On the Importance of Vertical Resolution in Certain Ocean General Circulation Models*

ANDREW J. WEAVER** AND E. S. SARACHIK

Joint Institute for the Study of the Atmosphere and Ocean, University of Washington, Seattle, Washington

17 July 1989 and 18 October 1989

ABSTRACT

In centered difference models of ocean circulation, two grid-point computational modes can be excited if grid Reynolds and Peclet numbers are greater than two. The Bryan–Cox General Circulation Model (GCM) is used to show the dramatic effect that this instability has on the equatorial thermohaline circulation. In many recent numerical calculations researchers have used 12 vertical levels. It is shown that this resolution produces an artificial cell at the equator when typical values of the vertical diffusivity and viscosity parameters are used. This artificial cell rotates counter to the primary cell driven by deep water formation at high latitudes, is driven by downwelling at the eastern boundary near the equator and is 40% the strength of the primary cell for the parameters used in the present study. When the vertical resolution is increased the cell vanishes. It is suggested therefore that higher vertical resolution should be used in Bryan–Cox GCM deep-ocean modeling studies when current values of the vertical diffusivity and viscosity parameters are used.

1. Introduction

In the past several years high speed computers have become more readily available. It is now feasible to spin up coarse resolution models for several thousand years until equilibrium is reached. As a consequence, researchers have become very interested in modeling the role of the ocean in climate (e.g., F. Bryan 1986; Manabe and Stouffer 1988; Toggweiler et al. 1989a,b).

Probably the most widely used primitive equation model is the Geophysical Fluid Dynamics Laboratory (GFDL) Bryan–Cox Ocean General Circulation Model. There has been much analysis done on the sensitivity of this model to various parameters (e.g., Bryan and Cox 1967; Cox and Bryan 1984; Bryan and Lewis 1979; Meehl et al. 1982; F. Bryan 1987; Cummins et al. 1989). The purpose of this note is to examine the sensitivity of the thermohaline circulation reproduced by the model to vertical resolution.

In many recent modeling studies there have been very curious features observed in the solutions at the equator. Most noteworthy of these is a cell which shows up in the meridional overturning streamfunction, centered at a depth of around 3000 m (Cox 1975; Takano 1981; Cox and Bryan 1984; Bryan and Sarmiento 1985; F. Bryan 1987; Suginoara and Aoki 1989). This cell is driven by intense downwelling at the eastern boundary, is of comparable magnitude to the primary cell driven by deep water formation at high latitudes and flows in opposite sense to this primary cell. Furthermore, it is associated with much weaker cells of oscillating sense at shallower depths. There have also been other curious features that have shown up in equatorial regions of model simulations. For example, Suginoara and Fukasawa (1988) found that their eastern equatorial meridional flow changed sign at each of their five levels. When they increased their resolution to nine levels they found that the flow changed signs eight times. Similarly, F. Bryan (1986) obtained a series of strong overturning cells of alternating signs on the equator. He argued that these were due to inertial instability of the cross-equatorial flow.

In this note we show that all the above observations are due to insufficient vertical resolution in the models. Since the same resolution has been used in the climatic studies cited above, and as computer limitations prevent the use of significantly higher resolution, it is important that the effects of increased resolution be understood.

The rest of this note is split into two sections. In the next section we describe the numerical model briefly, while in section 3 we present and discuss our results.

2. Description of the numerical model

In this study we employ the widely used Cox (1984) version of the Bryan–Cox Ocean General Circulation Model, which is based on the primitive equation model described by Bryan (1979) and Bryan and Lewis
The finite difference scheme used in the model has been described by Bryan (1969) and Cox (1984), while the polynomial approximation to the equation of state employed by the model is detailed by Bryan and Cox (1972). In this study the horizontal grid spacing is 2° by 2° and the model basin is everywhere 5000 m deep. The model domain consists of a 60° wide Southern Hemisphere basin extending from 70°S to the equator.

