2, 3, \ldots, m, \) and \(j = 1, 2, 3, \ldots, n\) denote the position in the south–north and west–east sense, respectively, and \(k\) denotes the isobaric level numbered consecutively upwards). Dependence of a variable or operator on latitude and/or longitude and/or pressure level is determined by an appropriate combination of subscripts \(i, j, \) and \(k,\) respectively. The zonal mean of variable \(X = X_{ijk}\) is a function of latitude and pressure level and is given by

\[
[X]_k = \frac{1}{n-2} \sum_{j=2}^{n-1} X_{ijk}. \tag{A1}
\]

The eddy component of this variable is a function of latitude, longitude, and pressure level and is given by

\[
(X)_{ijk} = X_{ijk} - [X]_k. \tag{A2}
\]

The area average of variable \(X\) over the computational region is

\[
[X]_k = \frac{1}{n-2} \frac{d}{\sin(\phi_{m-1} + d/2) - \sin(\phi_2 - d/2)} \times \sum_{i=2}^{m-1} \sum_{j=2}^{n-1} X_{ijk} \cos \phi_i. \tag{A3}
\]

where \(\phi = \phi_i\) is the latitude and \(d\) is the horizontal grid distance.

The difference between Eqs. (A1) and (A3) defines a quantity that is a function of latitude and pressure level

\[
(X)_{k} = [X]_{ik} - [X]_k. \tag{A4}
\]

A finite-difference symbol along a meridian is defined as

\[
\Delta_{[X_{ijk}]_{a}} = X_{ijk} - X_{alj}. \tag{A5}
\]

Similar symbols are used for the other indices \(j\) and \(k\) in the zonal and vertical sense, respectively.

The four energy forms within the pressure layer \(p_{k-1}, \) \(p_{k+1} (p_{k-1} > p_{k+1})\) are

\[
A_Z = \frac{\Delta_t [p_k^k + 1 [(T)_{ik}]^k]}{2 [\sigma]_k}, \tag{A6}
\]

\[
A_E = \frac{\Delta_t [p_k^k + 1 [(T)_{ijk}]^k]}{2 [\sigma]_k}, \tag{A7}
\]

\[
K_Z = \frac{\Delta_t [p_k^k + 1 [u]_{ik}^k + [v]_{ik}^k]}{2 g}, \tag{A8}
\]

and

\[
K_E = \frac{\Delta_t [p_k^k + 1 [(u)_{ijk}^k + (v)_{ijk}^k]]}{2 g}, \tag{A9}
\]

where \(T\) is the temperature, \(u\) and \(v\) the eastward and northward wind components, \(g\) the magnitude of the acceleration of gravity, and \(\sigma\) the isobarically averaged static stability parameter

\[
[\sigma]_k = \frac{g T_{ijk} - p_{uk} \Delta_t [T_{jk}]_{ik}^{k+1}}{c_p R \Delta_t [p_k^k + 1]_j}, \tag{A10}
\]

where \(R\) is the gas constant for air and \(c_p\) the specific heat at constant pressure.

When a clockwise flow of energy corresponding to Fig. 1 is adopted, the four energy-conversion terms take the form

\[
C(A_Z \rightarrow K_Z) = \frac{\Delta_t [p_k + 1} {g p_k} R [(T)_{ik}]_{ij} (\omega)_{ik} k], \tag{A11}
\]

\[
C(A_E \rightarrow K_E) = \frac{\Delta_t [p_k + 1} {g p_k} R [(T)_{ijk}]_{ijk} (\omega)_{ijk} k], \tag{A12}
\]

\[
C(A_Z \rightarrow A_E) = \frac{\Delta_t [p_k + 1} {2 r [\sigma]_d} \Delta_t [(T)_{ijk}]_{ijk} (\omega)_{ijk} k] \tag{A13}
\]

\[
+ \frac{[(T)_{ijk}]_{ijk} p_{uk} [\sigma]_k^{k+1}}{p_{uk}^{k+1}} [(\omega)_{ijk} k] \tag{A14}
\]

