On the Catalytic Role of High Baroclinic Modes in Eddy-driven Large-Scale Circulations

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ABSTRACT

This paper investigates the tridimensional consistency of the resolution of eddy scales in simulating large-scale flows. The generation of classical wind-forced, eddy-driven double-gyre circulation is investigated with a multilayered quasi-geostrophic model. Six-layers on the vertical have been chosen to assure the convergence of the baroclinic instability. Emphasis is on the resolution of the high baroclinic modes and its effects on the dynamics of the midlatitude jet. Several eddy-resolving experiments, identical except for the horizontal resolution, which can be low (20 km) or high (10 km), are compared. In every experiment, the scales associated with the first and second baroclinic modes are well resolved, but those associated with the third and higher baroclinic modes are so only in 10-km experiments. For a better illustration of the importance of the high vertical modes, three-layer experiments having configurations equivalent to that of the six-layer experiments have been conducted. Note that in the three-layer experiments, all the baroclinic radii of deformation that are explicitly present (the first and second) are well resolved with both 10-km or 20-km grid resolutions.

The major effect of fine resolution is to amplify the inertial mode and consequently increase the penetration scale of the midlatitude jet (by almost 40% in six-layer cases, and only 20% in three-layer cases), whereas the other indexes related to the large-scale flow are not modified by more than 10%. Our analyses show that fine (10-km) resolution produces a larger excitation of the inertial mode, and reverses the effect of numerical hyperviscosity on the stability of the jet stream, (less subgrid-scale dissipation yields a longer jet, the contrary happens when the resolution is 20-km). These latter effects are significantly more important when the vertical resolution is six-layered.

Global and local energetics indicate that the coupling between the layers is more efficient in high-resolution experiments, due to a better representation of the dynamics related to vortex stretching. Energy transfer rates show much larger amplitudes for the instability and rectification processes, which affect the large-scale flow toward a stronger jet and recirculation. Wavenumber spectra for all vertical modes show that when the resolution is six-layered, the fine resolution yields a better representation of the energy and enstrophy cascades.

The physics missing in 20-km grid simulations can be cast in terms of a scale-dependent eddy viscosity that is negative at large-scales. The analysis of the effects of negative viscosity demonstrates the dynamical impact of the resolution of the high vertical modes; the low-resolution experiment shows a deficit in negative viscosity at large wavenumbers, because the truncation of the 20 km grid interferes with the stratified inverse cascade. This artificially damps the large scales in the low-resolution experiments, and reduces the barotropic inertial circulation.

In conclusion, the high baroclinic modes appear to play a catalytic role in eddy-driven circulations: despite their low kinetic energy level, they are strongly involved in energy transfers and are essential pathways for determining the large-scale response of turbulent ocean models. However, our calculations show that if the fine horizontal resolution significantly modifies the quantitative impact of the second and third baroclinic modes on the mean circulation, the contribution of the fourth and fifth baroclinic modes remains negligible. Therefore, the tridimensional consistency of model resolution appears to be of crucial importance in the simulation eddy-driven large-scale flows, but the number of vertical modes that require a fine resolution seems to be limited.

1. Introduction

The ability of eddies to generate large-scale features in wind-driven ocean basin circulations was first demonstrated by Holland and Rhines (1980), and there has since been a large body of research on the dynamics of midlatitude jets and their recirculations. Marshall (1984) showed how the meridional transfer of vorticity,

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functional relations for the jet parameters. They also investigated the transition to barotropic instability.

Similar parameter studies are difficult to carry out with multilayered eddy-resolved general circulation models (EGCMs) because of the long integration time they require to reach a statistically steady state and the wide range of parameters that control the flow. Holland and Schmitz (1985) examined the physical processes that govern the zonal penetration of midlatitude jets in quasi-geostrophic numerical models. They demonstrated that a balance exists between the instability processes that restrict the penetration of the current, and the inertial processes that favor the zonal extension of the jet across the basin. They concluded that all stabilizing and destabilizing effects (vertical resolution, stratification, friction, forcing) act together to give a penetration scale peculiar to each experiment. The importance of horizontal resolution was not examined in this study.

A major problem specific to stratified flow is yet to be investigated: the global consistency between vertical and horizontal resolutions of quasi-geostrophic flows. Indeed, stratification introduces short horizontal scales that are linked to the internal radii of deformation; their dynamical importance has still to be quantified. For a given stratification, consistency should require that the three-dimensional grid resolves all scales that are quantitatively important to the dynamics, e.g., those which participate quantitatively to fluxes of energy and enstrophy within the stratified flow. This issue has been partially investigated in the context of stratified quasi-geostrophic turbulence by Hua and Haidvogel (1986). Their numerical experiments confirmed Charney's (1971) conjecture of the possibility of three-dimensional isotropization of the flow, once the vertical coordinate is rescaled by the scale of the stratification. This can be restated in a slightly different manner: the spatial spectrum of the stratified flow was shown, in Hua and Haidvogel (1986), to be a function only of a three-dimensional wavenumber \( K \) such that

\[
K^2 = k_h^2 + k_n^2
\]

where \( k_h \) is the horizontal wavenumber and \( k_n \) is the inverse of the radius of deformation of the \( n \)th vertical mode under consideration. Therefore, one possible way to obtain a three-dimensional consistent resolution is to require that three-dimensional scales that are within a factor of seven of the spectral peak of the flow be resolved by the three-dimensional grid. For the purpose of illustration, let us consider the case where the major process which produces turbulence is baroclinic instability. In that case the spectral peak is close to \( k_h = 1/\sqrt{2} k_1 \) where \( k_1 \) is the inverse of the first baroclinic radius of deformation. Therefore, the three-dimensional grid should resolve three-dimensional wavenumbers which are seven times larger than the spectral peak, namely \( K_{\text{max}}^2 = (49/2) k_1^2 \), i.e., it should resolve purely barotropic scales with \( k_n^2 = (49/2) k_1^2 \), or purely vertical scales with \( k_n = 0 \) and \( k_n = (49/2) k_1 \sim 5k_1 \). Both criteria are satisfied with the six-layer quasi-geostrophic model we use in the present study, where the highest baroclinic mode is the fifth one (and \( k_5 \approx 5k_1 \), see Table 1), if the horizontal grid is such that it resolves the fifth baroclinic radius of deformation.

Vallis and Hua (1988) investigated the importance of model resolution on the spectral distribution of explicit eddy viscosity. They showed quantitatively how the missing subgrid scales of coarse-resolution runs could seriously affect the inverse energy cascade, thereby modifying the largest scales of the flow.

In the present study, we reconsider the double-gyre problem with a multilayered, quasi-geostrophic EGCM forced by the classical antisymmetrical steady wind, with an emphasis on the resolution of the high baroclinic modes and its effects on the dynamics of the detached jet and eddy circulation. With this objective, several flat-bottom double-gyre, quasi-geostrophic EGCM simulations having different vertical and horizontal resolutions will be compared. The basic vertical resolution of the model has six-layers, and has been chosen since such a number appears to be necessary for the convergence of the baroclinic instability of a mean shear (Hua and Haidvogel 1986). Two horizontal grid resolutions, 20 and 10 km, are used to investigate the importance of the resolution of the high baroclinic modes, which are characterized by small radii of deformation. However, for a better illustration of the importance of the vertical resolution (henceforth the role of the highest baroclinic modes), every six-layer experiment has been redone in a three-layer configuration.

2. Model and parameters

a. Model

The vertical structure and exponential vertical density profile of the model in its six-layer configuration are shown in Fig. 1a. The depth of the six-layers, the jump in density at every interface, and corresponding internal radii of deformation are shown in Table 1. All physical and numerical parameters that define the principal numerical experiments discussed in the present study are also given. The values of the reduced gravity are such that the base of the main thermocline lies at the third interface, at the depth of 1050 m. The vertical eigenmodes, \( F_i(z) \), associated with the stratification are shown in Fig. 1b. Mode 1 is the barotropic one. The first three baroclinic modes (modes 2 to 4) are surface intensified. Note that mode 4 is maximum at the third interface where the first baroclinic mode is almost zero. Modes 5 and 6 may have large amplitudes under the main thermocline.

