Direct Atmospheric Forcing of Geostrophic Eddies

PETER MÜLLER

The Center for Earth and Planetary Physics, Harvard University, Cambridge, MA 02138

CLAUDE FRANKIGNOL

Department of Meteorology, Massachusetts Institute of Technology, Cambridge 02139

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ABSTRACT

To assess the role of direct stochastic wind forcing in generating oceanic geostrophic eddies we calculate analytically the response of a simple ocean model to a realistic model wind-stress spectrum and compare the results with observations. The model is a continuously stratified, $\beta$-plane ocean of infinite horizontal extent and constant depth. All transfer and dissipation processes are parameterized by a linear scale-independent friction law (Rayleigh damping). The model predictions that are least sensitive to this parameterization, the total eddy energy and the subsurface displacement, are in good agreement with observations in mid-ocean regions far removed from strong currents. Properties that depend crucially on the parameterization of nonlinearities and topographic effects are not well reproduced. Observed coherences and seasonal modulations provide direct evidence of wind forcing at high frequencies where motions have little energy. Direct evidence at the more energetic low frequencies will be difficult to detect because the expected coherences are small. Altogether, the present results suggest that direct wind forcing may well be the dominant forcing mechanism for central ocean eddies.

1. Introduction

Quasi-geostrophic motions in the ocean have time scales of a week or more and space scales from a few tens of kilometers to basin scales. These eddies have widely varying energy levels. The most energetic eddies are found near and caused by strong currents like the Gulf Stream or Kuroshio. Far away from strong mean currents, the eddy field is of smaller amplitude and more homogeneous. Various generation mechanisms for this mid-ocean eddy field have been proposed. They include open ocean baroclinic instability (Gill et al., 1974; Robinson and McWilliams, 1974), radiation from rings (Flierl, 1977) or meanders (Harrison and Robinson, 1979), mean flow interaction with topography (Bretherton and Karweit, 1975), and direct wind forcing (e.g., Philander, 1978). Most of these process studies considered single deterministic Fourier components and neglected the broad band and stochastic nature of the interacting fields. Thus, early studies of direct wind forcing (e.g., Veronis and Stommel, 1956; Phillips, 1966), which calculated the response to simple deterministic forcing patterns, generally concluded that transient wind stress forcing is negligible because of the mismatch of the dominant space-time scales and propagation direction of atmospheric and oceanic disturbances. Such a deterministic approach can, however, be misleading and a statistical treatment may lead to different results (Frankignoul and Hasselmann, 1977).

Recently, the authors have attempted to estimate more quantitatively the oceanic response to direct stochastic atmospheric forcing. Using a realistic broadband wind stress spectrum and integrating the effect of the wind forcing over the whole wave-number-frequency range of oceanic eddies, Frankignoul and Müller (1979a, hereafter FM) showed that stochastic wind forcing was more important than previously thought. They found that the energy input rate is of the order $10^{-3}$ W m$^{-2}$ in midlatitudes, which is smaller but comparable to the energy transfer rate between the mean atmospheric and oceanic circulations, and sufficient to maintain the eddy field in regions of weak eddy activity. The forcing of geostrophic eddies by the atmospheric pressure and buoyancy flux is negligible (Frankignoul and Müller, 1979a,b). To test further the hypothesis of wind forcing we calculate in this paper the energy level, space-time scales, coherences and modulation of the oceanic response from a simple analytical linear ocean model. These more detailed predictions can then be compared with oceanic observations.
It should be emphasized that the statistical properties of the oceanic response depend not only on the intensity and structure of the wind stress field but also on the internal dynamical processes which transfer and dissipate energy. The main transfer processes are nonlinear interactions and bottom scattering which cannot be parameterized simply in analytical studies. Furthermore, the main dissipation processes have not been clearly identified. In numerical models one generally uses lateral diffusion of the Laplacian or biharmonic type and (or) bottom friction, but there is little or no observational support for these parameterizations. In view of these difficulties and uncertainties we have chosen to parameterize transfer and dissipation by a simple linear scale-independent friction mechanism, referred to here as Rayleigh damping. Bottom friction and horizontal diffusion are also considered but disregarded because they lead to an infinite or overdamped baroclinic response. We are aware of the shortcomings associated with the parameterization of non-local (in wavenumber-frequency space) and nonlinear processes by a simple linear damping law. However, some features of the oceanic response are fairly insensitive to such parameterization, which also simplifies sensitivity studies.

Our results suggest that direct atmospheric forcing is sufficient to maintain the central ocean eddy field. Unfortunately, the oceanic response turns out to be characterized by only a few signatures that can be detected in observational data. Additional evidence may come from a detailed comparison of observed eddy spectral properties with the results of baroclinic eddy-resolving numerical ocean models which explicitly simulate nonlinearities and topographic effects. Still, the result of such a comparison will depend critically on the parameterization of the dissipation mechanisms in the model. A first step toward this alternative modeling approach was recently taken by Willebrand et al. (1980) who calculated the response of a more realistic but homogeneous ocean to four years of routinely observed winds. As discussed below, their results are very similar to the barotropic response discussed in this paper.

2. The oceanic model

The quasi-geostrophic response of the ocean to wind stress forcing is described by the potential vorticity equation. In the $\beta$-plane approximation it takes the general form (e.g., FM)

$$\begin{align*}
\partial_t (\partial_\alpha \partial_\alpha + \partial_3 f_o N^{-2} \partial_3) \phi + \beta \partial_\beta \phi &= D_v \\
\partial_\beta (N^2 + g \partial_3) \phi &= - \rho_o^{-1} g N^2 f_o^2 - \epsilon_{\alpha \beta} \partial_\alpha \tau_\beta + D_\alpha, \quad \text{at} \quad x_3 = 0
\end{align*}$$

$$D_v = -R(\partial_\alpha \partial_\alpha + \partial_3 f_o N^{-2} \partial_3) \phi$$

$$D_\alpha = -R(N^2 + g \partial_3) \phi$$

Choosing a damping coefficient $R$ is equivalent to selecting a decay time for the eddy field. We choose $R = 5 \times 10^{-8} \text{ s}^{-1}$ which corresponds to an amplitude decay time of 200 days (or an energy decay time of 100 days). This choice seems conservative\(^5\).

\(^5\) Since the energy level of the oceanic response is inversely proportional to the damping coefficient $R$, readers will find it easy to scale our results in accordance to their own bias.
since it implies a strong damping of baroclinic motions. For comparison we will also discuss two alternative linear models of the transfer and dissipation processes. In the first model, referred to as lateral diffusion, we assume that all field variables diffuse laterally with the same diffusion coefficient $A$. The source terms in the potential vorticity equation are obtained by replacing $R$ by $-A\partial_x\partial_y$ in (2.6). In the second model we use linear bottom friction. The source terms are

$$D_0 = -rN^2f_0^{-3}h\partial_x\partial_y\phi(-h)$$

(2.7)

and $D_r = D_s = 0$, where $r$ is the friction coefficient. Numerical values for $A$ and $r$ will be chosen to give the same energy level or decay time as Rayleigh damping.

