Sensitivity of a Spectrally Filtered and Nudged Limited-Area Model to Outer Model Options

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ABSTRACT

Numerical filters required to control spatial computational modes in a limited-area model (LAM) that uses the unstaggered A grid are developed and tested over the complex topography of the Great Basin of the western United States. The filters are founded upon Fourier expansions of forecast deviation fields and function equally effectively for both periodic and aperiodic local structures. Unlike other spatial filters, the approach used here avoids any direct contamination of larger scales. Provided that the shortest resolved wavelength of two grid intervals is removed, the results do not depend strongly on the range of filtered short waves or on the type and order of horizontal space difference approximations.

This approach leads naturally to methods in which the large scales predicted by an ambient outer model can be directly incorporated within the complete domain of the inner LAM, rather than just through conditions applied at the lateral boundaries of the LAM. This technique has some similarities to methods used both in operational regional models in Japan and in recent regional research models at the National Centers for Environmental Prediction (formerly National Meteorological Center) of the United States. Several methods to incorporate the large scales into the LAM are evaluated in a winter storm case study and in an ensemble of seven forecasts.

1. Introduction

Weather prediction by numerical methods utilizes a set of differential equations that are solved subject to specified initial and boundary conditions. For global forecasts, the only boundary conditions required are at the top and bottom of the three-dimensional forecast domain. Accurate specification of the bottom boundary can become particularly important and cumbersome above complex terrain, although terrain effects sometimes reduce the sensitivity of atmospheric predictions to details of the initial state (e.g., Vukicevic and Errico 1990; Paegle et al. 1990).

For regional and smaller-scale models, the specification of lateral boundary conditions is also very important. One of the most common methods of modifying the lateral boundaries was developed by Davies (1976, 1983), hereafter known as Davies nudging. The goal of the present study is to quantify the relative importance of uncertainties inherent in the range of methods used to impose boundary conditions. The nesting strategies currently used in most limited-area models (LAMs) can be divided into four categories:

1) boundary condition from outer model plus Davies nudging (one-way interacting),
2) boundary condition plus large internal scales from outer model (one-way interacting),
3) abruptly changing grid size (two-way interacting), and
4) continuously varying grid size.

The one-way interacting boundary condition (method 1) is the most common method for current operational models. It is simple to implement and provides the LAM with some knowledge of the large-scale features near the LAM boundaries. The disadvantage of this method is that the largest scale features resolved by the LAM may be better predicted by the global model and Davies nudging only crudely allows this. This shortcoming has prompted research into method 2, which has been tested in a regional spectral model at the National Centers for Environmental Prediction (NCEP, formerly National Meteorological Center) by Juang and Kanamitsu (1994).

The Nested Grid Model (NGM) and several research models, including the Penn State (PSU)—NCAR Mesoscale Model (MM5) (Grell et al. 1993) and the Colorado State University Regional Analysis and Modeling System (CSU RAMS) (Pielke et al. 1992), use a nested grid approach of the third type. This technique


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allows interaction of the local and large-scale model predictions. Criticisms of this method are that the coding is complicated and the computational dispersion changes abruptly where the resolution changes, although this has been reduced in recent versions of the MM5 (Smolarkiewicz and Grell 1992).

Variations of method 4 have been implemented in both spectral and variable resolution finite element models. Courtier and Geleyn (1988) describe the conformal spectral transformation used in the French global model. The University of Utah global model incorporates Fourier series in longitude, finite elements in latitude, and rotates the polar axis to provide high local resolution (Paegle 1989; Yang 1992). The disadvantages of these methods are that initialization and objective analysis must be performed on a hemispheric or global domain, even though the area of forecast interest may be much smaller. They also require larger computational resources and are not always practical alternatives to a LAM.

An extended implementation of the University of Utah LAM (hereafter referred to as the Utah LAM) is used in the present study. The model is described by Paegle and McLawhorn (1983) and Waldron (1994) and summarized in section 2.

The principal application of this model to date has been to simulate the three-dimensional circulation over the intermountain west. The grid size used is roughly 12 km. At such resolutions, the terrain varies in height over one to two km in just a few grid points, and the detailed structures of the forecast fields are dominated by topographic influences. Among these influences, the upslope mountain wind and downslope drainage wind are especially conspicuous modulators of the circulation and precipitation forecasts. Rigling et al. (1992) and Horel and Gibson (1994) discuss model skill for prediction of downslope windstorms and precipitation, respectively, over Utah. Nicolini et al. (1993) describe model skill and sensitivity over the Great Plains.

The correct simulation of slope circulations requires careful treatment of the turbulent boundary layer. A level 2.5 turbulence treatment is used in which variables such as the Richardson number can be more accurately calculated on the model's nonstaggered grid. In section 3, some numerical problems of the nonstaggered grid approach are presented and resolved.

A new procedure of incorporating the large-scale forcing is presented in which the largest scales within the Utah LAM are influenced by an "outer" model, whereas the evolution of smaller-scale features is dominated by local or "inner" model dynamics. The scales are separated by a Fourier decomposition and nudged in spectral space. Section 3 describes a one-dimensional shallow-water model and includes tests of the model's sensitivity to Fourier filtering, diffusion, and boundary conditions. The spectral method of large-scale forcing, following method 2, is developed and tested with the shallow-water model in section 4.

Initial conditions from a winter storm case study by Gibson (1993) and Horel and Gibson (1994) are used to compare the Utah LAM's performance with several different lateral boundary and nudging schemes. They include Davies boundary nudging and spectral nudging of various wavenumbers. Section 5 shows a root-mean-squared comparison of the Utah LAM forecast results and a spectral analysis of selected forecast fields.

The final goal of the research is to perform a statistical comparison of the relative importance of the possible outer model choices for initial and boundary conditions relative to the nudging scheme selected. Two ensembles of cases are run using Davies nudging. One ensemble is initialized from and uses the NGM for the large-scale forcing; the second uses the ETA analyses and forecasts. The ETA analyses and forecasts are then used in a third ensemble incorporating the spectral nudging scheme tested in section 5. The root mean square differences between ensembles 1 and 2 quantify the impact of different initial and boundary conditions, whereas differences between ensembles 2 and 3 quantify the effect of the type of boundary conditions. These results are presented in section 6.

