A Mechanistic Model of Ocean Interdecadal Thermohaline Oscillations*

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ABSTRACT

Oceanic interdecadal thermohaline oscillations are studied in the context of a conceptual two box model. In this mechanistic model, two essential processes associated with convection, subsurface advective heating and surface freshening, are modeled in terms of horizontal relaxation processes, with the surface relaxation timescale shorter than the subsurface one.

The model can oscillate with interdecadal periods within a certain parameter regime, and captures the fundamental characteristics of interdecadal oscillations: a clockwise trajectory in the $T-S$ plane. The dependence of the amplitude and period of the oscillation on different model parameters is explored. It is shown that there are two important parameters in the model: 1) $p$, the ratio of the timescale of the surface freshening process to that of the subsurface advective heating process, and 2) $q$, the ratio of the saline forcing to the thermal forcing in maintaining the vertical halocline and inverted thermocline structure. Oscillatory solutions exist when $p < q < 1$. The period of the oscillation increases with $q$ and decreases with $p$. When $p$ is small enough, the period of the oscillation is mostly determined by $q$. The amplitude of the oscillation increases with the period.

1. Introduction

This paper investigates what determines the period of interdecadal thermohaline oscillations, a question that has not been thoroughly explored. In a recent study of interdecadal thermohaline oscillations in an ocean general circulation model (OGCM), we showed that the ocean model can be in a state of interdecadal oscillations under steady forcing, and we gave a physical mechanism for these oscillations (Yin and Sarachik 1994; and references therein). In this paper, the mechanism is conceptualized in a two box model, where the parameter range may be more easily explored.

Welander (1982, 1986) carried out a comprehensive simple model study of ocean thermohaline oscillations. In a vertical two box model with unequal surface restoring times for temperature and salinity, he showed that the model can enter into a state of self-sustaining oscillations. The oscillation is basically a flip-flop type, in which vertical convection occurs periodically, homogenizing the temperature and salinity in the two boxes. Subsequent relaxation processes bring the system back to a state in which convection can occur again. Since the surface restoring timescale for salinity is longer than that for temperature, the trajectory in the $T-S$ plane is counterclockwise. Welander’s work built the foundation for flip-flop type thermohaline oscillations.

In this paper, Welander’s flip-flop idea is extended in a vertical two box model with specified surface freshwater flux (restoring surface boundary conditions were used in Welander’s study). It is shown that within the appropriate parameter regime, the reformulated model can oscillate with interdecadal periods and its main features are similar to those in an OGCM (Yin and Sarachik 1995). We also explore the period of the oscillation in the parameter space and show that the period is mainly determined by the vertical thermohaline structure and the ratio of the timescale of the surface advective freshening process to that of the subsurface advective heating process. The remainder of the paper is organized as follows: in section 2, we will give a brief description of the model; oscillatory solutions of the model and its parameter dependence are investigated in sections 3; in section 4 we summarize and conclude the paper.

2. Model formulation

In a previous study (Yin and Sarachik 1995), we described an advective and convective mechanism for interdecadal oscillations in an OGCM: horizontal advective heat transports from subtropical regions warm up the subsurface water in subpolar regions which enhance convection; convection in turn induces surface cyclonic and equatorward flows,

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which, together with horizontal diffusion and surface freshwater input, advect subpolar freshwater into convective regions to weaken or suppress convection. The periodic strengthening and weakening of convection caused by subsurface advective warming and surface-freshening processes in the subpolar region show up as interdecadal oscillations in the model. The main characteristics of these oscillations are that the timescale for the surface freshening is shorter than that for the subsurface heating and the trajectory in the $T$--$S$ plane is clockwise (see Figs. 1 and 2). This paper presents a box model based on the physical mechanism just described.