The three-layer configuration of the model has been defined to be as equivalent as possible to the six-layer
second internal radii of deformation are identical to those of the six-layer case (see Table 1). The vertical eigenmodes associated with the three-layer configuration are shown in Fig. 1c. Mode 1 is the barotropic one. The first two baroclinic modes (modes 2 to 3) are surface intensified and show a vertical structure very similar to their homologues of the six-layer model.

The governing equations are the quasi-geostrophic, nonlinear potential vorticity equations for every layer:

\[
\frac{\partial}{\partial t} \nabla^2 \psi_i = J(\nabla^2 \psi_i + f, \psi_i) + f_0 (w_{i+1/2} - w_{i+1/2}) / H_i
\]

\[
- A_s \nabla^2 \psi_i - \delta_{i,N} R \nabla^2 \psi_i + \delta_{i,1} \text{curl}\tau / H_i,
\]

\( i = 1, N \) layers, \( N = 3 \) or 6 \hspace{1cm} (1a)

coupled with the continuity equation applied at every interface,

\[
\frac{\partial}{\partial t} (\psi_{i+1} - \psi_i) = J(\psi_{i+1} - \psi_i, \psi_{i+1})
\]

\[
+ g'_{i+1/2} w_{i+1/2} / f_0, \hspace{1cm} i = 1, N - 1 \) interfaces. \hspace{1cm} (1b)

The notations are as follows: \( H_i \) is the constant thickness of \( i \)th layer \( (i = 1, N) \); \( \psi_i \) are the streamfunctions in the various layers shown in Fig. 1. At the interface separating layers \( i \) and \( i + 1 \), the vertical velocity is \( w_{i+1/2} \), the streamfunction is defined by \( \psi_{i+1/2} = (H_{i+1} \psi_i + H_i \psi_{i+1}) / (H_i + H_{i+1}) \), and the deviation of the interface, positive upward, is \( h_{i+1/2} = f_0 (\psi_{i+1} - \psi_i) / g'_{i+1/2} \), with \( g'_{i+1/2} \) being the reduced gravity. In each layer, vorticity is dissipated by biharmonic lateral friction (or hyperviscosity) with coefficient \( A_s \). Energy is dissipated in the lower layer by linear bottom friction with coefficient \( R \); \( J \) is the jacobian operator; \( f = f_0 + 2\beta(y - y_0) \) is the variable coriolis parameter \( (y_0 \) refers to the mid-latitude of the basin); and \( \tau \) is the wind stress.

Equations (1) are finite-difference integrated on a rectangular basin whose horizontal and meridional dimensions are \( L_x = 3600 \) km and \( L_y = 3200 \) km, respectively. Slip boundary conditions are used. The forcing is a steady sinusoidal wind stress \( (\tau_0 = 0.60 \text{ dyn cm}^{-2}) \) and drives a two-gyre ocean. The numerical code is from Holland, and details of the model formulation and the integration of the equations can be found in Schmitz and Holland (1986).

b. Experiments

We ran several six-layer experiments with different horizontal resolutions in order to investigate the effect of resolution of the high baroclinic modes on large-scale circulation.

The reference experiment is such that the six-layer, quasi-geostrophic model is integrated on a coarse horizontal grid of 20-km resolution, in the basic config-
Table 1. Model parameters of all experiments with results on the jet penetration scale.

<table>
<thead>
<tr>
<th>For every experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basin size: Zonal $L_x = 3600$ km</td>
</tr>
<tr>
<td>Coriolis parameter: $f_0 = 9.3 \times 10^{-5}$ s$^{-1}$</td>
</tr>
<tr>
<td>Bottom friction: $R = 10^{-7}$ s$^{-1}$</td>
</tr>
<tr>
<td>Meridional $L_y = 3200$ km</td>
</tr>
<tr>
<td>$\beta = 2 \times 10^{-11}$ m$^{-1}$ s$^{-1}$</td>
</tr>
<tr>
<td>Steady wind stress: $\tau_0 = 0.60 \times 10^{-4}$ m$^2$ s$^{-2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For every six-layer experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stratification (layer number):</td>
</tr>
<tr>
<td>Layer thickness (m):</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>Reduced gravity (10$^{-3}$ m s$^{-2}$):</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>Radius of deformation (km):</td>
</tr>
<tr>
<td>38.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For every three-layer experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stratification (layer number):</td>
</tr>
<tr>
<td>Layer thickness (m):</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>Reduced gravity (10$^{-3}$ m s$^{-2}$):</td>
</tr>
<tr>
<td>16.2</td>
</tr>
<tr>
<td>Radius of deformation (km):</td>
</tr>
<tr>
<td>38.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference of experiment</td>
</tr>
<tr>
<td>6HI10</td>
</tr>
<tr>
<td>6LO2</td>
</tr>
<tr>
<td>3HI10</td>
</tr>
<tr>
<td>3LO20</td>
</tr>
<tr>
<td>6HI10-2</td>
</tr>
<tr>
<td>3HI10-2</td>
</tr>
<tr>
<td>3LO20-2</td>
</tr>
</tbody>
</table>

The coefficient of hyperviscosity, $A_4 = 4 \times 10^{10}$ m$^4$ s$^{-1}$, has been chosen to get a midlatitude jet that reaches the middle of the basin. Note that the 20-km grid is of the order of the second baroclinic radius of deformation (18.7 km), and that the characteristic length scales of modes 4 to 6 are not well resolved. This experiment is referred to as 6LO2; 6 stands for the number of layers, LO for low-resolution, and 20 for 20-km grid).

Six-layer, high-resolution experiments, with a horizontal grid of 10 km, close to the characteristic scale of mode 6 (9.2 km), have been run. The dynamics and interactions between all modes should be correctly resolved with such a grid.

The first high-resolution experiment is referred to as 6HI10; 6 stands for the number of layers, HI for high-resolution, and 10 for 10-km grid. The model is in exactly the same configuration as for 6LO2, except for a fine horizontal grid of 10 km. In particular the coefficient of hyperviscosity is not changed ($A_4 = 4 \times 10^{10}$ m$^4$ s$^{-1}$). Experiment 6HI10 will be used as the reference for the high-resolution cases throughout the study, and is extensively compared with the low-resolution experiment 6LO20 because as discussed below, those two experiments are in identical frictional regimes.

We ran a second high-resolution experiment (referred to as 6HI10-2) in which the model is in a configuration similar to 6HI10, except for a biharmonic friction coefficient reduced by a factor of 16 ($A_4 = 0.25 \times 10^{10}$ m$^4$ s$^{-1}$). Since biharmonic friction is scale-selective (the damping time-scale increases as $k^4$), it is necessary in 10-km experiments to reduce the friction coefficient by a factor $2^4$ in order to obtain a subgrid scale dissipation equivalent to that of the reference 20-km experiment. Surprisingly, the midlatitude jet showed in that case an increase in zonal penetration that is even larger than in more viscous cases, which suggests a significant modification of the effects of the lateral friction in the high-resolution cases.

Three-layer experiments have also been run, with configurations similar to the six-layer experiments we have described above. Therefore, experiments 3LO20, 3HI10, and 3HI10-2 are the three-layer homologues of the six-layer experiments 6LO20, 6HI10, and 6HI10-2, respectively (see Table 1). We shall also refer to a 20-km resolution, three-layer experiment 3LO20-2 having a reduced lateral friction ($A_4 = 0.25 \times 10^{10}$ m$^4$ s$^{-1}$). Note that in three-layer experiments, all the radii of deformations explicitly present are well resolved with both 10-km or 20-km grid resolution.

Every experiment needs about 15 years of integration (with a time step of two hours for 20-km resolution, reduced to one hour for 10-km resolution) to reach a state of statistical equilibrium. For each case, a further 2500-day run then is conducted, from which the sta-
tistics of the experiments are calculated. Therefore, in the following, mean quantities are time-averaged over 2500 days. The results of the experiments are described in section 3. Diagnostic comparisons are presented in section 4. They identify the significant contribution of the high baroclinic modes in the establishment of largescale circulation and demonstrate the importance of a consistent tridimensional resolution in eddy-resolving ocean modeling. The computational cost of one six-layer, fine-resolution experiment is 80 hours of CRAY-2 cpu.

