Evaluation of the Kinetic Energy Approach for Modeling Turbulent Fluxes in Stratocumulus

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ABSTRACT

The modeling of vertical mixing by a turbulence scheme on the basis of prognostic turbulent kinetic energy (E) and a diagnostic length scale (l) is investigated with particular emphasis on the representation of entrainment. The behavior of this E-l scheme is evaluated for a stratocumulus case observed in the Atlantic Stratocumulus Transition Experiment, and a comparison is made with the results of large eddy simulation models for the same case. It appears that the E-l model is well capable of reproducing the main features of vertical mixing and entrainment. This is the case with a high vertical grid spacing of 25 m and a short time step of 1 s, even with a relatively simple formulation for the turbulent length scale. However, the model results degenerate rapidly on coarse temporal and spatial resolution. For time steps on the order of 1 min it is shown that the process-splitting time integration scheme (in which the tendencies due to turbulent diffusion and radiation are computed independently) results in a too cold cloud top, a too large buoyancy flux, and a too high entrainment rate. For a vertical grid spacing of the order of 200 m (as commonly used in operational models) the model does not behave well either. At such resolution, entrainment appears to be predominantly related to the Eulerian (gridbox) representation of the cloud and not to the physics of the turbulence scheme. This gridbox representation of the cloud prevents the cloud from descending due to prevailing large-scale subsiding motion, and therefore generates an entrainment rate that balances the subsidence rate. An unphysical dependency of the entrainment rate on the subsidence rate results. A general conceptual model for this behavior is presented. Finally, the relevance of these results for large-scale atmospheric models is discussed.

1. Introduction

Present-day atmospheric general circulation models (GCMs) have large difficulties simulating a realistic amount of boundary layer clouds in upwelling regions of the subtropical ocean (see, e.g., Browning 1994). The underestimation of the cloud cover in these regions, and its large influence on the radiation budget, motivated researchers to organize extensive measuring campaigns, like the Atlantic Stratocumulus Transition Experiment (ASTEX; Albrecht et al. 1995; Bretherton and Pincus 1995; Bretherton et al. 1995; Roode and Duynkerke 1997). Based on the measurements during ASTEX, several test cases were defined to be run with large eddy simulation (LES) models and single column models (SCMs) of GCMs. These cases provide a good opportunity to study the performance of the cloud-turbulence schemes used in GCMs in comparison with the output of LES models and the observations. The idealized case that is studied in this paper is defined in the European Cloud Resolving Modeling (EUCREM) model-intercomparison project, and encompasses a 3-h simulation of nocturnal stratocumulus (Duynkerke et al. 1999, hereafter DU99).

Cloud-top entrainment has a large influence on the time evolution of cloud-topped boundary layer (see, e.g., Bretherton 1997). Due to entrainment the cloud top rises (causing a cooling of the cloud top) and warm and dry air is mixed into the boundary layer. The modeling of entrainment and the modeling of boundary layer clouds are therefore strongly related. There are a few different approaches of modeling entrainment in GCMs. In some atmospheric models the entrainment fluxes are explicitly prescribed. These models make use of simple parameterizations of the entrainment fluxes (e.g., Beljaars and Viterbo 1998). In other models the mixing profiles are extended slightly beyond cloud top into the inversion to generate entrainment (e.g., van Meijgaard and van Ulden 1998). There are also turbulence schemes that use prognostic equations for the turbulence in order to generate entrainment. One of the most popular (because it is relatively simple and...
computationally inexpensive) of these turbulence schemes is the 1.5-order $E$--$l$ scheme. This $E$--$l$ turbulence closure contains a prognostic equation for the turbulent kinetic energy $E$ combined with a diagnostic length scale $l$. In this scheme, the transport of boundary layer $E$ into the inversion is responsible for entrainment. More advanced schemes based on the turbulent kinetic energy (TKE) equation may also use a prognostic equation for the length scale (e.g., Mellor and Yamada 1982; Yamada and Kao 1986; Duynkerke and Driedonks 1987). In this study we investigate the simplest branch of the TKE schemes: the $E$--$l$ scheme with diagnostic length scale.

Atmospheric GCMs are usually run with a coarse spatial and temporal resolution, typically with only a few levels in the atmospheric boundary layer and time steps of several tens of minutes. The use of such a coarse vertical resolution may strongly affect the simulation of the entrainment processes, since these processes occur on a very fine scale at the interface between the cloudy boundary layer air and the clear inversion air. Furthermore, the time representation may lead to large numerical errors too, in particular near the cloud top where the tendencies due to radiation and entrainment are large. It is therefore possible that, although the $E$--$l$ turbulence closure scheme contains the relevant physics to describe entrainment, its numerical representation on the coarse spatial and temporal resolution does not lead to realistic results (see also Bechtold et al. 1996; van Meijgaard and van Ulden 1998). This issue, however, has not received much attention yet. To investigate this issue, we organized this paper in two parts. At first it is illustrated that a relatively simple $E$--$l$ scheme gives realistic entrainment rates at a high vertical resolution and with a small time step (section 4). Then how the simulation of entrainment is affected by the coarse spatial and temporal resolution that is currently used in GCMs (sections 5 and 6) is investigated.

In more detail, the content of this paper is as follows: section 2 provides a case description and the LES results for the case. A description of the $E$--$l$ scheme is given in section 3, and results of this scheme on high resolution are given in section 4. In section 5a we illustrate the possible influence of the temporal resolution in a run using a 30-s time step, but retaining the high vertical resolution. In section 5b we then investigate the influence of the spatial resolution by operating the model at the European Centre for Medium-Range Weather Forecasts (ECMWF) vertical resolution of 31 levels. The dependency of the results on the vertical resolution is studied further in section 6. In particular we concentrate on the relation between entrainment and large-scale subsidence. In that section a simple conceptual picture is given of how the Eulerian gridbox representation of the cloud may lead to entrainment. Conclusions are in section 7.

2. Case description and LES results

a. Case description

For our investigations we consider a simulation of nocturnal stratocumulus. This simulation is defined in the EUCREM model-intercomparison project (DU99), and is based on measurements of flight RF06 during ASTEX (Albrecht et al. 1995; Bretherton and Pincus 1995; Bretherton et al. 1995; Roode and Duynkerke 1997). The case consists of a 3-h long simulation, starting at 0700 UTC 13 June 1992 with idealized initial profiles and large-scale forcings. The basic characteristics of the simulation are given here; more specific information on the case is given in DU99.

