Driving Mechanism of Band Structure of Mean Current over the Continental Shelf

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ABSTRACT

A two-layer numerical model is used to investigate the continental-shelf circulation forced by western boundary currents along shelf edges. The East China Sea (see Fig. 1) with the Kuroshio along the shelf edge provides a good example for the issue. Probably the simplest idea is a branching of western boundary currents by linear friction at the sea floor. Consider a thought experiment whose momentum equation (without the stratification, and without time dependency) is described as

\[ \hat{k} \times \mathbf{u} = -\rho_0 \nabla P - r_b \mathbf{u}. \]  

(1.1)

Here \( \hat{k} \) is a unit vector in the local vertical direction, \( \mathbf{u} \) \( = (u, v) \) is the current velocities, and \( r_b \) is the coefficient of bottom friction; otherwise, notation is standard. For instance, Minato and Kimura (1980) adopt the above formulation to explain the generation of the Tsushima Warm Current, a branch of the Kuroshio. Cross differentiating (1.1), we obtain

\[ J(D^{-1}, \psi) = \nabla \cdot (r_b D^{-1} \nabla \psi), \]  

(1.2)

where \( D \) is the depth, \( \psi \) is the (volume transport) streamfunction, and \( J(A, B) \) is the Jacobian \( = A_y B_x - A_x B_y \). Equation (1.2) provides a straightforward interpretation for the branching. The left-hand side of (1.2) states that the streamfunction is “advected” by the characteristic velocity whose “streamfunction” is \( fD^{-1} \), that is, the geostrophic contour. The right-hand side states that the streamfunction is “diffused” with a diffusivity of \( r_b D^{-1} \). Given a western boundary current along the shelf edge, streamfunctions have to be “diluted” toward the shallow shelf because of momentum transfer by the dissipation on the right-hand side of (1.2). If this is the case, it is likely that weak currents in the same direction as the western boundary current are found over the shelf.

It is interesting that the above speculation contradicts observational facts. Katoh et al. (2000) provide the most cogent result. Based on shipboard acoustic Doppler current profiler (ADCP) observations over the East China Sea shelf in summer, they elucidate detided current distributions as shown in Fig. 1. As expected, intense Kuroshio currents are revealed along the shelf edge (i.e., 200-m isobath). However, the currents do not weaken gradually toward the shallow portion, but form alternating bands (hereinafter: “band structure”) over the shelf. A band of intense currents is identified along the isobath of 100 m. The currents are called the Kuroshio Branch (Katoh et al. 2000). Of more interest than the Kuroshio Branch are countercurrents between the Kuroshio and Kuroshio Branch, which are unlikely to be explained with the branching mechanism mentioned above. There remain uncertainties in the mean current field on the shelf of the East China Sea because long-term observations have been rarely conducted; Katoh et al. (2000) provided daily mean currents on each observation day in summer. Nevertheless, looking at con-
The path of the Kuroshio is shown by the gray curve schematically. Also shown is the bottom topography. Contour interval is 1 km except for contour lines of 100 and 200 m. (bottom) Daily mean currents obtained by the shipboard ADCP within the square (after Katoh et al. 2000). Trajectories of satellite-track drifters are also shown by solid curves; see Katoh et al. (2000) for details.

Is the band structure a physically plausible solution in shelf circulations? What is the physical process that gives rise to the alternating bands of mean current on the shelf? They are questions of particular interest in the present study. Momentum of the western boundary current has to be transferred onto the shelf in a different way than the linear dissipation in (1.1). In the present study, special attention is given to momentum transfer by eddies, especially by frontal eddies (or frontal waves) that are also ubiquitous in western boundary currents.

Section 2 provides a detailed description of the momentum transfer by eddies. In this study, the generation of the band structure will be demonstrated by a two-layer numerical model. A model description is presented in section 3. Results of the modeling are shown in section 4. A physical interpretation for the generation of the band structure is provided in section 5. Section 6 gives a summary.

2. Momentum transfer by eddies

The vertical momentum transfer by eddies has been introduced by Rhines and Young (1982), Masuda and Uehara (1992), and Masuda and Mizuta (1995) to drive the abyssal circulation. The process is called the “effect by geostrophic eddies” by Rhines and Young (1982), and “diffusive stretching” by Masuda and Mizuta (1995). Gent and McWilliams (1990) also argue the transfer in a zonally uniform channel (see their appendix). Furthermore, Greatbatch (1998) provides a generalized discussion. For a straightforward interpretation, we herein demonstrate the vertical momentum transfer by eddies in a two-layer model. Although eddies cause horizontal momentum transfer as well, the vertical transfer is crucial to generate the band structure over the shelf, as will be shown later (see sections 5a and 5b).

We start with the vorticity and continuity equations in the two-layer model. They are expressed as

\[ \frac{\partial \xi_k}{\partial t} + \nabla \cdot (u \xi_k) + f \nabla \cdot u = 0 \quad \text{and} \quad (2.1) \]

\[ \frac{\partial h_k}{\partial t} + \nabla \cdot (u h_k) = 0, \quad (2.2) \]

where a subscript \( k \) denotes layers (upper layer = 1), \( h \) is the layer thickness, and \( \xi \) is the relative vorticity defined as \( \partial u/\partial x - \partial v/\partial y \). Friction and horizontal viscosity are neglected in (2.1) for simplicity, although they are included in numerical models in subsequent sections.

