Vorticity and Vertical Circulation at an Ocean Front*

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ABSTRACT

Density and velocity data with 4-km horizontal resolution from a survey of a front during FASINEX (Frontal Air-Sea Interaction Experiment) are combined to describe the structure of the top 300 m of the ocean. The geostrophic velocity field is derived and is used to examine the relative importance of stratification, relative vorticity, and twisting terms in Ertel's potential vorticity Q. Tenfold isopycnal changes in Q are found across a horizontal scale of only 10 km. These changes are confined to isopycnals that outcrop from the seasonal thermocline into the mixed layer. The ageostrophic velocity field is quantified by solution of the omega equation, and vertical velocities of up to 40 m day⁻¹ are found. Small (40 km) surface-trapped (top 200 m) eddies are found to play a crucial role in the transport and effective diffusion of properties across the thermocline out of the mixed layer.

1. Introduction

The dynamics of vertical circulation at oceanic fronts has important consequences for the transport of properties across the thermocline from the surface layer of the ocean into its interior. This assertion is the theme of a series of papers by Woods and colleagues (Woods et al. 1977; MacVean and Woods 1980; Woods 1985; Woods et al. 1986; Leach 1987; Bleck et al. 1988; Woods 1988; Fischer et al. 1989) and is analogous to the vertical circulation across atmospheric fronts that can transfer boundary-layer air across the tropopause (e.g., Sawyer 1956; Hoskins and Bretherton 1972). On frontal scales, vertical motion is a consequence of conservation of potential vorticity (Hoskins et al. 1985), which forces vertical velocities whenever the vorticity of the fluid changes, as, for example, when large-scale confluence moves fluid into a frontal jet (e.g., Woods et al. 1986). This situation has been modeled (e.g., Hoskins and Drachici 1977; Bleck et al. 1988; Onken and Woods 1989), but it has been very difficult to measure or infer vertical velocities in the seasonal thermocline of the ocean until recently because of crude observational techniques and the small velocities involved (10⁻⁵–10⁻³ m s⁻¹) relative to those induced by surface waves (up to 1 m s⁻¹) and internal waves and tides (10⁻³–10⁻¹ m s⁻¹) (e.g., New and Pingree 1990).

Despite the observational limitations, Voorhis and Bruce (1982) anticipated many of the conclusions of this paper in a remarkably perceptive analysis of XBT and surface drogue data collected in March 1977 from the same region as the data to be described here. Voorhis and Bruce estimated vertical velocities of 3 × 10⁻⁴–5 × 10⁻⁴ m s⁻¹ at a depth of 100 m at the base of a winter well-mixed layer. More recently, Bower (1989) tracked isopycnal RAFOs floats following Gulf Stream meanders and observed vertical excursions of up to 400 m in 5 days (10⁻³ m s⁻¹); Eriksen et al. (1991), using vertical current meters during the Frontal Air-Sea Interaction Experiment FASINEX (Stage and Weller 1985, 1986), observed excursions of up to 100 m in 8 h (4 × 10⁻³ m s⁻¹). Such large velocities, of order 100 m day⁻¹, will locally double the depth of a 100-m-deep mixed layer in a day and drastically alter the stratification of the thermocline beneath. The objective of this paper is to quantify and interpret the vertical velocities, using modern survey tools.

The paper describes density and velocity data collected during a cruise on RV Oceanus that took place in February 1986 in the Sargasso Sea southwest of Bermuda (Fig. 1). The cruise, led by R. Weller, was part of FASINEX. The area was chosen for frontal studies because it is known to be an area where surface fronts are frequently observed, both from in situ measure-
ments (Voorhis and Hersey 1964; Katz 1968; Voorhis 1969) and AVHRR imagery (Halliwell and Cornillon 1988). Our data consist of eight sections across a front (Fig. 1a) collected during a 3.5-day survey with a towed profiling CTD known as a SeaSoar (Pollard 1986) and a ship-mounted acoustic Doppler current profiler (ADCP) (Regier 1982). The dataset is introduced in Pollard (1986) and Pollard and Regier (1990) and was the first, to our knowledge, in which both density and velocity fields could be three-dimensionally resolved to scales of a few kilometers, sufficient to resolve the dynamical features of a front (but see Lillibridge et al. 1990 in which a similar survey is discussed).

The mesoscale context in which the survey was set is shown in Fig. 2, from Böhm and Cornillon (1987), taken two days after completion of the survey superimposed on the figure. A 100-km-wide tongue of warm (light colored) water is seen penetrating northward across 28°N at 71°W, with a similar tongue of colder water penetrating southward to the west of and outside our survey area. These features are reminiscent of those described by Leetmaa and Voorhis (1978). The mesoscale dynamics of the area were well studied during the Mid-Ocean Dynamics Experiment (MODE) (e.g., McWilliams 1976; Voorhis et al. 1976).