2. Numerical model

The three-dimensional Utah LAM used in sections 5 and 6 of this study is based on the boundary layer model described by Paegle and McLawhorn (1983). It utilizes a terrain following vertical coordinate with 17 levels over an atmospheric depth of 11 km. The vertical levels are at 0, 1, 10, 100, 300, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000, 10 000, and 11 000 m above terrain height. The model is hydrostatic and the dynamical equations are as in Paegle and McLawhorn (1983), except the anelastic version of the continuity equation is used, instead of the Boussinesq approximation, and the model is extended to three dimensions. A number of enhancements of the Utah LAM physics have been implemented over the past decade. These are discussed at length by Waldron (1994).

a. Initial conditions

The Utah LAM is initialized from regional model (NGM or ETA) data. Input variables include the zonal and meridional wind components (u, v), temperature (T), height (z), and specific humidity (q) on pressure levels at 50-mb spacing, with a horizontal grid spacing on the order of 80 km. The data are linearly interpolated to the Utah LAM's horizontal grid before being entered into the model. The basic-state height z0b and temperature T0b are calculated for each pressure surface. At the height z0b, corresponding to the average elevation of the pressure surface p0b, a perturbation pressure is computed from

\[ p' = g \frac{p_0(z - z_{0b})}{RT} \]  \hspace{1cm} (1)
Cubic splines are used to interpolate $p_{hs}$, $T_{hs}$, $p'$, the $u$ and $v$ wind components, and $q$ vertically to terrain-following coordinates. The basic-state density field $\rho_{bs}$ is computed by the equation of state:

$$\rho_{bs} = \frac{P_{hs}}{RT_{bs}}.$$  \hspace{1cm} (2)

The initial perturbation density field $\rho'$ is then calculated from the hydrostatic equation in the form

$$\rho' = -\frac{1}{g} \frac{\partial p'}{\partial z},$$  \hspace{1cm} (3)

and the temperature perturbation from

$$T' = T_{bs} \left[ \left( \frac{p'}{p_{bs}} \right) - \left( \frac{p'}{p_{bs}} \right) \right].$$  \hspace{1cm} (4)

The potential temperature $\theta$ is calculated from the Poisson equation.

The initial conditions from the regional model can be specified by the large-scale analysis or a forecast, with forecasts 6 and 12 h after the initial time used to provide large-scale input to the LAM as discussed in the next section. It should be emphasized that the basic-state variables, as presently defined and incorporated in all equations, are functions of elevation above sea level rather than above the local surface. This reduces pressure gradient errors near steep terrain relative to those found in some applications of $\sigma$ coordinates. One disadvantage of this approach is that the basic-state variables depend not only on height but also on the horizontal coordinate, and three-dimensional arrays must be stored. More complete initialization would require assimilation of data in a four-dimensional mode as done by Stauffer and Seaman (1990, 1994) and Stauffer et al. (1991) for LAMs.

b. Boundary conditions

Studies with previous versions of the Utah LAM (e.g., Paegle and Geisler 1986; Berri and Paegle 1990) have used steady Dirichlet lateral boundary conditions for the dynamical forecast variables. Over long time simulations, or when the meteorological situation involves rapidly moving synoptic features, the model forecast is degraded by continuing to use constant boundary conditions. An important component of the present study is to alter the boundary conditions such that the boundary values of the zonal and meridional wind components, potential temperature, and specific humidity are adjusted to linearly interpolated values from the NGM or eta forecasts at each time step. A version of the model with perimeter “nudging” of the wind and temperature fields, but retaining the initial boundary condition for specific humidity, was run on a nearly daily basis for a 1-yr period ending during fall 1993. This version of the model was used by Gibson (1993) for a winter storm case study over northern Utah. Since then, the model has been run every day using a version that also incorporates nudging of the specific humidity field on the perimeter.

A variation of this scheme is to apply Davies nudging to a small region along the lateral boundaries, rather than only nudging the perimeter fields. This was found to smooth some gradients near the boundaries, while generating little change in the interior of the domain. Tests of Davies nudging on a one-dimensional shallow water model will be presented in the next section and applied to the Utah LAM in section 5.

The pressure at the upper boundary is important because the hydrostatic equation is used to integrate the pressure downward from the model top. An updated pressure field for the model top is obtained from the time-interpolated regional model forecasts, following the same procedures as used for the lateral boundary nudging. A common problem with upper boundaries is the reflection of upwardly propagating waves off the top boundary and back into the domain. A radiation upper boundary condition, based on Klemp and Durran (1983), was added to the model to diminish this problem.

c. Vertical coordinate

The most common method of including the effect of topography is to use the $\sigma$ vertical coordinate system. As the horizontal resolution and terrain slope increase, the pressure gradient terms of the momentum equations tend to be computed from small differences between two large quantities when the $\sigma$ coordinate is used. Phillips (1957) and Paegle and McLawhorn (1983) discuss the problems for pressure-based and height-based terrain following coordinates, respectively. They demonstrate that the difficulties are due to a near cancellation of two individually large terms that contribute to the horizontal pressure gradient force in the vicinity of steep topography. This cancellation, which arises from the hydrostatically balanced portion of the thermodynamic state, cannot be entirely ensured in numerical approximations. Truncation errors related to the horizontal pressure gradient were significantly reduced in the nonhydrostatic PSU–NCAR MM5 by defining the model’s mass variables as perturbations to a hydrostatic base state (Stauffer 1994, personal communication).

This problem has led to alternative formulations for the vertical coordinate, such as the $\eta$ coordinate (Mesinger et al. 1988) used in the eta model of the NCEP. A potential deficiency of the $\eta$ coordinate is that it produces isolated horizontal domains on the lower levels of the model within areas of high topography. The isolated domains make it difficult to incorporate spectral methods or Fourier expansions that require periodic behavior at lateral boundaries. It is also difficult to uniformly resolve the boundary layer in areas of high terrain when the $\eta$ coordinate is used.
d. Topography and domain

The Utah LAM domain that is used in this study is shown in Fig. 1a along with the terrain. The horizontal latitude–longitude grid contains $65 \times 65$ grid points, spaced $0.133^\circ$ latitude by $0.133^\circ$ longitude. Elevations in this domain range from less than 500 m above sea level, near Lake Mead on the southern boundary, to over 3500 m above sea level, in the Uinta Mountains and in the Wind River Range. Steep topography gradients exist along these mountains and also along the Wasatch Front. While this grid spacing is not adequate to resolve individual canyon circulations, it does allow development of many meteorological features common in regions of complex terrain such as downslope winds.