Here a variable \( A \) is divided into the mean and eddy parts as follows:

\[ A = \bar{A} + A', \quad (2.3) \]

where an overbar denotes a long-term (much longer than the time taken for an eddy to pass over) mean, and a prime denotes an eddy part, that is, an anomaly from the mean part. Substituting (2.3) into (2.2) and averaging temporally, one finds

\[ \nabla \cdot (u \bar{h}_k) + \nabla \cdot \bar{u} h'_k = 0. \quad (2.4) \]

Introducing a horizontal diffusivity \( \kappa \) of the layer thick-
ness, we are able to convert the second term of (2.4) into a down-gradient Fickian diffusion as
$$\mathbf{u}_l^t \cdot \nabla \bar{h}_l = -\kappa_l \nabla \bar{h}_l. \tag{2.5}$$

The above parameterization has been used in many studies on the eddy-induced transport (e.g., Gent and McWilliams 1990). The diffusivity $\kappa_l$ represents eddy activity in the $l$th layer. Substitution of (2.5) into the second term of (2.4) yields
$$\nabla \cdot (\mathbf{u}_l \bar{h}_l) = \nabla \cdot (\kappa_l \nabla \bar{h}_l). \tag{2.6}$$

We now return to the vorticity equation [(2.1)]. As procedures for the continuity equation, substituting (2.3) into (2.1), and averaging temporally, one finds
$$\nabla \cdot (\mathbf{u}_l \bar{\zeta}_l) + \nabla \cdot \mathbf{u}_l^t \bar{\zeta}_l + f \nabla \cdot \mathbf{u}_l = 0. \tag{2.7}$$

The second term on the left-hand side can be rewritten as $-\nabla \cdot (\lambda \nabla \bar{\zeta}_l)$, where $\lambda$ is a horizontal viscosity of the vorticity. Such a horizontal viscous term represents horizontal vorticity (hence, momentum) transfer by eddies. The horizontal viscosity is neglected again for simplicity. Thereafter, eliminating the divergence of $\mathbf{u}_l$ in the third term of (2.7) by (2.6), we have a vorticity equation in the upper layer as follows:
$$\nabla \cdot (\mathbf{u}_l \bar{\zeta}_l) + \mathbf{U}_l \cdot \nabla (f \bar{h}_l^{-1}) = -f \bar{h}_l \nabla \cdot (\kappa_l \nabla \bar{h}_l), \tag{2.8}$$

where $\mathbf{U}_l = (\mathbf{u}_l \bar{h}_l, \mathbf{v}_l \bar{h}_l)$ is the volume transport of the mean current in the upper layer. Assuming geostrophy, we are able to rewrite gradients of the upper-layer thickness on the right-hand side of (2.8) as
$$\nabla \bar{h}_l = \left[ \frac{f}{g'} (\mathbf{v}_l - \mathbf{v}_\sigma), \frac{f}{g'} (\mathbf{u}_l - \mathbf{u}_\sigma) \right], \tag{2.9}$$

where $g'$ is a reduced gravity in the two-layer model, and a subscript $g$ denotes the geostrophic part of mean currents. Substitution of (2.9) into the right-hand side of (2.8) leads to a final form of the vorticity equation in the upper layer as follows:
$$\nabla \cdot (\mathbf{u}_l \bar{\zeta}_l) + \mathbf{U}_l \cdot \nabla (f \bar{h}_l^{-1}) = -\frac{1}{\rho_0 \bar{h}_l} \text{curl} \mathbf{u}_l, \tag{2.10}$$

where $\mathbf{u}_l = f^2 \rho_0 \kappa_l g' \left[ (\mathbf{v}_l - \mathbf{v}_\sigma), (\mathbf{u}_l - \mathbf{u}_\sigma) \right]$. Here $\mathbf{u}_l$ is the “eddy stress” (Greatbatch 1998) acting on the upper layer of the two-layer model. In a similar way, the vorticity equation in the lower layer is
$$\nabla \cdot (\mathbf{u}_2 \bar{\zeta}_2) + \mathbf{U}_2 \cdot \nabla (f \bar{h}_2^{-1}) = -\frac{1}{\rho_0 \bar{h}_2} \text{curl} \mathbf{u}_2, \tag{2.11}$$

where $\mathbf{u}_2 = f^2 \rho_0 \kappa_l g' \left[ (\mathbf{v}_l - \mathbf{v}_\sigma), (\mathbf{u}_l - \mathbf{u}_\sigma) \right]$. Terms on right-hand sides of (2.10) and (2.11) denote the vertical vorticity (hence, momentum) transfer by eddies. In line with Masuda and Mizuta (1995), we hereinafter refer to the processes as the “diffusive stretching.” In addition, we refer to the terms multiplied by $\rho_0 \bar{h}_l$ as the “diffusive stretching term.” If diffusivities in two layers are identical (i.e., $\kappa_1 = \kappa_2$), the diffusive stretching term in one layer becomes the mirror image of another. The terms will be estimated directly by a two-layer numerical model in subsequent sections.