The solution of the potential vorticity Eq. (2.1) is constructed by decomposing the streamfunction into its vertical and horizontal normal modes. For a horizontally infinite ocean of constant depth we write

$$\phi(x,t) = \sum_{n=0}^{\infty} \int d^2k a_n(k,t)\phi_n(x) \exp(ik_a x),$$

(2.8)

where $a_n(k,t)$ are the normal mode amplitudes, $\exp(ik_a x)$ the horizontal and $\phi_n(x)$ the vertical eigenfunctions. If we substitute (2.8) into the potential vorticity equation, multiply by $\phi_n(x)$ and integrate over the water column, we obtain the equation of motion for the normal mode amplitude

$$\partial_t a_n + i\omega_n a_n + d_n a_n = -i \frac{\phi_n(0)}{\rho_0} \frac{\epsilon_{ab}k^a\tau^b}{k^2 + R_n^{-2}},$$

(2.9)

where

$$\omega_n = -\frac{\beta k_1}{k^2 + R_n^{-2}}$$

(2.10)

is the eigenfrequency of planetary Rossby waves and $R_n$ the $n$th Rossby radius of deformation. The damping coefficient $d_n$ is given by

$$d_n = |R|, \text{ for Rayleigh damping}$$

$$\frac{k^2}{k^2 + R_n^{-2}}, \text{ for lateral diffusion.}$$

(2.11)

For bottom friction the different modes are coupled

$$d_n a_n = rh \frac{k^2}{k^2 + R_n^{-2}} \sum_{m=0}^{\infty} a_m \phi_m(-h) \phi_n(-h).$$

(2.12)

However, if we assume for simplicity that different modes are statistically independent we may substitute

$$d_n = rh \phi^2(-h) \frac{k^2}{k^2 + R_n^{-2}}.$$  

(2.13)

The three damping coefficients are shown in Fig. 1 as a function of mode and wavenumber. Rayleigh damping is independent of both wave and mode number. The damping coefficient for lateral diffusion is independent of mode number and increases with horizontal wavenumber. The damping coefficient for bottom friction also increases with horizontal wavenumber but decreases with mode number. For $k \gg R_n^{-1}$ bottom friction acts like Rayleigh damping.

The normal mode amplitude equation (2.9) is easily solved by Fourier transformation in time

$$a_n(k,\omega) = \frac{1}{\omega - \omega_n + id_n} \frac{\phi_n(0)}{\rho_0} \frac{\epsilon_{ab}k^a\tau^b(k,\omega)}{k^2 + R_n^{-2}}.$$  

(2.14)

The solution shows the typical resonance behavior with a peak at those frequencies and wavenumbers which satisfy the dispersion relation for Rossby waves; due to frictional effects the resonance peaks are finite.

The fluctuations in the atmospheric windstress are deterministic (annual and diurnal variations) and stochastic (day-to-day variability of the weather). In this paper, we calculate the response to the stochastic component and describe the forcing and response fields by their wavenumber-frequency spectra. The wind stress cross-spectrum $F_{ab}(k,\omega)$ is defined by

$$F_{ab}(k,\omega) = \langle \tau_a(k,\omega)\tau_b^*(k',\omega') \rangle,$$  

(2.15)
where cornered brackets denote an ensemble mean and the asterisk the complex conjugate.

Because of the orthonormal properties of the vertical eigenmodes, depth-integrated quantities take a particularly simple form. We shall consider the depth-integrated total energy spectrum $E_{00}(k, \omega)$ defined by (see FM)

$$E_{00}(k, \omega) = \frac{1}{2} \rho_0 \int_{-h}^{0} dx_3 (u_n u_n + N^2 \xi^2) + \frac{1}{2} \rho_0 g \xi^2$$

which describes the distribution of the total energy among wavenumber, mode number, and frequency. The total energy is given by

$$E_{tot} = \frac{1}{2} \rho_0 \int_{-h}^{0} dx_3 (u_n u_n + N^2 \xi^2) + \frac{1}{2} \rho_0 g \xi^2$$

$$= \sum_{n=0}^{\infty} d^2 k \omega E_{00}(k, \omega).$$

Substituting our normal mode solution (2.14) into (2.16) we find

$$E_{00}(k, \omega) = \frac{1}{2} \rho_0^{-1} \phi_n^2(0)$$

$$\times \frac{1}{(\omega - \omega_n)^2 + d_n^2 k^2 + R_n^{-2}} F_{00}(k, \omega),$$

which relates the oceanic energy spectrum to the windstress spectrum. Spectra of other oceanic variables can be constructed similarly.

For simplicity we consider an ocean with constant buoyancy frequency $N_0$. The vertical eigenfunctions are then given by $\phi_n(x_3) = (1/h)^{1/2}$ and $\phi_n(x_3) = (2/h)^{1/2} \cos(N_0 x_3 + h f_0 R_n)$ for $n = 1, 2, 3, \ldots$, and the Rossby radii by $R_n = (ghf_0^2)^{-1/2}$ and $R_n = N_0 h n \pi f_0$. In all our theoretical estimates we use (unless stated otherwise) the values $h = 5 \times 10^3 \text{m}$, $f_0 = 7 \times 10^{-3} \text{ s}^{-1}$ and $\beta = 2 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$, representative of the MODE region ($28^\circ \text{N}$, $70^\circ \text{W}$). For the Brunt-Väisälä frequency we choose $N_0 = 2 \times 10^{-3} \text{ s}^{-1}$ to reproduce the observed MODE value $R_1 \sim 47 \text{ km}$. The values for $R_n$, $n \geq 2$, are then reproduced within 20% but the surface amplitudes $\phi_n(0)$ of all baroclinic modes are systematically underestimated by a factor of 2 or more (see FM). We correct for this underestimation by doubling all baroclinic amplitudes.

3. The wind-stress model

In this paper we consider oceanic quasi-geostrophic fluctuations with periods larger than a few days and wavelengths from 50 to 4000 km. In this scale range the wavenumber-frequency spectrum $F_{ab}(k, \omega)$ of the wind stress can neither be calculated from routine meteorological observations (the observational grid is too coarse) nor be inferred from observations at island stations and ocean weather ships. Reviewing the available observations FM suggested that in midlatitudes.

$$F_{ab}(k, \omega) = \frac{1}{2} F_r(0) \frac{S_r(k)}{2 \pi k} \left( \delta_{ab} - \frac{k_a k_b}{k^2} \right)$$

(3.1)

is a reasonable zeroth order representation of the wind stress spectrum in the wavenumber range of oceanic eddies and for periods larger than 10–20 days.

The model spectrum (3.1) is white in frequency space. The (one-sided) white noise level $F_r(0)$ can directly be inferred from observations at island stations and ocean weather ships. As illustrated in Fig. 2, a representative value at midlatitudes is

$$F_r(0) = 10^4 \text{N}^2 \text{m}^{-4} \text{Hz}^{-1},$$

(3.2)

corresponding to the wind-stress spectrum at Bermuda ($32^\circ \text{N}$, $65^\circ \text{W}$).