The initial profiles are well mixed in the boundary layer with the liquid water potential temperature $\theta_l = 288$ K, and the total water content $q_r = 10.7$ g kg$^{-1}$. The inversion, with a jump in $q_r$ of 1.5 g kg$^{-1}$ and a jump in $\theta_l$ of 5.5 K, is located at 700 m (Fig. 1). A horizontal divergence of $1.5 \times 10^{-3}$ m$^2$ s$^{-2}$ is imposed, which corresponds to a large-scale subsidence rate of 2.25 cm s$^{-1}$ at 1500 m. Except for the subsidence there are no other large-scale forcings such as the horizontal advection of heat and moisture. At the surface a latent heat flux of 55 W m$^{-2}$ and a sensible heat flux of 13 W m$^{-2}$ are prescribed; thus there are no interactive surface fluxes. There is little wind shear in the boundary layer. In the simulations the effect of drizzle has been neglected. The LES runs have been performed on a domain of 3.2 km $\times$ 3.2 km $\times$ 1500 m (l $\times$ w $\times$ h). The single column models use a vertical column only (with no explicit horizontal extension, although the cloud and turbulence schemes used in the SCMs are designed to represent some area average). With these initial profiles and forcing the LES models and single column models were integrated 3 h in time, and results have been diagnosed after 2 and 3 h of integration.

b. Results of large eddy simulation models

A short description of the results of the LES models for the present stratocumulus test case is given here. For more results of the LES models we refer the reader to DU99. For our purpose, the outcome of the LES runs is considered to represent reality, although we recognize that also LES models may be subject to errors when small-scale mixing processes near the cloud top are considered (see, e.g., Bretherton et al. 1999; Lewellen and Lewellen 1998).

The LES models typically show a significant entrainment at cloud top; the cloud top rises about 100 m in 3 h time. This rate of cloud-top rise is in agreement with the measurements made during flight RF06 (Roode and Duynkerke 1997). The cloud is 400–500 m thick, after 3 h extending approximately from 300 (cloud base) to 800 m (cloud top). The cloud fraction is (near) 100% in the upper 400 m of the cloud with an approximately linear decrease to zero near cloud base. The inversion
height $z_i$ (defined as the level at which $q_l$ equals 9.4 g kg$^{-1}$) and also the cloud top rises from 700 to about 800 m in 3 h time. The entrainment rate, defined as the rate at which $z_i$ rises minus the large-scale vertical velocity, is 2.0 cm s$^{-1}$ during the third hour.

In Fig. 1 we show for two LES models the profiles of $u_l$, $q_t$, and $q_l$ (liquid water content), and the turbulent fluxes of $q_t$ and $\theta_i$, all averaged during the third hour of the simulation. We have chosen to show the results by Chlond and vanZanten (in DU99) because they show the typical results of the LES models, and also the typical difference among the LES model results (only one other model differs significantly from the results presented here, and simulated less entrainment). A slow warming and drying of the boundary layer is seen in both LES models. This warming is the net result of the entrainment flux at the cloud top and the sensible heat flux at the surface (both warming) and the longwave radiative flux (cooling). The drying results from entrainment (drying) and surface evaporation (moistening). The profiles of $\theta_i$ and (in particular) $q_l$ are well mixed in the cloud, but show small gradients in the subcloud layer. This indicates that the turbulence is less developed in the subcloud layer, and that the cloud and the subcloud layer are close to decoupling [decoupling means that there is no (strong) interaction between the turbulence at the surface and the turbulence in the cloud]. The entrainment flux of $q_l$ is approximately 95 W m$^{-2}$, and the minimum in the flux of $\theta_i$ is approximately $-80$ W m$^{-2}$.

Shear production of TKE is small, except near the surface. The buoyancy flux shows a typical structure with a linear decrease in the dry subcloud layer, a jump at the cloud base and a nearly constant value in the cloud layer, and a sharp minimum in the entrainment zone. This minimum of the buoyancy flux in the entrainment zone is strongly dependent on the subgrid scheme of the LES models, the details of the longwave cooling profile, and the numerical grid. As a consequence relatively large differences occur among the LES results. However, the resulting overall entrainment rate (signified, e.g., by the buoyancy flux in the cloud, the time evolution of boundary layer properties, and the cloud-top height) is not very sensitive to these. It is
noted that the TKE is nearly constant in the whole boundary layer with a sharp local maximum near cloud top and at the surface. This peak results from the horizontal velocity variance \( \langle u'^2 + v'^2 \rangle \), while the vertical velocity variance \( \langle w'^2 \rangle \) does not show such a peak near cloud top (see DU99).

3. Model description

The cloud-turbulence scheme used in the SCM model is based on the physical package used in ECHAM4 (Roeckner et al. 1996). The cloud scheme is basically the scheme by Sundqvist (1978). This scheme has prognostic equations for both water vapor \( q_v \) and liquid water \( q_l \). The cloud fraction is diagnosed as a function of the relative humidity. We used the modified version of the ECHAM4 condensation scheme, which is described extensively in Lenderink et al. (1998). This modified version is less time step dependent than the original ECHAM4 code, and gives (in accordance with the LES results) a 100% cloud cover and a much more spatially coherent cloud structure in the case of stratocumulus (Lenderink et al. 1998).

Following the EUCREM case description, we use a simple radiative scheme that accounts for the longwave cloud-top cooling only. This scheme computes the longwave radiative flux from the cloud liquid water path (LWP) according to

\[
F_{\text{longwave}}(z) = F_{\text{top}} e^{-a \text{LWP}(z)},
\]

where \( a = 130 \text{ m}^2 \text{ kg}^{-1} \), \( F_{\text{top}} = 74 \text{ W m}^{-2} \) is the longwave radiative flux at the cloud top, and LWP(\( z \)) is the integral of the liquid water from level \( z \) to the cloud top. The shortwave radiation has been neglected, since this is mainly a nighttime case.