3. Model description

To provide a straightforward interpretation for the current circulation induced by eddies, a rectangular ocean (1200 km $\times$ 700 km) with a shelf edge is adopted in this study. Figure 2 shows the topography of the model ocean. We use Cartesian coordinates, where the $x$ and $y$ axes are in the along-shelf and cross-shelf directions,
respectively. Hereinafter, the positive and negative x directions are referred to as the downstream and upstream directions, respectively. The shelf edge is located at 200 km (i.e., \( y_0 = 200 \) km) from the right boundary. The depth \( D \) of the shelf slope is modeled as

\[
D(y) = 300 - 20 \tan \{\alpha(y - y_0)\}, \quad \text{where} \\
\alpha = \begin{cases} 
0.0064864 & \text{for } y (\text{km}) > y_0 \\
0.064864 & \text{for } y (\text{km}) < y_0.
\end{cases} \tag{3.1}
\]

Except where otherwise stated, the unit in (3.1) is meters. The dimension and topography roughly represent the East China Sea. The model domain is surrounded by free-slip sidewalls except two open boundaries (from \( y = 0 \) to \( y = 300 \) km at both ends), where a periodic boundary condition is applied to avoid the generation of disturbances around the boundaries; all variables at one end are utilized for boundary conditions at another end. The model is on an \( f \) plane because the model domain is relatively narrow, and because the topographic \( \beta (O(10^{-12}) \text{ cm}^{-1} \text{ s}^{-1}) \) over the shelf is one order of magnitude larger than the planetary \( \beta \) in midlatitude.

We divide the model ocean into two active layers. Gaussian-shaped currents with an \( e \)-folding scale of 50 km are initially given only in the upper layer (see Fig. 2). The current velocity is maximal (=100 cm s\(^{-1}\)) at the shelf edge. The initial condition of the interface displacement is determined by geostrophy such that the lower layer is at rest. The currents represent a western boundary current along the shelf edge. For convenience, we refer to the currents set initially as the western boundary current. A part of currents runs on the shelf as Kuroshio currents of Fig. 1. As shown in Fig. 2, the slope of the interface is steeper than the bottom slope of the shelf. The steep slope of the interface results in baroclinic instability, a major cause of the frontal wave in western boundary currents [e.g., James et al. (1999) for the Kuroshio in the East China Sea and Xue and Mellor (1993) for the Gulf Stream in the South Atlantic Bight].

At both upstream and downstream ends of the model domain, we set sponge layers (see stippling of Fig. 2) in which variables are restored to their initial conditions within one day. Note that the periodic boundary condition is applied simultaneously. When disturbances and/or eddies excited in the model enter the sponge layer, they are absorbed into the layer. Part of the disturbances at the downstream boundary (\( x = 1100 \) km) might propagate along the left sidewall as the Kelvin wave. Another sponge layer is set along the left sidewall to remove the disturbances.

As mentioned above, model results in the sponge layers are restored to initial conditions. Hence, inflow and outflow conditions are maintained only in the upper layer during the calculation. Our model is driven only by the conditions.

Governing equations are described in the appendix.

In integrating the equations numerically, as expected, we find frontal waves developing in the currents along the shelf edge. After some time, an equilibrium state with eddies will be reached, and it is this state on which we will focus. Parameters used in the model are listed in Table 1.

### 4. Results

We first show an experiment without friction at both the bottom and layer interface. Influences of friction are investigated separately in section 5d. Were it not for the diffusive stretching term of (2.11), long-term mean currents are never driven in the lower layer, because interfacial friction is absent and because a state of no motion is prescribed in the lower layer of sponge layers around both open boundaries.

Figure 3 shows potential vorticity fields in the upper layer from the beginning of the calculation. Also shown is the interfacial displacement averaged in the \( x \) direction. As expected, frontal waves with a wavelength of 200 km or more are revealed in the western boundary current on day 25. Four wave crests are identified at \( x = 180, 400, 600, \) and 800 km, respectively; the portion where offshore (shelf) water extends toward the shelf (offshore) is referred to as the crest (trough) in this study. The waves show the folded-wave pattern that is a characteristic of mature frontal waves (e.g., Lee et al. 1981). The waves on day 25 grow systematically as they go downstream. However, the equilibrium state has not been accomplished yet. In integrating the model for a long time, one finds the potential vorticity field very different from the field on day 25.

On day 2025, a double-frontal structure of the potential vorticity is visible on the shelf. One is a front for the western boundary current along the shelf edge, and another is located at \( y = 400 \) km. The interfacial displacement also shows the double-frontal structure. Frontal waves are found between two fronts (e.g., a wavy pattern around \( x = 500 \) km, \( y = 200–400 \) km). Isolated water masses originating from the shelf and offshore regions are coexisting between two fronts. We count day 2000 as day 0 in the following descriptions because results before reaching the equilibrium state are not focused in this study. Figure 4 shows a space–time plot of the upper-layer potential vorticity along \( y = 250 \) km. Results from “day 0” to “day 250” are depicted here. The potential vorticity varies periodically because frontal waves propagate with a phase speed of 12 cm s\(^{-1}\). The period is around 25 days. The periodic variation after day 0 clearly shows the equilibrium state.

### Table 1. Parameters used in this study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coriolis parameter ( f (\text{s}^{-1}) )</td>
<td>6.38 \times 10^{-4}</td>
</tr>
<tr>
<td>Horizontal viscosity ( \nu (\text{cm}^2 \text{s}^{-1}) )</td>
<td>2 \times 10^4</td>
</tr>
<tr>
<td>Reduced gravity ( g’ (\text{cm}^2 \text{s}^{-1}) )</td>
<td>2.0</td>
</tr>
<tr>
<td>Grid spacing ( \Delta x, \Delta y (\text{km}) )</td>
<td>5.0</td>
</tr>
</tbody>
</table>
Fig. 3. (top) Horizontal distributions of the potential vorticity on days 0, 25, and 2025. The same tone pattern is used for the potential vorticity in the three panels. Sponge layers are omitted. Two broken lines show the area where currents are imposed initially. (bottom) The interfacial displacement averaged in the $x$ direction. Broken curve represents the initial condition of the interface.