3. Results of the simulations

a. Mean flow

For both low-resolution and high-resolution cases, in three- or six-layer configurations, the mean flows exhibit the robust features characteristic of this type of wind-driven model, and found over a wide range of model parameters (Holland 1985). The mean streamfunctions for layers 1, 3, and 6 of experiments 6LO20 and 6HI10, which are shown in Fig. 2, are a good illustration.

The upper-layer mean flow presents the usual double gyre pattern. The interior Sverdrup circulation feeds a narrow western boundary current which, at middle latitude, turns into a strong eastward jet, penetrating towards the middle of the basin along the line of zero wind stress curl. The flow exhibits an inertial recirculation of limited meridional extent, and the jet weakens to the east to join onto the interior circulation. Therefore, the circulation has two meridional characteristic length scales, a tight recirculation (inertial) scale and a broader, basin-sized, wind-driven gyre scale. At middepths, the flow is constrained by the upper layer circulation and still exhibits a double-gyre pattern (with a smaller meridional extent) with a thin eastward jet and a tight recirculation. In the bottom layers, the double-gyre pattern is meridionally confined to the tight inertial recirculation and is a signature of the barotropic component of the flow. The flow patterns which characterize the deep circulation are the two counter-rotating eddy-driven gyres to the north and the south of the main inertial gyres.

Before we begin to investigate the effect of the grid resolution, let us look briefly at the vertical resolution, and compare the mean flow of the reference six-layer experiment 6LO20 with its three-layer homologue 3LO20. The upper-layer mean flow of 6LO20 is shown in Fig. 2a and is almost identical to that of 3LO20 (not shown). Both show an identical length for the midlatitude jet \( (L_p = 1800 \text{ km}) \) (Table 1), a mean velocity in the jet of 98 cm s\(^{-1}\), and a mean transport slightly above 40 Sv \( (Sv = 10^8 \text{ m}^3 \text{ s}^{-1}) \) (Table 2). A comparison layer-by-layer is not obvious for the lower layers because of different vertical resolutions. It is thus more informative to consider vertical modes. Table 3 shows that in these two experiments, the first and second baroclinic modes are almost identical in amplitude and total kinetic energy. Besides the absence of the high baroclinic modes in 3LO20, the most significant differences appear in the barotropic mode (Figs. 3a,b), where transport is almost 25% smaller in 3LO20 in the jet area (see Table 2). This latter remark holds in the comparison of high-resolution experiments 3HI10 and 6HI10 (their respective mean barotropic streamfunctions are shown in Figs. 3c,d), indicating that in a general way the increase in vertical resolution modifies the barotropic mode consequently more than the first two baroclinic modes (see Table 3). Note that when the resolution is 20 km, a larger number of vertical modes does not change the length of the jet, while it does so when the resolution is 10 km. The high baroclinic modes (3 to 6) thus appear to be of great importance in the establishment of the barotropic circulation.

Let us now investigate the effect of the horizontal grid resolution. The mean barotropic streamfunctions obtained in our experiments are displayed in Figs. 3. They show that the eastward penetration of the midlatitude jet is drastically increased by the fine resolution, and that this effect is even larger as the number of vertical modes is higher. The plots of Fig. 2 illustrate a remarkable difference in the penetration scale of the jet between 6LO20 and 6HI10. We define this scale, \( L_p \), as the distance from the western boundary where the detached jet joins onto the interior Sverdrup circulation. This length is taken as the zero contour of the inertial circulation in the bottom layer. Table 1 shows the penetration scales we obtained in the different experiments: \( L_p \) is almost 40% larger in 6HI10 than in 6LO20. In homologous three-layer experiments (3HI10 and 3LO20), the increase of \( L_p \) due to the increase in horizontal resolution is near 20%. The fact that this increase is twice larger when the vertical resolution is six-layered, indicates that a grid-resolution that is consistent with the horizontal scales of the high baroclinic modes produces a more inertial circulation. Tables 1, 2, and 3 summarize the characteristics of the large-scale circulation obtained in our experiments. Except for the penetration scale and the amplitude and energy of the barotropic mode, all other indexes related to the large-scale mean flow are not modified by more than 10% by the change in horizontal resolution. However, it is worth mentioning that the mean transport is larger at all levels in 20-km experiments, and that the maximum velocity in the jet is larger in the surface layers in 10-km experiments. Therefore, the horizontal and vertical shear of the jet are larger in 10-km experiments. The width of the jet (250 km) and the inertial meridional lengthscale (280 km) are identical in all experiments.

To sort out the effects of dissipation and resolution changes, we now consider experiments with lower fric-
Fig. 2. The time-mean streamfunctions for layer 1, 3 and 6. (a) Case 6LO20. (b) Case 6HI10. Full (dashed) contour lines indicate positive or zero (negative) values. Units are in m² s⁻¹. Contour interval is 10,000 for layer 1, 6,000 for layer 3 and 3,000 for layer 6. The penetration scale of the jet $L_p$ is 1,800 km in 6LO20, and 2,500 km in 6HI10.
Table 2. Mean flow characteristics in the different experiments.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Barotropic</th>
</tr>
</thead>
<tbody>
<tr>
<td>6LO20</td>
<td>98.0</td>
<td>60.0</td>
<td>38.0</td>
<td>26.0</td>
<td>16.1</td>
<td>14.8</td>
<td>25.3</td>
</tr>
<tr>
<td>6HI10</td>
<td>110.0</td>
<td>65.0</td>
<td>40.0</td>
<td>26.5</td>
<td>16.1</td>
<td>14.2</td>
<td>27.3</td>
</tr>
<tr>
<td>3LO20</td>
<td>98.0</td>
<td>45.0</td>
<td>12.0</td>
<td></td>
<td></td>
<td></td>
<td>21.8</td>
</tr>
<tr>
<td>3HI10</td>
<td>115.0</td>
<td>50.0</td>
<td>12.7</td>
<td></td>
<td></td>
<td></td>
<td>24.2</td>
</tr>
</tbody>
</table>

Mean transport (10^9 m^3 s^-1) in the midlatitude jet

<table>
<thead>
<tr>
<th>Experiment</th>
<th>43</th>
<th>33</th>
<th>25</th>
<th>22</th>
<th>39</th>
<th>55</th>
<th>217</th>
</tr>
</thead>
<tbody>
<tr>
<td>6LO20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6HI10</td>
<td>41</td>
<td>31</td>
<td>23</td>
<td>20</td>
<td>35</td>
<td>49</td>
<td>199</td>
</tr>
<tr>
<td>3LO20</td>
<td>42</td>
<td>49</td>
<td>76</td>
<td></td>
<td></td>
<td></td>
<td>167</td>
</tr>
<tr>
<td>3HI10</td>
<td>40</td>
<td>44</td>
<td>72</td>
<td></td>
<td></td>
<td></td>
<td>156</td>
</tr>
</tbody>
</table>

The biharmonic friction coefficient is reduced by a factor of 16 to obtain a subgrid scale dissipation equivalent to that of the 20-km experiments discussed above. The general belief in this type of numerical dissipation is that a reduction in hyperviscosity destabilizes the jet with regard to barotropic and baroclinic instabilities, which results in a shorter jet (Holland and Schmitz 1985). This is indeed what happens in 20-km experiments; for example, experiment 3LO20-2, which is similar to 3LO20 except for a lower friction, has a penetration scale reduced by 30% (Table 1).

In the parameter range used in the present study, this later effect appears to be quite different in high-resolution cases. In the three-layer configuration, the reduction of the biharmonic friction seems to have little effect on the jet penetration; Lp is almost identical in 3HI10 and 3HI10-2. In the six-layer configuration the effect is reversed and the penetration scale is increased by the reduction of the friction coefficient; Lp is almost 10% larger in 6HI10-2 than it is in 6HI10 (see Table 1 and Fig. 3).