The turbulence scheme consists of an \( E-L \) closure, which uses a prognostic equation for the TKE \( E \) in combination with diagnostic length scale. The eddy viscosity/diffusivity is parameterized by

\[
K_{m,h} = c \Lambda_{m,h} \sqrt{E},
\]

with \( c = 0.516 \) (Mailhot and Benoit 1982), the length scale \( \Lambda_{m,h} \) (defined below), and the velocity scale \( \sqrt{E} \).

In the ECHAM4 model the turbulent length scale \( \Lambda_{m,h} \) depends on the moist gradient Richardson number \( R \) (Roeckner et al. 1996). It can be shown that in the limit of no wind shear, this turbulent length scale becomes unbounded, which is physically incorrect. This resulted in our case in instabilities of the TKE equation, and instabilities of the cloud cover [see Lenderink et al. (1998) for details].

In this study we therefore use a different formulation of the turbulent length scale, as originally described in Brinkop and Roeckner (1995). In the case of unstably stratified conditions \( N^2 \leq 0 \), the mixing length for momentum \( \Lambda_m \) equals \( l \), where \( N^2 \) is the Brunt–Väisälä frequency (computed from \( q_v \) and \( \theta \) and the cloud fraction) and \( l \) is the Blackadar (1962) length scale

\[
\frac{1}{l} = \frac{1}{\kappa z} + \frac{1}{\lambda_0},
\]

with \( \lambda_0 = 300 \text{ m} \) and \( \kappa = 0.4 \) is the von Kármán constant. In the case of stably stratified conditions \( N^2 > 0 \) the turbulent mixing length for momentum is given by

\[
\frac{1}{\Lambda_m} = \frac{1}{l} + \frac{1}{c_e \sqrt{E/N}}
\]

The constant \( c_e \) influences the mixing length in stable stratifications, and therefore it affects the entrainment at the top of the atmospheric boundary layer. Accordingly, \( c_e \) is referred to as the entrainment constant (Brinkop and Roeckner 1995). For both stable and unstable stratifications, the mixing length for heat \( \Lambda_h \) is determined by \( \Lambda_m/Pr \), with the Prandtl number \( Pr = 0.77 \).

The prognostic equation for the turbulent kinetic energy \( E \) reads as

\[
\frac{\partial E}{\partial t} = - \left( \frac{u'w'}{\partial z} \right) - \left( \frac{v'w'}{\partial z} \right) + \frac{g}{\theta_w} \frac{w' \theta_e}{\partial z} + \frac{\partial}{\partial z} \left( w' E + \frac{w' p'/\rho} - \epsilon, \right.
\]

with on the right-hand side (A) shear production, (B) buoyancy production, (C) vertical redistribution of \( E \) (pressure correlation and turbulent transport), and dissipation \( \epsilon \). (For ease of notation, we omitted here and in the subsequent the bars for the mean variables \( u, v \) and \( \theta_e \).) The dissipation rate is set to \( \epsilon = \frac{E^{3/2}}{15 l} \) (Roeckner et al. 1996). The vertical redistribution of \( E \) is parameterized by eddy diffusion of \( E \) with a diffusion constant \( K_e = K_m \), which gives \( (\partial E/\partial z)(K_e \partial E/\partial z) \). Shear production is also computed by eddy diffusion, yielding \( K_m((\partial u/\partial z)^2 + (\partial v/\partial z)^2) \). To account for water/vapor phase changes on the buoyancy, the buoyancy flux is computed from the turbulent fluxes of \( q_v \) and \( \theta_e \) (see Cuijpers and Duynkerke 1993); the latter turbulent fluxes are also computed by eddy diffusion.

4. Model results

To keep the numerical errors small and to enable a one-to-one comparison with the LES model results, we ran a single column model with the physics described in section 3 on the same high vertical resolution as the LES models; that is, a uniform grid size of 25 m until 1500 m. In addition, we used a small time step of 1 s in order to keep the numerical errors related to the time discretization also small.

Following Brinkop and Roeckner (1995) we first used \( c_e = 0.4 \). Figure 2 shows the profiles of \( \theta_e, q_v \), and \( q_l \), the turbulent fluxes and the TKE (budget), all averaged over the third hour of the integration. Comparing this
figure with the LES results in Fig. 1 it is seen that the inversion after 3 h is located about 40 m higher in the SCM than in the LES models. In addition, the cloud liquid water content is about 0.1 g kg$^{-1}$ lower than in the LES results. Both indicate that the entrainment rate in the SCM is higher than in the LES model. This is again confirmed by computing the mean entrainment rate during the third hour, which turned out to be 2.3 cm s$^{-1}$ (cf. 2.0 cm s$^{-1}$ for the LES).

To get a better agreement between the output of the LES model and of the SCM, a value of $c_e$ of 0.1 is found more appropriate. This value gives good correspondence with the LES results with respect to the entrainment rate, the buoyancy flux in the cloud, and the mean boundary layer TKE. In addition, we performed runs for a dry convective boundary layer driven from the surface. With a surface heat flux of 100 W m$^{-2}$, the entrainment flux (defined as the minimum of the heat flux) was 17 W m$^{-2}$. This is close to the commonly accepted value of 20% of the surface flux (e.g., Stull 1976), though some LES models seem to generate somewhat more entrainment (e.g., vanZanten et al. 1999). This indicates that $c_e = 0.1$ is typical for different boundary layers, and is not just a tuning constant that has to be adjusted for each case separately. From now on, this value of $c_e$ will be used as default in our runs, and results for this value are plotted in Fig. 3.

Comparing the SCM results in Fig. 3 to the LES results in Fig. 1, it is seen that gradients in $\theta_i$, $q_i$, and $q_l$ are somewhat too high in the boundary layer, in particular in the subcloud layer. This suggests that the turbulent mixing is underestimated, which is probably due to an underestimation of the turbulent length scale $l_0$ in Eq. (3). The simulated cloud liquid water is close to the LES results. Also the inversion is located at a realistic height after 3 h in comparison with the LES. The entrainment rate is 2.1 cm s$^{-1}$ during the last hour. Focusing on the TKE (budget) it is seen that the boundary layer TKE simulated by the SCM is slightly high compared to the LES results. Furthermore, it lacks the peak near the cloud top. On the other hand, the TKE distribution in the SCM is similar to the vertical velocity variance ($\overline{w'^2}$) in the LES models (not shown). The buoyancy flux profile in the SCM shows the same shape as in the LES. In the cloud the buoyancy flux is slightly lower, and the negative peak near cloud top is overestimated. (It is noted however that the LES results also show a large difference near the cloud top, which is due to the difference in the unresolved, subgrid buoyancy flux in the LES models.) Figure 4 shows the time evo-
lution of the cloud cover. A solid, approximately 400-m thick cloud layer is seen, which rises from 700 m to 800 m in 3 h time.