where \(r\) is the mean radius of the earth and \(\omega\) is the vertical velocity (in terms of the total time derivative of pressure).
The combined horizontal and vertical fluxes of energy are

\[
B_A = \frac{1}{2\sigma_k} \{ \Delta_k[p_k]\beta d \sum_{i=2}^{m-1} \Delta[(2(T)_{ik}(T)_{ijk}u_{ijk} + (T)^2_{ik}u_{ijk})]^{\frac{3}{2}} + \Delta_k[p_k]\beta \Delta_i[2(\omega)_{ijk}(T)_{ijk}]^{\frac{3}{2}} + \Delta_k[p_k][\cos \phi_i (T)^2_{ijk}v_{ijk}]^{\frac{3}{2}} - \Delta_k[2(\omega)_{ijk}(T)_{ijk}]^{\frac{3}{2}} + [\omega]_{ijk}(T)^{\frac{3}{2}} \} (A15)
\]

\[
B_E = \frac{1}{2\sigma_k} \{ \Delta_k[p_k]\beta d \sum_{i=2}^{m-1} \Delta_i[(T)^2_{ijk}u_{ijk}]^{\frac{3}{2}} + \Delta_k[p_k]\beta \Delta_i[\cos \phi_i (T)^2_{ijk}v_{ijk}]^{\frac{3}{2}} - \Delta_k[(T)^2_{ijk}\omega_{ijk}]^{\frac{3}{2}} \} (A16)
\]

\[
B_Z = \frac{1}{2g} \{ \Delta_k[p_k]\beta d \sum_{i=2}^{m-1} \Delta_i[u_{ijk}(u_{ijk}^2 - (u)^2_{ijk} + v_{ijk}^2 - (v)^2_{ijk})]^{\frac{3}{2}} + \Delta_k[p_k]\beta \Delta_i[\omega_{ijk}(u_{ijk}^2 - (u)^2_{ijk} + v_{ijk}^2 - (v)^2_{ijk})]^{\frac{3}{2}} \} (A17)
\]

\[
B_F = \frac{1}{2g} \{ \Delta_k[p_k]\beta d \sum_{i=2}^{m-1} \Delta_i[u_{ijk}(u_{ijk}^2 + v_{ijk}^2)]^{\frac{3}{2}} + \Delta_k[p_k]\beta \Delta_i[\omega_{ijk}(u_{ijk}^2 + v_{ijk}^2)]^{\frac{3}{2}} \} (A18)
\]

Here \(\alpha = -1/(\sin(\phi_{m-1} + d/2) - \sin(\phi_{2} - d/2))\) and \(\beta = \alpha/(n-2)\).

The contributions to \(K_Z\) and \(K_E\) from the pressure work at the boundaries take the form

\[
B_{FZ} = \frac{1}{g} \{ \Delta_k[p_k]\beta d \sum_{i=2}^{m-1} \Delta_i[u_{ijk}(\Phi_{ijk} - (\Phi)^2)]^{\frac{3}{2}} + \Delta_k[p_k]\beta \Delta_i[\cos \phi_i (\Phi)^2_{ijk}]^{\frac{3}{2}} - \Delta_k[(\omega)_{ijk}(\Phi)^2_{ijk}]^{\frac{3}{2}} \} (A19)
\]

\[
B_{FE} = \frac{1}{g} \{ \Delta_k[p_k]\beta d \sum_{i=2}^{m-1} \Delta_i[(\Phi)^2_{ijk}]^{\frac{3}{2}} + \Delta_k[p_k]\beta \Delta_i[\omega_{ijk}(\Phi)^2_{ijk}]^{\frac{3}{2}} - \Delta_k[2(\omega)_{ijk}(\Phi)_{ijk}]^{\frac{3}{2}} \} (A20)
\]

where \(\Phi\) is the geopotential.

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