Figure 3 shows the structure of the two box model used in this study. The surface heat flux is parameterized by restoring $T_1$, the temperature of the top box, to a reference atmospheric temperature $T_a$ with a restoring time constant $k_r^{-1}$. The downward surface freshwater flux is $F$, and its equivalent upward salinity flux is $F_s$. In order to explore the model in the simplest possible context, both the surface horizontal freshening and subsurface horizontal advective heating will be modeled as relaxation processes. Thus, surface salinity is restored to a constant $S_0$ with a time constant $k_r^{-1}$, and subsurface temperature is restored to a constant temperature $T_0$ with a time constant $k_r^{-1}$. Since the OGCM result indicates that the halocline structure is mainly determined by the surface freshening, we fix the salinity value in the bottom box as $S_{20}$. The vertical exchange between the two boxes is also modeled as a relaxation process, with the time constant depending on the static stability of the two boxes. Let $\rho_1$ and $\rho_2$ be the densities of the top and bottom boxes. When $\rho_1 - \rho_2 > 0$, the time constant is small (large $k$) because convection acts rapidly and, when $\rho_1 - \rho_2 \leq 0$, the time constant is large (small $k$) since mixing is mainly due to slow vertical advection and diffusion. After making these assumptions, the governing equations for the temperature and salinity, $T_1$, $T_2$, and $S_1$, can be written as the following:

$$\frac{dT_1}{dt} = -k_0(T_1 - T_a) - k(\rho)(T_1 - T_2),$$

$$\frac{dT_2}{dt} = -k_r(T_2 - T_0) - c k(\rho)(T_2 - T_1),$$

and

$$\frac{dS_1}{dt} = -F_s - k_s(S_1 - S_0) - k(\rho)(S_1 - S_{20}).$$

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**Fig. 1.** Time series of temperature difference between 25 and 300 m ($^\circ$C, dashed) and salinity (PSU, solid) difference between 25 m and 500 m over the 50-year synchronous integration of a sector ocean general circulation model that is in a state of interdecadal oscillations. Four water columns are respectively at longitudes 35$^\circ$ (a), 40$^\circ$ (b), 45$^\circ$ (c), and 50$^\circ$ (d) in a slab around 56$^\circ$N where large convective variations occur. Note that the timescale for salinity is shorter than that for temperature.
We use a linear equation of state:

$$\rho = -\alpha(T_1 - T_2) + \gamma(S_1 - S_2),$$  \hspace{1cm} (4)

because, as will be confirmed later, the nonlinearity is not essential for oscillatory solutions in the model. Here $\rho = \rho_1 - \rho_2$, $k(\rho) = k_2$ for $\rho > 0$, and $k(\rho) = k_1$ for $\rho \leq 0$, with $k_2 \gg k_1$; $\alpha$ and $\gamma$ are the thermal expansion and salt contraction coefficients; $c = d_1/d_2$, the vertical thickness ratio of the two boxes (see Fig. 3).

Note that $k_0^{-1}$ is on the order of a few months if $d_1$ is on the order of $50-10^2$ m, while $k_1^{-1}$ and $k_2^{-1}$ are both on interdecadal timescales since they are the horizontal advective timescales. Here $k_1^{-1}$ is the vertical exchange timescale, which is on the order of $10^2-10^3$ years. Thus, $k_0 \gg (k_1, k_2}$, $k_1 \ll (k_1, k_2)$, and $k_2 \gg (k_0, k_1, k_2)$.

To explore the nature of any oscillatory solutions and to understand how the period of the oscillation depends on the parameters in the system, we first nondimensionalize Eqs. (1)–(4). Defining

$$\hat{T}_1 = T_1 - T_a, \quad T = T_1 - T_2, \quad S = S_1 - S_{20},$$

$$Q_T = T_0 - T_a, \quad Q_S = F_1/S_{20} - S_0,$$

$$p_0 = k_0/k_T, \quad p = k_T/k_S, \quad and \quad q = \frac{\gamma Q_S}{\alpha Q_T},$$  \hspace{1cm} (5)