Above, we showed the results for \( c_e = 0.1 \) and \( c_e = 0.4 \). Comparing these two results, it appears that the change in \( c_e \) over a factor of 4 only makes a relatively small impact on the entrainment rate. On the other hand, there is a more significant impact on the buoyancy production and, consequently, on the TKE. This relative intensity of the entrainment rate to \( c_e \) is due to the following (compensating) mechanism. The buoyancy flux can be expressed into the turbulent fluxes of \( q_t \) and \( u_l \) by

\[
p_c w' \theta_v' = \alpha \rho_c w' \theta_f' + \beta \rho_L w' q_f',
\]

with \( \epsilon = \theta_c / L = 0.12 \), and \( \alpha \) and \( \beta \) are constants mainly depending on whether or not the air is saturated (Cuypers and Duynkerke 1993). In the cloud, and with the typical temperature and humidity for this case, we have \( \alpha = 0.5 \) and \( \beta \epsilon = 0.4 \). The contributions of the \( q_t \) flux and the \( \theta_f \) flux to the buoyancy flux are therefore of the same order, but of opposite sign. Because the rate at which the \( \theta_f \) flux decreases (becomes more negative) is larger than the rate at which the \( q_t \) flux increases, a net buoyancy flux decrease results for increasing \( c_e \). This causes a decrease in boundary layer TKE and therefore an indirect decrease in entrainment. This counteracts the direct increase in entrainment resulting from the increase of the turbulent length scale. This feedback is dependent on the jumps of \( \theta_f \) and \( q_t \) across the inversion. For a sufficiently large \( q_t \) jump, the increase in \( q_t \) flux becomes larger than the increase in \( \theta_f \) flux, and this negative feedback becomes a positive, destabilizing feedback. In that case more entrainment gives an increase in buoyancy forcing, thus destabilizing the cloud (Randall 1980; Deardorff 1980).

5. Results on operational resolution

a. Time resolution

The results in section 4 were obtained with a time step of 1 s, which kept the numerical errors related to the time discretization small (this was confirmed by runs employing even smaller time steps). In practice, however, much longer time steps are used in GCMs. To study the possible impact of this we performed model integrations using much longer time steps, but still with the same high vertical resolution.

Figure 5a shows the results for the turbulent fluxes and the TKE (budget) obtained with a time step of 30 s. Comparing this figure with Fig. 3 (obtained with the time step of 1 s), it shows that TKE is now too large.
and peaks near cloud top. This is the result of the buoyancy forcing, which has a much too large peak near cloud top. As a result of the large TKE, the entrainment rate is also too large. The cloud cover (not shown) reaches a height of 800 m after 1.5 h, which is twice as fast as in the reference run.

An analysis of this finding reveals that it is related to the heat balance at the cloud top. At the cloud top there is cooling due to the longwave radiation and warming due to entrainment. With the present quasi-stationary conditions these two counteracting processes are approximately in balance. However, the correct equilibrium balance is only established for small time steps. This is illustrated in the appendix with a simple example describing the balance between radiation and diffusion. For this example we show that the process splitting time integration scheme leads to a cool equilibrium balance when the radiative cooling is large. It is shown that this occurs because 1) the turbulent diffusion is computed (over) implicitly (for numerical stability reasons) and 2) the radiation and diffusion are computed independently, based on the values of the prognostic variables of the previous time step. The time integration scheme then converges to [inserting $(\Delta y)_u = 0$ in Eq. (21)]

\[
\theta_{\text{cl um}} = \theta_{\text{eq}} + ab \Delta t, \tag{7}
\]

where $\theta_{\text{eq}}$ is the exact, analytical equilibrium liquid water potential temperature, $b$ is the radiation tendency, and $a$ is the (over)implicit factor of the turbulent diffusion scheme. This equation demonstrates that only in the limit of small time steps ($\Delta t \to 0$) or using a fully explicit diffusion scheme the exact equilibrium balance is obtained. For $a > 0$, the cloud top at the numerical equilibrium state is too cold ($b$ is negative) and the stratification is therefore too unstable. Using $a = 1.5$ (overimplicit, which is the default in the model) this leads, even with a relatively small time step of 30 s, to significant errors in temperature profile at the present high resolution. Because the model uses this stratification to compute the buoyancy flux, the latter becomes too large and this then feeds back into the TKE equation.

In the appendix we also derive that the exact equilibrium state at time $T$ is given by

\[
\theta_{\text{cl um}}(T) = \theta_{\text{eq}} + ab \Delta t (\Delta t)^{a-1} + \text{const},
\]

where $\theta_{\text{eq}}$ is the equilibrium state at zero time, $b$ is the radiation tendency, and $a$ is the (over)implicit factor of the turbulent diffusion scheme. This equation demonstrates that only in the limit of small time steps ($\Delta t \to 0$) or using a fully explicit diffusion scheme the exact equilibrium balance is obtained. For $a > 0$, the cloud top at the numerical equilibrium state is too cold ($b$ is negative) and the stratification is therefore too unstable. Using $a = 1.5$ (overimplicit, which is the default in the model) this leads, even with a relatively small time step of 30 s, to significant errors in temperature profile at the present high resolution. Because the model uses this stratification to compute the buoyancy flux, the latter becomes too large and this then feeds back into the TKE equation.
librium state is obtained if the turbulent diffusion scheme works on an updated liquid water potential temperature field \( \theta_l \) given by

\[
\theta_l^n = \theta_l^{n+1} + \alpha b \Delta t,
\]

where \( \theta_l \) is the value of \( \theta_l \) at the previous "full" time step \( n \) (full means after all processes have been taken into account). Note that the implicit factor \( \alpha \) occurs in this equation. Using this time integration scheme in the SCM, much better results are obtained. The results of this updated integration scheme, again with a time step of 30 s, are shown in Fig. 5b. The results are now very similar to the results in Fig. 3 obtained with a time step of 1 s.

b. Vertical resolution

Having investigated the dependency of the model results to the time discretization, we now evaluate the model performance on operational vertical resolution. For this purpose we used the ECMWF 31-level resolution with 7 levels in the lower 2000 m of the atmosphere. The thickness of the levels ranges between 60 m near the surface and 500 m at a height of 1500 m. Because a rise in the cloud top of 100 m in 3 h time cannot be resolved with the present resolution, we extended the simulation period to 18 h. The time step in these runs was 20 s, and the "updated" time integration scheme [Eq. (8)] was used.