Fig. 4. Space–time plot of the potential vorticity along $y = 250$ km. The same tone pattern as in Fig. 3 is used. Boldface broken line indicates the phase speed of the frontal wave.

4, the signal is very weak from $x = 0$ to 200 km because undisturbed water is injected from the upstream boundary (i.e., $x = 0$ km), and because the fastest-growing wave in the model does not grow sufficiently until it reaches the location around $x = 200$ km. In addition, the variation is interrupted around $x = 700$ km. This means that the mature frontal eddies break in the downstream portion and that new eddies grow there.

Snapshots of the along-shelf current in the upper layer are shown in Fig. 5 every 4 days during 24 days (around one period of the frontal wave). As observed by Katoh et al. (2000) on the shelf of the East China Sea, alternating bands of the along-shelf current are always revealed on the shelf. Although currents in the sponge layer are not shown here, they are restored to the initial (i.e., inflow–outflow) condition in the sponge layer. To show the band structure of currents clearly, we obtain mean currents by averaging results from day 0 to day 250 (i.e., 10 times of the wave period). The mean currents are shown in Fig. 6. Initial currents are removed from panels of the upper-layer and vertically averaged currents to emphasize mean currents excited in the course of the calculation. The mean currents take the form of the band structure both in the upper and lower layers. Countercurrents appear along the left fringe of the western boundary current. In addition, currents with the same direction as the western boundary current are revealed at $y = 400$ km. It is likely that the currents correspond to the Kuroshio Branch on the shelf of the East China Sea. Furthermore, in both the upper and lower layers, countercurrents are visible along the center axis of the western boundary current and along the right boundary of the model domain.
Figure 5. Snapshots of the along-shelf current in the upper layer every 4 days from day 4 to day 24. Sponge layers are omitted. Stippling has been chosen to emphasize countercurrents (toward the negative $x$ direction). Contour interval is 20 cm s$^{-1}$.

The current speed on the shelf is comparable to the observational results obtained by Katoh et al. (2000). Upper-layer currents are stronger than lower-layer currents on the shelf. Bottom friction is absent in the model, and so the vertical shear of currents must be caused by baroclinicity.

We again emphasize that mean currents are never driven in the lower layer of the model unless the diffusive stretching term works. This is because interfacial friction is absent in the model, and so the vertical shear of currents must be caused by baroclinicity.

5. Discussion

a. How does the diffusive stretching work?

The diffusive stretching term contains the unknown mixing coefficient $\kappa$. Bryan et al. (1999) mention that the direct evaluation of the mixing coefficient is infeasible even if (2.5) is used. This is because the left-hand side of (2.5) contains a nondivergent (rotational) component (Marshall and Shutts 1981); note that (2.4) requires a divergence of the left-hand side of (2.5). In the present application, the diffusive stretching term is rewritten as

$$-\text{curl} \tau_x = \rho_0 f \nabla \cdot (\overline{u} \overline{h})_t,$$

(5.1)

by (2.5) and (2.9). The overbar denotes a temporal mean during 250 days, and the prime denotes an anomaly from the temporal mean. The right-hand side of (5.1) is evaluated in the numerical model.
Before evaluating the right-hand side of (5.1), we depict snapshots of the divergence of \( \mathbf{u}_i \mathbf{h} \) along with potential vorticity fields to find a relationship between the diffusive stretching and frontal eddies. Figure 7 shows contour lines of the potential vorticity of \( 1.0 \times 10^{-8} \text{ cm}^{-2} \text{ s}^{-1} \) every 8 days and shows the horizontal distribution of the divergence of \( \mathbf{u}_i \mathbf{h} \) on the same day. On day 178, large divergences are revealed around the upstream portion of a wave crest. As the wave grows further (see a panel of day 186), a negative area is intensified at a trough of the frontal wave. On day 194, when the wave crest is elongated upstream, the magnitude of the divergence decreases around the frontal wave. When the frontal wave decays afterward, the area with the large divergence disappears (not shown). The space–time plot of the divergence of \( \mathbf{u}_i \mathbf{h} \) along \( y = 250 \text{ km} \) is shown in Fig. 8. Periodicity is similar to that of Fig. 4 showing the passage of frontal waves. Similar periodicity means that the magnitude of the divergence increases whenever growing frontal waves pass over.

The temporal average of the divergence is calculated from day 0 to day 250. Using the average, we are able to estimate the diffusive stretching term on the right-hand side of (5.1). Figure 9 shows horizontal distributions of diffusive stretching terms in the upper and lower layers. As shown in (2.10) and (2.11), the terms are regarded as forcing terms in the vorticity equations. Large values appear around the shelf edge along which the frontal wave propagates. In the upstream portion of the model (\( x < 200 \text{ km} \)), the magnitude of the diffusive stretching term is small because the growth of the frontal wave is insufficient around the entry of the domain. Apart from the portion around the entry, the magnitude \( [O(10^{-7}) \text{ dyn cm}^{-2}] \) on the shelf is 10 times as large as the typical magnitude \( [O(10^{-8}) \text{ dyn cm}^{-2}] ; \) Han et al. 1995) of the wind stress curl over the East China Sea. However, the magnitude of the diffusive stretching term is not homogeneous in the along-shelf direction because the magnitude depends on the phase of frontal wave (see Fig. 7). Hence, as shown in Fig. 6, the band structure of mean currents is also inhomogeneous in the along-shelf direction because of the inhomogeneous distribution of the term.