and introducing scaling

$$\hat{T}_1, \hat{T} \sim Q_T; \quad S \sim Q_S; \quad t \sim k_T^{-1};$$

$$\rho \sim \alpha Q_T; \quad k_1, k_2, k_3 \sim k_T,$$

we obtain the following

$$\frac{dT}{dt} = -1 - (p_0 - 1)\hat{T}_1 - [1 + (1 + c)k(\rho)]T,$$  \hspace{1cm} (7)

$$\frac{dS}{dt} = -\frac{1}{p} - \left[\frac{1}{p} + k(\rho)\right]S,$$  \hspace{1cm} (8)

$$\frac{d\hat{T}_1}{dt} = -p_0\hat{T}_1 - k(\rho)T, \quad and$$

$$\rho = -T + qS.$$  \hspace{1cm} (9)

Note that $\hat{T} = k \ll 1$ for $\rho \leq 0$ and $k = k_2 \gg 1$ for $\rho > 0$. Clearly, there are six parameters in this system: $p_0, p, q, k_1, k_2$, and $c$. Note that the time is scaled by the subsurface heating timescale. For a water column with a halocline and inverted thermocline structure, $Q_S > 0$ and $Q_T > 0$. From now on, all variables will be nondimensional.

3. Oscillatory solutions

System (7)–(10) has two equilibria for $\bar{k} = k_1$ and $\bar{k} = k_2$:
Since $p_0 \gg 1$, $k_2 \gg 1$, $k_1 \ll 1$, $c = O(1)$ and $p = O(1)$, (13) can also be simplified as $p/c < q < 1$, which is depicted in Fig. 5. Note that the density structures shown in Fig. 4 are given by (11), but Fig. 4b corresponds to the parameter regime defined by (13), whereas Fig. 4a is within the parameter regime excluded by (13).

Condition (13) is a sufficient one for the system (7)–(10) to have a limit cycle solution. Specifically, when (13) is satisfied, the relaxation process ($k = k_1 \ll 1$) always bring the system from a state of $\rho \leq 0$ to that of $\rho > 0$. Then convection ($k = k_2 \gg 1$) starts to occur and quickly restores the system back to a state of $\rho \approx 0$.

Note that the convection process is much faster than the relaxation process since $k_1 \ll 1$ and $k_2 \gg 1$. For simplicity, we can take $k_3 = \infty$. Then the convective step lasts only a very short time $p_0^{-1} \ll 1$ (see the appendix for related analysis). Therefore, only the final state of the convective step is necessary for calculating

\[
\tilde{T} = -1/[1 + (c + 1/p_0)\tilde{k}],
\]
\[
\tilde{T}_1 = -\frac{\tilde{k}}{p_0} \tilde{T},
\]
\[
\tilde{S} = -1/(1 + p\tilde{k}),
\]

and
\[
\tilde{\rho} = 1/[1 + (c + 1/p_0)\tilde{k}] - q/(1 + p\tilde{k}).
\] (11)

A simple linear stability analysis shows that these two equilibria are stable, which is physically understood since there are only relaxation processes in this system. As expected, at equilibrium (if it can be achieved), there is a halocline and inverted thermocline structure: $\tilde{T} < 0$ and $\tilde{S} < 0$, and the surface heat flux is upward (heat loss): $\tilde{T}_1 > 0$ [see Eq. (5)].

Figure 4a shows the two equilibria schematically in terms of density. One ($\tilde{k} = k_1$) is in an advective and diffusive balance (no convection), and the other ($\tilde{k} = k_2$) is in a state of constant convection.

It is clear that the system will oscillate as a limit cycle if none of the two equilibria shown in Fig. 4a can be realized; that is, if

\[
\tilde{\rho}(k_1) > 0 \quad \text{and} \quad \tilde{\rho}(k_2) < 0.
\] (12)

Figure 4b shows schematically a density structure that does not support any equilibria and thus satisfies (12). The condition for oscillatory solutions can be derived from (11) and (12) as the following:

\[
\frac{1 + pk_2}{1 + (c + 1/p_0)k_2} < q < \frac{1 + pk_1}{1 + (c + 1/p_0)k_1}.
\] (13)
the interdecadal oscillation period $\tau \sim O(1)$ since this final state provides an initial condition for the relaxation process.