By comparison, the terrain specified by the NMG and eta models, within the LAM domain, is shown in Figs. 1b and 1c, respectively. The grid size of the operational runs of both of these models is on the order of 80 km. The eta model has also been tested at the NCEP with a 40-km grid spacing. Figure 1d shows the topography used in that version of the eta model.

3. Computational modes and spatial filtering

Staggered grid treatments require either the temperature to be interpolated to the wind grid points or the wind to be interpolated to the grid points that specify the temperature. Such interpolations can produce spurious results above complex terrain where adjacent grid points may represent very different surface conditions or elevations. Purser and Leslie (1988) demonstrate that the interpolations also offset the truncation error advantages of staggered grids and show that superior nonstaggered approaches can be constructed.

A principal difficulty with nonstaggered (“A grid”) treatments is that computational horizontal modes develop, and these can become particularly troublesome.
when rapidly propagating gravity waves are present. In earlier boundary layer implementations of the Utah LAM (e.g., Paegle and Mclawhorn 1983; Astling et al. 1985; Paegle and Geisler 1986; McCorcle 1986), such problems were not encountered. The model depth in those applications was on the order of only 2 km, so that the resolved gravity waves were relatively shallow, with slow propagation rates, and strong boundary layer dissipation eliminated pronounced computational problems.

Strong spatial computational modes developed in deep versions of the Utah LAM that included most of the troposphere. These modes, which have a characteristic spatial scale of approximately two grid intervals, reflected strongly from lateral boundaries and dominated the solution rather quickly, especially near the boundaries. Integrations of models on the A grid require some sort of spatial filter to remove spurious computational modes. In the present study, tests were performed with horizontal filters of the sort that are sometimes used in other nonstaggered treatments. The relatively low-order filters used in those tests were not very effective and also significantly damped the longer waves, containing physically relevant structures. Other models have successfully used nonstaggered grids with high-order spatial filters. The Goddard Institute of Space Sciences model (Kalnay-Rivas et al. 1977; Kalnay-Rivas and Hoitsma 1979) employs such grids and later implementations of that model have used the 16th-order filter developed by Shapiro (1970). Purser and Leslie (1988) implement a filter with highly selective and flexible response based on Purser’s (1987) analysis.

The multipoint filtering approach was eventually abandoned in this research in favor of a Fourier filter that selectively removes all waves of a given wavelength range. This approach works quite well, and two studies of the Utah LAM using this technique have already been documented (Nicolini et al. 1993; Horel and Gibson 1994). We have not performed systematic comparisons of spectral filtering with recently advanced higher-order multipoint filters. Our reasons to implement spectral filters are based on practical considerations. A Fourier transform is required every time step by the boundary methods discussed in section 4. The addition of a filter to this transform is straightforward and economical. Additionally, such transforms allow much higher accuracy approximations for horizontal derivatives than does any gridpoint approach although this advantage remains to be exploited here.

The goal of the present section is to explain the implementation of the spectral filter, outline potential concerns, and demonstrate through a series of simple numerical experiments its utility. In this section the tests are performed with a one-dimensional shallow water model that is embedded in a nested approach analogous to the full LAM implementation. In the nested approach, the “outer” model forecasts are analogous to the NGM or eta model forecasts that provide lateral boundary conditions for the “inner” local model forecasts.

\[ \frac{\partial u}{\partial t} = -g \frac{\partial \eta}{\partial x}, \]

\[ \frac{\partial \eta}{\partial t} = -H \frac{\partial u}{\partial x}, \]

where \( u \) is the horizontal wind field, \( \eta \) is the height field, \( g \) is gravity, \( H \) is the scale height (set at 8 km). The field is initialized using a cosine wave, whose wavelength is half the inner domain size. The propagation speed of the longwave field,

\[ c = (gH)^{1/2}, \]

equals 280 m s\(^{-1}\) for present parameter selection. The outer domain covers 257 points, including the cyclic boundary points. For the present grid spacing of 30 km, the outer domain spans a distance of 7680 km, and the wave pattern propagates across this region in approximately 7.6 h.

The inner domain is initialized from the corresponding points in the global model, and the forecast equations have an additional forcing term

\[ \frac{\partial u}{\partial t} = -g \frac{\partial \eta}{\partial x} + A \sin[k(x - c_t t)], \]

\[ \frac{\partial \eta}{\partial t} = -H \frac{\partial u}{\partial x} + B \sin[k(x - c_t t)], \]

where \( k \) is the wavenumber of the locally forced wave. In the local domain, the forced solution propagation speed is one-tenth that of the long wave pattern. The wavelength of this inner wave is 16 grid intervals, or 480 km. Four complete wavelengths of the forced wave fit inside the inner domain, which covers 65 grid points. The finite-difference treatment of the forecast equations uses simple, centered, three-level approximations for spatial and temporal derivatives. These can be shown to generate both physical and computational
spatial modes, which tend to be more numerous and severe for the present case of a nonstaggered A grid.

In analogy to the typical one-way interactive boundary condition approach of most local models (see section 4), no feedback of the local forecast is allowed upon the outer domain evolution. Thus, the lateral boundary conditions of the local region must adequately include forcing by the outer domain and also minimize spurious internal reflections of the inner solution. The following section describes the effectiveness of Fourier filtering and Laplacian diffusion in the case that the wave propagating through the outer domain has zero amplitude, so that the lateral boundary conditions of the inner region are simply zero.