Although frequency spectra of atmospheric fluctuations are observed to start falling off at periods smaller than 20 days, the energy density in the wavenumber range of oceanic eddies remains approximately constant down to periods of a few days. This is demonstrated in Fig. 3 left, which shows zonal wavenumber spectra of the geostrophic wind for different frequencies as estimated by Pratt (1975) from objectively analyzed geopotential height.
NMC data. As the frequency increases, the energy density decreases at low wavenumbers but remains virtually unchanged in the eddy range. For the wavenumbers of interest in this paper the atmospheric spectra are independent of frequency down to a period of a few days.

The model spectrum (3.1) is symmetric (no preferred propagation direction) in wavenumber space. This is consistent with observations for periods larger than about 10 days (Fig. 4). Both the symmetry in wavenumber space and the whiteness in frequency space arise because, at synoptic and smaller space scales, the low-frequency variability of the atmosphere is mainly the white noise extension of the short time-scale weather fluctuations (Hasselmann, 1976).

For simplicity the wind stress is also assumed to be isotropic (no excess energy in one of the components), which is a reasonable approximation in most of the oceanic eddy scale range. The dependence on the wavenumber magnitude is described by the normalized wavenumber spectrum

\[
S_s(k) = \begin{cases} 
\frac{3}{4} k_b^{-1}, & \text{for } 0 \leq k \leq k_b \\
(k/k_b)^2, & \text{for } k_b \leq k \leq k_c \\
(k/k_c)^{-2}, & \text{for } k_c \leq k \leq k \leq \infty \\
0, & \text{for } k_c \leq k \leq \infty 
\end{cases}
\]

where \( k_b = 2\pi/5000 \text{ km}^{-1} \) is the characteristic wavenumber of the baroclinic instability in the atmosphere and \( k_c \) a cutoff wavenumber which need not be specified as long as it is larger than the characteristic wavenumbers of the oceanic eddy field (i.e., \( k_c \gg 2\pi/50 \text{ km}^{-1} \)). The model also assumes that the wind-stress is linearly related to the wind and the wind geostrophically to the atmospheric pressure. As discussed in FM, the model (3.3) is consistent with the available tropospheric data. This is illustrated in Fig. 3 right, which shows the zonal wavenumber spectrum of the low-frequency wind stress estimated from (3.1) and (3.3). The agreement in spectral shape is good, taking into account the large loss of variance of the objectively analyzed NMC data at high wavenumbers (Julian and Cline, 1974). In (3.3), the spectral slope has
been extrapolated to much higher wavenumbers than are resolved in synoptic observations. This representation includes fronts and other mesoscale phenomena. Observational evidence for a power-law behavior like that of (3.3) at wavelengths smaller than a few hundred kilometers recently has been discussed by Gage (1979).

In reality the wind-stress fluctuations are neither statistically stationary nor homogeneous. During fall and winter the wind-stress variance is larger than during spring and summer, with the wavenumber structure essentially unchanged. Spatial inhomogeneities are most pronounced in the meridional direction where the rms amplitude of the wind-stress may vary by as much as a factor of 3 over 20° of latitude (cf. Fig. 11). The wavenumber structure also seems to be weakly dependent on latitude. At synoptic periods (∼3–10 days) the atmospheric fluctuations become strongly asymmetric in the zonal direction (cf. Fig. 4). At the cyclone scale (zonal wavenumbers around 6) nearly all fluctuations propagate eastward. Toward larger wavenumbers and smaller and higher frequencies the zonal asymmetry decreases (Pratt and Wallace 1976, Frankignoul and Müller 1979a). For some of the calculations in Section 5 these properties have to be taken into account. This will be done by modifying (3.1) in simple ways. Zonal asymmetry in the propagation direction at synoptic frequencies will be modeled by multiplying (3.1) by the factor

\[1 + a \cos \theta,\]

where \(\theta\) is the direction of the wavenumber vector. For \(a = \frac{1}{2}\), the averaged ratio of eastward to westward propagating wind-stress variance is 3, a characteristic value of the asymmetry observed by Pratt and Wallace (1976). Inhomogeneities will be represented by allowing the pressure Fourier components to vary slowly with latitude. In general, the changes in the oceanic response are small.

4. Model results

Here we discuss the basic properties of our model oceanic response, its energy, its space and time scales, its coherence with the atmosphere, and its space-time variability.

a. Energy

The total energy is obtained by summing the energy spectrum \(E^E_{\text{tot}}(k, \omega)\) over all modenbbers and integrating over all wavenumbers and frequencies. The wavenumber integration is restricted to the interval from \(k_m = (2\pi/4000) \text{ km}^{-1}\) to \(k_M = (2\pi/50) \text{ km}^{-1}\). The frequency integration should include all frequencies larger than \(f_o\). However, the resonance structure of the energy spectra makes the integration insensitive to any high-frequency limit, provided it is larger than all Rossby wave frequencies. We therefore use \(-\infty\) and \(+\infty\) as the limits for the frequency integration. For the discussion it is useful to divide \(E_{\text{tot}}\) into the total barotropic energy, \(E^B_{\text{tot}} = \sum_{n=1}^{\infty} E^B_n\), and total baroclinic energy, \(E^C_{\text{tot}} = \sum_{n=1}^{\infty} E^C_n\). For Rayleigh damping the integration of the energy spectrum (2.18) yields

\[E^B_{\text{tot}} = \frac{3\pi}{16} \rho_0 h \int_0^k \frac{k_b 1}{k_m R} \, d\omega,\]

\[E^C_{\text{tot}} = \frac{3\pi}{16} \rho_0 h \int_0^k \frac{k_b 0.53}{k_m R} \, d\omega.\]

The total energy is inversely proportional to the damping coefficient \(R\). This dependence is much less dramatic than suggested by the deterministic analysis of Harrison (1979). For \(R = 5 \times 10^{-8}\) s\(^{-1}\) we find

\[E^B_{\text{tot}} = 3 \times 10^9 \text{ J m}^{-2}\]

\[E^C_{\text{tot}} = 6 \times 10^9 \text{ J m}^{-2}\]

\[E_{\text{tot}} = 9 \times 10^9 \text{ J m}^{-2}\]

If half of the baroclinic energy is kinetic, the depth averaged rms velocity and vertical displacement corresponding to (4.3) will be \(\langle u_d \rangle \sim 0.05 \text{ m s}^{-1}\) and \(\langle z^2 \rangle^{1/2} \sim 17 \text{ m}\). Since the vertical displacement depends little on the stratification (it varies like \(N^{-1/2}\) in the WKB approximation) this last figure is convenient for comparisons with observations.

\[A \text{ good assumption at low frequencies (Willebrand 1978).}\]
As discussed in FM, the barotropic response may be slightly overestimated because of loss of zonal symmetry at high frequencies, but it is fairly insensitive to the wind-stress model. On the other hand, the baroclinic response is sensitive to the high wavenumber slope of the wind-stress field: using a $k^{-3}$ slope instead of $k^{-2}$ in (3.3) reduces the baroclinic energy by a factor of 8.