At the equator a symmetric boundary condition is used, while at the lateral walls no-slip and insulating boundary conditions are applied. The bottom is assumed to be impermeable and insulating. Bottom friction, which is assumed to be quadratic in velocity with a 10° turning angle (Gill 1982) is also added. At the surface the model employs the rigid lid approximation. We specify a surface windstress which has only a zonal component and is a function only of latitude. This windstress is given by the simple analytic expression (F. Bryan 1987)

$$\tau^\lambda(\phi) = 0.2 + 0.8 \sin(6\phi) - \frac{1}{2} \left[ 1 + \tanh(10\phi) \right]$$

and captures the major features of the observed zonally averaged zonal windstress. Here $\lambda$ is the longitude and $\phi$ is the latitude in radians. This surface windstress is illustrated in Fig. 1a. A linear damping condition with time constant 25 days (F. Bryan 1987) is used for both temperature and salinity at the surface. The zonally uniform reference temperatures and salinities were obtained by averaging the Atlantic Ocean basin sea surface data of Levitus (1982) over both the Northern and Southern Hemispheres. These reference values are illustrated in Figs. 1b and 1c.

The vertical eddy viscosity was everywhere set to 1.0 cm$^2$ s$^{-1}$, while the vertical diffusivity was allowed to increase with depth from $A_{TVS} = 0.3$ to $A_{TVB} = 1.3$ cm$^2$ s$^{-1}$ according to the formula (Bryan and Lewis 1979; Toggweiler et al. 1989a,b)

$$A_{TV} = \left( A_{TV2500} + \frac{C_T}{\pi} \tan^{-1} \left[ 4.5 \times 10^{-3} \right] \right) \times (z - 2.5 \times 10^3) \text{ cm}^2 \text{ s}^{-1},$$

where $A_{TV2500} = 0.8$ cm$^2$ s$^{-1}$ is the vertical diffusivity at a depth of 2500 m and the range between the top and bottom is given by $C_T = 1.0$ cm$^2$ s$^{-1}$. The horizontal eddy viscosity and diffusivity were set to $5 \times 10^8$ and $1 \times 10^7$ cm$^2$ s$^{-1}$, respectively, over the whole domain. A parameterization which allows lateral diffusion to occur along isopycnal surfaces (Cox 1987) was not included although the parameterization to compute vertical diffusion implicitly (Cox 1984) was included.

The model was started from a resting homogeneous state with uniform temperature of 5°C and salinity of 32.5 ppt. To speed up the convergence of the model the acceleration techniques of Bryan (1984) were used. The barotropic vorticity and baroclinic velocity equations were integrated with a timestep of 1/2 hour, whereas the tracer equations for temperature and salinity were integrated with a timestep that varied exponentially with depth (Bryan 1984). At the deepest level the timestep was three times its value at the surface level. For the first 438 years, when convection was intense, the tracer equation timestep was set to 2 days at the surface. This was increased to 4 days for all subsequent integrations.

In Experiment 1 we used 12 levels in the vertical. The levels were chosen to be the same as those used by Bryan et al. (1975, 1982), F. Bryan (1986, 1987), Manabe and Stouffer (1988) and Toggweiler et al.
(1989a,b) and are indicated in Table 1. The model was integrated forward in time for 2628 years at the surface (7885 years at the bottom) by which time a steady state configuration had been reached. There were no trends in the salinity, temperature or overturning streamfunction fields. A new 19 level model (Experiment 2) was then initialized by interpolating vertically the steady state solution from Experiment 1 onto a finer grid. This new experiment was integrated forward in time for a further 1752 years at the surface (5257 years at the bottom) until a new steady state was reached. Once more there were no trends in the salinity, temperature or overturning streamfunction fields at this time. The same interpolation procedure was then used to initialize a 33 level model with the steady state data from the 19 level model. This very fine vertical resolution model (Experiment 3) was integrated for the same amount of time as Experiment 2. A steady state was once more reached.

The depths of each level used in the three experiments are shown in Table 1, while the steady state vertical distribution of the mean temperature and salinity are shown in Table 2. Table 3 gives the length of integrations for the three experiments.