It is interesting that the distribution pattern of the diffusive stretching term in one layer is the mirror image of another despite the evaluation undertaken independently in each layer. This fact means that the distribution pattern of the mixing coefficient in the upper layer is similar to that in the lower layer (i.e., \( \kappa_1 \approx \kappa_2 \)). Thereby, the diffusive stretching exchanges the vorticity and momentum of mean currents (in strict terms, the geostrophic part of mean currents) between layers as if interfacial friction works. The vertical momentum transfer induced by eddies therefore breaks the state of no motion in the mean current field of the lower layer.

By averaging the divergence of \( \mathbf{u}_i \mathbf{h} \) temporally, we obtain the diffusive stretching term that causes the vertical vorticity (hence, momentum) transfer. Thus, the
periodic variation of the divergence (see Fig. 8) indicates that the vertical momentum transfer also occurs on the shelf slope intermittently. This result suggests that topographic Rossby waves are excited on the shelf slope. Next, we will investigate the spinup of mean currents by the topographic Rossby waves.

b. Driving mechanism of band structure

The suggestion in the previous section is that topographic Rossby waves are excited by frontal eddies through the diffusive stretching on the shelf slope. To confirm the generation of the waves, space–time plots of the vertically averaged current in the $x$ direction are depicted in Fig. 10. Three lines at $y = 250$, 350, and 450 km are chosen here. Temporal means are removed from the figure. A periodic variation with the downstream phase propagation is revealed along the line at $y = 250$ km. The period and phase speed are similar to those in Fig. 4. It is apparent that the variation results from the propagation of frontal waves. However, signals propagate in the Rossby wave sense in the shallow portion (see panels of 350 and 450 km).

A three-dimensional power spectrum of the vertically
averaged current is calculated to detect the topographic Rossby wave more clearly. We analyze the spatiotemporal variation of the vertically averaged current in the $x$ direction within the shaded square (128 and 64 grids in the $x$ and $y$ directions) shown in the left panel of Fig. 10. Results of the time integration extended until day 1024 are used to ensure fine resolution in lower frequencies. The initial condition of the current (i.e., the western boundary current) is removed from the current field. In addition, by fitting by least squares a plane ($ax + by + c$) to the horizontal distribution of the current, we remove a two-dimensional linear trend every time step. Furthermore, a linear trend of the temporal variation of the current is removed at each grid, too. Thereafter, the three-dimensional fast Fourier transform (Press et al. 1992) is adopted to obtain the power spectrum. Signals propagating both in upstream and downstream directions are coexisting in the shaded square of Fig. 10, so that the three-dimensional power spectrum might be contaminated in such a statistically nonuniform area. Nevertheless, it is expected that topographic Rossby waves are detected clearly in the spectral analysis because the intense signal of the frontal wave is restricted only around the shelf edge in the downstream portion.

Figure 11 shows the power spectrum on each wavenumber plane with the same frequency. The power is normalized by the maximum value on each plane. The maximum value on each plane is shown by a bar chart in the lower left of the figure. Dispersion curves of the topographic Rossby wave are also shown on each wavenumber plane. Fitting an exponential profile of the depth $D$,

$$D = D_0 e^{2\alpha r},$$

(5.2)

to the shelf slope of our model, we utilize the dispersion relation obtained by Buchwald and Adams (1968). The dispersion relation of the topographic Rossby wave is

$$\omega = \frac{-2\alpha f k}{k^2 + l^2 + \lambda^2},$$

(5.3)

where $\omega$ is the frequency and $k$ and $l$ are the wavenumbers in the $x$ (along shelf) and $y$ (cross shelf) directions,
Fig. 10. Space–time plots of the vertically averaged current in the $x$ direction along $y = 250$, 350, and 450 km (along broken lines in the left panel). The temporally averaged value at each location is removed. Contour interval is $5 \text{ cm s}^{-1}$. Stippling represents negative values. Two solid lines in the left panel show the area where currents are imposed initially. A shaded square represents the area in which the three-dimensional power spectrum is obtained (see Fig. 11).
Fig. 11. Three-dimensional power spectrum of the vertically averaged current in the $x$ direction. (right) The power spectrum on each wavenumber plane with the same frequency. Energy is normalized by the maximum value on each wavenumber plane. Periods are shown in the upper right on each plane. Broken curves represent the dispersion relation of the topographic Rossby wave. Wavenumber planes with the period longer than 24.4 days (around the forcing period) are shown here. (left) The maximum value on each wavenumber plane. Prominent peaks are marked by letters $a$, $b$, $c$, and $d$ in the bar chart. Numerals under the letters denote frequencies (day$^{-1}$). Numerals in parentheses represent periods in day.
respectively. The power spectrum on each wavenumber plane shows that, in general, high energy is detected around the dispersion curve of the topographic Rossby wave.