The diffusion, when used, is applied most strongly in buffer zones adjacent to the lateral boundaries. The maximum value of the diffusion coefficient is $2.25 \times 10^7$ m$^2$ s$^{-1}$ on the boundary and decreases linearly over five grid points to $1 \times 10^3$ m$^2$ s$^{-1}$. The latter value is applied through the interior of the domain.

b. Spatial filtering

Three tests were done without interaction with the outer model. The local forecast was initialized with 0 amplitude and evolved with the local forcing. The boundaries were maintained at 0. The first test is without Fourier filtering or diffusion, the second retains the Fourier filter but no diffusion, and the third has diffusion, as described previously, but no Fourier filter. Figures 2a–c depict the three tests after 12 h of simulation time. The computational noise obscures the signal due to the physically forced, wavenumber 4 pattern, in the first and third runs. Only in the second run, with the Fourier filter, are the computational modes suppressed. From this it is concluded that Fourier filtering is an efficient way to minimize the computational wave noise that occurs as a consequence of lateral boundary reflection and the use of the A grid. However, that approach is facilitated by the fact that the lateral boundary conditions are zero in the present case, insuring periodic behavior for which the Fourier transform works well. In more general applications, the inner model will not have periodic lateral boundary conditions, and the method must be modified.

In the case that the lateral boundary conditions are not periodic, it is useful to note that the computational mode difficulties discussed above are due to the forecast evolution within the local domain. If the difference field of this forecast evolution from the outer model forecast is taken, the result is zero on the boundaries, where the inner model and outer model values are the same, and the resulting periodic field, which contains all of the evolution predicted by the local model, can be Fourier filtered. Although this field is periodic, its derivatives are not, so it is not evident a priori that the resolution of the problems will be as effective as in the previous section. The results of this subsection demonstrate the effectiveness of the method and the sensitivity to diffusion and Davies nudging.

The "diffusion" curve in Fig. 3 depicts the effects of adding Laplacian diffusion to the inner model forecast equations. Each inner model forecast is made with
lateral boundaries set by the outer model and Fourier filtering applied to the difference field to remove forecast wavelengths between $2\Delta x$ and $4\Delta x$. The addition of strong diffusion near the boundaries does not seem to significantly affect either the phasing or amplitude of the waves generated by the inner domain forcing, but it does shift the forecast closer to the outer domain forecast. In all cases, Fourier filtering of the $2\Delta x - 4\Delta x$ waves from the difference fields maintains stability by controlling the computational modes.

The Fourier filters are apparently effective in eliminating computational modes arising in a local $A$ grid model. However, they do not filter out the reflection of physical modes from the lateral boundaries of the local forecast domain. This is ordinarily done with highly diffusive sponge layers adjacent to the lateral boundaries together with explicit nudging of the local, inner model solution toward the values provided by the outer model in a thin forecast region adjacent to the perimeter of the inner forecast domain.

In the present case, Eqs. (8) and (9) are modified to

$$
\frac{\partial u}{\partial t} = -g \frac{\partial \eta}{\partial x} + A \sin[k(x - c_1 t)] - D(u - u_{in}) \quad (10)
$$

$$
\frac{\partial \eta}{\partial t} = -H \frac{\partial u}{\partial x} + B \sin[k(x - c_1 t)] - D(\eta - \eta_{in}), \quad (11)
$$

where $D$ is the nudging factor, and $u_{in}$, $\eta_{in}$ represent the outer model prediction. The nudging factor decreases as the lateral boundary increases by a maximum on the lateral boundaries calculated by

$$
D_{max} = \frac{1}{4\Delta t}, \quad (12)
$$

where $\Delta t$ is the forecast time step. The Davies nudging profile chosen has an exponential decrease from the maximum value, to the interior value, over four grid points. The curve labeled “Davies” in Fig. 3 compares the inner model forecast using the Davies nudging profile on the lateral boundaries with previous results. Davies nudging, similar to increased boundary diffusion, shifts the average of the locally forced waves toward the solution from the outer domain, without changing either the phase or amplitude of the waves. It does, however, dampen the amplitude of the waves near the boundaries where the nudging is strongest.

c. Numerical schemes

The effectiveness of the Fourier filter was further tested by modifying the application to remove only wavelengths of $2\Delta x$ and in the range between $2\Delta x$ and $3\Delta x$. The results (not shown) suggest that the extraction of wavelengths in the range of $2\Delta x - 4\Delta x$ eliminates obvious difficulties associated with the $A$ grid and reflecting lateral boundary conditions. They also suggest that less severe filtering (i.e., only of wavelength $2\Delta x$ or of wavelengths $2\Delta x - 3\Delta x$) is also sufficient, although the spatial computational modes of the present nonstaggered grid can be shown to cover the entire wavelength range from $2\Delta x$ to $4\Delta x$.

Most numerical models use higher-order horizontal space differencing than the second-order differencing of the present tests. Such treatments can be shown to have a broader range of computational spatial modes. All model integrations in later sections use second-order horizontal differencing. For completeness, however, we demonstrate the utility of this filtering technique for fourth-order finite-difference and finite element spatial treatments. Three horizontal difference schemes are compared, using centered time differencing in all cases. Prior tests of Eq. (5) used a second-order finite-difference quotient approximation:

$$
\frac{u^{n+1}_i - u^{n-1}_i}{2\Delta t} = -g \left[ \frac{\eta^{n+1}_i - \eta^{n-1}_i}{2\Delta x} \right]. \quad (13)
$$

The equivalent fourth-order finite-difference quotient approximation is written as

$$
\frac{u^{n+1}_i - u^{n-1}_i}{2\Delta t} = -g \left[ \frac{4}{3} \left( \eta^{n+1}_i - \eta^{n-1}_i \right) \right] - \frac{1}{3} \left( \eta^{n+2}_i - \eta^{n-2}_i \right). \quad (14)
$$
and the finite element expression is
\[
\frac{u_i^{n+1} - u_i^n}{2\Delta t} = -g \left( \frac{\partial \eta}{\partial x} \right)_i
\]
(15)
where \( \frac{\partial \eta}{\partial x} \) is obtained by solving
\[
a_i \left( \frac{\partial \eta}{\partial x} \right)_{i-1} + b_i \left( \frac{\partial \eta}{\partial x} \right)_i + c_i \left( \frac{\partial \eta}{\partial x} \right)_{i+1} = \frac{\eta_{i+1} - \eta_{i-1}}{2}
\]
(16)
for \( \frac{\partial \eta}{\partial x} \), where
\[
a_i = \frac{x_i - x_{i-1}}{6}
\]
(17)
\[
b_i = \frac{x_{i+1} - x_{i-1}}{3}
\]
(18)
\[
c_i = \frac{x_{i+1} - x_i}{6}
\]
(19)

Analogous changes were made to approximate Eqs. (6), (8), and (9). The graphical comparison of the resultant forecasts is shown in Fig. 4 and indicates that the proposed Fourier filtering is effective for higher-order space-differencing approximations as well.