3. Discussion

To illustrate the most dramatic effects of increased vertical resolution we examine the meridional overturning streamfunction, which is generally thought of as a manifestation of the thermohaline circulation. Consider the continuity equation in spherical coordinates,

$$u_\lambda + (v \cos \phi)_{\phi} + a \cos \phi w_\phi = 0,$$  \hspace{1cm} (3)

where $\lambda$ is longitude, $\phi$ is latitude and $a$ is the radius of the earth. The zonal integration of (3) from the western boundary $\lambda_w$ to the eastern boundary $\lambda_e$ gives

$$\int_{\lambda_w}^{\lambda_e} (v \cos \phi)_{\phi} d\lambda + a \cos \phi \int_{\lambda_w}^{\lambda_e} w_\phi d\lambda = 0,$$  \hspace{1cm} (4)

since $u = 0$ at $\lambda = \lambda_w$ and $\lambda = \lambda_e$. As (4) is non-

<table>
<thead>
<tr>
<th>Table 1. Vertical spacing used in the three experiments. All depths are in meters.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model layer</strong></td>
</tr>
<tr>
<td><strong>Depth of middle</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>14</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>17</td>
</tr>
<tr>
<td>18</td>
</tr>
<tr>
<td>19</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>21</td>
</tr>
<tr>
<td>22</td>
</tr>
<tr>
<td>23</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>26</td>
</tr>
<tr>
<td>27</td>
</tr>
<tr>
<td>28</td>
</tr>
<tr>
<td>29</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>31</td>
</tr>
<tr>
<td>32</td>
</tr>
<tr>
<td>33</td>
</tr>
</tbody>
</table>
Table 2. Steady state hemispheric mean temperatures and salinities at model layers for all three experiments. Refer to Table 1 for the depth of each layer.

<table>
<thead>
<tr>
<th>Model layer</th>
<th>Experiment 1 (12 levels)</th>
<th>Experiment 2 (19 levels)</th>
<th>Experiment 3 (33 levels)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean T (°C)</td>
<td>Mean S (ppt)</td>
<td>Mean T (°C)</td>
</tr>
<tr>
<td>1</td>
<td>18.578</td>
<td>34.675</td>
<td>18.576</td>
</tr>
<tr>
<td>2</td>
<td>16.963</td>
<td>34.758</td>
<td>17.152</td>
</tr>
<tr>
<td>3</td>
<td>14.481</td>
<td>34.700</td>
<td>15.359</td>
</tr>
<tr>
<td>4</td>
<td>11.702</td>
<td>34.470</td>
<td>13.617</td>
</tr>
<tr>
<td>5</td>
<td>8.043</td>
<td>34.105</td>
<td>11.516</td>
</tr>
<tr>
<td>6</td>
<td>4.620</td>
<td>33.777</td>
<td>8.978</td>
</tr>
<tr>
<td>7</td>
<td>3.017</td>
<td>33.629</td>
<td>6.429</td>
</tr>
<tr>
<td>8</td>
<td>2.503</td>
<td>33.577</td>
<td>4.446</td>
</tr>
<tr>
<td>9</td>
<td>2.432</td>
<td>33.570</td>
<td>3.319</td>
</tr>
<tr>
<td>10</td>
<td>2.425</td>
<td>33.570</td>
<td>2.830</td>
</tr>
<tr>
<td>11</td>
<td>2.422</td>
<td>33.570</td>
<td>2.367</td>
</tr>
<tr>
<td>12</td>
<td>2.416</td>
<td>33.572</td>
<td>2.565</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td>2.542</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td>2.533</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td>2.528</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td>2.525</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
<td>2.524</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td>2.523</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td>2.521</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basin mean</td>
<td>3.762</td>
<td>33.691</td>
<td>3.839</td>
</tr>
</tbody>
</table>

Divergent we may define the meridional overturning streamfunction $\Phi$ by

$$-\Phi_z = \int_{-\infty}^{\infty} va \cos \phi d\lambda,$$

where $v_{\infty}$ is the mean surface current, $a$ is the angular velocity of the Earth, $\cos \phi$ is the cosine of latitude, and $\Phi$ is the meridional overturning streamfunction. Figures 2a and 2b illustrate $\Phi$ midway through and at the end of Experiment 1, respectively. At the southern boundary deep water is formed through surface cooling and this water sinks to the bottom. Throughout most of the rest of the basin there is uniform weak upwelling. Towards the equator and near the surface Ekman pumping drives an additional cell. The very peculiar feature mentioned in the introduction is ap-

Table 3. Time in years of the model integration corresponding to the iteration counter ITT referred to in Figs. 2–4.