We shall quantify vertical velocities using the omega-equation in the form developed by Hoskins et al. (1978). The forcing terms in this equation are calculated from horizontal second derivatives of the geostrophic velocities, so considerable care is needed to derive the basic density and velocity fields. For this reason, these fields are described in detail in sections 2 and 3. The vorticity balance is described in section 4 before the omega equation is solved in section 5.

2. The three-dimensional density field

Pressure, temperature, and conductivity were sampled by a NBIS CTD towed behind the ship in a SeaSoar following a sawtooth path between the surface and about 300 m, with a complete cycle every 3 km (Pollard 1986). The eight legs (Fig. 1a) are about 16 km apart. After initial reduction and editing (Pollard et al. 1986) the data were gridded onto a 10 m × 4 km grid by averaging all values within ±5 m and ±5 km of each grid point. Horizontally adjacent grid points 4 km apart are thus not independent, but the 10-km averaging is necessary to smooth the internal wave field. Density sections for all eight legs are contoured in Fig. 3. The averaged data are sufficiently smooth to allow calculation of dynamic height and hence geostrophic shear between adjacent columns. Because of the 10-km overlap, the geostrophic velocity field is effectively
calculated between two columns of water, each averaged over 4 km and centered 10 km apart.

Figure 3 shows that the isopycnals that outcrop at the surface front (Fig. 1a) slope downward to the south to 100–150 m in the seasonal thermocline. The maximum frontal slope is $7 \times 10^{-3}$. There are considerable variations in the thickness, defined here as the separation between any pair of isopycnals 0.1 kg m$^{-3}$ apart. In leg 1, for example, there is a striking increase in thickness between 25.5 and 25.6 kg m$^{-3}$ at the southernmost end of the section. It can be seen again on leg 3, and on closer examination on legs 2 and 4 also. Thus, there is an elongated tube of weakly stratified water at least 50 km long (east–west), 20 km wide, and up to 50 m high. Another example is between 25.2 and 25.3 kg m$^{-3}$ at about 27.9°N on legs 5, 6, and 7.

How are these elongated tubes created? An explanation may be found by examining the thickness increase (Fig. 3, shaded) between 25.7 and 25.8 kg m$^{-3}$ around 28.6°N on leg 8. On legs 2 to 5 the 25.7 kg m$^{-3}$ isopycnal outcrops at the front, but 25.8 lies at the top of the seasonal thermocline. Thus, the mixed layer north of the front has density in the range 25.7–25.8 kg m$^{-3}$ and appears to be pulled down under the front at 28.2°N. On legs 5, 6, and 7 the outcropping isopycnal 25.7 kg m$^{-3}$ dips down again at 28.6°N before outcropping finally at about 28.8°N. Figure 1a shows that the double outcrop is the signature of an eddy straining water lighter than 25.7 kg m$^{-3}$ northward between legs 7 and 8 (Fig. 1b), then eastward as far as leg 5.

The tongues of weak stratification in the thermocline are thus caused by small (40-km diameter), shallow (50–100 m), anticyclonic eddies carrying frontal water over the top of the mixed layer to the north, capping the weakly stratified water and trapping it in the thermocline. Several such eddies are marked with asterisks in Fig. 1. There is evidence for vertical motion in the depths of the weakly stratified regions, several of which are found between 150 and 200 m. The mixed layer is only about 100 m deep on both sides of the front (e.g., leg 4), so mixed-layer water appears to have been carried down 50 m or more. Pollard (1986) found high values of oxygen in the weakly stratified tongues, confirming their origins in the aerated mixed layer. The eddies can develop rapidly. On 17 February, while RV Oceanus was surveying leg 2, her sister ship RV Endeavor was working a CTD section at the longitude of leg 8. At that time the front lay east–west along 28.2°N. The eddy has thus developed within 3 days. We shall examine the dynamics of these features once the velocity field has been derived.

3. The three-dimensional velocity field

RV Oceanus was equipped with two-chain Loran C, GPS, and a 150 KHz RDI Acoustic Doppler Current Profiler (ADCP). In this section, we derive smoothed, absolute velocities from these instruments, compare the ADCP-derived vertical shear with geostrophic vertical shear, and derive a quasigeostrophic velocity field by combining ADCP and SeaSoar data.

GPS and Loran positions were logged once per minute on a PDP 11/34 computer, part of the SeaSoar system. Whenever GPS 3-satellite positions were available, they were used in preference to Loran positions (Pollard 1986). Because Loran was the primary method of navigation, GPS positions were adjusted to minimize the (Loran–GPS) offsets at the beginning and end of each GPS period by subtracting a constant value from GPS latitude and longitude for that period. Offsets were up to 1 km, differing from day to day depending on the mean position of the ship, and reflect the spatial drift of Loran signals from absolute (GPS) position.