4. Boundary schemes

a. Approaches

The basic LAM or inner model forecast equation can be expressed in a highly schematic form as
\[
\frac{dQ}{dt} = L(Q),
\]
(20)
where \( Q \) may be a single forecast variable or a vector of forecast variables and \( L(Q) \) represents local influences tending to change \( Q \).

Juang and Kanamitsu (1994) propose decomposing \( Q \) in a nested model into two components:
\[
Q = Q_i + Q_o,
\]
(21)
where \( Q_o \) represents all scales resolved by the outer model and \( Q_i \) represents all other smaller scales that are resolved only by the inner model. Therefore, the resulting forecast equation can be written as
\[
\frac{dQ_i}{dt} = L(Q) - L(Q_o),
\]
(22)
Note that \( L(Q_o) \) is obtained from the outer model forecast, even within the inner model domain. An advantage to this method is that \( Q = Q_o \) on the inner model boundary, thus \( Q_i = 0 \) on the boundary, and a spectral expansion of \( Q_i \) may be possible.

In the Juang and Kanamitsu (1994) formulation expressed by Eqs. (21) and (22), the forecast variable, \( Q_i \), is a deviation variable; whereas in the Utah LAM, the forecast variable is the complete field \( Q \). The next two sections describe the incorporation of the large-scale forcing by Davies nudging and spectral nudging.

Davies nudging in the LAM can be expressed by modifying Eq. (20) to include a nudging term, \(-K(Q - Q_o)\), on the right-hand side of the forecast equation for \( Q \):
\[
\frac{dQ}{dt} = L(Q) - K(Q - Q_o),
\]
(23)
where \( K \) decreases with increasing distance from the lateral boundary, so that the transition to the boundary value is accomplished smoothly. This nudging method is applied in many models that use one-way interacting boundary conditions and apparently performs quite well, as shown for the shallow-water example in section 3b.

We now suggest a modification of Davies and Juang and Kanamitsu’s methods, which allows partial incorporation of each in a way that depends upon the scale of the forecast field. The modification is to use the equation
\[
\frac{dQ}{dt} = L(Q) - \sum_{|m| < M^*} \sum_{|n| < M^*} K_{mn} (A_{mn} - A_{mn}^*) e^{ik_{mx} x} e^{ik_{ny} y},
\]
(24)
where $M''$ is defined as the highest wavenumber, defined relative to the inner grid, that is well predicted by the outer model, and where $A''_{nm}$ are Fourier coefficients of a field that is defined as $Q_e$ within the local forecast domain and 0 on its boundaries, and $A_{nm}$ are similar Fourier coefficients obtained by incorporating the solution $Q$ given by the LAM within the LAM domain and values of zero at its perimeter. The result is an equation where the nudging is done selectively in the wavenumber domain.

If $K_{nn}$ is sufficiently large, and if $M'' = (\text{local model domain length})/(2 \times \text{outer model grid size})$, then Eq. (24) allows the inner solution to be everywhere affected by the outer model forecast, while if $K_{nn} = 0$ and $L(Q)$ retains the Davies nudging of Eq. (23), then Eq. (24) is equivalent to the classical method of one-way interacting boundary conditions. If $K_{nn}$ varies with $m$ and $n$ then the basic benefits of each method can be retained, by selectively nudging the longer waves toward values forecast by the large-scale model and allowing the shorter waves to evolve following the LAM treatment. This is accomplished by making $K_{nn}$ large for long waves—that is, $(m, n \approx M'')$—and small for shorter scales—that is, $(m, n \approx M'')$.

This method should be superior to one-way interacting boundary conditions because it allows for the possibility that some scales within the LAM may be better predicted by the larger-scale outer model. The flexibility of the method is appealing. However, it should be noted that the Fourier coefficients $A_{nm}$ and $A''_{nm}$ represent the transforms of fields that drop abruptly to 0 values at lateral boundaries. The resulting spectra may contain high amplitudes in short waves, particularly when the outer solution is not periodic on the inner domain. The method may nonetheless work well because the Fourier contributions at high wavenumbers may cancel in Eq. (24), but the effectiveness of the approach needs to be demonstrated. This is done in the next section where the spectral decomposition and selective nudging are tested using the one-dimensional model discussed in section 3.

b. Spectral nudging in the shallow-water model

Forecasts with spectral nudging are sensitive to the amount of the spectrum that is nudged. Because the local forcing of the shallow water model of section 3 produces a wavenumber 4 pattern, nudging wavenumbers 0–4 or greater should essentially eliminate the effects of the local forcing. Thus, to depict the effects of the spectral nudging, Fig. 5 compares cases nudging wavenumbers 0–2 and wavenumbers 0–8 to a test without spectral nudging. All tests use Fourier filtering to remove wavelengths between $2\Delta x$ and $4\Delta x$, the Davies nudging profile used in the previous section, the spectral nudging factor specified by $K_{nn} = 1/4\Delta t$ for wavenumbers within the nudged range, and $K_{nn} = 0$, for wavenumbers outside of the nudged range.

Nudging wavenumbers 0–8 virtually eliminates the effect of the local forcing from the inner model, reproducing the outer model forecast, and nudging wavenumbers 0–2 dampens the amplitude of the local forcing while leaving the phase unchanged. The outer model forecast is distinctly aperiodic over the inner domain in Fig. 5. Nevertheless, the spectrally nudged solutions do not show any pronounced short-wave anomalies.