An approximate solution of the relaxation process is given in (A5). In deriving (A5), approximations $p_0 \gg 1$, $p = O(1)$, $c = O(1)$, $k_1 \ll 1$, and $k_2 = \infty$ have all been used. The period of the oscillation $\tau$ is determined by

$$\rho|_{t = \tau} = 0,$$

where $\rho$ is the density during the relaxation step. Using (10), (14) can be written as

$$T|_{t = \tau} = qS|_{t = \tau}.$$

Here $T$ and $S$ are similarly the temperature and salinity during the relaxation process. Substituting (A5) into (15), we obtain

$$1 - \exp(-\tau) + \frac{1 + c}{cp_0} \exp(-p_0\tau) = q \left[ 1 - \exp\left(-\frac{\tau}{p}\right) \right],$$

which is the equation that determines the oscillation period $\tau$. Since $p_0 \gg 1$, $\tau$ mainly depends on

$$q = \frac{\gamma Q_s}{\alpha Q_T},$$

and

$$p = \frac{k_T}{k_s}.$$  

Figure 6 shows $\tau$ in the $q-p$ plane for $p_0 = 100$ and $c = 1$ as determined by (16). The period of the oscillation increases with $q$ and decreases with $p$. For $p \ll 1$, $\tau$ is mainly determined by $q$ and approaches infinity as $q$ goes to 1. This can be understood physically since it takes a longer time to reach convective critical condition $\rho > 0^+$ when $p$ is smaller or when $q$ is larger.

Note that the oscillation period shown in Fig. 6 (where $c = 1$) is the time between two consecutive convective events. Here $q = p/c$ and $q = 1$ are bifurcation lines of the model, across which the behavior of the model changes. Transition from $q = (p/c)^-$ to $q = p/c$ corresponds to a change from an oscillation state sketched in Fig. 4b to an equilibrium in Fig. 4a at $k_2$ (constant convection), while from $q = 1^-$ to $q = 1$, it corresponds to a transition from oscillations in Fig. 4b to equilibria in Fig. 4a at $k_1$ (no convection). Following each convective event, $T$ and $S$ are changed to nonzero values by the relaxation process. But when the model parameters are very close to $q = (p/c)^-$, these changes become very small, and the periods are also near zero. Thus the oscillation becomes indistinguishable from a steady state. The peculiarity of this transition from an oscillation with near infinite frequency and near zero amplitude to an equilibrium across the bifurcation line $q = (p/c)^-$ is related to the density structure of the system. As $k_2$ approaches infinity, the densities shown both in Figs. 4a and 4b become zero, indistinguishable from each other.

The oscillation time periods shown in Fig. 6 are scaled by the subsurface heating timescale, and that they are interdecadal since $k_T^{-1}$ is the subsurface horizontal advective heating timescale (the time it takes for the water column to reach a convective critical state.

![Figure 5](image1.png)

**Fig. 5.** Parameter regime of limit cycle states, $p < q < 1$, in $q-p$ plane.

![Figure 6](image2.png)

**Fig. 6.** Time periods between two consecutive convective events, shown in the same parameter regime as in Fig. 5. Here $p_0 = 100$, $k_1 = 0$, and $c = 1$. Note that $q = p$ and $q = 1$ are the bifurcation lines of the model, across which the behavior of the model changes from oscillations to equilibria. The transition across $q = p$ is from an oscillation with near infinite frequency and near zero amplitude to an equilibrium.
due to subsurface warming). It is interesting that if $k_r^{-1}$ is taken to be the subsurface diffusive heating time, these periods will be on the order of $10^3$ years. In deep decoupling thermohaline oscillations, subsurface heating during the phase of collapsed oscillations is diffusive, so the period of the oscillations is millennial (Winton and Sarachik 1993).