To conclude on mean flow, it appears that modes 2 and 3 (first and second baroclinic modes) are rather identical in all experiments, (the larger values found in 10-km experiments for the basin-averaged mean kinetic energy must be weighted by the length of the jet). This indicates that they are correctly calculated by the different horizontal and vertical resolutions used in the different experiments, and that the presence of the higher baroclinic modes (modes 4 to 6) has little influence on the mean baroclinic circulation related to these modes. On the contrary, it appears that the high baroclinic modes are important to the mean barotropic circulation, the barotropic mode being always smaller (in amplitude, transport, and kinetic energy) in three-layer experiments.

The combined effects of the vertical and horizontal resolution on the mean circulation are summarized in Table 3. Comparison of 3HI10 with 3LO20 shows that the increase in horizontal resolution (from 20 to 10 km) when the vertical resolution is three-layer, only produces a slight growth of the mean kinetic energy of the barotropic mode (30%), which is certainly related to a more inertial jet, since the jet is longer and the amplitude of the mode is not changed. Comparison between 3LO20 and 6LO20 indicates that an increase in vertical resolution (from three to six layers), when the horizontal resolution is 20 km (and does not resolve all the scales introduced by high baroclinic modes), also produces a larger mean barotropic mode (larger amplitude and 45% larger kinetic energy), but seems to have little influence on the penetration scale of the jet, which is not changed. Modes 2 and 3 are correctly calculated by the 20-km resolution and are almost unchanged. But the comparison between 3HI10 and 6HI10 indicates that, when the horizontal resolution is 10 km (and thus resolves all the characteristic length scales), an increase in the number of vertical modes drastically increases the barotropic mean circulation, (longer jet, larger barotropic mode, and 135% larger mean kinetic energy). This demonstrates the importance of the contribution of the higher modes in determining the mean barotropic circulation and the penetration of the jet.

b. Transients in six-layer experiments

The vertical distribution of the mean kinetic energy (MKE) and eddy kinetic energy (EKE) in the axis of the midlatitude jet is shown in Fig. 4. Those plots illustrate fairly well the difference in the penetration scale. For example, in 6HI10, eddy energy levels are significant (8 cm^2 s^-2) in the eastern limit of the basin due to the longer jet. Larger MKE values in the surface layers in the 10-km resolution are simply due to the

Table 3. Mean flow characteristics of the vertical modes F(z) in the different experiments.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>3LO20</th>
<th>3HI10</th>
<th>6LO20</th>
<th>6HI10</th>
<th>6HI10-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum amplitude of the mean streamfunction in the southern gyre (m^2 s^-1)</td>
<td></td>
<td></td>
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larger mean velocities at the surface (see Table 2). The EKE values are somewhat equivalent throughout the domain, indicating similar eddy activity in all experiments. The distribution of EKE shows oscillations related to the inertial and barotropic nature of the mid-latitude jet.

FIG. 3. The mean barotropic streamfunction for several experiments listed in Table 1. Contour interval is 3000 m$^2$ s$^{-1}$ for all plots.
The instantaneous streamfunctions displayed in Fig. 5 also reveal interesting features. In 6HI10, besides the increased length of the free jet, which is the major characteristic of the high-resolution cases, the jet is straighter and less distorted, with less meandering activity, than in 6LO20. In both experiments, formation of mesoscale rings by pinch-off of a meander has been observed (Fig. 5a), but most eddies are generated in the final part of the jet (Fig. 5b) and in the subgyre recirculation. This is consistent with an inertial jetstream as described by Le Provost and Veron (1987) for the barotropic model: the destabilization of the jet, in the case of strongly nonlinear flows, occurs in the recirculation, whereas in the case of weakly nonlinear flows, it is the meanders of the jet that become unstable and might result in a situation more favorable to ring generation by pinch-off. In 6HI10 the mesoscale rings are more sharply delineated and their lifetime is several times longer than in 6LO20. This is because when the resolution is low, the eddies are more often absorbed by the meandering of the jet. We thus expect the eddy-mean flow interaction to be quite different in both experiments.

Phase diagrams of the streamfunction field along the axis of the free jet are shown in Fig. 6. They reveal a bimodal distribution of the jet instabilities. We can see that during long periods the jet is rather calm in the first half, showing standing inertial oscillations. This occurs, for example, in 6HI10 approximately between day 0 and day 150 and between day 300 and day 480. It is remarkable that in both low- and high-resolution experiments these oscillations have the same wavelength (about 700 km) and present the same barotropic nature. In contrast, there are periods where the jet is rather unsteady, with propagating meanders that often result in the pinch-off of a mesoscale ring near the end of the jet. This happens in 6HI10 between day 450 and day 550 and in 6LO20 between day 1 and day 60. Such signatures in the phase diagrams are related to an eastward sloping of the contours. This leads to a ring generation, which can be followed in the sequences chosen in Fig. 5.

This section has demonstrated that the tridimensional resolution is of fundamental importance in the excitation of the large scale barotropic mode. In particular, the high baroclinic modes contribute significantly to the building of the amplitude of the external mode, and the horizontal resolution used to compute those internal modes is essential in the determination of the penetration of the barotropic jet. In the next section, we investigate the physical processes which are the most improved by this increase in resolution and can be responsible for strengthening the inertia of the flow.
Fig. 5. Instantaneous pictures of the upper layer streamfunction (three plots at eight-day intervals).
(a) Case 6L020 (204 km resolution). (b) Case 6H110 (181 km resolution). The contour interval is 15,000 m$^2$ s$^{-1}$. 
Fig. 6. Time evolution of zonal sections ($x-t$ plot) of the upper and lower layer streamfunctions. (a) Case 6LO20 (20 km) and (b) case 6HI10 (10 km). Contour interval is 15 000 m$^2$ s$^{-1}$ in layer 1, and 6000 m$^2$ s$^{-1}$ in layer 6. The vertical lines show the inertial wavelength.

4. Diagnostics comparisons

a. Inertia of the mean jet

As noted for the phase diagrams of Fig. 6, there are, both in 6LO20 and 6HI10, long periods during which the midlatitude jet shows standing barotropic oscillations. Another illustration of these oscillations can be obtained by computing the work done by the Reynolds stresses, which is a conversion between the mean and the eddy flows:

$$H_t \tilde{\psi} J(\nabla^2 \tilde{\psi}, \tilde{\psi}).$$

We consider this term a conversion term because it appears with the opposite sign in the two pointwise
equations for the kinetic energy of the mean flow and the eddy flow (Barnier 1986). The overbar denotes a time averaging over 2500 days, and the prime denotes perturbation from the mean. In Fig. 7 this quantity has been spatially averaged over the width of the jet (whose meridional structure is displayed in Fig. 10c) and is plotted as a function of the zonal coordinate. It is clear that the jet is a place where strong, standing barotropic waves are present, and the wavelength and the number of oscillations are independent of the resolution. Note that the amplitudes of the energy transfers are much larger in 6H110. This was also noticed in three-layer experiments and will be discussed below, but it already indicates that the energy transfer rates are considerably increased in high-resolution cases. Note that in the bottom layers (Fig. 7b) the work done by the Reynolds stresses is always negative in 6H110 in the second half of the jet, thus indicating a strong rectification of the mean flow there. This feature is characteristic of the 10-km resolution, since it is found in 3H110 but not in 6LO20 and 3LO20.

In the case of a barotropic steady jet, the wavelength of the inertial oscillations can be estimated by

\[ \lambda_i = 2\pi(U_i/\beta)^{1/2}, \]

with \( U_i \) being the velocity of the jet, and the number of oscillations is a function of the balance between the advection of relative vorticity and the diffusive processes (Pedlosky 1987). Here, for \( U_i \) we use the maximum barotropic velocity given in Table 2 and obtain a value of \( \lambda_i = 700 \text{ km} \) in 6LO20 and \( \lambda_i = 720 \text{ km} \) in 6H110. This scale is reported on the plots of Figs. 6 and 7 and is consistent with the wavelength exhibited by the observed oscillations. This is a strong indication that the jet is very inertial. The same comment applies to three-layer experiments, with inertial wavelengths close to 700 km.