Applying this technique to the Utah LAM is complicated by the three-dimensional nature of the model and the many scale interactions. Because the grid size, terrain, and vertical resolution near the surface differ greatly between the Utah LAM and the regional model, and the regional model forecasts are only available at 6-h intervals, it is hypothesized that the lowest model levels can be best forecast without nudging from the outer or regional model. Results for tests with three different spectral nudging schemes are compared to tests without spectral nudging in the next section.

5. Utah LAM forecasts

The Department of Meteorology at the University of Utah currently receives the eta and NGM model forecasts produced at the NCEP at resolutions of 80 km.
These model forecasts are used to provide the “outer” model predictions incorporated for initial and boundary conditions of the Utah LAM described in section 2. The Utah LAM is currently run daily in real time with horizontal grid spacing of approximately 25 km. The domain is centered over Utah and is approximately 800 km on a side, resolved by 32 grid intervals in each horizontal direction. This representation allows 16 waves in each direction, ranging in wavelength from approximately 50 km to about 800 km. The shortest wavelengths, between approximately 50 and 100 km, are filtered explicitly to control spatial computational modes as discussed in sections 2 and 3. The remaining resolved waves have wavelengths between approximately 100 and 800 km, corresponding to wavenumbers in the range of 1–8. Selected forecasts are evaluated and compared with observations by Horel and Gibson (1994).

The version of the Utah LAM used in this study and described in section 2 is run with 64 grid points, in each horizontal dimension, over the same domain. Thus, the grid size is approximately 12 km and wavelengths between approximately 50 and 800 km are resolved, after applying the Fourier filter. This corresponds to wavenumbers between 1 and 16. The shortest wave resolved by the NGM and eta model, run at 80 km grid size, corresponds approximately to wavenumber 5 on this LAM grid. The spectral nudging methods described in the previous section can be invoked by nudging spectral wave components to the NGM values for wavenumbers less than 6.

The initial data is from a winter storm studied earlier by Gibson (1993) and Horel and Gibson (1994). This case was chosen for a detailed model comparison because the rapid movement of a cutoff low through the domain is a difficult feature for a small-scale model to
forecast; yet the larger, regional models cannot ade-
quately forecast the interactions with the complex ter-
rain of this area. The Utah LAM simulations in this
section focus on the period from 0000 to 1200 UTC 7
January 1992. Figure 6 depicts the NGM analysis and
shows the strong temperature gradient that was present
at the beginning of the simulation.

a. Comparison of large-scale forcing

The NGM analysis and its 6 and 12 h forecasts are
used as the outer model forecast for this series of four
tests. Figure 7a shows the NGM 12-h forecast of the
500-mb streamlines and isotherms, horizontally inter-
polated to the LAM grid. Figure 8 depicts the observed
hourly wind profile from Dugway, Utah (located at
DPG in Fig. 1a), and Fig. 9a shows a time–height cross
section corresponding to the location of DPG for the
NGM forecast, horizontally interpolated to the LAM
grid and temporally interpolated to 1-h intervals. The
NGM forecast represents the midtropospheric trough
passage between 0100 and 0300 UTC well but does not
forecast the development of the northwesterly low-
level jet that began at 0500 UTC. The vertical velocities
for the interpolated NGM forecast were very small.

In the first LAM case (not shown), only the perim-
eter grid points are nudged by the evolving NGM fore-
cast, linearly interpolated to the LAM each time step.
In the second, and subsequent experiments, the lateral
boundaries utilize Davies nudging. The differences in
the forecast fields in the interior part of the domain
were very slight, thus only the results from the Davies
nudging test will be shown. Figures 7b and 9b depict
the 12-h 500-mb forecast and DPG time–height cross
section, respectively, from the Davies nudging test. The
streamlines and isotherms show considerably more
structure, especially near high topography, than the
NGM forecast even at 500 mb. The trough passage
does not show a sharp signal in the cross-section winds
in this experiment, although a strong center of vertical

![Image](image_url)

**Fig. 7.** The 500-mb temperatures and streamlines for 12-h forecasts valid 1200 UTC 7 January 1992, overlaid on the LAM topography for (a) NGM, (b) LAM with Davies nudging, (c) LAM with spectral nudging of wavenumbers 0–5, and (d) LAM with spectral nudging of wavenumbers 0–2. The contour interval for temperature is 1°C.
velocity is forecast at the time of trough passage and the development of the low-level jet is better depicted.

For the last two cases, spectral nudging is used, but confined to the domain at and above 3 km AGL to allow greater interaction of the forecast fields with the topography of the LAM. The third case nudges wavenumbers 0–5, as defined on the LAM domain, similar to the approach of Juang and Kanamitsu (1994) in which all scales resolved by the larger-scale model affect the LAM internally. As mentioned earlier, although wavenumbers 0–5 of the LAM are long enough to be resolved by the NGM, the shortest of these waves are strongly influenced by diffusion and dissipation terms within the NGM. Therefore, in the fourth case, following the method developed in section 4c, wavenumbers 0–2 from the outer model are used to nudge the LAM. Figures 7c,d and 9c,d show the 500-mb fields and time–height cross sections for these runs, respectively. Both of these cases show more structure than the NGM in the 500-mb fields and a better forecast of the low-level jet at DPG, while also having a more clearly defined trough passage than the Davies nudging experiment. This is evident in the wind shift between 0100 and 0300 UTC at the 600 through 500-mb levels. All of the LAM experiments produced a strong vertical velocity response, probably due in part to the small grid size of the LAM; although the spectral nudging appears to reduce the magnitude.

Figure 10a displays 12-h accumulated precipitation predicted for the period from 0000 to 1200 UTC 7 January by the NGM, interpolated to the LAM grid. Figures 10b–d display the results of the LAM with Davies nudging, spectral nudging of wavenumbers 0–5, and spectral nudging of wavenumbers 0–2, respectively. There is clear sensitivity to resolution (cf. panel a to the others), but relatively small sensitivity to nudging.