Figures 7 and 8 depict the oscillatory solutions (A5) and the corresponding trajectories in the $T$--$S$ plane at four points a–d in the $p$--$q$ plane as marked in Fig. 5. The amplitude of the oscillation increases with the period, and approaches 1 when the period is large enough. The trajectories are all clockwise, with convection bringing $T$ and $S$ to zero, and the subsequent relaxation restoring it to the critical state of $\rho = 0^\circ$. A comparison of Figs. 1, 2, 7, and 8 reveals that this simple conceptual model captures the essential characteristics of interdecadal oscillations in the OGCM of Yin and Sarachik (1995).

4. Summary and concluding remarks

In this paper, the dependence of the period of ocean interdecadal thermohaline oscillations on the model parameters is explored within the context of a two box model. This simple box model is formulated based on an advective and convective mechanism of oceanic interdecadal thermohaline oscillations as discussed in a study with an OGCM (Yin and Sarachik 1995). The two essential processes of subsurface horizontal advective heating and surface freshening tend to maintain a halocline and inverted thermocline structure in the subpolar region; this vertical structure of the water column is so vulnerable to convection that periodic convection can occur to vertically homogenize the temperature and salinity, and as a consequence, flip-flop interdecadal oscillations occur.

The idea of flip-flop thermohaline oscillations was originally proposed by Welander in a vertical two box model with larger surface restoring time constant for salinity than for temperature (Welander 1982). In this paper, Welander’s idea has been extended to a two vertical box model, in which there is a downward surface freshwater flux and a surface heat flux parameterized by restoring the temperature of the top box to a reference value. Since the main purpose is to gain insight into what determines the period of the oscillations, subsurface heating and surface freshening are parameterized as relaxation processes, with restoring time constant for surface salinity shorter than that for subsurface temperature. The salinity value in the lower box is fixed at a constant value since the vertical halocline structure is mainly determined by surface freshening (Yin and Sarachik 1995).

The physical characteristics of the simple two-box model depend primarily on two parameters: 1) $p$, the
ratio of the timescale of the surface advective freshening process to that of the subsurface advective heating process and 2) $q$, the ratio of saline to thermal forcings in determining the halocline and inverted thermocline structure. There are two stable equilibria in this system: one entirely without convection and the other with continual convection. When $p < q < 1$, these two equilibria cannot be reached, and the system oscillates. The period of these oscillations increases with $q$ and decreases with $p$, but depends mainly on $q$ when $p$ is small. The amplitude of the oscillation also increases with the oscillation period. The trajectories in the $T-S$ plane are clockwise, and are in the same sense as those in an OGCM. The nature of these oscillations is flip-flop, or advective and convective.

In the two-box model, the period of the oscillations is basically the relaxation time between two consecutive convective events. Since this time period depends on the relative strength and effectiveness of subsurface heating and surface freshening in maintaining the halocline and inverted thermocline structure, the period of the oscillations is not simply the time given by advection over a distance. This distinguishes the advective and convective mechanism from the advective mechanism proposed by Weaver and Sarachik (1991). Of course, in an OGCM, surface freshening during the strong convective phase is enhanced by convection due to an increase in surface meridional salinity gradient (see Yin and Sarachik 1995), and this process is not included in the two-box model but it could be effectively parameterized by increasing $Q_s$ in (5). Note that although the oscillatory condition (13) can be simplified as $p/c < q < 1$ with realistic parameters, (13) may not be satisfied in the extreme that $p_0 \ll 1$. In this case, the surface restoring timescale is so long that the critical condition $p = 0^+$ cannot be reached, and the model will stay in an advective and diffusive equilibrium.

The conceptual model presented in this paper may be generalized. In their study of deep decoupling thermohaline oscillations, Winton and Sarachik (1993) and Winton (1993) have proposed a subsurface heating and surface freshening mechanism for the $10^3$ year oscillations. Although the dominant processes involved in maintaining the vertical halocline and inverted thermocline structure in interdecadal and millennial oscillations may be different (collapsed thermohaline overturning is necessary to introduce diffusive warming of the deep ocean in the deep decoupling oscillation, Winton and Sarachik 1993), hence a different manifestation of ocean circulations, the essence of these oscillations is flip-flop—that is, no equilibria can exist within the parameter regime.