In the case of lateral diffusion, the total barotropic and baroclinic energies are given by

$$E_{\text{tot}}^{bt} = \frac{\pi}{16} \frac{1}{\rho_0 h} F_s(0) \frac{k_b}{k_m^3} \frac{1}{A}$$

$$E_{\text{tot}}^{bc} = 4 \frac{\pi}{16} \frac{1}{\rho_0 h} F_s(0) \frac{k_b}{k_m^3} 0.043 \frac{1}{A}$$

The diffusion coefficient $A$ may be tuned to get the same energy decay time or total energy as with Rayleigh damping. This yields

$$A \approx 5 \times 10^{10} R = 2.5 \times 10^9 \text{ m}^2 \text{ s}^{-1},$$

a rather large value, comparable to the friction coefficients used in general circulation models of the ocean to parameterize both eddies and smaller scale motions.

For bottom friction, $E_{\text{tot}}$ is infinite since each baroclinic mode contains the same energy. This is due to the fact that both forcing and damping decrease proportionally to the modenumber. Bottom friction hence does not provide a sufficient damping mechanism for atmospheric forcing, and is not considered further.

b. Space scales

Here we consider the distribution of energy among the various mode and wavenumbers. We define baroclinicity as the ratio $E_{\text{tot}}^{bc}/E_{\text{tot}}$. For Rayleigh damping we find from (4.3)

$$E_{\text{tot}}^{bc}/E_{\text{tot}} \sim 0.6$$

independent of the value of $R$. For horizontal diffusion, the baroclinicity is smaller, about 0.15. Hence, the use of lateral diffusion in a numerical model will decrease the baroclinic response to stochastic windstress forcing (at least if strong bottom topographic effects are excluded).

The contribution of the first baroclinic mode to the total baroclinic energy is 0.38 for Rayleigh friction and 0.68 for lateral diffusion.

The horizontal space scales are described by the wavenumber spectra

$$E_{\text{tot}}^n(k) = \int_{-\infty}^{+\infty} d\omega E_{\text{tot}}^n(k, \omega)$$

$$= \frac{1}{4} \frac{1}{\rho_0 h} F_s(0) h \phi_2^2(0) \frac{k^2}{k^2 + R_s^{-2}} \frac{S_s(k)}{2\pi k} \frac{\pi}{d},$$

where $y = \pi k R_s$. The wavenumber magnitude spectra $E_{\text{tot}}(k)$ and $E_{\text{tot}}^{bc}(k)$ are shown in Fig. 5 for Rayleigh damping (heavy lines) and lateral diffusion (thin lines). For Rayleigh damping the barotropic response is concentrated at large scales and the baroclinic response at scales comparable or smaller than the first radius of deformation; the characteristic wavenumbers are given by

$$\langle k \rangle^{bt} = \frac{\int dkkE_{\text{tot}}^{bt}(k)}{\int dk E_{\text{tot}}^{bt}(k)} = (2\pi/900) \text{ km}^{-1}$$

$$\langle k \rangle^{bc} = \frac{\int dkkE_{\text{tot}}^{bc}(k)}{\int dk E_{\text{tot}}^{bc}(k)} = 2\pi/135 \text{ km}^{-1}$$

To interpret our spectral density plots, one should remember that a $k^{-1}$ spectrum indicates equal variance per fixed logarithmic interval.
For horizontal diffusion, the scales are larger, especially for the baroclinic response, $\langle k \rangle_{\text{bt}} \sim 2\pi/2700$ km$^{-1}$ and $\langle k \rangle_{\text{be}} \sim 2\pi/1100$ km$^{-1}$.

The partition of the total energy into kinetic energy

$$E_{\text{kin}}(k, \omega) = \frac{k^2}{k^2 + R_n^{-2}} E_{\text{tot}}(k, \omega),$$

and potential energy

$$E_{\text{pot}}(k, \omega) = \frac{R_n^{-2}}{k^2 + R_n^{-2}} E_{\text{tot}}(k, \omega),$$

also differs for the two friction laws. For Rayleigh damping we find

$$\frac{E_{\text{kin}}}{E_{\text{tot}}} \sim 0.5,$$

whereas the ratio is only 0.06 for lateral diffusion. This difference merely reflects the difference in horizontal scale, since most of the energy is potential for small ($k \ll R_n^{-1}$) wavenumbers and kinetic for large ($k \gg R_n^{-1}$) wavenumbers. These estimates show that the use of lateral diffusion leads to an unrealistic eddy field where the baroclinic energy would be concentrated at large scales in the form of potential energy. We therefore will not consider lateral diffusion any further.

c. Time scales

The frequency spectra

$$E_{\text{tot}}(\omega) = \int_{k_m \leq k \leq k_M} d^2 k E_{\text{tot}}(k, \omega)$$

cannot be evaluated analytically. In polar coordinates we transformed the integration over the direction $\theta$ into a contour integral along the unit circle and evaluated it by the calculus of residues. The remaining integration over $k$ was done numerically. The basic structure of the frequency spectra can, however, be inferred from general arguments distinguishing between resonant and off-resonant frequencies. For each mode, the resonance range has a high frequency limit $\omega_{n}^{\text{max}}$ determined by setting $k_z = 0$ and $k_1 = k_m$ (barotropic mode) or $k_1 = R_n^{-1}$ (baroclinic modes) in the dispersion relation (2.10). For our standard values we find $\omega_{n}^{\text{max}} = 1.3 \times 10^{-5}$ s$^{-1}$ ($2\pi/6$ days) and $\omega_{n}^{\text{max}} = 4.7 \times 10^{-7}$ s$^{-1}$ ($2\pi/155$ days). The spectral level is high for resonant frequencies $\omega \leq \omega_{n}^{\text{max}}$ and decays proportionally to $\omega^{-2}$ for off-resonant frequencies $\omega \gg \omega_{n}^{\text{max}}$. If $\omega_{n}^{\text{max}} \gg \Gamma$ there is a jump at $\omega_{n}^{\text{max}}$.

Fig. 6 shows the barotropic kinetic energy spectrum and its meridional and zonal components, characterized by a large drop at $\omega = \omega_{n}^{\text{max}}$. At very low frequencies the meridional component is much smaller than the zonal component, a consequence of the Rossby wave dispersion relation. Small values of $\omega_{0}$ require small values of $k_1$, hence there is little meridional energy in the resonant part of the low frequency motions. Note that the barotropic spectrum is very similar to that of the (flat-bottom) numerical simulation of Willebrand, et al. (1980); except that they find peaks at the basin mode frequencies. In our unbounded ocean, such peaks are smoothed out.

Fig. 7 shows the depth-integrated kinetic and potential energy spectra for the first three baroclinic modes. The spectra are white at low frequencies and change smoothly to the off-resonant $\omega^{-2}$ law at $\omega = \omega_{n}^{\text{max}}$.

d. Coherence

Since the oceanic flow is driven by the atmospheric wind stress, we expect oceanic and atmospheric variables to be coherent. However, two effects degrade the coherences.