ADCP data were initially logged over 30-s intervals, and binned into 4.1-m depth intervals. The method of repeated turns (Regier 1982; Pollard and Read 1989) was used to calculate a scaling factor (1.024) and misalignment angle (0.14°) by which the data were corrected for the present survey. We have subsequently calculated 5-min averages of ADCP-corrected velocities of the water relative to the ship and combined them with the ship's velocity to obtain a time series of water velocity (Fig. 4) further averaged over the entire ADCP profiles (4–250 m).

The daily patches of smooth velocity in Fig. 4 correspond to GPS periods, demonstrating that Loran is the major source of noise in the velocity estimates. To reduce this noise, we proceeded as follows. The Loran fixes were smoothed to reduce short period oscillations...
Fig. 3. Density sections for all eight legs. The legs are stacked as seen from the western end of the survey (bottom left to top right). The separation between two isopycnals (25.7 and 25.8 kg m$^{-3}$) is shaded to show the wide variations in thickness.

by calculating a 10-min running mean. The resultant ship's track is shown in Fig. 1a. Assuming that column-averaged velocity should change little from one 5-min average to the next (about 1 km apart spatially), we may first-difference the velocity time series and calculate the mean (east, north) velocity noise due to Loran as (13, 9) cm s$^{-1}$. This is still unacceptably large. To match the velocity fields with the 10-km smoothed
density fields, we further average the 5-min averaged velocities over 40 min. The noise level is thus reduced by a factor of 8 (not \( \sqrt{8} \), as the noise is in the Loran positions, not the derived velocities) to (2, 1) cm s\(^{-1}\). The resultant velocity vectors at a depth of 150 m are shown in Fig. 5a. Velocities within 10 min of the turns at each end of the north–south legs have been edited out, as they are biased by the 10-min smoothing applied to the Loran fixes.

Next, we compared the vertical shear in the 10-km–averaged ADCP velocities with the geostrophic shear derived from SeaSoar data (Fig. 6). The shears compare well in the depth range 100–150 m. Below 150 m, quite unrealistic shears are apparent in the ADCP data. In general, we noticed that shears below 150 m appeared to reflect shears above 150 m. The cause was eventually traced by RDI to an error in the tracking firmware once the return signal dropped below a threshold value. All ADCP data below 150 m were discarded.

The cause of the near-surface ageostrophic ADCP shear is more interesting. Moorings just south of the survey area (Weller et al. 1991; also Fig. 1) showed strong inertial oscillations (Fig. 7a) at a depth of 10 m, which had mostly died out at 160 m (Fig. 7b). This structure is typical of wind-driven inertial oscillations (e.g., Pollard 1980), which will in general have large horizontal scales in the mixed layer, comparable with the scale of the meteorological forcing field. A fast-moving atmospheric front on 15 February (Mundy 1987) probably initiated the present oscillations coherently over an area larger than the Oceanus survey area. Therefore, we removed geostrophic frontal shears from the ADCP data by subtracting the geostrophic shear between 30 and 150 m from the ADCP data between the same depths (Fig. 7c). The resultant time series (Fig. 7a) shows peaks that coincide with maxima in the current meter observed inertial oscillations.

Therefore, we conclude that the ageostrophic shear is caused by wind-driven inertial oscillations, which is undesirable when it is the geostrophic velocity field, and more seriously horizontal gradients in geostrophic velocity, that we wish to estimate. “Contamination” by inertial and tidal oscillations will be an inherent...
problem with all shipboard ADCP data (Saunders 1990). While short period (less than 1 h) or short-scale (less than a few kilometers) internal waves can be averaged out, tidal and inertial period oscillations with periods of order 12 h and horizontal scales greater than tens of kilometers cannot be removed without also removing the geostrophic component of the velocity field.

In the FASINEX survey, however, the SeaSoar density data can be used to reduce inertial contamination of the ADCP data. We use the ADCP data only to give a reference velocity field at 150 m (Fig. 5a), chosen to be the deepest level at which the ADCP data are good, in order to minimize inertial motions (Fig. 7b). We fit a streamfunction (dynamic height field) to the ADCP data at the chosen level to approximate a geostrophic (nondivergent) velocity field. The streamfunction \( \phi \) is obtained by solving

\[
\nabla^2 \phi = \text{curl} u
\]

by relaxation with the Dirichlet boundary conditions

\[
\frac{\partial \phi}{\partial n} = n \times k \cdot u
\]

where \( n \) is the unit vector perpendicular to the boundary and \( k \) is the unit vertical vector. The dynamic height fields at all other levels between 50 m and 250 m are then obtained from the SeaSoar density data (Fig. 8a, b) by adding the dynamic heights relative to 150 m at each 4-km grid point. Nondivergent velocity fields \( U = (\psi, \phi) \) are then recovered by central differencing, or sideways differencing on the boundaries.