Therefore, in both low- and high-resolution experiments, the nonlinear inertial mode is largely excited, but it penetrates farther east in 6H110. In our experiments, the horizontal scale associated with the biharmonic friction is

\[ \delta_M = (A_4/\beta)^{1/5} = 18 \times 10^3 \text{ m} \]

and must be compared to the inertial length scale for 6H110

\[ \delta_i = (U_i/\beta)^{1/2} = 117 \times 10^3 \text{ m}. \]

The ratio \( \delta_i/\delta_M \) which, in the steady theory, determines the number of oscillations of the inertial mode, is about six, which is larger than the three oscillations observed in both low- and high-resolution experiments. This ratio is even larger if \( \delta_M \) is calculated using bottom friction. Consequently the number of oscillations and the penetration of the jet in our simulations must be controlled by a damping mechanism different from biharmonic or bottom friction (namely the mesoscale eddies).

b. Energetics

1) Basin average

In numerical ocean simulations performed in a closed domain, basin-averaged energy diagrams are commonly used to synthesize the mean and eddy energy levels in the different layers and the vertical and horizontal transfer rates between the different energy
components, which are the mean kinetic (MKE) and mean potential (MPE) energies and the eddy kinetic (EKE) and eddy potential (EPE) energies (Holland 1978). In a steady state, the input and output for each type of energy cancel out except for small discrepancies due to a finite sample time, here 2500 days.

Such a box diagram, comparing three-layer experiments 3LO20 and 3HI10, is displayed in Fig. 8. For six-layer experiments, this type of diagram has 22 boxes, which makes it quite complicated to understand. Thus, we present diagrams, which synthesize the results in a form similar to that of three-layer experiments by regrouping several layers together. To make the six-layer diagrams comparable to a three-layer one, the six-layers have been grouped as follows: Layer 1 alone, layer 2 with layer 3, and layers 4, 5, and 6 all together, since this combination is the most consistent with the three-layer stratification. A diagram comparing 6LO20 and 6HI10 is shown in Fig. 9.

Several features common to Fig. 8 and Fig. 9 are worth noting: MKE and MPE values are always larger at every level in the high-resolution experiment (when compared to the equivalent low-resolution experiment). If the slightly larger mean velocities observed in the jet (Table 2) contribute to that difference, the other significant contribution is from the larger penetration of the jet. Inertial gyres have a larger zonal ex-

![Diagram](image)

**FIG. 8.** Basin-averaged energy diagrams for three layer experiments 3LO20 (bold), and 3HI10 (italic). Units are in m (m² s⁻²) for the energies (boxes) and in 10⁻⁴ m² s⁻² for the transfer rates (arrows). Negative numbers indicate transfers opposite to the direction of the arrows. All quantities are basin-averaged, mean (i.e., averaged over 2500 days) quantities. MKE, and EKE, stand for kinetic energy of the mean and the eddy flow in layer i; MPE, and EPE, stand for potential energy of the mean and eddy flow at interface i, respectively.

![Diagram](image)

**FIG. 9.** Basin-averaged energy diagrams for six layer experiments 6LO20 (bold), and 6HI10 (italic). Units are in m (m² s⁻²) for the energies (boxes) and in 10⁻⁴ m² s⁻² for the transfer rates (arrows). Negative numbers indicate transfers opposite to the direction of the arrows. All quantities are basin-averaged, mean (i.e., averaged over 2500 days) quantities. MKE, and EKE, stand for kinetic energy of the mean and the eddy flow in layer i; MPE, and EPE, stand for potential energy of the mean and eddy flow at interface i and j, respectively.

tent and therefore increase the basin-averaged value of the mean energies. EPE and EKE values are quite similar with both 10-km or 20-km resolution, indicating equivalent basin-averaged eddy activity. The dissipation of energy by bottom and lateral biharmonic friction is not changed by the resolution (see also Table 4). Therefore, experiments 6LO20 and 6HI10 have similar frictional regimes, the input energy being dissipated at 55% by the bottom friction and at 45% by the biharmonic friction.

Most interesting are the modifications of the energy transfer rates relative to the Reynolds stress work (barotropic transfer MKE to EKE) and buoyancy work (baroclinic transfer MPE to EPE). Those transfers are in proportion modified more by the increase in resolution when the vertical resolution has six modes. For example, transfer MPE₁ to EPE₁ is only 8% larger between 3LO20 and 3HI10, whereas it increases by more than 60% between 6LO20 and 6HI10.

Experiment 6HI10 reveals, on the one hand, significantly larger baroclinic transfers at all levels (0.38 in the upper layers compared with 0.23 for 6LO20) and, on the other hand, reduced barotropic transfers in the surface layers (0.71 against 0.80 in 6LO20). This is also found in three-layer experiments, but with a much smaller amplitude. Therefore, a tentative conclusion...
is that the fine grid allows the energy transfers related to the stretching of the interfaces (baroclinic transfers) to be better resolved, and consequently increased, and that the horizontal shear (barotropic) instabilities are reduced. The first inference seems acceptable and will be effectively demonstrated in the following subsections as a major improvement of fine resolution. However, the second inference regarding a possible inhibition of the barotropic energy transfers is surprising and questionable.

2) ZONAL AND MERIDIONAL AVERAGE

The diagrams of Fig. 8 and Fig. 9 can in fact be misleading, since they present energetics averaged over the whole basin, whereas dynamics in the model are far from being spatially homogeneous. It is therefore instructive to look at the spatial distribution of these transfer rates. As a result, we show in Figs. 10 and 11 the meridional structure of the zonally averaged baroclinic and barotropic energy transfer rates, at different layers and interfaces. To make these quantities comparable in both high- and low-resolution experiments, we need to compensate for the longer jet in the fine-resolution cases. Thus, transfers are multiplied by a dimensionless coefficient $\alpha$, which is $\alpha = 1$ in low resolution cases and $\alpha = L_p(\text{low})/L_p(\text{high})$ in high-resolution cases, the ratio of the penetration scale of the low resolution experiment to that of the high-resolution one. Note that in experiments with a long jet, this normalization reduces the values of the energy transfers outside the jet area since $\alpha < 1$. This has to be considered when analyzing the results.

Figure 10a displays the baroclinic transfer rates:

$$-\alpha f_0/g_{i+1} (\bar{\psi}_{i+1} - \bar{\psi}_i) \langle \nabla^2 \bar{\psi}, \bar{\psi} \rangle.$$  \hspace{1cm} (3)

This term is associated with eddy buoyancy fluxes and is a conversion between mean and eddy potential energies. Baroclinic instability (positive transfer) is concentrated on the two flanks of the eastward midlatitude jet and, to a lesser extent, at the inner edge of the return flows on each side of the jet. This is clearly shown by the double positive peak in the center of Fig. 10a for case 6HI10. This fine pattern is hardly noticeable in 6LO20 or in three-layer cases even with 10-km resolution (see Fig. 11), and is probably a contribution of the high baroclinic modes. Baroclinic instability is also significant in the surface layers in the westward large-scale Sverdrupian flow. The eddy-driven part of the circulation (negative transfers) clearly stands within the westward flow and at the outer edge of the return flow of the inner gyres. Figure 10a reveals that baroclinic instabilities (MPE to EPE) and baroclinic inverse transfers (EPE to MPE) are, in regard to the low-resolution case, almost doubled in the high-resolution experiment. This confirms the tendency shown by the box diagram of Fig. 9, and we suspect that the computation of the stretching of the interfaces, which involves all the baroclinic modes, is significantly improved by the fine resolution and results in more active transfer rates. This is true for modes 2 and 3, since the above remarks are also valid in three-layer experiments (see Fig. 11a), and must be true for modes 4, 5, and 6 in six-layer cases. Therefore, as it will be demonstrated in the next subsection, the nonlinear interactions involving the high baroclinic modes with the barotropic one must also have a larger amplitude in 6HI10 than in 6LO20, and they contribute to the amplification of the inertial mode.