Verifying precipitation over the 24-h period ending at 1200 UTC 7 January is displayed in Fig. 6a of Horel and Gibson (1994). This shows a local maximum in excess of 20 mm southeast of the Great Salt Lake, most of which fell in the forecast period displayed in Fig. 10. For the 12-h period of Fig. 10 Salt Lake City (located just southeast of the lake) received 5.6 mm and Hill AFB (just east of the lake) recorded 16 mm precipitation. There are no observations directly southwest of the Great Salt Lake, and it is not possible to validate the LAM there. In comparison to the other LAM, the forecast made with wave 0–2 nudging (Fig. 10d) has this center slightly further north and east, closer to the available measurements. Overall, these precipitation forecasts show little sensitivity to spectral nudging.

To further compare the cases, rms differences between pairs of tests are shown in Fig. 11 for the vector wind field at 700 mb and 100 m AGL. The strongest sensitivity in the present set of experiments is between the NGM and the Utah LAM simulation with Davies nudging demonstrating the importance of resolution enhancement. The differences between the perimeter and Davies nudging tests were relatively large near the lateral boundaries and near zero in the interior of the domain (not shown). Thus, the integrated rms impacts appear similar between this comparison and the comparison between the (0–2) wavenumber spectral nudging and Davies nudging, while the impact on the interior forecast is much greater in the latter case. Little difference is seen between the two spectral nudging cases. Although there is no spectral nudging below 3000 m, the (0–2)–(0–5) and (0–2) DAV curves are nonzero in Fig. 11b, due to vertical propagation.

b. Spectral analysis

Spectra of the forecast fields were computed following the procedure developed by Errico (1985) using two-dimensional Fourier decomposition. In a rectangular domain, this procedure requires periodic boundary conditions and regularly spaced data. The Utah LAM results are on a regular grid, but the lateral boundaries are not required to be periodic; the Fourier decompositions within the LAM are performed on difference fields, which are restrained to be periodic. Errico (1985) solves this problem by removing a linear trend field from the data before the spectra are calculated. The two-dimensional array of spectral coefficients is reduced to a simpler, directionally independent form by integrating around annuli with widths corresponding to the fundamental wavenumber of the grid. The width of the fundamental wavenumber \( \Delta k \) is calculated from
Figure 9. Hourly profiles of forecast horizontal winds (vectors in meters per second) and vertical velocities (contour interval is 20 cm s$^{-1}$) for DPG from (a) NGM, (b) LAM with Davies nudging, (c) LAM with spectral nudging of wavenumbers 0–5, and (d) LAM with spectral nudging of wavenumbers 0–2. The vertical axis is labeled in millibars, and time increases from right to left.

\[
\Delta k = \frac{2\pi}{(n_{\text{max}} - 1) \Delta x},
\]

(25)

where "\(n_{\text{max}}\)" is the maximum horizontal dimension and \(\Delta x\) is the horizontal grid size. For the present simulations, \(\Delta k = 8.9 \times 10^{-6} \text{ m}^{-1}\).

Figure 12a depicts the spectra for the kinetic energy field at 500 mb from the NGM analysis and forecasts, horizontally interpolated to the LAM domain, that were used to initialize and nudge the Utah LAM. The variance decreases nearly two orders of magnitude between wavenumbers 1 and 3 for both analyses and forecasts.
The NGM grid spacing is approximately 80 km, which corresponds to wavenumbers 0–5 on the LAM grid; the small amplitude variances for waves shorter than wavenumber 5 are introduced by the interpolation procedure. For the Davies nudging case (Fig. 12b), the slope of energy variance with increasing wavenumber is not nearly as steep as the NGM fields, which is due, at least in part, to the effects of the more complex topography in the Utah LAM.

Figures 12c and 12d are the kinetic energy spectra for the simulations nudging wavenumbers 0–5 and 0–2, respectively. As discussed in section 5a, the nudging in these two runs is imposed at and above 3 km; therefore, the nudging is more apparent at levels above 500 mb (not shown). Nudging is not done below 3 km because boundary layer treatment and resolution of the outer model may often be inconsistent with their LAM counterparts even for large horizontal scales. In both cases, the spectra resemble the NGM spectra for the nudged waves, then there is an increase in amplitude corresponding to the first unnudged wavenumber. This jump creates a local minimum of kinetic energy between wavenumbers 5 and 6 in the case of wavenumber 0–5 nudging, but only a discontinuity of slope in the case of wavenumber 0–2 nudging.

6. Sensitivity to outer model

The tests presented in this section are from an ensemble of seven forecast periods. Three simulations from the Utah LAM are done for each initial time; the NGM and eta analyses are each used to initialize and provide lateral boundary conditions for a series of tests using the Davies nudging scheme. Then the eta analyses are used to initialize and nudge a series of tests using the 0–2 wavenumber spectral nudging scheme. The NGM and eta forecasts are from the period of 5–11 October 1993, using the 0000 UTC runs. This pe-

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**Fig. 10.** Twelve-hour accumulated precipitation forecasts for the period from 0000 to 1200 UTC 7 January 1992 for (a) NGM forecast, (b) LAM forecast with Davies nudging, (c) LAM forecast with spectral nudging of wavenumbers 0–5, and (d) LAM forecast with spectral nudging of wavenumbers 0–2. Contour interval is 1 mm in panel (a) and 1 cm in panels (b), (c), and (d).
period was selected because it contained a mixture of active and benign weather conditions over the western United States. At the beginning of this seven-day period, the intermountain region was under the effect of a northwest tilted high pressure ridge and extremely flat thermal gradient. Midway through the week, the combination of moisture brought into the intermountain region, with the low pressure center moving east from California, and the cold temperatures and dynamics from a stronger low pressure center that moved southeast from Washington to northern Utah by 9 October 1993, caused heavy rainfall throughout the state. By the end of this period, the 500-mb low pressure center moved over southeastern Wyoming and a ridge was reestablished along the western Utah border, with heights continuing to rise on 11 October 1993. More details, including selected weather maps, are shown by Waldron (1994).

Since data frequency and density are insufficient for a detailed verification of the Utah LAM results for the domain as a whole, rms differences between ensembles of forecasts are used to quantify model sensitivity. Figures 13a and 13b show the rms differences in the vector wind fields due to varying the outer model (eta versus NGM). The LAM forecasts in both ensembles use the Davies nudging scheme on the lateral boundaries, without any spectral nudging. One trend evident in the vector wind field is that there is relatively weak amplification for this comparison of different outer model forecasts. That is, the general magnitudes of the rms differences show only a weak increasing trend.