Comparing \( u \) (150) (Fig. 5a) with \( U \) (150) (Fig. 5b), it can be seen that the ADCP velocity field has been smoothed. In particular, there is a tendency for \( u \) to oscillate from track to track (look at the north component of \( u \) along any east to west line), which is most probably the residual inertial oscillation at 150 m, because the time taken for \( Oceanaus \) to traverse each track was approximately half an inertial period.

The 250-m dynamic height field (Fig. 8a) reflects the underlying mesoscale eddy field visible in the infrared surface temperature field (Fig. 2). In the western half of the survey area, a warm mesoscale feature is moving water north-northwestward. In the eastern half, the flow turns eastward at 28.4°N along the line of the front. At 50 m (Fig. 8b) several small-scale eddylike features are apparent (compare with the features marked * on Fig. 1), which are only weakly apparent at 250 m. These \( O \) (50 km) eddies are thus shallow, baroclinic features, and it will be shown that they play a major role in the evolution of the near-surface stratification.

It has been necessary to take considerable care in deriving a geostrophic velocity field because our use of vorticity in section 4 and solution of the omega equations in section 5 depend entirely on horizontal gradients of the velocity field. The ADCP data cannot be directly used for this purpose, as they contain inertial motions that would severely bias gradients between adjacent tracks. With maximum inertial oscillations of 0.25 m s\(^{-1}\) and track separation of 16 km, horizontal gradients could be in error by up to \( 3 \times 10^{-3} \) s\(^{-1}\), or 0.4 \( f \). We estimate that the use of (a) a nondivergent velocity field at 150 m (Fig. 5), (b) density-derived dynamic heights relative to that level, and (c) gradients calculated between alternate tracks surveyed approximately an inertial period apart, will reduce the error by factors of 2, 2.5, and 2.5, respectively. On the other hand, we have shown that an eddy can develop in three days (section 2), so development within an inertial period will increase the error in the horizontal gradients of geostrophic velocity. Overall, we conclude that the error will be no more than 0.1 \( f \) and may be a small as 0.05 \( f \).

4. Vorticity

For adiabatic, frictionless motion, Ertel's potential vorticity

\[
Q = \rho^{-1}(2\Omega + \nabla \times u) \cdot \nabla \rho
\]

is conserved following a particle. Here \( \Omega \) is the earth's angular velocity, \( u \) is the water velocity relative to earth, and \( \rho \) is the water density. We approximate \( Q \) from our data as follows.

Let \( u = (u, v, w) \) and \( 2 \Omega = (0, f_1, f) \) in \( (x, y, z) \) coordinates with \( z \) vertically upward. Then

\[
\rho Q = (w_y - v_z)\rho_x + (f_1 + u_z - w_x)\rho_y + (f + \xi)\rho_z
\]

where \( \xi = (v_x - u_y) \) is the vertical component of relative vorticity.
Approximating \((u, v, w)\) by the geostrophic velocity \((U, V, 0)\), using the hydrostatic approximation and geostrophy to write
\[
(\rho_x, \rho_y) = -fg^{-1}(V_z, -U_z),
\]
and neglecting \(f_i\) (order \(10^{-4}\) s\(^{-1}\)) compared to \(u_c\) (order \(10^{-2}\) s\(^{-1}\)), (4.2) reduces to
\[
Q = -fg^{-1}N^2(1 + \xi f^{-1} - F),
\]
where \(N^2 = -g \rho \frac{\partial \rho}{\partial z}\) and \(F = (U_z^2 + V_z^2)/N^2\) is the Froude number. In (4.3), \(-F\) is the twisting term, expressing conversion of horizontal vorticity to vertical vorticity by the vertical shear. For gyre or mesoscale motions, \(\xi \ll f\) and \(F \ll 1\), and \(Q\) reduces to the Sverdrupian potential vorticity \(Q_s = fN^2/g\) (Woods 1985). On sub-mesoscales, we shall show that neither \(\xi f\) nor \(F\) is small everywhere.