The model described in this paper is a mechanistic one. It contains the essential physical processes of advective and convective interdecadal oscillations in an OGCM (Yin and Sarachik 1995) and gives a simplified interpretation of these oscillations.
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APPENDIX

Solution of Relaxation Processes

The governing equations for $T$, $S$, and $\tilde{T}_1$, derived from (7)–(9), can be written as the following:

\[
\frac{1}{p_0} \frac{d^2}{dt^2} \left[ \frac{T}{\tilde{T}_1} \right] + \left( 1 + \frac{1 + (1 + c)k}{p_0} \right) \frac{d}{dt} \left[ \frac{T}{\tilde{T}_1} \right] + \left[ 1 + \left( \frac{1}{p_0} + c \right) k \right] \left[ \frac{T}{\tilde{T}_1} \right] = \left[ \frac{-1}{kl/p_0} \right] \\
\frac{dS}{dt} = -\frac{1}{p} \left( \frac{1}{p} + k \right) S, \tag{A1}
\]

where $k$ is a constant: $k = k_1 \ll 1$ for relaxation step, and $k = k_2 \gg 1$ for convective step. For simplicity, we consider only one oscillation period. If $t = 0$ is defined to be the beginning of each process, initial conditions for the convective process can be written as

\[
T|_{t=0} = T_r, \\
\frac{dT}{dt} \bigg|_{t=0} = -1 - [1 + (1 + c)k_2]T_r, \\
\tilde{T}_1|_{t=0} = 0, \\
\frac{d\tilde{T}_1}{dt} \bigg|_{t=0} = -k_2 T_r,
\]

and

\[
S|_{t=0} = S_r, \tag{A2}
\]

where $T$, and $S$, are the values of $T$ and $S$ at the end of the relaxation process. Note that $\tilde{T}_1$ is zero at the end of the relaxation process, and this will become clear in the following.

The approximate solution to (A1) for the convective step, as $k_2 \rightarrow \infty$, can be written as

\[
T = T_r \exp\left[ -(1 + c)k_2 t \right] - \frac{1}{ck_2},
\]

\[
\tilde{T}_1 = \frac{-T_r}{1 + c} \exp\left( -\frac{c}{1 + c} p_0 t \right) + \frac{1}{p_0 c},
\]

and

\[
S = \left( S_r + \frac{1}{pk_2} \right) \exp(-k_2 t) - \frac{1}{pk_2}, \tag{A3}
\]

where $p_0 \gg 1$, $p = O(1)$, and $c = O(1)$ have been used. Note that the approximation involved in (A3) is only in the exponent, which is $O(p_0/k_2)$. Clearly, the convective step lasts a very short time $p_0^{-1} \ll 1$, and for calculating the interdecadal oscillation period $\tau \sim O(1)$, the final state of the convective step is adequate. Taking $t, k_2 \rightarrow \infty$ in (A3), we obtain

\[
T = 0, \quad S = 0, \quad \tilde{T}_1 = \frac{1}{p_0 c}. \tag{A4}
\]

Note that (A4) represents the final state of the convective step as $k_2 \rightarrow \infty$, and it is also the initial state of the relaxation process.

Using (A4) as the initial condition and taking into account of $k_1 \ll 1$, $p_0 \gg 1$, $p = O(1)$, and $c = O(1)$, the approximate solution to (A1) for the relaxation step ($k = k_1 \ll 1$) can be obtained as the following:

\[
T = \exp(-t) - 1 - \frac{1 + c}{cp_0} \exp(-p_0 t),
\]

\[
\tilde{T}_1 = \frac{1}{cp_0} \exp(-p_0 t),
\]

and

\[
S = \exp\left( -\frac{t}{p} \right) - 1. \tag{A5}
\]

Again the error involved in (A5) is only in the exponent, which is $O(p_0^{-1}, k_1)$. Note that (A5) is also an exact solution to (A1) with $k_i = 0$.

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