In the bar chart of the maximum value on each wavenumber plane, four spectral peaks with the relatively broad foot (marked by letters a, b, c, and d in the panel) are visible. A peak c with the period of 27 days is the most prominent among all peaks. The period is nearly identical to those of the frontal wave (see Fig. 4) and divergence of $u'h'_1$ (see Fig. 8). Hence, it is considered that the peak c is forced by the diffusive stretching. In addition, relationships among frequencies of four peaks satisfying

$$\omega_a + \omega_b = \omega_c \quad \text{and} \quad \omega_d = 2\omega_c \quad (5.4)$$

suggest that the peaks form two sets of a resonant triad of topographic Rossby waves [note that (5.5) can also form the resonant triad with different wavenumbers for c]. When resonant triads appear, that is, when Rossby waves are unstable by resonant interactions, energy of waves proceeds to flow toward lower frequency on average (e.g., Pedlosky 1987). In fact, the bar chart shows that values with frequencies lower than c are one order of magnitude larger than values with frequencies higher than c. When the energy shifts toward the lower-frequency limit, that is, $\omega \to 0$, the dispersion relation (5.3) requires $k \to 0$. Namely, the wavenumber vanishes in the along-shelf direction. Hence, a resultant wave is expressed as cos(ly), which denotes the alternating bands of mean current (band structure) in Fig. 6. The band structure spreads over the entire shelf (see Figs. 5 and 6) because of the Rossby waves with cross-shelf wavenumbers.

One may perceive that the above process resembles evolution of geostrophic turbulence (Rhines 1975). As shown in Fig. 12, the turbulence is transformed into Rossby waves by the inverse energy cascade. Thereafter, the band structure is formed by resonant interactions between the Rossby waves. Evolution of the geostrophic turbulence on the topographic $\beta$ plane is discussed by Vallis and Maltrud (1993) and LaCasce and Brink (2000). The present study demonstrates that the diffusive stretching by frontal waves excites topographic Rossby waves on the shelf slope. Thereafter, the band structure is formed over the continental shelf in the same way as described by Rhines (1975).

c. Baroclinicity

In the band structure of Fig. 6, the upper-layer currents are stronger than the lower-layer currents. As mentioned previously, in the absence of bottom friction, the vertical shear of currents results only from baroclinicity. The left panel of Fig. 13 represents the difference between the temporally averaged upper-layer thickness and the initial condition. The right panel shows along-shelf current velocities in the upper and lower layers; the initial condition is removed from upper-layer currents. The along-shelf currents are discontinuous at the shelf break ($y = 200$ km) where the depth changes abruptly. In fact, the currents vary smoothly when a uniform slope is adopted in the model (shown later in section 5c). The large vertical shear is revealed especially on the shelf. We herein note the areas where the upper-layer thickness increases. Superimposing the areas on the current field, we are able to find that the increase of the upper-layer thickness results in the large vertical shear. How does the upper-layer thickness increase? In other words, how is “warm water” carried onto the shelf?

We convert the continuity equation [(2.6)] into an advective–diffusive equation of the upper-layer thickness. The rigid-lid condition enables us to define the (volume transport) streamfunction as

$$u_1 h_1 + u_2 h_2 = -\frac{\partial \Phi}{\partial y} \quad \text{and} \quad v_1 h_1 + v_2 h_2 = \frac{\partial \Phi}{\partial x} \quad (5.6)$$

Variables are divided into the mean and eddy parts as in (2.3). Substituting these two parts into (5.6) leads to

$$\pi_1 \tilde{h}_1 + \pi_2 \tilde{h}_2 + \pi'_1 \tilde{h}'_1 + \pi'_2 \tilde{h}'_2 = -\frac{\partial \Phi}{\partial y} \quad \text{and}$$

$$v_1 h_1 + v_2 h_2 + v'_1 h'_1 + v'_2 h'_2 = \frac{\partial \Phi}{\partial x} \quad (5.7)$$

The geostrophic relation (2.9) is mostly valid even if the geostrophic part of currents is replaced with the mean part (not shown). Using (5.7) and geostrophic relation, we obtain

$$\pi_1 = -\frac{1}{D} \frac{\partial \Phi}{\partial y} - \frac{g'}{fD} \frac{\partial \tilde{h}_1}{\partial y} - \frac{1}{D} (\pi'_1 \tilde{h}'_1 + \pi'_2 \tilde{h}'_2) \quad \text{and}$$

$$\pi_1 = \frac{1}{D} \frac{\partial \Phi}{\partial x} + \frac{g'}{fD} \frac{\partial \tilde{h}_1}{\partial x} - \frac{1}{D} (v'_1 h_1' + v'_2 h_1'). \quad (5.8)$$

Substituting (5.8) into current velocities on the left-hand side of (2.6) yields
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Fig. 13. Relationship between the upper-layer thickness and vertical shear of the mean current in the \( x \) direction. (top left) The difference of the temporally averaged upper-layer thickness from its initial value. Contour interval is 10 m. Dashed contour is used for the negative value. Contributions of each term in (5.9) are estimated in the area surrounded by the boldface broken line (see Fig. 14). (top right) Distributions of the along-shelf mean current in the upper (solid curve) and lower (broken curve) layers. Stippling indicates the area where the upper-layer thickness is larger than the initial condition. Sponge layers are omitted. (bottom) The model topography.

\[
\mathbf{u} \cdot \nabla \bar{h}_1 = \nabla (\kappa_1 \nabla \bar{h}_1) + \frac{\overline{u_1}}{D} \nabla \cdot [(\kappa_2 - \kappa_1) \nabla \bar{h}_1] + \frac{\overline{u_1}}{D} \frac{\partial D}{\partial y}, \tag{5.9}
\]

where \( \mathbf{u} \) denotes

\[
[D^{-1}(\pi \overline{u_1} + \pi_2 \overline{u_2}), D^{-1}(\pi \overline{v_1} + \pi_2 \overline{v_2})], \tag{5.10}
\]

that is, mean parts of vertically averaged currents. Equation (5.9) represents the advective–diffusive equation of the mean part of the upper-layer thickness. The left-hand side represents the advection by the vertically averaged current. The first term on the right-hand side indicates the horizontal diffusion of the upper-layer thickness by eddies. The second term on the right-hand side represents the interfacial displacement induced by the lower-layer flow impinging on the bottom slope. The second term on the right-hand side is negligibly small because \( \kappa_1 \) is nearly equal to \( \kappa_2 \), as shown in section 5a. Magnitudes of each term are integrated within the area surrounded by the broken line in the left panel of Fig. 13.