The first one is destructive interference. In our model the Fourier transform of an oceanic variable $Y$ and an atmospheric variable $X$ are linearly related

$$Y(k, \omega) = T(k, \omega) X(k, \omega),$$

where $T(k, \omega)$ is the transfer function. The coherence as a function of frequency is then given by...
where $S_X(k, \omega)$ denotes the spectrum of the atmospheric variable. If $T$ is independent of $k$ the coherence will be one, otherwise destructive interference decreases the coherence. Table 1 lists the transfer functions between the oceanic fields $p$ $(x_3 = 0)$ (subsurface pressure), $u^n$ (velocity amplitude of the $n$th mode), and $\xi$ $(x_3 = 0)$ (subsurface displacement) and the atmospheric fields $\tau$ (wind stress curl), $\pi$ (wind stress), and $p_a$ (atmospheric pressure). Except for the coherence between the subsurface displacement and the wind-stress curl, all transfer functions depend on $k$.

The second effect which degrades the coherence arises from the fact that only wavenumbers within the internal $[k_m, k_N]$ are assumed to force the ocean, whereas all wavenumbers contribute to observed frequency spectra. The coherence is hence uniformly reduced by a factor $\gamma$ which is the square root of the fraction of the atmospheric variance within the eddy range. For our atmospheric model spectrum (3.1) this factor is $\gamma = 0.5(k_b/k_m)^{1/2} \sim 0.36$ for coherences with the atmospheric pressure and $\gamma = (0.75k_b/k_m)^{1/2} \sim 0.77$ for coherences with the wind stress. The wind stress curl has most of its variance at small scales, and the factor $\gamma$ depends sensitively on whether or not these small scales contribute to the observed wind stress curl frequency spectrum.

Figs. 8–10 show the numerically calculated coherences and phases for our simple homogeneous and symmetric wind stress model spectrum (3.1). The coherences $u_1^n - p_a$, $\rho - \tau_1$, $\xi_1^n - \tau_2$, $u_2^n - \tau_1$, $\xi - \tau_1$, $\xi - \tau_2$ and $u_1^n - \text{curl} \tau$ vanish identically since their transfer function is an odd function of either the meridional or zonal wavenumber. The absolute value of the coherence with the wind stress curl depends on the unspecified degradation factor $\gamma$. The coherences with the atmospheric pressure and wind stress are generally small. Only the coherences $p - p_a$, $u_1^n - \tau_1$, and $u_2^n - \tau_2$ rise to a value larger than 0.2 in the off-resonant range $\omega \gg \omega_n^{\text{max}}$. It has been argued that low coherences must be expected in the resonant range since waves propagate away from their forcing region. This argument is not correct in our homogeneous ocean model, e.g., the coherences $u_2^n - \text{curl} \tau$ are larger in the resonant range. It is the amount of destructive interference of different wavenumbers which determines the value of the coherence.

e. Depth dependence

Our model predicts the energy spectrum of each mode. This gives a measure of the depth distribution of the oceanic response. Observations usually have a vertical resolution too sparse for a modal decomposition and yield spectra at fixed depths. These spectra are difficult to calculate within our approach, except for the vertical displacement spectrum in the off-resonant range $\omega \gg \omega_1^{\text{max}}$ which is given by (FM)

![Graph showing model frequency spectrum of the depth-integrated potential (solid lines) and kinetic (dashed lines) energy for the first three baroclinic modes.](attachment:image.png)
Table I. Transfer functions between subsurface pressure \( p(0) \), velocity amplitude of \( n \)th mode \( u^{(n)} \), subsurface displacement \( \xi(0) \), wind stress curl, wind stress \( \tau \), and atmospheric pressure \( p_a \), except for a (irrelevant) positive real constant.

<table>
<thead>
<tr>
<th>( p(0) )</th>
<th>( \tau_a )</th>
<th>( p_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{i}{k^2 + R_a^{-2}} ) ( \frac{1}{\omega - \omega_a + iR} )</td>
<td>( \frac{i}{k^2 + R_a^{-2}} ) ( \frac{1}{\omega - \omega_a + iR} )</td>
<td>( \frac{ik^2}{k^2 + R_a^{-2}} ) ( \frac{1}{\omega - \omega_a + iR} )</td>
</tr>
<tr>
<td>( u_x^{(n)} )</td>
<td>( \frac{\epsilon_{ab} k_a}{k^2 + R_a^{-2}} ) ( \frac{1}{\omega - \omega_a + iR} )</td>
<td>( \frac{\epsilon_{ab} k_a}{k^2 + R_a^{-2}} ) ( \frac{1}{\omega - \omega_a + iR} )</td>
</tr>
<tr>
<td>( \xi(0) )</td>
<td>( \frac{i}{\omega + iR} )</td>
<td>( \frac{ik^2}{\omega + iR} )</td>
</tr>
</tbody>
</table>

\[
S_x(k, \omega; x_a) = \frac{1}{\rho_0 f_0^2} \frac{\sinh^2(x_a + h)k^2 \pi R_1/\omega}{\omega^2 \sinh^2 k R_1} \times \epsilon_{ab} \epsilon_{by} k_{a} k_{y} F_{ab}(k, \omega), \quad (4.16)
\]

The response is strongly trapped near the surface with an approximate \( \epsilon \)-folding length of \( h(\langle k \rangle R_1)^{-1} \approx 700 \text{ m} \).

At the surface the vertical displacement is given for all frequencies by

\[
S_x(x_a = 0)(k, \omega) = \frac{1}{\rho_0 f_0^2} \frac{1}{\omega^2 + R^2} \epsilon_{ab} \epsilon_{by} k_{a} k_{y} F_{ab}(k, \omega), \quad (4.17)
\]

which simply reflects the forcing, independently of stratification. The variance is

\[
\langle \xi^2(x_a = 0) \rangle = \frac{3\pi}{8} \frac{1}{\rho_0 f_0^2} F_{0}(0) k_{b} k_{m} R^{-1}, \quad (4.18)
\]

which corresponds to an rms displacement of 34 m for our standard set of parameters.

f. Modulation

The atmospheric wind stress field is neither statistically homogeneous nor stationary. For weak inhomogeneity and non-stationarity (i.e., inhomogeneity scales larger than the periods and wavelengths) we can still define a local spectrum but this becomes a slowly varying function of space and time. The response of the ocean is governed in this case by the radiation balance equation (see the Appendix for a derivation)

\[
(\partial_t + 2R + v_a \partial_x)E(k, \omega; x, t) = Q(k, \omega; x, t), \quad (4.19)
\]

where

\[
v_a = -\frac{2 \omega k_a}{k^2 + R_a^{-2}} - \frac{\beta}{k^2 + R_n^{-2}} \delta_{ab}. \quad (4.20)
\]

In (4.19), both the oceanic energy spectrum \( E \) and the atmospheric forcing spectrum \( Q \) are slowly varying functions of space and time. On the resonance surface \( \omega = \omega_n(k) \), \( v \) becomes the group velocity of planetary Rossby waves. To gain insight into the inhomogeneity and non-stationarity of the response fields we solve (4.19) for two different cases, for a seasonally modulated and for a strongly localized wind-stress spectrum.