Fig. 7. The eastward component of velocity at three moorings near the survey region (Fig. 1) is shown for current meters at depths of (a) 10 m (b) 160 m. The ADCP shear between 30 m and 150 m for the 10-km averaged data is shown in (c), for the period of the survey, and the difference between the ADCP shear and the geostrophic shear (30 to 150 m) derived from the SeaSoar is superimposed on (a). Frontal shears are thereby eliminated or reduced, and inertial periodicity in the ageostrophic shear becomes apparent.
An alternative expression for $Q$,

$$Q = \frac{f + IV}{\Delta \rho} \frac{\Delta \rho}{\rho} \quad (4.4)$$

is obtained by calculating the relative vorticity $IV$ isopycnally. Thus, $IV = [V_x - U_y]_{is-const}$ and

$$IV = \xi - Ff. \quad (4.5)$$

Also, $\Delta \rho$ is the separation (thickness) in meters of two isopycnals $\rho \pm \Delta \rho/2$. Rossby (1940) defined $IV$ as the isentropic vorticity (Haynes and McIntyre 1987).

Contours of $Q$ on legs 1 and 4 are shown in Fig. 9. The contour interval is 0.2 potential vorticity units (PVU), where we define 1 PVU to be $10^{-9}$ m$^{-1}$ s$^{-1}$, for ease of reference. Aspects of the partitioning of $Q$ between thickness $\Delta \rho$ and isentropic vorticity $IV$ (4.4) may be inferred from Fig. 9. The three hatched bands, each of thickness 0.1 kg m$^{-3}$, also identify six isopycnic surfaces (compare Fig. 3). The positively marked near-vertical bands bounded by thick lines show where $IV$ is positive (cyclonic). Figure 10 shows maps of properties in the shallowest of the three hatched bands in Fig. 9 (25.45 kg m$^{-3}$). Examination of Figs. 10c and 10d shows a good correlation between isopycnic salinity variations (a well-measured conservative property) and isopycnic potential vorticity variations, which improves our confidence in our estimation of $Q$.

Examination of Fig. 9 shows that $Q$ ranges from less than 0.2 PVU in the mixed layer and below 200 m to over 1.4 PVU in the thermocline just below the base of the mixed layer (where stratification is strongest). The isopycnic surfaces 26.2 kg m$^{-3}$ and deeper nearly follow $Q$ contours (compare Fig. 3 and Fig. 9), showing that there is little variation of $Q$ on isopycnic surfaces with density greater than 26.2 kg m$^{-3}$. On the shallower hatched bands, which lie within the thermocline, there is considerable variation in $Q$ (see also Fig. 10d). This variation arises because isopycnals within the thermocline outcrop into the mixed layer at the front (Fig. 10a). Wind mixing intensifies potential vorticity gradients on outcropping isopycnals by increasing the stratification just below the base of the mixed layer and decreasing the stratification where the isopycnal enters the mixed layer itself.

The greatest values of $Q$ lie in the stippled blobs (Fig. 9) 20–40 m high and 10–30 km wide in the north–south direction, which are shown (Fig. 10d) to be elongated east–west tongues up to 100 km long. Within these tongues, $Q$ is partitioned between thickness and isentropic vorticity. In Fig. 9a, for example, the stippled region of $Q > 0.8$ PVU around 27.7$^\circ$N lies almost entirely within a band of cyclonic vorticity ($f + IV$ maximum) with the hatched regions pulled together ($\Delta \rho$ minimum). The maximum isopycnic change in $Q$ from 0.15 to 1.5 PVU (Fig. 10d) occurs in a north–south distance of only 10 km, across which $IV$ changes.
from $-0.74 f$ to $+0.26 f$ (Fig. 10e), a factor of 5 change in absolute vorticity from $0.26 f$ to $1.26 f$. Therefore, the thickness must have changed by a factor of 2 in such a fashion as to enhance $Q$ when the relative vorticity is cyclonic. Hoskins et al. (1985) show that the vorticity and stratification (inverse thickness) must always reinforce each other in a potential vorticity anomaly. The lens-shaped regions of high $Q$ in Fig. 9 may thus be viewed as regions of anomalous cyclonic vorticity, and the induced flow field around them will be similar to that derived by Thorpe (1985).

The tongues of minimum $Q$ (Fig. 10d) are likewise regions of anticyclonic relative vorticity and weak stratification. The origin of these tongues from anticyclonic eddies has already been shown in section 2. Hence, the eddies are also acting to strain isopycnic potential vorticity variations into bands of alternately high and low values (Fig. 10d), which will induce vertical velocities whose magnitude we shall quantify in section 5.