The corresponding difference graphs between the Davies nudging scheme and spectral nudging of wave-numbers 0–2, both using the eta model as the outer model forecast, are shown in Figs. 13c and 13d. Since both series of Utah LAM forecasts in this comparison use the eta model as an outer model forecast, the initial differences are 0. The largest change is in the first 1–3 h at all levels, with most of the curves becoming essentially flat after 6 h of simulation time.

Comparing the two series of graphs shows that changing the outer model appears to have a slightly larger impact on the resulting LAM forecasts than changing the nudging scheme, but the relative magnitude of the changes is sufficiently similar to conclude that both are significant effects.

7. Summary and conclusions

The overall goal of this study has been to explore methods of boundary specification and initialization in a highly resolved limited-area model for applications over complex terrain. A step toward this goal is the development of a modeling approach that adequately resolves the topographic effects in a sufficiently economical approach to allow real-time implementation on local workstations. Cotton et al. (1994) describe local real data implementations of the CSU RAMS, and Warner and Seaman (1990) do this for the PSU–NCAR model.

The terrain-following coordinate and the high-resolution treatment of sloped turbulent boundary layers, based upon the method originally outlined in the study by Paegle and McLawhorn (1983), have special advantages compared to the more commonly used σ system derived from pressure coordinates. The advantages
include extraction of a basic-state thermodynamic field, which is a function of height above sea level rather than elevation above surface, and a reflection of topography in the height of the coordinate surface that does not increase with elevation. These benefits are especially important in reducing truncation error of pressure gradient calculations in deep implementations of the model around complex topography and have not been emphasized in prior publications, most of which incorporated a shallow boundary layer application of the model.

The eta model used at the NCEP has even less truncation error for pressure gradient calculations than does the Utah LAM, but it is easier to implement high-resolution sloped boundary layers in the present approach than in the eta coordinate.

Experiments presented in section 3 show that Fourier filters are effective in controlling the computational modes that result from use of an unstaggered grid. The Fourier filter can be easily implemented in cases of nonperiodic lateral boundary conditions by applying it to the forecast deviation produced by the inner (LAM) model relative to the outer model state over the LAM domain.

The filtering approach leads naturally to a method that allows imposition of the large-scale fields throughout the inner model (LAM) domain. The method is inherently more complete than that permitted in more traditional approaches that impose outer model information only at the boundary or in a thin region adjacent to the boundary. The method suggested in section 4 is to nudge the LAM toward the outer model forecast in wavenumber space. This suggestion, which appears to be original in this context, leads to a formalism that is analogous to limited-area spectral models such as those described by Tatsumi (1986), Segami et al. (1989), and Juang and Kanamitsu (1994).

The approach suggested in section 4 utilizes the forecast fields of the outer model to specify the background state with respect to which the deviation fields are calculated. The advantage of this relative to the methods described by Tatsumi (1986) and Segami et al. (1989) for the Japanese limited-area model is that these background fields do not need to be computed; they can...
simply be read in from time-interpolated outer model predictions. This benefit is also contained in Juang and Kanamitsu's (1994) approach. The method proposed in section 4 carries the additional flexibility that it may be incorporated between different model configurations.

The first experiments in section 5 simply evaluate the modifications produced by the enhanced resolution above the present topography imposing the outer model only at the lateral boundaries. This produces the largest forecast sensitivity produced of our experiments, underlining the critical role of adequate resolution of the lower boundary above complex terrain. Spectral decomposition of forecasts show that the small scales are generated within 1–3 h. The experiment nudging only wavenumbers 0–2 within the LAM domain produces a much more continuous spectrum than does nudging wavenumbers 0–5.
In view of the sparseness of verification data, section 6 focuses upon sensitivity questions in order to ascertain whether the Utah LAM is more sensitive to outer model options or to different methods of implementing the outer model predictions within the inner model. In the case of the 12-h predictions compared in section 6, the effect of the available outer model is slightly more important than is the effect of the method used to impose the outer model. However, since the differences are of similar magnitude, a clear priority, with regard to future research emphasis, has not been established.

Changing the outer model modifies initial conditions as well as boundary conditions, and this blurs the relative roles of boundary value influences and initial state influences in the comparison of the ensembles. However, it is evident that the Utah LAM must rely on a regional model for initial and boundary conditions and that the skill of the LAM forecasts is dependent on the accuracy of the regional model forecasts. Therefore, the spectral nudging method, which extracts more information from the regional model, should be preferable when the regional model forecast is good. A logical continuation of this research is to examine the strength of the nudging factor. Preliminary tests have already been done on making it dependent on height and it could also be varied in time—that is, stronger early in the forecast period and tapering off at later times.

The main purpose of the study has been to demonstrate the feasibility of spectral nudging within a limited-area model. As previously stated, the reasons to perform nudging are to permit external conditions to influence local evolution in a more uniform fashion than that allowed merely through boundary conditions, as in traditional approaches. A similar philosophy is inherent in the regional spectral models of Tatsumi (1986), Segami et al. (1989), and Juang and Kanamitsu (1994), which also allow global-scale information to permeate the local domain.

Nudging of larger internal scales may be beneficial in those cases where these scales are more directly influenced by global-scale structures than by smaller scales. This is the situation for advective processes such as those studied by Lorenz (1969). He shows that the evolution of a given wave scale is more sensitive to relatively larger scales than to relatively smaller scales. Spectral nudging provides a cleaner way to directly impose the potentially superior global model information on the larger internal scales than does boundary nudging.

The present example does not provide clear evidence for superior forecasts when using spectral nudging of larger-scale information across the entire finer-scale domain as compared to using standard methods to specify large-scale conditions at only the lateral boundaries (e.g., Davies nudging). It would be difficult to claim that a single example would prove the method's superiority, even if it were more successful. A larger set of carefully constructed numerical experiments is required to further probe the theoretically interesting question regarding relative sensitivity of local prediction to internal and external influences. Our ongoing research addresses this question.

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