In the first case we consider the equation

\[
(\partial_t + 2R)E(k, \omega; t) = Q(k, \omega)(1 + \gamma \cos \Omega t), \quad (4.21)
\]

where \( \Omega \) is the annual frequency. The solution is an annual modulation of the oceanic response

\[
E = (Q/2R)(1 + q \cos(\Omega t - \phi)), \quad (4.22)
\]

with a phase shift

\[
\phi = \tan^{-1}(\Omega/2R) \quad (4.23)
\]

and an amplitude

\[
q = \gamma \cos \phi. \quad (4.24)
\]

The phase shift and amplitude depend on the damping coefficient \( R \). For \( R \to 0 \), \( q \to 0 \) and \( \phi \to \pi/2 \) (6 months lag). For \( R \to \infty \), \( q \to 1 \) and \( \phi \to 0 \) (no lag). For our standard value \( R = 5 \times 10^{-8} \text{ s}^{-1} \) we find a reduction of the modulation by a factor of 0.45 and a lag of about 3 months.

The response to a strongly localized wind-stress spectrum can be modeled by the equation

\[
(2R + v_a \partial_x)E(k, \omega; x) = Q_0(k, \omega)\delta(x), \quad (4.25)
\]

which has the solution

\[
E = \frac{Q_0}{v} \delta(x \eta_a) \theta(x \eta_a) \exp(-\frac{2R}{v} e_a x_a), \quad (4.26)
\]

where \( v = (u_v^2 v^{-1})^{1/2} \), \( e_a = u_v v^{-1} \) and \( \eta_a = \epsilon_{ab} u_{by} v^{-1} \).

In the direction of the group velocity the solution decays with an \( \epsilon \)-folding scale \( L = v/2R \). Since most of the energy is concentrated close to the resonance modes we may substitute \( \omega = \omega_n(k) \) in the expression for the group velocity \( v \). For fixed \( k \) the group velocity then describes a circle with center \( a_n = [-\beta R_n^{-1} k^2 + R_\eta^{-1}]^2, 0 \) and radius \( r_n = \beta k^2 / (k^2 + R_\eta^{-1})^2 \). For the barotropic mode, \( a_0 \ll r_0 \), and
Fig. 8. Model coherences and phases of subsurface pressure \( p \) \((x_2 = 0)\), subsurface displacement \( \xi \) \((x_3 = 0)\), and zeroth and first mode meridional velocity amplitude \( u^{(0)}_2, u^{(1)}_2 \) with atmospheric pressure \( p_n \). The model coherences \( u^{(n)}_2 - p_n \) vanish identically. A positive phase means that the oceanic variable leads the atmospheric pressure.

Group velocity and e-folding scale are approximately independent of direction. We find

\[
L_0 = \frac{v}{2R} \approx \frac{\beta}{2Rk^2} \approx 4200 \text{ km,} \tag{4.27}
\]

where we substituted \( k = 2\pi/900 \text{ km}^{-1} \) as the characteristic wavenumber of the barotropic response [cf. (4.9)]. This analysis suggests that, at least at small wavenumbers, the barotropic response to local forcing is smoothed over ocean wide scales. Thus, the local barotropic response should be related to atmospheric fields averaged over the ocean basin.

The situation is different for the baroclinic response which has a characteristic wavenumber \( \langle k \rangle_{bc} \sim R_1^{-1} \). Substituting this value, we find \( a_n \sim r_n \) which implies that the baroclinic energy cannot propagate in the eastward direction and is trapped within the distance

\[
L_{bc}^2 = \frac{\beta R_1^2}{4R} \approx 220 \text{ km} \tag{4.28}
\]
in the westward direction. The meridional trapping scale is
\[ L_{bc}^m = \frac{\beta R_0^2}{8R} \approx 110 \text{ km}. \] (4.29)

The baroclinic response is hence strongly local and should be estimated using local values for the atmospheric fields and oceanic parameters. This analysis also suggests that the intensification of the baroclinic response due to wave reflection at the western boundary is limited to a narrow region of width \( L_{bc}^m \).

In summary, our model predicts a barotropic response with an rms velocity \( \sim 3.5 \text{ cm s}^{-1} \) which is concentrated at ocean wide scales and around the highest possible Rossby wave frequencies, with little energy in the off-resonant frequency range. The baroclinic response is characterized by \( u \sim 3.5 \text{ cm s}^{-1} \) and \( \xi \sim 17 \text{ m} \), and is strongest at scales smaller and equal than the first Rossby radius of deformation. Most of the energy is at Rossby wave frequencies, but there is also a substantial off-resonance response. The barotropic response is global, the baroclinic response local, and little seasonal modulation is expected. The coherences with the atmospheric pressure and wind stress are generally small, except at high off-resonant frequencies.

Some of the model predictions seem fairly reliable, because they do not depend crucially on our admittedly poor parameterization of transfer and
dissipation processes. This is the case for the overall eddy energy and the near surface displacement spectrum which are determined by the forcing and the overall energy decay time. On the other hand the predicted space-time scales are the least reliable because they are strongly controlled by nonlinearities and topography. Nonlinear interactions redistribute energy among the different space and time scales. Finite-amplitude bottom topography and bottom slopes may also considerably modify the response and cause frequency spectra to vary considerably over short distances. This is well documented for a homogeneous ocean model by Willembrant et al. (1980). In a stratified ocean, topography leads (even in the linear approximation) to mode coupling and an associated energy exchange among modes: in spin-up experiments Anderson et al. (1979) found a modulation of the barotropic mode by the baroclinic modes leading to a longer time scale of the barotropic response.

5. Comparison with data

a. Energy level

Since the oceanic eddy field is more inhomogeneous than the wind stress field, a comparison of the geographical variability of the two energy levels indicates where direct atmosphere forcing

---

**Fig. 10.** Model coherences and phases with wind-stress curl. The model coherences $u_2^{(0)} - \nabla \times \tau$ vanish identically. The coherence is given in units of $\gamma$, the unspecified square root of the fraction of the wind stress curl variance in the wave-number range from $2\pi/4000$ to $2\pi/50$ km$^{-1}$. 
cannot be the dominant generation mechanism for the eddy field. In Fig. 11, the rms amplitude of the large-scale surface wind stress in the North Atlantic, estimated by Willebrand (1978) from sea level data, is compared with the rms vertical displacement of the 15°C isotherm, estimated by Dantzer (1977) from expendable bathythermograph data. Although the XBT data are sparse in many regions and the mean depth of the 15°C isotherm varies widely from 100 to 600 m, the maps clearly show that the large intensification of the baroclinic eddy energy in the Gulf Stream current, extension and recirculation regions cannot result from direct atmospheric forcing. Much of this high eddy activity is associated with rings, meanders and other products of eddy-mean flow interactions which can be thought of as being superimposed on a smoother background eddy field. Indeed, Kim and Rossby (1979) showed that the subtraction of Gulf Stream rings from XBT data yields rms vertical displacements of less than 50 m. This figure is comparable to central ocean values and only slightly larger than our model predictions based on values characteristic of the MODE-Bermuda region (a depth-averaged rms displacement of 17 m and a near-surface rms displacement of 34 m). Thus, the XBT data are not inconsistent with the hypothesis that a substantial part of the background eddy field is atmospherically driven. Current measurements (Schmitz 1978; Richman et al. 1977) offer the same picture: the eddy field is much too energetic in the Gulf Stream region to be atmospherically driven, but central ocean amplitudes are comparable with our model predictions (a depth averaged rms velocity of 5 cms⁻¹). This is illustrated in Fig. 12 (after R. Dickson, private communication) which summarizes the avail-
Fig. 12. Distribution of kinetic energy (in cm$^2$ s$^{-2}$; after Dickson, 1980) at 0–800 m and 3800–4300 m depth over the North Atlantic.
able estimates of eddy kinetic energy near the surface and at large depth. Note that the MODE region (near 28°N, 70°W) is at the margin of the region where direct atmospheric forcing might be important.