By plotting isentropic vorticity, we have amalgamated the effects of relative vorticity $\zeta$ and the twisting term expressed by $\Phi$ (4.3), which is separately contoured in Fig. 10f. The Froude number is maximum along the line of the frontal jet, not only because vertical shears are largest there, but also because the jet is moved...
Fig. 10. Maps of (a) pressure, (b) geostrophic velocity, (c) salinity, (d) potential vorticity $Q$, (e) isopycnic relative vorticity $\nabla \theta / f$, and (f) Froude number $F$, on the surface $\rho = 25.45$ kg m$^{-3}$, which lies in the center of the uppermost hatched bands in Fig. 9.
by the anticyclonic eddies to overlie weakly stratified surface water. Where IV reaches its minimum of −0.74 (Fig. 10e), Fig. 10f shows that $F$ attains a maximum of 0.43. The twisting term is thus contributing more to the isentropic vorticity than the vertical component of relative vorticity, whose maximum anticyclonic value is therefore $-0.31/(1.5)$ from 4.5. Another consequence of maximum Froude number, or minimum Richardson number $Ri=F^{-1}$, is that the most likely regions for mixing are where weakly stratified mixed-layer water lies beneath the frontal jet (Toole and Schmitt 1987). The geostrophic velocity field on the isopycnal surface 25.45 is shown in Fig. 10b. The velocity vectors are not parallel to the pressure contours, from which one of two inferences can be drawn. One possibility is that the front is not in equilibrium, but is progressing northward. The second possibility is that there are vertical velocities that carry particles upward along the sloping isopycnals, compensating the cross-isobaric horizontal flow. Evidence to partially support the second hypothesis can be seen by comparing Fig. 10a with Fig. 10d. Where the frontal slope is greatest (28.0°–28.2°N), there are two places where the 0.6 PVU contours cross the front, marked with asterisks on Fig. 10d. Comparison with the eddies marked in Fig. 1a suggests that wisps of high (>0.6 PVU) $Q$ are being carried up the frontal slope by the eddies. If the front is stationary, a vertical velocity of $50 \times 10^{-5}$ m s$^{-1}$ may be inferred (Pollard and Regier 1990; Eriksen et al. 1991).

We may further infer that vertical velocities are maximum at a depth somewhere between 100 m and 200 m. Thickness variations on two isopycnal surfaces are shown in Fig. 11. The contours appear very similar, but where thickness is maximum on one surface, it is minimum on the other. Vertical velocity must therefore reach a maximum up (down) somewhere between the two surfaces, thus compressing (stretching) isopycnals on the upper surface and stretching (compressing) them on the lower surface.

Similar inferences about vertical velocities can be drawn from Fig. 9. Note how the areas of cyclonic vorticity tend to lie in vertical bands. If a water parcel is moved sideways, by the eddies say, to an area where the relative vorticity is different, it quickly adjusts its own relative vorticity to match the surroundings. If it did not, vertical shears would develop. To conserve potential vorticity, the water parcel must therefore adjust its thickness, requiring a vertical velocity. We have shown that, in the cyclonic anomalies, thickness decreases to enhance $Q$. Below such an anomaly, the relative vorticity remains cyclonic (Fig. 9), but the isopycnal potential vorticity variations are small. Hence, the thickness has increased to compensate the absolute vorticity increase.

All these indications of ageostrophic circulation are in accordance with theory (Woods 1985; Hoskins et al. 1985) and are shown schematically in Fig. 12. Suppose that a front is maintained by large-scale confluence A. As water moves toward the front on the anticyclonic side B, its absolute vorticity decreases, and the thickness...
5. The vertical velocity field

Following Hoskins et al. (1978), we may use the quasigeostrophic approximation to obtain the omega equation for the vertical velocity in terms of spatial gradients of the geostrophic velocities. The derivation in the oceanographic case is as follows. Partitioning the horizontal velocity \((u, v)\) into geostrophic \((U, V)\) and ageostrophic \((u_a, v_a)\) components,

\[
(u, v) = (U, V) + (u_a, v_a) \tag{5.1}
\]

and approximating the rate of change following a particle \((D/Dt)\) by

\[
\frac{Dg}{Dt} = \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \tag{5.2}
\]

reduces the horizontal momentum equations to

\[
\dot{u}_a = \frac{-D_g V}{D t}, \quad \dot{v}_a = \frac{D_g U}{D t} \tag{5.3}
\]

to first order on an \(f\) plane. Using the hydrostatic approximation, the vertical momentum and buoyancy equations reduce similarly to

\[
N^2 w = -\frac{1}{\rho} \frac{D_g}{D t} \frac{\partial p'}{\partial z} \tag{5.4}
\]

where the reduced pressure \(p'\) is related to \((U, V)\) by

\[
(U, V) = \frac{1}{\rho f} \left(-\frac{\partial p'}{\partial y}, \frac{\partial p'}{\partial x}\right). \tag{5.5}
\]