The time evolution of the cloud cover as obtained with \( c_e = 0.1 \) is shown in Fig. 6. It shows that the cloud top remains at the same level during the whole integration period. A jump in the cloud base occurs shortly before 6 h, and the cloud disappears after 13 h. We plotted in Fig. 7 the fluxes of \( q_t \) and \( \theta_l \) during the third hour of the simulation. In comparison with the high-resolution run, the \( q_t \) flux is reasonable but the \( \theta_l \) flux is much too small, in particular near the top of the boundary layer. In addition, the right panel of Fig. 7 shows that the TKE in the bulk of the cloud is much too large; the TKE in the entrainment zone, however, seems rather low. The differences in the buoyancy profile are even more significant. It appears that the typical structure of the buoyancy flux as obtained on high resolution (both in the LES models as in the SCM) has almost dissipated now.

The underestimation of the rise in the cloud top and the overestimation of the buoyancy flux indicate that the (numerical) scheme does not generate sufficient cloud-top entrainment. Therefore, we tried different, higher values of \( c_e \) to compensate for this underestimation. Using \( c_e = 0.4 \) as in Brinkop and Roeckner (1995) we obtained virtually identical results as before. To illustrate the sensitivity to \( c_e \) more distinctly we show in the lower panels of Fig. 7 the results obtained with a more extreme value of \( c_e \); that is, \( c_e = 2 \). It turns out that even an increase of \( c_e \) by more than one order of magnitude does not result in large differences with the reference run; all profiles, including the TKE, are very close to those obtained with \( c_e = 0.1 \). Only in the inversion at 800 m there are small differences; the buoyancy flux is more negative, and the TKE is lower. Also the time evolution of the cloud cover is very similar to the run using \( c_e = 0.1 \), with the cloud dissappearing after 13 h.

From these experiments we conclude that on coarse resolution the description of the turbulent length scale in the inversion is of little importance. This contrasts with the high-resolution runs where there is a modest sensitivity to the turbulent length scale, in particular with respect to the buoyancy flux and the TKE. Apparently, there are other factors that determine the behavior of the coarse-resolution model. From the experiments above it seems that in the coarse-resolution model the cloud top tends to be "locked in" at one level. The cause of this behavior is studied in the next section, and since the time development of the cloud top is the
FIG. 7. Fluxes of \( q_t \) and \( \theta_u \), and TKE (budget) on low resolution. (a) and (b) The results with \( c_e = 0.1 \); (c) and (d) the results with \( c_e = 2.0 \). For reference, [in (a) and (c)] we also plotted the turbulent fluxes obtained on high resolution with \( c_e = 0.1 \).

FIG. 8. Dependency of entrainment rate on the subsidence rate for the high-resolution model (crosses). The \( x \) axis shows the subsidence rate at 700 m (approximately the cloud top). The dashed line corresponds to a one-to-one relation between entrainment and subsidence, as found approximately on low resolution.

6. Entrainment and subsidence

a. Model results

To establish the relation between entrainment and large-scale subsidence, and the dependency of this relation on the vertical resolution, we performed several additional experiments with the SCM on both high and coarse vertical resolution. Our initial expectation was that the subsiding motion would merely displace the (conserved) cloud-top interface downward, and would therefore not directly interfere with the entrainment process.

To investigate whether our hypothesis holds, we ran the SCM (with \( c_e = 0.1 \)) for different values of the subsidence rate. To keep the numerical errors small we used the high vertical resolution as in section 4. For these runs we prescribed a large-scale divergence of \( 0.0 \times 10^{-5}, 0.5 \times 10^{-5}, 1.0 \times 10^{-5}, 1.5 \times 10^{-3} \) (standard), and \( 2.0 \times 10^{-3} \) m\(^2\) s\(^{-2}\), respectively. As a first measure of the entrainment rate we computed the rise of the inversion (approximately rise in the cloud top) minus...
the large-scale subsidence rate at the inversion height. (As noted before, the inversion is defined as the level at which the total water $q_t$ equals 9.4 g kg$^{-1}$.) Figure 8 shows the entrainment rate obtained with different values of the subsidence rate. In all runs the entrainment rate turned out to be close to 2.1 cm s$^{-1}$, and the entrainment rate is therefore basically independent of the subsidence rate, thus confirming our hypothesis.

As a second measure of entrainment we show the turbulent flux of $q_t$. The entrainment flux (i.e., maximum of the turbulent $q_t$ flux near the inversion) is approximately equal to the product of the entrainment rate times the jump of $q_t$ at the inversion (see, e.g., vanZanten 1999). The turbulent flux is shown in Fig. 9 for three different values of the subsidence rate. We also show the flux that results from the integration with height of the tendency due to vertical advection (subsidence). The latter is denoted below as the dynamical flux, and represents the downward forcing by the large-scale subsiding motion. Comparing these runs for different values of the subsidence rate (but all with $c_e = 0.1$) it is shown that the entrainment flux is very similar for all runs. This confirms again that entrainment is nearly independent of the subsidence rate. In addition to the runs with $c_e = 0.1$, we plotted the results of one run with $c_e = 10^{-4}$ (and no subsidence). Using this value for $c_e$ practically suppresses the "physics" leading to entrainment in the $E-l$ scheme, and this run is therefore denoted the "no entrainment physics" run. (Here, "physics" is referring to representation of physics by the $E-l$ scheme, and is added to distinguish from the entrainment generated by numerics as discussed below.) The $q_t$ flux for this run clearly shows that there is no entrainment now, which confirms the influence of the turbulent length scale in the representation of the entrainment by the $E-l$ scheme. (It is also noted that the dynamical flux divergence is small in the boundary layer, and the time evolution of boundary layer $q_t$ is therefore dominated by the flux divergence of the turbulent flux.)