The situation is analogous in the North Pacific. In the western part, close to the Kuroshio region, the eddy activity is too high to be explained by direct wind forcing; in the central and eastern parts the smaller eddy energy levels are consistent with our estimates (see Fig. 15).

b. Space-time scales and depth dependence

The scales of the observed eddy field vary greatly with depth, location and variable.

In the MODE region, spatial correlation function estimates of the temperature field suggest a dominant baroclinic wavenumber increasing from $2\pi/600$ km$^{-1}$ in the thermocline to $2\pi/200$ km$^{-1}$ at large depth (Richman et al., 1977). The scale of the barotropic currents seems to be somewhat larger. Wave fits to MODE-O data (where it is rather successful) indicate the presence of two barotropic waves with $k = 2\pi/450$ km$^{-1}$ and $k = 2\pi/1100$ km$^{-1}$ (McWilliams and Flierl, 1976). Pressure records (Brown et al., 1975) have an even larger scale comparable to that of atmospheric fields. All these figures are not unlike our model predictions $\langle k \rangle^b \sim 2\pi/135$ km$^{-1}$, $\langle k \rangle^e \sim 2\pi/900$ km$^{-1}$ and $\langle k \rangle^v \sim 2\pi/2700$ km$^{-1}$ for the pressure field.

The dominant time scale in the thermocline was not resolved in MODE where much of the energy is at periods longer than 100 days (Schmitz, 1978). In intermediate and deep water, the energy is concentrated at periods of 50–100 days, possibly because barotropic motions are more important at depth. Richman et al. (1977) have estimated spectra for the barotropic and first baroclinic modes from two years of current and temperature data at 500, 1500 and 4000 m. In Fig. 13, these are compared with our model predictions. The energy level of the barotropic spectrum is reproduced correctly but the predicted sharp cutoff between resonant and off-resonant motions is not observed. The shape of the baroclinic spectra is well reproduced, but the energy level is underestimated by a factor of 2 or 3, which is, however, within the uncertainty of our calculation. Further east (near 28°N, 55°W), observed (Richman et al., 1977) and predicted energy levels match closely.

The observed depth dependence of temperature data in MODE is consistent with WKB scaling (Richman et al. 1977). This is in contrast to our prediction of a surface trapped baroclinic response at off-resonant frequencies.

The comparison with MODE data is inconclusive.
Our model predicts the right amplitudes and space scales but not the correct time scales and depth dependence. The concentration of the barotropic variance at high frequencies and the associated separation between barotropic and baroclinic time scales is not observed. These discrepancies are not unexpected. Indeed, numerical simulation of the MODE data suggests that the eddy properties are mainly controlled locally by bottom topography and nonlinear interactions, regardless of their energy sources and sinks (Owens and Bretherton, 1978; Owens, 1979).

At POLYMODE Array III, cluster C (55°W, 15°N), the eddy activity is comparable to that in MODE. In Fig. 14 we compare two observed displacement (or temperature) spectra (after Keffer et al., 1979) with the model subsurface displacement spectrum obtained from (4.17). The model spectrum has been calculated for the local values $f_0 = 3.8 \times 10^{-5}$ s$^{-1}$ and $F_r(0) = 2 \times 10^2$ N$^2$ m$^{-3}$ Hz, using $6 \times 10^{-2}$°C m$^{-1}$ for the mean vertical temperature gradient. The agreement is good and strongly supports our representation of the atmospheric forcing spectrum.

At site D (39°N, 70°W) the current data show a strong barotropic component (Thompson, 1977) and the observed kinetic energy frequency spectra compare excellently with our barotropic prediction (Fig. 15). This has also been noted by Willebrand et al. (1980). The observed length scale at site D is, however, much smaller (Thompson, 1977) than predicted by our theory, presumably due to the scattering at small scale topography and reflection at the nearby continental rise. Both these processes change predominantly the wavenumber but less so the frequency structure of oceanic eddies.

Wavenumber spectra of the near-surface displacement in the western and the central North Pacific, estimated after Bernstein and White (1977), are compared with the model prediction in Fig. 16. Here, we used $f_0 = 8.7 \times 10^{-5}$ s$^{-1}$, $F_r(0) = 2 \times 10^4$ N$^2$ m$^{-3}$ Hz and a mean vertical temperature gradient of $2 \times 10^{-2}$°C m$^{-1}$. As stated above, the agreement is good for the central part, but direct wind forcing is clearly insufficient to explain the larger baroclinic eddy activity in the western part of the ocean. Further evidence for direct wind forcing in the central North

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**Fig. 14.** POLYMODE array III, cluster C temperature spectra at 161 and 194 m depth (heavy lines; after Keffer et al., 1979) and model subsurface temperature spectrum (thin lines).
The variance spectrum of the subsurface pressure is shown in Fig. 17. The variance in the period range 3–200 days is about (10 mb)$^2$. Also shown are bottom pressure spectra of Brown et al. (1975). In our model most of the pressure variance is barotropic and the pressure field can be assumed to be depth independent. The spectrum is then given by

$$S_p(\omega) = \int d^2k \frac{\phi^2(0)}{h} \frac{f_0^2}{(\omega - \omega_0)^2 + R^2} \times \frac{\epsilon_{\alpha\beta}\epsilon_{\gamma\delta}k_\alpha k_\gamma}{(k^2 + R_0^{-2})^2} F_{\beta\delta}(k, \omega), \quad (5.2)$$

and has the variance

$$(p^2) = \frac{\pi}{8} \frac{\phi^2(0)}{h} F_0(0) \frac{k_\beta}{k_m^3} \frac{1}{R} \approx (9 \times 10^2 \text{ Nm}^{-2})^2 \approx (9 \text{ mb})^2. \quad (5.3)$$

The agreement in spectral level is reasonable but the spectral slopes differ significantly. It should be noted that the annual signal has not been removed from the Bermuda spectrum and dominates the lowest frequency estimate.

c. Coherences

Our model predicts small coherences between most oceanic and atmospheric variables, except at high off-resonant frequencies. This is consistent with some of the observations reviewed in FM and with the findings of Brown et al. (1975) who observed a significant coherence between bottom pressure fluctuations in the MODE region (4 months of data) and the atmospheric pressure at high frequencies.

For a more detailed comparison we analyzed eight years (1953–60) of sea level, wind and atmospheric pressure observations taken at Bermuda. These data have been previously investigated by Wunsch (1972). The sea level data are converted here into subsurface pressure by adding the isostatic contribution of the atmospheric pressure

$$p = \rho_0 g \zeta + p_a. \quad (5.1)$$

Fig. 15. Site D kinetic energy spectra at 1000 and 2000 m depth (heavy lines; after Thompson, 1977) and model barotropic response (thin line).