Using (5.5) to eliminate the time derivatives between (5.3) and (5.4), we obtain

\[
\frac{\partial}{\partial x} (N^2 w) - f^2 \frac{\partial u_a}{\partial z} = 2f(V_x U_z + V_y V_z) = Q^x \tag{5.6}
\]

\[
\frac{\partial}{\partial y} (N^2 w) - f^2 \frac{\partial v_a}{\partial z} = -2f(U_x U_z + U_y V_z) = Q^y. \tag{5.7}
\]

Finally, we use continuity of the ageostrophic velocity field \((u_a, v_a, w)\) to obtain the omega equation

\[
f^2 \frac{\partial^2 w}{\partial z^2} + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) (N^2 w) = \frac{\partial Q^x}{\partial x} + \frac{\partial Q^y}{\partial y}. \tag{5.8}
\]

In principle, (5.8) can be solved for \(w\), given suitable boundary conditions, because the right-hand side is a function only of spatial gradients of \((U, V)\), which we have calculated in section 3. However, we found that \(x\) derivatives tend to be significantly smaller than \(y\) derivatives, which allows significant simplification (see also Hoskins and Draghici 1977).

Assume that variations alongfront \((\partial/\partial x)\) are much smaller than those across-front \((\partial/\partial y)\). The continuity
Fig. 13. Ageostrophic streamfunctions $\psi$ for legs (a) 2 and (b) 4. Regions of negative values are stippled. Arrows indicate the sense of circulation, such that high values of $\psi$ are on the left of the direction of travel.

The equation then allows us to define an ageostrophic streamfunction $\psi$ by

$$v_y = -\psi_z, \quad w = \psi_y. \quad (5.9)$$

Instead of solving the full omega equation (5.8), we can save one level of differentiation by inserting (5.9) into (5.7) to obtain

$$\frac{\partial}{\partial y} \left( N^2 \frac{\partial \psi}{\partial y} \right) + f^2 \frac{\partial^2 \psi}{\partial z^2} = Q^\gamma. \quad (5.10)$$

The Laplacian-like operator on the left-hand side is amenable to solution by relaxation in any $y$-$z$ plane with the assumption that there is no inflow ($\psi = 0$) across the boundaries; that is, $w = 0$ at the surface and 275 m (the deepest level for which we have data) and $v = 0$ on the north and south boundaries. These are defensible boundary conditions if the front lies east-west across the center of the survey area, as it does, and is a shallow feature. The solution for $w$ will tend to be parabolic in $z$ (5.8), and we have inferred that vertical velocities are maximum about 100–150 m deep (Fig. 11), so $w = 0$ at 275 m is not seriously unrealistic.

Fig. 14. Relative amplitudes of the second derivative terms in the omega equation (5.8) for (a) leg 1, 100 m (b) leg 4, 150 m confirm that the alongfront $(x)$ term is significantly weaker than the acrossfront and vertical terms.
These tests indicate that the solutions are consistent with the approximations, giving solutions that appear realistic except possibly close to the northern and southern boundaries. Therefore, let us examine their physical characteristics more closely.

The ageostrophic streamfunctions (Fig. 13) closely resemble the circulation inferred qualitatively from a potential vorticity argument (Fig. 12). We find, however, not one but a series of circulations corresponding to the banded structure in the potential vorticity field (Fig. 16), which in turn relates to the alternating cyclonic and anticyclonic shear. We shall discuss this further in section 6.

Figure 13 shows that vertical velocities are maximum at depths between 50 and 150 m. The streamfunction weakens rapidly below that depth, so it would be little changed if the lower boundary condition were applied slightly shallower or deeper, rather than the arbitrary depth of 275 m dictated by the maximum depth to which the SeaSoar was programmed to dive. Thus, we believe the depth variations of vertical velocity are well resolved. Figure 15a thus shows close to the maximum vertical velocities.

The omega equation calculation confirms our earlier estimate that vertical velocities are of order $10^{-4}$ m s$^{-1}$, with maxima of $4 \times 10^{-4}$ m s$^{-1}$, approximately 40 m day$^{-1}$. These amplitudes are an order of magnitude larger than those inferred by Leach et al. (1987), but in line with the estimates of Eriksen et al. (1991) and Voorhis and Bruce (1982). The importance of such large velocities has been discussed by Pollard and Regier (1990). Ageostrophic cross-frontal velocities are shown in Fig. 15b. They reach amplitudes of several cm s$^{-1}$, but do not show the banded structure of Fig. 15a. The streamfunction (Fig. 13) clarifies this difference. The presence of several adjacent circulation cells leads to cross-frontal flow that is positive (toward the main front) at depths shallow than about 100 m, and negative below that. The sense of this circulation is such as to reduce the frontal slope, thus counteracting the increase in baroclinicity caused by the large-scale confluence (Bleck et al. 1988). We believe this effect can be seen in Fig. 3, where the slope of the outcropping front decreases from leg 4 to leg 6. Large-scale confluence is greatest on leg 4 (Fig. 1b), and a strong ageostrophic circulation has been induced (Fig. 13b). On leg 6, 30 km farther west, we are effectively seeing the consequences of the ageostrophic circulation at a later time. The frontal slope has diminished (Fig. 3, leg 6), with surface waters drawn northward, as described in section 2. Figure 15b shows that the northward velocities are greatest on leg 4, and have virtually ceased by leg 6, so the maximum northward displacement of the surface water is 20–40 km.