We then repeated the same runs with the model on...
the coarse resolution as in section 5. Because the inversion is not well resolved on this coarse resolution, the first measure of the entrainment rate is not well defined. In all runs the cloud top and the inversion remain at the same height during the whole time integration period. This implies that the entrainment rate approximately balances the large-scale subsidence rate. To inspect the differences more precisely, we study the flux profiles of $q_t$, which are shown in Fig. 10. The dependency of the turbulent flux on the subsidence rate is now evident. At a height of 500 m, this difference amounts up to 60 W m$^{-2}$. In contrast, in the case of no subsidence, the results for the “no entrainment physics” run ($c_e = 10^{-4}$) are rather close to the model results with entrainment physics ($c_e = 0.1$). A similar comparison (results not shown) for the runs with subsidence yielded similar results; in each case, the difference between the flux profiles was smaller than 10 W m$^{-2}$. The turbulent fluxes therefore mainly depend on the subsidence rate, and do not seem to depend much on the “physics” of the turbulence scheme—this all contrasts with the model behavior on high resolution. By adding the turbulent flux to the dynamical flux, linear flux profiles until 800 m are obtained for all runs on coarse resolution. Thus the boundary layer is well mixed up to the cloud top at 800 m, and there is a uniform drying (or moistening) of the whole boundary layer. Moreover, it shows that the upward moisture flux at 500 m approximately balances the dry air that is advected downward into the cloud at 800 m [the difference between these fluxes are related to the uniform drying (moistening) of the whole boundary layer].

The dependency of entrainment on the subsidence is not only restricted to the coarse resolution. In some cases, it can also be found with the high 25-m resolution. To illustrate this we ran the model with the standard subsidence rate, but now without entrainment physics ($c_e = 10^{-4}$). We expected the cloud to be advected downward due to the subsiding motion, but the cloud top again remained “locked in” during the whole time integration instead. Computing the entrainment rate from the rise in the inversion, we got a mean entrainment rate during the third hour of 1.0 cm s$^{-1}$. This is exactly the subsidence rate at the level of the inversion. In this case, entrainment and subsidence therefore balance exactly. This behavior will be explained in part b of this section. To illustrate here that this is indeed a resolution effect, and not a property of the “physics” of the turbulence scheme, we repeated the same experiment with a resolution of 2.5 m near the inversion. Indeed, the entrainment rate now decreased significantly to 0.3 cm s$^{-1}$.

b. A conceptual model

The dependency of entrainment on the large-scale subsidence (as found predominantly on low resolution) can be described with a simple conceptual model. With this model it is illustrated that the numerical, Eulerian gridbox representation keeps the cloud top “locked in,” and is therefore responsible for this behavior. Below we will refer to this behavior as “numerical entrainment.” This numerical entrainment has the same effect as entrainment caused by the physics of the turbulence scheme; that is, warm and dry inversion air is mixed into the boundary layer, and the cloud-top rises relative to the backgroundsubsiding motion (although the absolute cloud top always remains at the same height).

To illustrate the mechanism of this numerical entrainment we consider the $q_t$ budget in a cloud as sketched in Fig. 11. For simplicity we assume that the cloud is fully turbulent, which is characterized by large values of the turbulent diffusion constant $K$ in the whole cloud. It is assumed that there is no turbulent exchange (no entrainment) between the cloudy and the clear air. The initial situation is sketched in Fig. 11a; the cloud with high $q_t$ is located in the lower two grid boxes. In this situation, the numerical representation (as sketched in Fig. 11c) is rather good. The use of an upstream differencing scheme for the advection (as is used in the SCM simulations) leads to the correct advective flux, and the computation of the turbulent flux based on the
Next, we consider situation b, a short time later. The cloud has now been advected a small distance \((w\Delta t)\) downward. Because we assumed above that there is no entrainment between the clear and cloudy air, there is no process that changes \(q_t\) in the cloud and no gradient of \(q_t\) is created. The turbulent flux at interface (2) in the cloud should therefore be zero. However, the numerical representation (as sketched in Fig. 11d) of this problem is rather different. Because the middle grid box now consists of a mixture of low \(q_t\), clear air and high \(q_t\), cloudy air, the grid box mean \(q_t\) will be lower than the inclow value of \(q_t\), and a small gradient in the cloud at interface (2) is created. Because fast mixing occurs in the cloud, this small \(q_t\) gradient leads to a large turbulent moisture flux. Numerical entrainment therefore occurs because the turbulence in the cloud immediately supplies sufficient moisture to the upper cloudy grid box to compensate for the drying due to the subsidence.

The turbulent mixing timescale for the middle box is \(l\Delta z/K_2\), where \(l\) is a typical length scale associated with the \(q_t\) gradient across interface (2) (limited by the grid spacing \(\Delta z\)), and \(K_2\) is the turbulent diffusion constant at interface (2). The advective timescale for the same grid box is \(\Delta z/w\). Therefore, the turbulent flux balances the advective flux across interface (1) as long as \(lw/K_2 \ll 1\). Using typical values, \(w = 0.01 \text{ m s}^{-1}\), \(l = 100 \text{ m}\), and \(K_2 = 10 \text{ m}^2\text{s}^{-1}\), it is seen that this condition is in practice nearly always fulfilled. The numerical entrainment flux therefore balances the subsidence flux.