<table>
<thead>
<tr>
<th>Iteration counter</th>
<th>Experiment 1 (12 levels)</th>
<th>Experiment 2 (19 levels)</th>
<th>Experiment 3 (33 levels)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uppermost level</td>
<td>Lowest level</td>
<td>Uppermost level</td>
</tr>
<tr>
<td>120 000</td>
<td>876</td>
<td>2628</td>
<td>2738</td>
</tr>
<tr>
<td>280 000</td>
<td>2628</td>
<td>7885</td>
<td>4381</td>
</tr>
<tr>
<td>290 000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>440 000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>450 000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>600 000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 2. Meridional overturning streamfunction (Sverdrups; 1 Sv = 10^6 m^3 s^-1) for Experiment 1 after (a) 120 000 timesteps; (b) 280 000 timesteps. Refer to Table 3 for the corresponding time intervals of integration.

parent in the deeper levels near the equator. The counterclockwise cell develops around the 120 000 timestep (refer to Table 3) and intensifies until the steady state (Fig. 2b) is reached. In this steady state there is no longer any trend in any of the fields and the magnitude of the counterclockwise cell is about 40% that of the primary cell. It is interesting to note that Fig. 2a looks very similar to the solution of F. Bryan (1987) in his case with $A_{TV} = 0.1 \text{ cm}^2 \text{s}^{-1}$. In this case he initialized his model with the final solution obtained from his $A_{TV} = 0.5 \text{ cm}^2 \text{s}^{-1}$ run and ran it for a further 800 years. Above the strong counterclockwise equatorial cell are a series of much weaker cells with oscillating sense. Inspection of vertical velocity plots (not shown) indicates that the strongest spurious cell is driven by a spurious downwelling at the eastern boundary (as noted by F. Bryan 1987). At the eastern boundary and near the equator very unusual features are also observed in the temperature and salinity fields. There is an artificial
source of anomalous warm, saline water at a depth of about 1000 m and deeper down there is an artificial source of cold, less saline water, in violation of the second law of thermodynamics. The latter statement follows since the deep eastern equatorial waters are slightly cooler and fresher than the deep waters being formed at 70°S. This anomalous water then disperses across the basin in a ripple-like manner. Similar features were noticed by Gerdes et al. 1989.

In order to test that these artificial features are insensitive to the symmetric boundary condition that is used at the equator we conducted an additional experiment. In this experiment we flipped the steady state circulation of Experiment 1 about the equator and used this as an initial condition for a two hemisphere integration (i.e., 70°N to 70°S). This new experiment was run for a further 20,000 timesteps (209 years at the surface; 657 years at the bottom) with the same forcing (now symmetric about the equator). At the end of the experiment the meridional overturning had not changed from its initially symmetric condition. This leads us to the conclusion that the anomalous features are not due to the boundary condition at the equator.

To show that the equatorial cells are not physically real we ran the 19 level model of Experiment 3. Almost immediately (Fig. 3a) the strange features at the equator disappeared. The model took over 1000 years of integration to settle down (Table 3) to the steady state of Fig. 3b. The increased vertical resolution not only eliminated the spurious equatorial cells but also caused the basin mean temperature to increase somewhat (Table 2). Similarly, the primary overturning deep water formation cell increased in maximum strength from 13 Sv (Fig. 2b; Sv = 10^8 m^3 s^{-1}) to about 15 Sv (Fig. 3b). The increased vertical resolution (Table 1) also resolved the thermocline much better and the contours of meridional streamfunction are now generally far smoother (Fig. 3b). Both the spurious downwelling and the anomalies in the salinity and temperature fields, mentioned above, were eliminated.

To test that the steady state of Fig. 3b was insensitive to our initialization procedure (interpolation of the steady state solution of Experiment 1), an additional test experiment was run. In this experiment we once more used the interpolation scheme from the 12 to 19 levels but also added some significant initial noise into the T, S, u and v fields. Although there were very slight differences in the fields during the first 100 years of integration, there were no differences at all in the steady state solution.