The actual particle paths are helical. As a water parcel rises and moves across front at a speed of, say, 0.03 m s$^{-1}$, it is advected along the front by the geostrophic jet at up to 0.7 m s$^{-1}$. 
6. Discussion

Our findings are in complete agreement with those of Voorhis and Bruce (1982), who inferred in remarkable detail the structure of frontogenesis from XBT surveys, surface temperature maps, and drogued floats in the same location as FASINEX, in March 1977. They found shallow-intensified eddies, with wavelengths of order 500 km, straining the surface temperature field into elongated (100 km) tongues of alternating warm and cool water. They found that the eddies developed in a few days, and inferred ageostrophic circulations similar to those we have described, with vertical velocities of 30 to 50 m day$^{-1}$ and crossfront velocities of 3 to 5 km day$^{-1}$. They found the largest vertical velocities 100 to 200 m deep. They tracked a drogue whose relative vorticity changed from 0.3$f$ (cyclonic) to $-0.3$f$ (anticyclonic) in 60 h.

Voorhis and Bruce provide explanations for several of our findings. It is noteworthy that the small-scale eddies are typically 20–50 km in diameter. Voorhis and Bruce use Griffiths and Linden's (1981) laboratory experiments to infer that these are the fastest growing disturbances.

Leetmaa and Voorhis (1978) explored the connection between mesoscale motions and frontogenesis, and found that the front lay on the boundary of a mesoscale cyclonic eddy similar to that in Fig. 2. While the energy source for the initial disturbance is the available kinetic energy in the current shear (Griffiths and Linden 1981), the growing disturbance can extract potential energy from the large-scale fields created by winter cooling and deep mixing. This occurs in our observations as warm, light water rises and cools and heavier water sinks in the ageostrophic circulations.

Are the vertical circulations reversible? In principle, as we have described them, they are. We have invoked no mixing, and the stratification can alternately increase and decrease as water parcels move into regions of decreased or increased relative vorticity. In practice, the circulations are associated with 50-km eddies that are instabilities growing in time and being strained into bands by the alongfront flows. It would take a parcel weeks to complete one of the circuits in Fig. 13. In less than one week, it is likely that it will enter a region where the Froude number is large enough (Fig. 10f) for shear-driven mixing to occur (Toole and Schmitt 1987). Alternatively and probably more commonly, air–sea interaction on that time scale will mix or re-stratify the top 50–100 m, disrupting the front where it outcrops to the surface. The large and mesoscale confluences will sharpen the front, and the vertical circulations will reform.

More generally, over periods of weeks there will be a continuous interplay between air–sea interaction and mesoscale eddies, both modifying the structure of the upper ocean. The vertical circulations are continuously transporting some water downward to depths below the influence of surface forcing and simultaneously lifting other water upward, shallowing the thermocline to a depth where it is within range of surface forcing and can be either mixed by the wind or further stratified by surface heating. The net effect of the circulations is thus to diffuse properties across the thermocline.
The magnitude of the effective diffusion coefficient and its parameter dependence remain to be determined. They will depend not only on the season, stratification, and strength and frequency of wind-mixing and heating events, but also on the scales and strength of the mesoscale eddy field. From the magnitude of the circulations we have deduced, it seems likely that eddy-driven diffusion will be considerably more effective than that due to internal wave breaking in many circumstances.

When the property being diffused is biomass, there is the additional complication that the doubling time of phytoplankton may be of order a few days and is strongly dependent on the depth. We have observed streamers of high biomass, which must have originated in the top few tens of meters, extending downward along isopycnals to depths of over 200 m (Pollard et al. 1990; Read et al. 1991). The streamers are perhaps 10 km wide and 10–30 km apart, consistent with the circulations we have described. The vertical circulations may thus be central to understanding of the patchiness and rate of growth of spring blooms. The streamers extend deepest where the stratification is weak (but not zero) at the beginning of spring. If the energy source for the circulations is the potential energy available in the water column at the end of winter (Voorhis and Bruce 1982), they may also be central to a proper physical description of the spring restratification of the upper ocean (Woods 1985).

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