A more general way to understand the mechanism of “numerical entrainment” is by considering what happens with the dry air that is advected into the top of the cloud. Because gridbox mean values of \(q_t\) are considered in a Eulerian representation, this dry air is immediately distributed over the whole grid box, and in the next step communicated with the cloud below. On a coarse resolution, the latter process is very fast because the turbulent diffusion constant at the interface (2), \(K_2\), is representative for the turbulent bulk of the cloud. The communication of the dry inversion air with the bulk of the cloud is therefore very fast, and the entrainment rate is therefore high. On high resolution, the communication becomes slower because the interface (2) is now located near the top of the cloud, and \(K_2\) is therefore also representative for the top of the cloud. This explains why the model eventually, at a resolution of 2.5 m, converges to a different (and more physical) solution in which the cloud is advected downward by the vertical velocity. Convergence is therefore dependent on the scaling behavior of the turbulence near the top of the cloud. This is reflected by the dependency of \(K_2\) on turbulence a distance \(\Delta z\) below cloud top.

To confirm once more that the existence of turbulence at interface (2) is indeed essential, we performed the following, rather artificial experiment. According to our hypothesis, numerical entrainment should disappear when the turbulence at interface (2) is suppressed. To achieve this we displaced the radiative cooling one level (25 m) downward. In accordance with our expectations, this eliminated a large part of the numerical entrainment and the entrainment rate decreased to 0.3 cm s\(^{-1}\).

One may argue that numerical entrainment is due to the diffusive properties of the upward differencing scheme for the vertical advection. Such a scheme leads to additional diffusion of the order of \(w\Delta z/2\), which has qualitatively the correct properties since the numerical diffusion increases with increasing subsidence rates. In Figs. 9 and 10 we already plotted the flux profiles (labeled as dynamical in those plots) generated by the vertical advection scheme. These flux profiles are computed from the actual advective tendencies, and therefore include the diffusive property of the upstream scheme. Comparing high resolution with coarse resolution, differences of the order 20 W m\(^{-2}\) are visible. These are much too small to explain the behavior we observe. Considering that the downward advection of dry air increases with decreasing resolution, the sign of the diffusive property is also wrong—the upward scheme advocates properties of the upstream region (and therefore higher in the inversion and thus dryer) downward. With coarse resolution drying at the cloud top is actually higher and yields large buoyancy fluxes (because the buoyancy flux is a strong function of the cloud fraction) and thereby sustains the turbulence. Nevertheless, although the cloud water physics play a significant role, they are not essential. This is confirmed in runs with a smoke cloud. A smoke cloud has the same radiative properties as a real cloud (with longwave cloud top cooling), but does not contain water phase changes (Lilly 1968; Bretherton et al. 1999). Experiments with a smoke cloud gave results similar to the ones presented above.

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7. Discussion and conclusions

In this paper we evaluate the kinetic energy approach with a diagnostic length scale (E–l approach) for modeling vertical turbulent fluxes in a realistic stratocumulus case. On high vertical resolution and with a small time step, this approach is well able to model the vertical fluxes and entrainment for the present case. With the same model also a reasonable (although slightly low) entrainment rate for a surface-driven dry convective boundary layer is obtained. This does not mean that the E–l scheme gives realistic entrainment rates for all types of boundary layers; for instance, Lenderink et al. (1999) show for a smoke-cloud topped boundary layer that the E–l scheme tends to overestimate entrainment in the (extreme) case of a very thin (order 10 m) radiative layer at the top of the cloud.

We find that the modeled turbulent fluxes are strongly dependent on both time and spatial discretization. This is in particular true for entrainment. For the time discretization, we find that even relatively small time steps may lead to large errors. This is the case when using a process splitting time integration scheme, in which the tendencies due to the separate physical processes are computed independently. When this process splitting scheme contains both an explicit (the radiation) and an implicit (the turbulent diffusion) part, it leads to a too cold cloud top and a too large buoyancy flux near cloud top, and a significant overestimation of the entrainment rate.

For the spatial discretization, we find that the E–l scheme is not able to generate sufficient entrainment on the coarse resolution that is currently utilized in large-scale models. However, in those cases “numerical entrainment” plays a large role. This numerical entrainment is associated with the Eulerian formulation of the model. Such a formulation does not conserve the downward advection of the cloud top by the large-scale subsidence. The entrainment rate caused by this numerical process balances the large scale subsidence, and keeps the cloud top “locked in” at one level. On coarse resolution this process dominates the solution. In these runs, the entrainment rate is in close balance with the subsidence rate, and is nearly independent of the physics represented by the E–l scheme. A strong dependency of the entrainment rate on the subsidence rate results. On high resolution, numerical entrainment only plays a role in the runs for which we explicitly suppressed certain entrainment physics of the E–l scheme. In the other and more realistic cases, the entrainment rate is almost independent of the large-scale subsidence.

On coarse resolution, numerical entrainment leads to a, sometimes even significant, improvement of the results. The reason for this is that numerical entrainment now compensates for the lack of entrainment by the low-resolution representation of the “physics” of the E–l scheme. However, in runs without subsidence this compensation does not occur and, in those cases, the entrainment rate is seriously underestimated. In this paper we did not investigate the cause of this lack of entrainment in great detail, but it seems likely that it is related to an underestimation of transport of TKE into the inversion. This transport is modeled at present by downgradient turbulent diffusion. On high resolution a sharp TKE gradient near the inversion is seen, which causes a significant transport of TKE into the inversion. On coarse resolution, the sharp TKE gradient is not resolved, and the transport of TKE is therefore significantly reduced. Therefore, it is worth investigating other formulations for the transport of TKE, including non-local terms (see, e.g., Cuijpers and Holtslag 1998; Holtslag 1998).

One may question to what extent the numerical problems described above are generic, and could also be expected to play a role in large-scale atmospheric models. The mechanism of numerical entrainment seems to apply generally because it is only related to the formulation of the cloud in Eulerian grid boxes, which do not explicitly keep track of the cloud top. As such, it is not dependent on details of the discretization of advection and turbulent diffusion. Therefore we expect a similar behavior with other turbulence schemes too. The underestimation of entrainment on coarse resolution, however, is likely to be more model dependent. In particular, the formulation of the transport of the TKE seems to be crucial, but also the inability at low resolution to resolve the sharp gradient in temperature and moisture at the inversion could play a significant role. Finally, the results concerning the time integration are probably most model dependent. Nevertheless, schemes that contain both explicit and implicit parts are commonly used in atmospheric general circulation models (see, e.g., Beljaars 1991), and it is good to notice that these schemes may easily lead to the wrong equilibrium balance. The consequences of the mismatch however may be strongly model dependent.