Figure 14 shows that three terms mostly determine the mean part of the upper-layer thickness. The advection and diffusion (denoted by letters \( a \) and \( b \) in the figure) increase the upper-layer thickness while the upper-layer thickness is reduced by upwelling caused by the lower-layer flow impinging on the bottom slope. Thereby, it is concluded that warm water that enhances baroclinicity is carried onto the shelf through the advection by the mean current and, equivalent, through horizontal diffusion by eddies.

d. Friction

The foregoing investigation has neglected friction at both the bottom and layer interface for simplicity. However, in this study, our attention is given to mean currents on the shallow shelf where friction is nonnegligible. We here examine the effect by friction on the band structure.
Bottom friction that may weaken the band structure is included in the two-layer model (see the appendix for its formulation). Time integration is conducted in the same manner as in previous sections. A bottom friction coefficient $r_b$ is evaluated as $0.5(f \mu/2)^{0.5}$, where $\mu$ denotes a vertical viscosity. The viscosity is chosen from 1 to 10 cm$^2$ s$^{-1}$ as typical values on the shelf.

Figure 15 shows distributions of the vertically averaged current in the $x$ direction at $x = 400$ km. Results of experiments with vertical viscosities of 0 (the same as that in Fig. 6), 5, and 10 cm$^2$ s$^{-1}$ are depicted in the figure. The band structure is maintained even if we choose the large viscosity. Hence, it is considered that the band structure is a robust structure in mean currents on the shelf with bottom friction.

In addition, a numerical experiment with interfacial friction is conducted. Although the figure is not shown, we discover that the band structure appears on the shelf whenever frontal eddies are generated in the model.

e. Bottom topography

Last, we investigate how the bottom topography alters the band structure of mean current. In general, steep bottom slopes in the same direction as interfacial slopes stabilize baroclinic instability that is a main cause of the frontal wave (e.g., Kubota 1977). Thus, the experiment demonstrates how the band structure depends on the activity of the frontal wave.

Three uniform bottom slopes (cases 1, 2, and 3 in the upper panel of Fig. 16) are adopted in the experiments. The shelf edge is removed in the experiments for simplicity. The maximal speed of the western boundary current is reduced to 50 cm$^2$ s$^{-1}$ to prevent the interface from touching the bottom. Friction at both interface and bottom is neglected. The procedure and other parameters for the experiments are the same as those in previous sections.

The middle three panels of Fig. 16 show horizontal distributions of the diffusive stretching term in the upper layer. The diffusive stretching term becomes large in the western boundary current on the gentle slope that activates the frontal wave. This means that the forcing of the mean current is intensified on the gentle slope. Thereby, as shown in the lower panel of Fig. 16, alternating bands of mean current strengthen on the gentle slope.

In addition, the above experiments enable us to deduce a relationship between the band structure and location of the western boundary current. When an axis of western boundary current migrates toward the shallow shelf, the gentle slope of the shelf activates frontal waves. Thus, the band structure of mean current also strengthens much more. However, the band structure weakens when the axis moves onto the steep shelf slope. Besides, we are able to deduce a relationship between the band structure and magnitude of the western boundary current. The slope of the interface becomes steep when the current speed increases. Hence, as is the case on the gentle slope, the band structure strengthens over
FIG. 16. Results of three experiments with different bottom slopes. (top) The bottom topography in cases 1, 2, and 3. Also shown is the initial displacement of the interface. (middle) Horizontal distributions of the diffusive stretching terms in the upper layer in cases (left) 1, (center) 2, and (right) 3. Contour interval is $0.5 \times 10^{-7}$ dyn cm$^{-3}$. Dashed contour is used for the negative value. Sponge layers are omitted. For case 1, contour lines of $\pm 0.2 \times 10^{-7}$ dyn cm$^{-3}$ are added. Two solid lines show the areas where currents are imposed initially. (bottom) Distributions of along-shelf mean currents. Vertically averaged currents at $x = 400$ km are depicted here. Solid, broken, and dotted lines represent mean currents in cases 1, 2, and 3, respectively.
the continental shelf when the western boundary current is intensified.

6. Summary

This study demonstrates how mean currents on the shelf are forced by the western boundary current along the shelf edge. A solution is the generation of alternating bands of mean currents, which are observed on the shelf of the East China Sea. The scenario for the generation of the band structure is summarized below.

The western boundary current is accompanied by frontal waves, and so vertical momentum transfer occurs through diffusive stretching. Topographic Rossby waves are excited on the shelf slope by vertical momentum transfer. Thereafter, energy of the waves proceeds to flow toward lower frequency by resonant interactions between the topographic Rossby waves. The along-shelf wavenumber also vanishes to satisfy the dispersion relation of the topographic Rossby waves. As a consequence, alternating bands of mean current are found over the continental shelf.

The above scenario means that either a fine resolution sufficient to reproduce frontal waves or an adequate parameterization for the diffusive stretching by frontal waves is required in a realistic modeling of continental shelf circulations.