The 19 level solution portrayed in Fig. 3b still is not entirely pleasing. One is left with the puzzling question as to what the weak cell in the deep waters near the equator is. The answer to this is once more insufficient vertical resolution. This conclusion is drawn from our 33 level run (Table 1; Experiment 3). Figure 4a shows how the aforementioned cell disappears almost immediately when increased vertical resolution is incorporated into Experiment 2. The final steady state meridional overturning streamfunction for Experiment 3 is illustrated in Fig. 4b. The field is now smooth everywhere. Once more the basin mean temperature has increased slightly while the strength of the primary cell has diminished somewhat to about 14 Sv.

Since one of the most important roles of the ocean in the global climate system is its poleward transport of heat, it is important to understand the possible effects of vertical resolution on this transport. Figure 5 illustrates the poleward heat transport at the steady state of all three experiments and also in the middle of Experiment 1. There is very little difference between the poleward heat transport midway through the 12 level run (○) and at the end of the higher resolution 19 (●) and 33 (▼) level runs, except near the equator where the artificial cells are forming. As time progresses, the poleward heat transport drops significantly in Experiment 1 over the middle latitudes (+). This follows since the effects of the growing equatorial computational mode disperse polewards and cause a significant change in the meridional overturning streamfunction. The increased resolution from 12 to 19 levels eliminates the oscillating surface cells and the corresponding poleward heat transport drops significantly near the equator. A further increase in vertical resolution has little effect on the poleward heat transport. Thus, sufficient resolution to eliminate the near-surface equatorial cells is important in obtaining realistic heat transports in midlatitude as well as equatorial regions.

In spatially centered difference numerical schemes, a condition which is sufficient to eliminate a two grid point computational mode is that the grid Peclet and Reynolds numbers be less than 2 (see the Appendix). In most modeling studies care is taken to satisfy the condition on the horizontal Reynolds number. Since there is a strong geostrophic coupling between the velocity and density fields at most latitudes, a computational mode in the density field tends to be suppressed by the lateral viscosity acting on the velocity field (Bryan et al. 1975). Thus, the horizontal diffusion coefficient can be much less than the value required for $A_{MH}$. However, little attention is paid to the conditions on the vertical Reynolds and Peclet numbers.

In our experiments the maximum horizontal grid spacing across $u$, $v$ ($T$, $S$) boxes was $\Delta_{MH}(\Delta_{TH}) = 220$ km, while the horizontal eddy viscosity (diffusivity) was constant at $A_{MH} = 5 \times 10^8$ cm^2 s^{-1} ($A_{TH} = 1 \times 10^7$ cm^2 s^{-1}). Thus, with a typical horizontal velocity scale of $U = 10$ cm s^{-1}, the horizontal grid Reynolds and Peclet numbers are:

$$Re_H = \frac{U\Delta_{MH}}{A_{MH}} = 0.44, \quad Pe_H = \frac{U\Delta_{TH}}{A_{TH}} = 22. \quad (7)$$

Thus, for all three experiments we satisfy the grid Reynolds number criterion on the horizontal viscosity coefficient. For reasons discussed above it is not nec-
necessary to satisfy the condition on the horizontal diffusion coefficient. 

In all three experiments the vertical eddy viscosity was set to $A_{MV} = 1.0 \text{ cm}^2 \text{ s}^{-1}$. If we take a typical vertical diffusivity at 2500 m to be $A_{TV} = 0.8 \text{ cm}^2 \text{ s}^{-1}$ [see Eq. (2)] and a vertical velocity of $4 \times 10^{-5} \text{ cm s}^{-1}$ (a typical interior value away from the deep water formation regions and the area of spurious eastern boundary downwelling near the equator), then the vertical grid Reynolds and Peclet numbers for the three experiments are,

$$\text{Re}_V = \frac{W \Delta V}{A_{MV}} = 3.2, \quad \text{Pe}_V = \frac{W \Delta V}{A_{TV}} = 4.0,$$

for Experiment 1, (8)

$$\text{Re}_V = 1.6, \quad \text{Pe}_V = 2.0, \quad \text{for Experiment 2}, \quad (9)$$

$$\text{Re}_V = 1.0, \quad \text{Pe}_V = 1.3, \quad \text{for Experiment 3}, \quad (10)$$

where we have used vertical grid spacing values of $\Delta V = 800 \text{ m}, 400 \text{ m}$ and $250 \text{ m}$, for Experiments 1, 2 and 3, respectively.