We think that many of the numerical aspects demonstrated in this paper could apply generally to numerical weather prediction models and climate models. These numerical aspects have a significant influence on the model simulations. Most seriously, the strong relation on coarse (operational) resolution between the entrainment rate and the subsidence rate, provides a strong, though unrealistic coupling between the large-scale dynamics and the existence of boundary layer clouds. This might well be related to the general problems GCMs have with the representation of boundary layer clouds (e.g., Browning 1994).

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APPENDIX

Time Integration Scheme

Here we illustrate with a simple example how a process splitting scheme may lead to the wrong equilibrium balance. This occurs if the scheme contains both explicit and (over) implicit parts. In that case, the numerical solution converges to a state that is dependent on the time step. Our analysis also suggests a simple modification to the time integration scheme that removes this dependency.

For our investigations we consider the following simple equation

\[
\frac{\partial y}{\partial t} = F(y) + b, \tag{A1}
\]

where \( F \) is a linear function of \( y \) (can be a vector) and \( b \) is a constant. One may think of \( y \) being the (liquid water potential) temperature, \( F \) being the diffusive operator (turbulent mixing), and \( b \) being the temperature tendency due to radiation. This equation therefore states the time evolution of a system governed by (longwave) radiative cooling and entrainment warming caused by turbulent mixing. We are mainly interested in the (quasi) stationary state of this system, as is approached in a slowly (compared to the radiative and turbulence time-scale) evolving system. The analytical and exact equilibrium stationary state of Eq. (A1) is given by

\[
y_{\text{anal}}^{\text{eq}} = -F^{-1}(b). \tag{A2}
\]

Below we investigate whether or not this stationary state is approached by time integration of the discretized system, and we assume the following format of the time integration scheme:

\[
y_{n+1} = y_{n} + \Delta y^{F} + \Delta y^{h}, \tag{A3}
\]

where \( \Delta y^{F} \) is the increment due to \( F \), and \( \Delta y^{h} \) is the increment due to the constant forcing \( b \), and \( n \) and \( n + 1 \) denote the previous and new full time step, respectively. This format corresponds to the format used in many numerical weather prediction models, in which the tendencies due to the different physical processes are computed in separate routines. Assuming that the integration of \( b \) can be done explicitly yields

\[
\Delta y^{h} = b \Delta t. \tag{A4}
\]

However, because of numerical stability reasons (see, e.g., Beljaars 1991), the increment due to \( F, \Delta y^{F} \), has to be done (over) implicitly; that is,

\[
\frac{\Delta y^{F}}{\Delta t} = \alpha F(y_{0} + \Delta y^{F}) + (1 - \alpha)F(y_{0}), \tag{A5}
\]

where \( y_{0} \) is the field where \( F \) is working on, and \( (y_{0} + \Delta y^{F}) \) the provisional new value of \( y \) computed by the \( F \) scheme. The factor \( \alpha \) is the degree to which the scheme is implicit. [If the tendency due to \( F \) is computed based on \( y_{n} \) only (i.e., \( y_{n} = y^{n} \)), the procedure above is denoted by process splitting. Both the tendency due to \( F \) and \( b \) are now computed independently based on the previous (time step \( n \)) value of \( y \). This is the scheme presently used in the SCM.]

When \( F \) is assumed to be a linear operator, Eq. (A5) can be written as

\[
\Delta y^{F} - \alpha \Delta t F(y_{0}) - \alpha \Delta t F(\Delta y^{F}) = (1 - \alpha)\Delta t F(y_{0}). \tag{A6}
\]

This is equivalent to

\[
\tilde{F}(\Delta y^{F}) = \Delta t F(y_{0}) \tag{A7}
\]

with

\[
\tilde{F}(x) = x - \alpha \Delta t F(x) \tag{A8}
\]

and yields the formal solution

\[
\Delta y^{F} = \Delta t \tilde{F}^{-1}[F(y_{0})]. \tag{A9}
\]

The time integration scheme is now given by

\[
y_{n+1} = y_{n} + \Delta t \{ b + \tilde{F}^{-1}[F(y_{n})]\}. \tag{A10}
\]

In equilibrium there is no change in \( y \) from one time step to the next; that is, \( y_{n+1} = y_{n} \). Using this in Eq. (A10) and using the definition of \( \tilde{F} \) [in Eq. (A8)] yields

\[
\tilde{F}(\Delta y^{F}) = \Delta t \tilde{F}^{-1}[F(y_{n})]. \tag{A11}
\]

If we express \( y_{n} \) in terms of \( y^{n} \) plus an update \((\Delta y)_{n}\),

\[
y_{n} = y^{n} + (\Delta y)_{n}, \tag{A12}
\]

and using that the discretized system has converged to a stationary state for which \( y^{n} = y^{n+1} = y_{\text{num}}^{\text{eq}} \), we get for this state

\[
y_{\text{num}}^{\text{nl}} = y_{\text{anal}}^{\text{eq}} + ab \Delta t - (\Delta y)_{n}. \tag{A13}
\]

In this derivation we used in Eq. (A11) that \( F \) is a linear operator and inserted the analytical equilibrium solution \( y_{\text{num}}^{\text{eq}} \) for \( -\tilde{F}^{-1}(b) \).

From Eq. (A13) it is clear that with a process splitting scheme, that is, \((\Delta y)_{n} = 0\), a time step–dependent equilibrium solution is obtained. A simple way to cure this time step dependency is obtained by taking \((\Delta y)_{n} = ab \Delta t \). In that case the equilibrium solution \( y_{\text{num}}^{\text{nl}} \) obtained by time integration equals the analytical equilibrium solution. This means that, in order to get the correct equilibrium solution by time integration, the turbulent diffusion should work on the temperature field that is updated with \( ab \Delta t \). Note that the overimplicit factor occurs in this update.

Another way to circumvent this time step dependency is used in the ECMWF model. For the radiative–diffusive equation, this "fractional steps" procedure first computes the radiative tendency explicitly. This radiative tendency is then passed to the diffusion routine, and
the diffusion routine now solves the whole system including the radiative tendency implicitly (A. C. M. Beljaars 1998, personal communication; Beljaars 1991).

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