Figure 10b shows the work done by the Reynolds stresses:

$$\alpha H \langle \bar{\psi}, \nabla^2 \bar{\psi} \rangle.$$  \hspace{1cm} (4)

Unlike the box diagrams of Fig. 9, which indicate a global reduction of the barotropic energy transfers in the high-resolution case, Fig. 10b shows that, in fact, the increased resolution leads to an amplification of the horizontal shear instabilities by a factor of two in the upper layers and a spectacular increase (by a factor of five) of the rectification processes in the lower layer in the region of the midlatitude jet. This latter feature is rather important since rectification helps to build the inertial barotropic recirculation. Again, the latter comments apply to three-layer experiments (Fig. 11b), but with a smaller amplitude.

Thus, the apparently equivalent barotropic transfer rates (MKE to EKE) given by the basin-averaged box diagrams for the upper layers are, in fact, overall figures from two opposite, very active pathways, one from the mean flow to the eddies and the other from the eddy flow to the mean, and they may balance each other out. In Fig. 9, the smaller values obtained in 6HI10 (as compared to 6LO20) for the work done by the Reynolds stresses in the upper layers are the result of more effective transfers from the eddies to the large-scale mean flow. The small negative values given for the barotropic energy transfer rates in the lower layers in 6HI10 (almost zero) are, in fact, the result of an even balance between the shear instability of the jet and a very effective rectification of the lower layer mean flow by the eddies, both processes having a much larger amplitude than in 6LO20. This analysis is also valid if the vertical resolution is three-layered but with less strength, which stresses the quantitatively important contribution of the high baroclinic modes.

As a result global and local energetics indicate that the coupling between the layers is more efficient in the high-resolution experiments, and this is certainly due to better representation of the dynamics related to the vorticity stretching. This means that with a 10-km resolution the horizontal scales associated with the vertical motions (i.e., the scales of the high baroclinic modes) are now better resolved, and consequently their contribution to the large scale circulation is modified. The much larger rectification means that the large-scale flow is affected, and the result is a stronger inertial jet.
Fig. 10. Six-layer experiments. Meridional distribution of the zonal average of the energy transfer rates normalized by the penetration scale (in km) of Table 2. Units are in $10^{-3}$ m (m$^2$ s$^{-2}$) s$^{-1}$. (a) Baroclinic transfer (from MPE to EPE) at the first and fourth interfaces. (b) Barotropic transfer (Reynolds stresses, from MKE to EKE) in the first and sixth layers. Full line is for 6LO20 and dashed line for 6HI10. (c) The meridional profile of the mean zonal velocity obtained in experiment 6HI10 in layer 1 and layer 6 at 720 km from the western boundary. Units are in m s$^{-1}$. 

penetration scale if the grid is 20 km, but slightly increases it when the resolution is 10 km (see Table 1). Quantitatively, high-resolution experiments appear to be less sensitive to changes in hyperviscosity coefficient. Globally, an increase of $A_4$ by a factor of two in 20-km experiments has the same effect on the jet penetration scale as a reduction of $A_4$ by a factor of 16 in 10-km experiments; $L_p$ increases by about 10 to 15%.

Figure 12 compares the basin-averaged energetics of 6HI10 and 6HI10-2, two high-resolution experiments which only differ by their lateral friction coefficient ($A_4$ is 16 times smaller in 6HI10-2, see Table 1). Obviously these experiments are in different frictional regimes: in 6HI10 the hyperviscosity dissipates 45% of the input energy, and this amount is only 15% in 6HI10-2 (see also Table 4). The differences in eddy energy levels (EKE and EPE are doubled in the low friction case) are simply due to the different frictional regimes. When the friction is reduced (case 6HI10-2), the eddies are hardly damped by the hyperviscosity, and they transfer most of their kinetic energy downward where it is dissipated by bottom friction. This results in larger eddy kinetic energy and larger downward energy transfer rates at all levels (Fig. 12). However, both experiments present similar mean kinetic energy (MKE) levels, and their penetration scales differ by less than 10% (Table 1). Since the barotropic inertial jet results from eddy dynamics, the fact that the mean flow is only slightly changed by the reduction of friction is an indication that in 10-km resolution the grid is fine enough so the

c. Lateral friction

As was mentioned in section 3a, the effects of the biharmonic friction on the midlatitude jet change according to the horizontal grid resolution. Qualitatively, a decrease in friction coefficient $A_4$ largely reduces the

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**Fig. 11.** Three-layer experiments. Meridional distribution of the zonal average of the energy transfer rates normalized by the penetration scale (in km) of Table 2. Units are in $10^{-5}$ m (m$^2$ s$^{-2}$) s$^{-1}$. (a) Baroclinic transfer (from MPE to EPE) at the first interface. (b) Barotropic transfer (Reynolds stress, from MKE to EKE) in the first layer. Full line is for 3LO20 and dashed line for 3HI10.

**Fig. 12.** Basin-averaged energy diagrams for six layer experiments 6HI10 (italics), and 6HI10-2 (bold). Units are in m (m$^2$ s$^{-2}$) for the energies (boxes) and in $10^{-5}$ m (m$^2$ s$^{-2}$) s$^{-1}$ for the transfer rates (arrows). Negative numbers indicate transfers opposite to the direction of the arrows. All quantities are basin-averaged, mean (i.e., averaged over 2500 days) quantities. MKE$_i$, and MPE$_i$, stand for the kinetic and potential energy of the mean flow, and EKE$_i$ and EPE$_i$, for the kinetic and potential energy of the eddy flow, summed over layers/interface $i$ and $j$, respectively.
friction does not interfere with rectification processes (i.e., the domain where numerical explicit viscosity is active does not overlap that of the inverse baroclinic energy cascade). Note that in Fig. 12, energy transfers to $E_{PE}$ and $E_{KE}$ are not in equilibrium for 6H110-2. Since this experiment has been integrated over a very long period (over 25 years), we believe that the problem comes from undersampling; we saved the model results every four days and we may underestimate the variance of the eddy energy transfers that are related to the fastest dynamical features, like the transfer from $E_{PE}$ to $E_{KE}$ in the upper layer. (The lower friction peculiar to experiment 6H110-2 reduces the time scale characteristic of these exchanges, which might be less than our sampling time.)

Our belief is that the vertical and horizontal resolution modifies the role of viscosity in its control of the balance between inertia and instability processes. The friction has two opposite effects on the midlatitude jet: it has a global damping effect on the jet and prevents inertia to bring the jet across the basin, but it also damps the jet instabilities and tends to increase the jet penetration. With a 20-km resolution, independent of the vertical resolution, a decrease in friction favors the instability processes, which become more active, and the result is a shorter jet. In 10-km experiments, the balance is changed to favor inertia, in ways that are quantitatively different according to the vertical resolution. In the case of three vertical modes, a decrease in friction does not change the balance and the length of the jet is not modified. But in the case of six vertical modes, the better resolution of the dynamics related to the vortex stretching shifts the balance toward inertia, and a reduced friction produces a longer jet. Note that in experiment 6H110-2, the jet is slightly longer than in 6H110, although the global eddy kinetic energy and the energy transfer rates relative to barotropic instability are doubled at all levels (see Fig. 12, which compares basin-averaged energy diagrams of experiments 6H110 and 6H110-2). Thus, because of a reduced hyperviscosity, the jet is much more turbulent in 6H110-2, but contrary to 20-km grid experiments the shear instabilities do not reduce the jet penetration, and the lower friction allows the jet to go forward.

d. Three-dimensional resolution and negativie viscosity effects

The aim of this section is to rationalize the two main differences observed in the comparison of the energetics in cases 6H110 and 6LO20, namely that both the instability level and the rectification of the mean flow by the eddies is stronger in the high-resolution case, although the eddy energy levels are roughly the same for both runs.

The main idea put forward below is that, somewhat surprisingly, these differences are not due to a purely horizontal effect, but to three-dimensional subgrid scales of the stratified turbulent flow, which are unresolved in 6LO20 but have an important role in the nonlinear energy and enstrophy transfers.