Pacific has been discussed by White et al. (1980), who found a significant correlation between the wind stress curl and the vertical displacement field at 300 m depth for two latitudes (35° and 45°N) along 160°W.

Fig. 16. Zonal wavenumber spectrum of the vertical displacement at 300 m depth in the North Pacific, computed from zonal XBT sections between 35° and 40°N (after Bernstein and White, 1977). Heavy dashed line: Pacific west of 170°W; heavy solid line: Central Pacific east of 170°W; thin line: model spectrum of the subsurface displacement.
The coherences and phases between the subsurface pressure and the atmospheric pressure for the Bermuda and MODE data and for our model are shown in Fig. 18. Consistent with our prediction the observed coherences are significant (at the 95% confidence level) at high frequencies, but not at low frequencies. This pattern is not an artifact of the frequency averaging and represents direct evidence of atmospheric forcing at high frequencies. In the frequency range of significant coherence the MODE phases are scattered and the Bermuda phases are 180° out of phase with our prediction.

The predicted phase does not change significantly when the asymmetry of the wind field is taken into account, and agrees with the phase given by Willebrand et al. (1980, Fig. 7) for their numerical experiment. We have been unable to find an explanation for this discrepancy, which might be due to island effects.

The observed coherences and phases between the subsurface pressure and the wind stress components at Bermuda are shown in Fig. 19. Also shown is our model prediction for the north component. If we assume an averaged ratio of eastward to westward propagating wind stress variance of 3 at high frequencies, the predicted coherences and phases change to the values indicated by the heavy arrow. Again, we qualitatively reproduce the coherences and find a phase discrepancy of about 180°.

The predicted coherence with the east component is zero and is not affected by zonal asymmetries. A meridionally inhomogeneous wind stress field, however, would yield a nonzero coherence.

Recently, Koblinsky and Niiler (1980) have shown that at POLYMODE Array III, cluster C (15°N, 55°W) the wind stress is coherent with the currents throughout the water column at periods from 4–40 days, as predicted for off-resonant forced barotropic motions.

d. Seasonal modulation

A seasonal modulation of the central Pacific displacement data, shown in Fig. 16, was detected by FM. This is surprising because most of the baroclinic variance is expected to be at low subseasonal frequencies. Indeed a search for a seasonal modulation in Levitus and Oort's (private communication) XBT data (comparing winter and summer variance at standard depths and locations) was without success. However, we do expect a seasonal modulation.
of the high-frequency response. To investigate this modulation at Bermuda we have separated the atmospheric and subsurface pressure data into seasons and repeated the spectral analysis for each season separately (8 realizations, except for fall where only 7 years of data were available). Wind-stress spectra for fall–winter and spring–summer are presented in Fig. 20 together with the corresponding spectra for the subsurface pressure. There is a pronounced difference in the wind-stress spectra: the difference in the two subsurface pressures is smaller but significant at high frequencies, and consistent with the prediction given in Section 4f. However, the predicted phase lag (3 months) is not observed (the winter–spring and summer–fall spectra show no significant differences).

6. Summary and conclusions

To assess the role of direct wind forcing in generating oceanic quasi-geostrophic eddies, we have estimated the response of a simple ocean model to stochastic wind-stress forcing, using a statistical representation of the forcing and response fields. The ocean model is a stratified β-plane ocean of infinite horizontal extent and constant depth. All transfer and dissipation processes are parameterized by a linear friction law with a scale independent damping coefficient \( R = 5 \times 10^{-8} \text{ s}^{-1} \) (Rayleigh damping). Both the friction law and the value of the damping coefficient seem reasonable for purely barotropic motions. They correspond to bottom friction with an amplitude e-folding time of 200 days. The parameterization scheme is presumably less appropriate for the complicated transfer and unknown dissipation processes in a baroclinic ocean.

Our model predicts a depth-averaged rms horizontal velocity and vertical displacement of 5 cm s\(^{-1}\) and 17 m. Atmospheric forcing might hence be the major energy source of oceanic eddies in mid-ocean regions far removed from strong currents, where fluctuations of this order are observed. Our model also predicts qualitatively the vertical displacement or temperature field near the surface which basically implies that our forcing function is good. For deeper spectra the agreement is less satisfactory because of nonlinear interactions, topography and other dynamical effects. Observed coherences and seasonal modulations provide direct evidence of atmospheric forcing at high frequencies. If our simple ocean model is reasonably correct it will be difficult to establish direct evidence of atmospheric
forcing in the more energetic low-frequency range since the expected coherences are small.

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APPENDIX

Derivation of the Radiation Balance Equation

Here we derive the radiation balance equation (4.16), describing the response to weakly inhomogeneous and instationary forcing using a two-scale approximation. The normal mode equation may be written (see Section 2)

\[
(\partial_t + R)(1 - R_n^2\partial_{\omega} \partial_{a_n})a_n(x, t) - \beta R_n^2 \partial_t a_n(x, t) = f(x, t).
\]

We assume that both the forcing term and the normal mode amplitude can be decomposed into rapidly varying Fourier components whose amplitudes vary slowly with time and position

\[
f(x, t) = \int d^2k d\omega f(k, \omega; X, T) \exp[i(k_\alpha x_\alpha - \omega t)]
\]

\[
a_n(x, t) = \int d^2k d\omega a_n(k, \omega; X, T) \exp[i(k_\alpha x_\alpha - \omega t)].
\]

Here \(X = \epsilon x\) and \(T = \epsilon t\) with \(\epsilon \ll 1\) are the slow space and time scales. Correct to first order in \(\epsilon\), the normal mode equation becomes
\[
\left( \frac{\partial}{\partial T} - i\omega + i\omega_n + R + v_a \frac{\partial}{\partial X_n} \right) a_n
\]
\[
= \frac{f}{1 + k^2R_n^2}, \tag{A3}
\]
where
\[
v_a = -\frac{2\omega k_a}{k^2 + R_n^2} - \frac{\beta}{k^2 + R_n^2} \delta_{n,1}. \tag{A4}
\]
We multiply (A3) by \(a_n^*\), add the complex conjugate and take the ensemble average. This yields the energy equation
\[
\left( \frac{\partial}{\partial T} + 2R + v_a \frac{\partial}{\partial X_n} \right) \langle a_n a_n^* \rangle
\]
\[
= \frac{\langle a_n^* f \rangle + \langle a_n f^* \rangle}{1 + k^2R_n^2}. \tag{A5}
\]
On the right-hand side, we substitute the zeroth order solution of (A3)
\[
a_n = \frac{1}{i(\omega_n - \omega) + R + \frac{\partial}{\partial X_n}} \frac{f}{1 + k^2R_n^2}. \tag{A6}
\]
The source term then becomes
\[
Q = \frac{2R}{\omega - \omega_n^2 + R^2 \left( 1 + k^2R_n^2 \right)^2} \langle f f^* \rangle, \tag{A7}
\]
where \(\langle f f^* \rangle\) is proportional to the slowly varying spectrum of the forcing function. Eq. (A5) together with (A7) is the radiation balance equation (4.16).

REFERENCES