Acknowledgments. The author expresses sincere thanks to Akira Masuda, whose lecture on eddy-related dynamics at Mount Kuji made this study possible. Thanks are extended to Osamu Katoh for permitting the use of his figure. Particular acknowledgement is made of the two reviewers for providing helpful comments on the manuscript. The Japan Society for the Promotion of Science supported this study through Grant-in-Aid for Scientific Research.

APPENDIX

Governing Equations of a Two-Layer Model

Governing equations of a two-layer model with the hydrostatic, rigid-lid, and Boussinesq approximations are

\[
\begin{align*}
\frac{\partial u_k}{\partial t} + u_k \frac{\partial u_k}{\partial x} + v_k \frac{\partial u_k}{\partial y} - f v_k &= -\frac{1}{\rho_0} \frac{\partial P_k}{\partial x} + X_k + A_k \nabla^2 u_k, \\
\frac{\partial v_k}{\partial t} + u_k \frac{\partial v_k}{\partial x} + v_k \frac{\partial v_k}{\partial y} + f u_k &= -\frac{1}{\rho_0} \frac{\partial P_k}{\partial y} + Y_k + A_k \nabla^2 v_k,
\end{align*}
\]

and

\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(u_i h_i) + \frac{\partial}{\partial y}(v_i h_i) = 0,
\]

where a subscript \(k\) denotes layers (uppermost \(k = 1\)), \(u\) and \(v\) are the currents in the \(x\) (along shelf) and \(y\) (cross shelf) directions, \(f\) is the Coriolis parameter, \(P\) is the pressure, \(A_k\) is the horizontal viscosity, \(\eta\) is the interface (upward) displacement, \(h\) is the layer thickness, and \(X_k\) and \(Y_k\) denote friction at both the layer interface and bottom in the \(x\) and \(y\) directions, respectively. When \(k = 1\),

\[
\begin{align*}
X_1 &= -r_x h_1^{-1} (u_1 - u_2) \quad \text{and} \\
Y_1 &= -r_y h_1^{-1} (v_1 - v_2),
\end{align*}
\]

and when \(k = 2\),

\[
\begin{align*}
X_2 &= -r_x h_2^{-1} (u_2 - u_1) - r_y h_2^{-1} u_2 \quad \text{and} \\
Y_2 &= -r_y h_2^{-1} (v_2 - v_1) - r_y h_2^{-1} v_2,
\end{align*}
\]

where \(r_x\) and \(r_y\) indicate coefficients of interfacial and bottom friction.

After vertically averaging (A.1), we cross-differentiate it to obtain a vorticity equation of the vertically averaged flow as

\[
\frac{\partial \xi}{\partial t} - \frac{\partial}{\partial x} \left( \frac{1}{D} \frac{\partial}{\partial y} \sum_{k=1}^{2} u_i \frac{\partial h_i}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{1}{D} \frac{\partial}{\partial x} \sum_{k=1}^{2} v_i \frac{\partial h_i}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{1}{D} \frac{\partial}{\partial y} \sum_{k=1}^{2} u_i v_i h_i \right) + \frac{\partial}{\partial y} \left( \frac{1}{D} \frac{\partial}{\partial x} \sum_{k=1}^{2} v_i^2 h_i \right) = -f J(\psi, D^{-1}) + g' J(h_1 D^{-1}, h_1)
\]

where \(g'\) is the reduced gravity and \(D\) is the depth, respectively; \(\psi\) is the (volume transport) streamfunction defined as

\[
\sum_{i=1}^{2} u_i h_i = -\frac{\partial \psi}{\partial y} \quad \text{and} \quad \sum_{i=1}^{2} v_i h_i = \frac{\partial \psi}{\partial x}
\]

and \(\xi\) is the vorticity expressed as

\[
\xi = \nabla \cdot \left( \frac{1}{D} \nabla \psi \right).
\]

Furthermore, we introduce variables

\[
\hat{u} = u_1 - u_2 \quad \text{and} \quad \hat{v} = v_1 - v_2,
\]

which are differences between velocities in the upper and lower layers. Taking the difference between (A.1) in the upper and lower layers, we obtain

\[
\begin{align*}
\frac{\partial \hat{u}}{\partial t} - \sum_{k=1}^{2} \left[ (-1)^k \left( u_i \frac{\partial h_i}{\partial x} + v_i \frac{\partial h_i}{\partial y} \right) \right] - f \hat{v} &= g' \frac{\partial \eta}{\partial x} + A_x \nabla^2 \hat{u} - \frac{r_D h_1}{h_2} \hat{u} + \frac{r_x u_1}{h_2}, \\
\frac{\partial \hat{v}}{\partial t} - \sum_{k=1}^{2} \left[ (-1)^k \left( u_i \frac{\partial h_i}{\partial x} + v_i \frac{\partial h_i}{\partial y} \right) \right] + f \hat{u} &= g' \frac{\partial \eta}{\partial y} + A_y \nabla^2 \hat{v} - \frac{r_D h_1}{h_2} \hat{v} + \frac{r_y v_1}{h_2}.
\end{align*}
\]
Equations (A.3), (A.4), and (A.5) are solved numerically using the leapfrog scheme with the Matsuno scheme every 25 time steps. Advection terms in (A.3) are replaced with the finite-difference scheme of Lilly (1965). We use the generalized Arakawa scheme (Grammeltvedt 1969) for advection terms of (A.5). Equation (A.4) is solved by applying the SOR method.

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