For both 6H110 and 6LO20, the length scales associated with the first and second baroclinic modes, $2\pi \times 38.8$ km and $2\pi \times 18.7$ km respectively, are well resolved. However, scales associated with the third and higher baroclinic modes are either marginally or badly resolved in case 6LO20. One common opinion is that these higher baroclinic modes capture quantitatively only a small fraction of the total energy of the flow and that their inaccurate resolution should have a lesser impact on the solution. The same argument is not valid, however, when one looks at energy fluxes, and the crucial point is that dynamical indexes such as the zonal penetration of the free jet, are the result of these fluxes. These observations can be related to the results of section 4b, which show that the Reynolds stresses have roughly doubled in 6H110, while the energy reservoirs have roughly the same content in both runs.

The energy fluxes of stratified turbulent flows depend on nonlinear interactions between vertical modes (Hua and Haidvogel 1986), and one important point is that low baroclinic modes can interact significantly with higher baroclinic scales of motion.

Fu and Flierl (1980) have explained these nonlinear interactions between vertical modes $F_i(z)$ in terms of the tensor $\epsilon_{(i,j,k)}$, multiplying the triadic interactions between vertical modes $i$, $j$, and $k$:

$$\epsilon_{(i,j,k)} = \int F_i(z)F_j(z)F_k(z)dz$$

(5)

where $i, j, k = 1, \ldots, 6$, and the chosen convention is that mode 1 corresponds to the barotropic mode. A nonzero value of $\epsilon_{(i,j,k)}$ means that advection of mode $j$ by mode $i$ may generate an energy transfer to mode $k$.

For a nonconstant Brunt–Väisälä profile, such as the exponential stratification profile chosen here, this tensor has a whole wealth of nonzero terms, much more so than for a constant Brunt–Väisälä profile. For instance, interactions of the first baroclinic mode with the second baroclinic mode, or self-interactions of the second baroclinic mode can have nonzero projections on the third baroclinic one. For our profile, this corresponds to the following values in the tensor:

$$\epsilon_{(234)} = 0.76$$

$$\epsilon_{(334)} = -0.90.$$


Moreover, self-interactions of the third baroclinic modes can project onto the first baroclinic mode, since
\[ \epsilon_{(244)} = -1.36. \]

It would be tedious to describe the values of the full tensor in detail, but the main result is that numerous possible paths exist for energy and enstrophy fluxes between low and high vertical modes, so that a resolution that discards a priori certain of these paths will affect the overall stratified turbulent cascades in a non-negligible way. This issue is further developed below from three different points of view.

A first assessment of the finite resolution impact on the vertical modes can be obtained from an examination of the vorticity spectra of the various modes for cases 6LO20 and 6HI10, displayed in Fig. 13. These spectra correspond only to fluctuating quantities. The time-mean has first been subtracted for both solutions in order to remove the spatial inhomogeneity contained in the time-mean flow. It should be noted that it is still awkward to interpret spatial spectra of inhomogeneous turbulence, since we saw in section 4b that the overall energetics corresponded to very different dynamics in different regions.

We have reported on the spectra of Fig. 13 the wavenumber associated with the characteristic lengthscale of hyperviscosity, \( L_A \), defined as the scale at which the damping of the direct enstrophy cascade by hyperviscosity begins to be important. This scale has no simple analytical expression since it depends on the strength of nonlinear interactions. [In a linear approximation we would have \( L_A = (A_4/\beta)^{1/5} \).] Therefore, in Figs. 13 and 14, we visually identified this scale by the wavenumbers \( k_A = 1/2\pi L_A \) where the slope in \( k^{-5} \) begins. In experiments where we used the standard biharmonic friction coefficient (6LO20 and 6HI10), \( L_A \) is close to the deformation radius of mode 3 (Fig. 13). Thus, the high baroclinic modes are the most affected by hyperviscosity. In experiments with reduced friction (\( A_4 = 0.25 \times 10^{10} \text{ m}^4 \text{ s}^{-1} \)), \( L_A \) is close to the radii of deformation of modes 5 and 6. The damping of the high baroclinic modes by lateral friction should be less important (Fig. 14b).

In the low-resolution case 6LO20, the vorticity spectrum (Fig. 13a) shows a strong decay (as \( k^{-6} \)) in the high-wavenumber band (to the right of \( k_A \)), where hyperviscosity is an active sink of vorticity. In the low-wavenumber band (left of \( k_A \)), the barotropic mode shows a slope close to \( k^{-3} \), when a \( k^{-2} \) slope would be more consistent with an expected \( k^{-4} \) energy spectrum obtained in direct simulations of the stratified turbulent energy cascade. Spectra of the baroclinic modes should present a maximum at scale close to \( 2\pi\sqrt{2} R_d \), \( R_d \) being their respective radii of deformation. It is the case for the first baroclinic mode. But the second and third modes show a rather flat spectrum before the sharp decrease in the band affected by hyperviscosity. The fourth and the fifth baroclinic modes present their peaks at a wavenumber very close to \( k_A \). Consequently, the spectra of Fig. 13a indicate that the high baroclinic modes are too severely damped by biharmonic friction and/or may not be correctly resolved by the 20-km
over, for the highest horizontal wavenumbers, the third baroclinic mode exceeds the barotropic one in quantitative importance, and this can be an important source of error since its scales of motions are only marginally resolved by the 20-km grid.

In the high-resolution case 6H110 (Fig. 13b), the barotropic mode shows in the inertial range a slope close to \( k^{-2} \) consistent with an observed \( k^{-4} \) energy spectrum (not shown). Its maximum (at \( k = 2 \times 10^{-3} \) cycle/km) is larger than in 6LO20. Spectra of the baroclinic modes have a larger amplitude in the vicinity of \( k_A \), yielding sharper peaks and indicating larger amplitude for all baroclinic modes near their characteristic length scale. Since these two experiments have identical friction, the differences are to be credited to the increase in horizontal resolution, which improves the calculation of the high baroclinic modes. Note that in the 10-km grid, three-layer experiment 3H110, the eddy vorticity spectrum (Fig. 14a) presents a \( k^{-5} \) shape in the inertial range, indicating that the turbulent energy cascade is affected by the absence of the high baroclinic modes.

In the high-resolution case with reduced friction 6H110-2 (Fig. 14b), the improvements noticed for 6H110 are drastically amplified since the scale of the hyperviscosity \( k_A \) is shifted toward higher wavenumbers. The increase of the baroclinic mode in the vicinity of \( k_A \) is around one order of magnitude and is much larger for the high baroclinic modes. The spectra of the barotropic mode is also larger and does show a \( k^{-2} \) slope in the inertial range.

Consequently, the increase in horizontal resolution improves the model representation of the stratified turbulent energy cascade by a better computation of the high baroclinic modes. However, we note that in all six-layer experiments the spectra of modes 5 and 6 are always an order of magnitude lower than those of the other modes, even in the vicinity of \( k_A \). Therefore, the dynamical role of those modes is probably of secondary importance, and the improvements of the high-resolution cases are certainly due for the most part to a better resolution of modes 3 and 4.

A second way of assessing the impact of unresolved scales is to independently check the necessary conditions of potential vorticity conservation by individual Lagrangian trajectories, by launching particles in the advecting fields of 6LO20 and 6H110. The results are reported in detail in Hua (1990) and can be briefly summarized as follows. For both runs, float trajectories were tracked using high-order precision interpolation schemes, so that the main sources of error in the trajectories were due to the differences in the Eulerian advecting fields, and were checked to be unaffected by interpolation errors. For case 6H110, all float trajectories are compatible with conservation of the initial potential vorticity within 5% for at least ten Lagrangian time scales, while for case 6LO20, numerous examples of nonconservation correspond to an error level of
100% in less than one integral time scale. Moreover, in the Lagrangian test, the origin of the error can _always_ be readily traced to errors due to the advection vortex stretching term along the Lagrangian trajectory. Thus, this Lagrangian analysis suggests an inconsistency in the resolution of the baroclinic part of the Eulerian flow field in simulation 6LO20.