**Fig. 3.** Meridional overturning streamfunction (Sw) for Experiment 2 after (a) 290 000 timesteps; (b) 440 000 timesteps. Refer to Table 3 for the corresponding time intervals of integration.
Equations (8)–(10) therefore give us a possible explanation for the oddities which were observed at the equator. In Experiment 1 a two grid point computational mode was excited. In Experiment 2, with higher vertical resolution, this instability was largely suppressed although the Peclet number criterion was only just satisfied [Eq. (9)]. For the highest resolution run (Experiment 3) both the vertical grid Reynolds and Peclet number criteria were satisfied and no computational mode was excited. We do not understand what controls the vertical and horizontal scale of the deep spurious overturning cell. It must certainly be linked to the artificial sources in the salinity and temperature fields observed at the eastern equatorial boundary since these produce artificial overturning.

Sufficient vertical resolution has thus been shown to be crucial in obtaining a physically meaningful equatorial response in simulations using centered difference ocean general circulation models. It is suggested that future large scale modeling studies should either use increased vertical resolution, larger vertical diffusion and viscosity parameters (large vertical viscosity parameters were used by Bryan and Lewis 1979; Manabe and Stouffer 1988; Toggweiler et al. 1989a,b), or indeed
different numerical schemes (such as those described by Gerdes et al. 1989) in order to suppress computational modes observed in equatorial regions.

Acknowledgments. The authors are indebted to M. Cox at GFDL, Princeton for providing them with a copy of the GFDL Bryan–Cox OGCM. This research was supported by cooperative agreement number NA85ABH000031 of the Joint Institute for the Study of the Atmospheric and Ocean (University of Washington), by an NSERC postdoctoral fellowship awarded to A.J.W., and by a grant from the NOAA Office of Climate and Atmospheric Research to the University of Washington Experimental Climate Forecast Center. The calculations were done on the University of Washington’s IBM3090-200.

APPENDIX

The Criterion on the Grid Reynolds and Peclet Number

To derive the condition which is sufficient (but not necessary) to eliminate the two grid point computational mode which may be present when centered finite differences are used, we consider the steady state form of Burger’s equation:
$U\xi_u = A\xi_{uv}$. \hspace{1cm} (A1)

Following Bryan et al. (1975) and Chen (1971) we express the derivatives in (A1) using centered differences to get

$$(\sigma - 2)\xi_{j+1} + 4\xi_j - (\sigma + 2)\xi_{j-1} = 0, \hspace{1cm} (A2)$$

where $\sigma = U\Delta/A$. The general solution to (A2) can be written in the form

$$\xi_j = \alpha + \beta\tau^j, \hspace{1cm} (A3)$$

with $\alpha$ and $\beta$ constants. The substitution of (A3) into (A2) gives the quadratic

$$(\sigma - 2)\tau^2 + 4\tau - (\sigma + 2) = 0, \hspace{1cm} (A4)$$

with two roots

$$\tau = 1, \quad \text{or} \quad \tau = \frac{(2 + \sigma)}{(2 - \sigma)}. \hspace{1cm} (A5)$$

A computational mode then exists if $\tau$ is negative since the second term in (A3) oscillates with sign at adjacent grid points. Thus we require $\sigma < 2$ or

$$\frac{U\Delta}{A} < 2. \hspace{1cm} (A6)$$

Equation (A6) therefore states that both the horizontal and vertical grid Reynolds and Peclet numbers must be less than 2. That is,

$$\text{Re}_H = \frac{U\Delta_{MH}}{A_{MH}} < 2, \quad \text{Re}_V = \frac{W\Delta_V}{A_{MV}} < 2,$$

$$\text{Pe}_H = \frac{U\Delta_{TH}}{A_{TH}} < 2, \quad \text{Pe}_V = \frac{W\Delta_V}{A_{TV}} < 2, \hspace{1cm} (A7)$$

where $A_{MH}(A_{MV})$ and $A_{TH}(A_{TV})$ are the horizontal (vertical) eddy viscosities and diffusivities, respectively. Here $U(W)$ is a characteristic horizontal (vertical) velocity, $\Delta_{MH}(\Delta_{TH})$ is the horizontal grid spacing between velocity (tracer) points, and $\Delta_{V}$ is the grid spacing in the vertical.

REFERENCES