Keeping in mind the enhanced inertia of the barotropic flow observed in 6HI10, a third quantitative evaluation of the impact of the unresolved scales on the barotropic flow can be obtained by computing the explicit eddy diffusivity induced by a 20-km resolution. This is done in the same way as Vallis and Hua (1988), by measuring the differences in the nonlinear interactions induced by different resolutions. The barotropic vorticity equation is

$$d(q_i)/dt + \sum_{i=1}^{6} \left[ \epsilon_{i,j} J(P_i, q_i) \right] = \text{Other terms} \quad (6)$$

where $q_i$ is the potential vorticity in mode $i$ and $P_i$ is the streamfunction in mode $i$. Because of the orthogonality property of the vertical modes, one has

$$\epsilon_{i,j} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$$

so that only self-interactions of each mode $i$ will contribute to the barotropic vorticity equation. Moreover, Eq. (6) can now be written in Fourier space for each horizontal wavenumber $k$, for both low and high resolutions. As in Vallis and Hua (1988), one can introduce an explicit eddy diffusivity, which is caused by missing subgrid scales in mode $i$ and which will modify the barotropic mode at wavenumber $k$:

$$\nu_i = - (J_{\text{high}}(P_i, q_i) - J_{\text{low}}(P_i, q_i))/k^4(P_i)$$

where $J_{\text{high}}$ and $J_{\text{low}}$ refer respectively to the 10-km and 20-km solutions, and the denominator in Eq. (7) represents the Laplacian of barotropic vorticity at wavenumber $k$. The numerical statistics for Eq. (7) cannot be computed directly by taking the differences of the Jacobians between runs 6HI10 and 6LO20, because of phase differences. Therefore, the following procedure was taken: for five different initial times, the 6HI10 field was extrapolated back onto a 20-km grid and was integrated forward for 200 days on the coarse grid. This was done for five statistically independent realizations and the differences between the solutions on the two grids were used to estimate Eq. (6).

Results are shown in Figs. 15a and 15b for the various vertical modes. The quantity plotted is $T_i(k) = 1/(k\nu_i(k))$, a time scale characteristic of the period necessary for mode $i$ to affect mode 1 through the inverse stratified turbulent cascade. One can immediately notice the well-known positive viscosity effect at the highest wavenumbers for all modes, which is usually mimicked by the use of a hypervisosity operator. The time scale associated with this positive viscosity is of 10 days for the self-interaction of the barotropic mode. However, the largest missing effect in the 20-km resolution is the lesser-known negative viscosity effect at the lowest wavenumber, which is clearly present in Fig. 15 for all modes (Starr 1968; Sivashinsky and Yakhot 1985, Kraichnan 1976). The time-scale for this negative viscosity effect can be quite short, and is for instance 20 days for mode 2, 30 days for mode 1 (self-interaction), and 1000 days for mode 4. Since the integration period needed to reach statistical equilibrium is much larger than 1000 days, this latter time scale is still significant.
for the physics of the establishment of the mean circulation. This negative viscosity is missing in the low-resolution grid case because the wavenumber truncation by the 20-km grid interferes with the stratified inverse cascade of quasi-geostrophic turbulence, the end points of which are the largest scales of the barotropic circulation (Rhines 1977). These largest scales are artificially damped in 6LO20, e.g., one observes a comparatively positive viscosity in 6LO20 on the barotropic inertial recirculation because the negative viscosity effect is missing.

In conclusion, the dynamical impact of unresolved stratified subgrid scales, through a misrepresentation of high vertical modes, can happen in two different geometries of low three-dimensional resolution, whenever baroclinic instability effects are important:

(i) in a low vertical resolution system such as a three-layer model, whatever the spatial size of the horizontal grid. This is why, despite a 20-km grid which resolves all explicitly present vertical modes in a three-layer model (Lozier and Riser 1989), the potential vorticity of individual Lagrangian particles will not usually be conserved.

(ii) in a high vertical resolution model with too coarse a horizontal grid, such as a six-layer model with a 20-km grid [cf. case 6LO20].

It remains to be determined what would be a sufficient three-dimensional resolution of quasi-geostrophic flows both in the general case and for flows where the barotropic inertial recirculation plays a lesser role, for instance in the presence of topographic roughness. One possible a posteriori check would be to test the conservation of Lagrangian vorticity by the Eulerian advecting field.

5. Conclusion

The generation of classical wind-forced, eddy-driven double-gyre circulation is investigated with a multilayered (three or six), quasi-geostrophic model, in order to investigate the tridimensional consistency of the resolution of eddy scales in simulating large-scale flows. Our numerical simulations have confirmed the importance of high baroclinic modes on eddy-driven large-scale circulation. The most obvious change due to fine resolution is the amplification of the resonant inertial mode, resulting in a considerable increase of the penetration scale of the midlatitude jet (almost 40% in six-layer cases). Moreover, an inversion of the effect of the hyperviscosity was observed; when smaller values are used, the penetration of the jet is slightly increased even though barotropic instabilities in the jet appear to be more active. (In 20-km experiments the larger shear instabilities that result from a lower biharmonic friction produce a shortening of the jet). Those effects, which are already significant when the vertical resolution is three-layered, are quantitatively (at least twice) more important when the vertical resolution includes six vertical modes.

The other quantities that characterize the large-scale circulation (mean transport, surface intensification of the flow, level of basin-averaged eddy kinetic energy, inertial oscillations and barotropic nature of the jet, etc.) are not modified by more than 10%. However, we noted that when the resolution is low (20 km), a larger number of vertical modes (six instead of three) is responsible for a significant increase in the amplitude of the barotropic mode, but has no effect on the inertia of the jet; whereas when the resolution is high (10 km) the better vertical resolution increases the length of the jet.

However, although the global reservoirs of energy (total kinetic and potential energies and related global energy transfer rates) are more or less equivalent in both low- and high-resolution experiments, a detailed analysis of the fluxes revealed that in the fine-resolution cases, this balance results from stronger local interactions between the mean flow and the eddies. The processes of destabilization and rectification of the mean currents are much more energetic locally, although their global balance is not drastically changed. Again we noticed that quantitatively the modification of the energy transfers is much larger when the model has six vertical modes.

Our analyses showed that the consistent tridimensional resolution of the six-layer, 10-km experiment yields a better representation of the inverse stratified cascade (with a $k^{-2}$ eddy vorticity spectrum), which involves interactions with the barotropic mode. The effective viscosity, related to the mean shear and the eddies, now controls the direct and inverse cascades, instead of the biharmonic friction, and allows a larger resonant excitation of the inertial mode.

The tridimensional consistency of an adequately high resolution of the eddy-scales is fundamental in simulating large-scale flows. A three-layer experiment contains three vertical modes, and a 20-km resolution gives a correct representation of all the baroclinic modes explicitly present in the model, although a 10-km resolution increases the inertia of the jet by a better representation of the work done by the Reynolds stresses. However, the number of baroclinic modes is not sufficient to represent nonlinear baroclinic energy transfer rates (yielding a $k^{-3}$ eddy vorticity spectrum instead of $k^{-2}$), since the higher baroclinic modes play a significant role in the energy cascades. In a six-layer experiment, truncation occurs after the fifth baroclinic mode, and the representation of the turbulent baroclinic processes is clearly improved. However, with a 20-km resolution, the high baroclinic modes are not correctly calculated, (yielding a $k^{-3}$ eddy vorticity spectrum, whereas with a 10-km resolution the spectrum is $k^{-2}$, in agreement with a $k^{-4}$ energy spectrum). Although those modes show low energy levels in coarse- and fine-resolution experiments, they act differently in
both cases on the fluxes. Since these energy fluxes determine the large-scale response, one can say that the high baroclinic modes, despite their low kinetic energy content, play a catalytic role in the eddy-driven circulation. Just as the right catalyst is essential for triggering a chemical reaction, they are essential for determining the large-scale response of turbulent ocean models. However, our study indicates that the number of vertical modes that require a fine resolution tends to be limited since even in high-resolution experiments, modes 5 and 6 appeared to have a relatively unimportant role compared to that of modes 3 and 4.

The novel characteristic of the detached jet, as revealed by the enhanced resolution, is its inertial nature. As in all resonance problems, the amplitude of the solution is controlled by the dissipation level. The eddies act as an effective sink, and it is then of crucial importance to resolve them in a fully tridimensional consistent way, without subgrid scale parametrization affecting the solution.

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